CDCC Calculations of Total Fusion of $^6,^7$Li with Targets $^{27}$Al, $^{28}$Si, $^{144}$Sm, $^{198}$Pt and $^{209}$Bi: Effect of Resonance States

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Calculations of total fusion cross sections of the weakly bound nuclei $^6,^7$Li with a variety of targets at energies around the Coulomb barrier are presented. Special emphasis is focused on the effects that resonance states, $l=2, J^\pi=3^+, 2^+, 1^+$ of $^6$Li, and $l=3, J^\pi=7/2^-, 5/2^-$ of $^7$Li, have on fusion. The study uses the Continuum-Discretized Coupled-Channel (CDCC) framework to calculate total fusion, from which the effects of couplings to resonance states are extracted. Resonance states with shorter half-lives have a more important effect on total fusion at energies below and around the Coulomb barrier, where incomplete fusion dominates. On the other side, resonance states with longer half-life, act as quasi-bound inelastic states, thus, these states have a more important role on complete fusion than on incomplete fusion.

KEYWORDS: CDCC, weakly bound, fusion, resonance states.

1. Introduction

Nuclear reactions of weakly bound projectiles with stable targets have been a subject of intense research in the last decade. In particular, fusion and breakup reaction processes induced by this type of projectiles have been studied theoretically and experimentally speaking [1-3]. Of the utmost importance are the effects of breakup of the projectile on other reaction. Due to the easiness with which weakly bound projectiles break up into several fragments, different fusion processes appear. Complete Fusion (CF) occurs when the whole projectile is captured by the target. In fact, complete fusion can happen in two different ways, i.e., direct when fusion takes place without a previous breakup, and sequential when all projectile fragments are captured by the target after breakup. Incomplete Fusion (ICF) occurs when not all fragments are captured, that is, some are captured while others escape to continuum states. Total fusion (TF) is then the sum of CF and ICF. Furthermore, processes in which, none of the fragments is captured contribute to the non-capture breakup mechanism (N CBu). At sub-barrier energies, reaction mechanisms involving weakly bound nuclei may be more complex. This is because breakup of short-lived projectile-like nuclei, after transfer of nucleons, is also an important process that may predominate over the direct breakup [4]. On the other hand, direct breakup of the projectile should have a significant effect on fusion at low energies, mainly by its contribution to ICF. Since ICF becomes increasingly relevant with...
decreasing incident energies, the direct breakup process inhibits CF. Theoretically speaking, the CDCC framework, [5-7] is commonly used to quantify the effect of continuum breakup states on other reaction mechanisms. However, the CDCC model has a serious drawback because an explicit calculation of CF and ICF is not possible without ambiguity, and only TF can be calculated. On the other hand, resonance states of weakly bound nuclei may have particular effects on both elastic scattering and fusion. In fact, the effects of breakup resonance \((l=2, f^*=3^+, 2^+, 1^+)\) and non-resonance continuum states of \(^6\text{Li}\) with several targets has been studied in Refs. [8, 9]. It is important to point out that, breakup resonance states of \(^6,7\text{Li}\) should produce very particular effects on fusion depending on their half-lives. That is, those states with long half-lives may behave as quasi-bound states, therefore, may remain bound during the incoming part of the trajectory to the target and contribute mainly to CF. If, the projectile breaks up during the outgoing part, would not any effect on fusion. On the contrary, resonance states with smaller half-lives may break up as the projectile approaches the target, therefore, have a more important role on ICF. Excitation to non-resonance breakup states, due to their prompt breakup may have some contribution to ICF, but mostly feed elastic breakup states [10]. In this work, CDCC calculations of the effects resonance states of the weakly bound nuclei \(^6,7\text{Li}\) in reaction with several targets at energies below and above the barrier energy are presented. Particular attention is paid to the importance of short-lived resonance states on total fusion at energies below the Coulomb barrier, where incomplete fusion dominates.

2. Importance of long- and short-lived resonance states on fusion.

Although, in the CDCC model is not possible to explicitly determine CF and ICF components of TF, it is still possible to extract information about the roles that resonance states of \(^6\text{Li}\) and \(^7\text{Li}\), have on CF and ICF. The roles should be dependent on the decay time of the resonances relative to the typical collision time of interaction \(\sim 10^{-21}\text{s}\). The calculations are performed following the procedure as in Refs. [8-10]. That is, to calculate the effect of a resonance or group of resonance states, couplings to these states are omitted from the coupled-channel radial equations,

\[
\left[\hat{t}_{R,K} + \hat{U}^{(j)}_{\beta\beta'}(R) - (E - \varepsilon_0 - \varepsilon_\beta)\right]F^{(j)}_{\beta\beta'}(R) = -\sum_{\beta', \beta''} \hat{U}^{(j)}_{\beta\beta'}(R) F^{(j)}_{\beta'}(R),
\]

(1)

\(\hat{U}^{(j)}_{\beta\beta'}\) are the diagonal \(\beta=\beta'\) and non-diagonal \(\beta\neq\beta'\) coupling matrix elements given by,

\[
\hat{U}^{(j)}_{\beta\beta'}(R) = \langle u_\beta | \hat{V}_{d(t)-T}(r_{d(t)-T}) + \hat{V}_{a-T}(r_{a-T}) | u_{\beta'} \rangle.
\]

(2)

Here, \(u_\beta(r)\) are the normalized square-integrable radial wave functions of discrete breakup states \(\beta\), and \(\hat{V}_{d(t)-T}(r_{d(t)-T})\), \(\hat{V}_{a-T}(r_{a-T})\) the fragment-target interaction potentials. The effect of a resonance/resonances state/s on fusion is obtained respect to the calculation with the full discretized space. Figs. (1a-1f) show the results for total fusion for \(^6\text{Li}\) with targets \(^{28}\text{Si}\), \(^{198}\text{Pt}\) and \(^{209}\text{Bi}\) for energies around and above the barrier. The solid-lines represent the calculations with the full discrete energy space, that is, couplings to resonance and non-resonance states are included. Resonance and non-resonance sub-spaces are those defined in Refs. [8-10]. The dashed-lines show the calculations when couplings to 2+ and 1+ and non-resonance states are considered but
those from the 3+ resonance are omitted. The dotted-lines represent the case when, couplings to the 3+ and non-resonance states are included but those from 2+ and 1+ resonances are not.

As observed in Fig. 1, when couplings from the 3+ resonance (half-life, $\tau \approx 2.74 \cdot 10^{-20}$ sec, longer than the typical collision time $\approx 10^{-21}$ s) and non-resonance states are considered (dotted-lines) but those to the 2+, 1+ are excluded, an enhancement is observed respect to the full discrete space calculation. This effect is similar to that of a bound inelastic excitation state of the projectile. On the other hand, the calculations in which, couplings to resonance states 2+ and 1+ (half-lives, $\approx 3.8 \cdot 10^{-22}$ s, $1.56 \cdot 10^{-22}$ s), and non-resonance states are considered but the 3+ is excluded (dashed-lines), also enhance fusion respect to the full space calculation, although, the enhancement is smaller than in the previous case. In fact, the enhancement decreases at the lowest energies. Therefore, in the latter case (2+, 1+ states included, 3+ excluded) for the heavier targets at low energies, the inclusion of the 3+ state has a negligible effect on fusion. Of course, in both of these calculations, when either the 3+ or both 2+, 1+ states are incorporated, the full space calculation is obtained (solid-lines). Therefore, the following conclusions are obtained at the lowest energies where ICF dominates over CF.

As the projectile approaches the target, the states 2+ and 1+ may break up and mainly feed the ICF process. The state 3+ may survive breakup and mainly contribute to CF. Similar calculations are shown in Figs. (2a-2f) for the projectile $^7$Li with targets $^{27}$Al, $^{144}$Sm and $^{209}$Bi, where the solid-lines correspond to the calculation with the full discrete space, the dashed- and dotted-lines when either, the 5/2 or 7/2 resonance is included (the other omitted) together with non-resonance states. The $^7$Li projectile shows resonance states 7/2 and 5/2 with half-lives $\approx 7.08 \cdot 10^{-21}$ s and $\approx 0.75 \cdot 10^{-21}$ s, which are of the same order as the collision time $\approx 10^{-21}$ s. Therefore, it is expected that the projectile once excited to these states, particularly to the 5/2 resonance, mostly feed the ICF process at low energies. This effect is shown in Figs. 2a-2c, where is particularly important for $^{209}$Bi. At higher energies, Figs. 2d-2f, the effect of either resonance approach to similar behaviors for the heavier targets.

![Fig. 1. Solid-lines represent total fusion cross section with the full discrete energy space. The dotted-lines correspond to fusion when the resonances 2+, 1+ are omitted and 3+ included while the dashed-lines represent the opposite situation.](image-url)
3. Summary and Conclusions

Through CDCC calculations of total fusion of the weakly bound projectiles $^6\text{Li}$ and $^7\text{Li}$ with several reaction targets, some information can be drawn about the importance of resonance states of the projectiles on CF and ICF components of TF. Those resonance states with longer half-lives than the typical collision time $t \approx 10^{-21}$ s, behave as quasi-bound inelastic states that contribute mainly to CF at low energies. If these states breakup during the outgoing part of their trajectory from the target do not have any effect on fusion. Those resonance states with similar or smaller half-lives than $t$, may break up as the projectile nears the target, and have a significant role on ICF.

References