Quantum Information Processing (2025) 24:9
https://doi.org/10.1007/s11128-024-04626-0

Limitation on the capacity of arbitrary bipartite pure entangled states for the unambiguous transmission of classical information

Lvjun Li[1]

Received: 13 July 2024 / Accepted: 15 December 2024

Abstract

The capacity of quantum states to send information is a fundamental and intriguing issue in quantum communication theory. This study explores a scenario where Alice intends to transmit n bits of classical information to Bob by sending m qudits, using an arbitrary bipartite pure entangled state with E qudits as the shared resource. We employ the unambiguous discrimination strategy in decoding and establish an upper bound on the success probability, which extends the findings of a previous study (Li in Phys Rev A 85:052304, 2012).

Keywords Quantum communication · Information capacity · Entangled state · Unambiguous discrimination

1 Introduction

Investigating the information capacity of quantum states is a fundamental and intriguing problem in the quantum communication theory. This problem can be described as follows. Alice intends to transmit n bits of classical information (represented by a random variable X over an n-bit message) to Bob, utilizing an entangled state as a shared resource. Let Q be an encoding of X, consisting of m quantum bits. Then Bob identifies the state by measuring the encoded Q (where Y represents any random variable obtained from this measurement). It is expected that a large amount of classical information can be encoded into significantly fewer quantum bits. However, Holevo's theorem formalizes this limitation, stating that the mutual information $I(X:Y)$ cannot exceed m.

It is well established that the information encoded in a quantum state can only be accessed through measurements, which may disturb the observed states. It is impossible to deduce a complete description of quantum superposition from measurements

---

✉ Lvjun Li
hnbcllj@163.com

[1] School of Computer and Information Engineering, Hanshan Normal University, Chaozhou 521041, China

of a quantum system. Additionally, it is known that non-orthogonal states cannot be perfectly discriminated [1–3], but there are two main probabilistic methods for discriminating them.

The first method is known as minimum error discrimination [1–12], which allows for some errors. To achieve minimum error discrimination between N mixed states, a measurement scheme involves N positive semidefinite operators that collectively form a complete set of measurements on the Hilbert space spanning all eigenvectors corresponding to the nonzero eigenvalues of the N mixed states.

The second method is referred to as unambiguous discrimination [13–22], which allows for a nonzero probability of inconclusive results while guaranteeing absolute correctness when successful. In order to achieve unambiguous discrimination of N mixed states, a measurement is designed with the capability to yield inconclusive results with some probability. However, if a definite answer is provided, it will be completely accurate. This type of measurement requires $N+1$ measurement operators, one of which will result in an inconclusive outcome.

In previous research exploring the capacity of quantum states to transmit classical information, this approach relied on minimum error discrimination. As we know, with EPR pairs as a resource, superdense coding [23], introduced by Bennet and Wiesner, enables the transmission of n classical bits with absolute fidelity using only $n/2$ quantum bits. However, if $m < n/2$, the perfect transmission of n classical bits using m quantum bits becomes impossible. Holevo [24] quantified the information carried by a finite quantum system, formalizing this inherent limitation. This boundary was later broadened by Cleve et al. [25] to encompass interactive communication protocols as well. Nayak and Salzman [26] derived an optimal limit on the number of quantum bits necessary for such transmissions, where the likelihood of accurately decoding a randomly chosen message is capped at $2^{2m}/2^n$. This applies to both scenarios: encoding messages using EPR pairs or arbitrary entangled states with ancillary qubits. More recently, Li [22] introduced an upper limit on the success rate of unambiguous decoding, set at $(2^{2m} - 1)/(2^n - 1)$. In this context, Alice aims to communicate n bits of classical information to Bob, randomly selected, using m quantum bits, with EPR pairs serving as the shared resource.

A natural question emerges: Is there a ceiling on the success probability when Alice aims to communicate n bits of classical information to Bob in a non-uniform randomized fashion by transmitting m qudits? Moreover, if they choose to replace EPR pairs with an arbitrary entangled state as their shared resource, and adopt unambiguous discrimination during the decoding process, how does this affect the potential success rate?

In this paper, we delve into a scenario wherein Alice endeavors to randomly transmit n bits of classical information to Bob by sending m qudits. This communication relies on bipartite pure entangled states serving as the shared resource. We consider the strategy of unambiguous discrimination during the decoding process and subsequently derive an upper bound for the success probability.

The remaining sections of this paper are organized as follows. Section 2 revisits essential lemmas and notations for clarity and reference. Section 3 delves into the exploration of the capacity bound for unambiguously transmitting classical information using bipartite pure entangled states. Section 4 provides some examples to illustrate the application of the theorem and discusses potential extensions. Lastly, Sect. 5 wraps up with some concluding remarks, summarizing our findings and offering open questions for future research.

2 Preliminaries

We now revisit two lemmas that will prove invaluable in subsequent sections of this paper.

Lemma 1 *[26] If* $0 \leq \lambda_i \leq 1$ *for all* i *and* $\sum_{i=1}^{N} \lambda_i \leq l$, *then it holds that* $max_\lambda \sum_{i=1}^{N} p_i \lambda_i \leq Pr(\{p_i\}, l)$, *where* $Pr(\{p_i\}, l)$ *represents the combined probability of the* l *most probable states within the set* $\{\rho_i : i = 1, \cdots, N\}$.

To explore the potential of quantum states in transmitting classical information, we can formulate the problem through the following hypothetical scenario: Imagine that Alice and Bob share arbitrary bipartite pure entangled states, denoted as $|\Phi_{d,E}^{AB}\rangle$, which consist of E qudits on both Alice's and Bob's sides. These states are mathematically represented as:

$$|\Phi_{d,E}^{AB}\rangle = \sum_{a \in \{0,\cdots,d-1\}^E} \sqrt{\lambda_a} |a\rangle_A |a\rangle_B. \tag{1}$$

Here, λ_a represents the probability amplitudes associated with the bipartite entangled states, and they sum to 1. Alice intends to communicate n bits of classical information, expressed as a random variable X encompassing an n-bit message, to Bob. To achieve this, she applies a unitary transformation U_x to her part of the entangled state and subsequently transmits m out of the E qudits to Bob with a priori probability p_x(where $x \in \{0, 1\}^n$). Upon receiving these qudits, Bob possesses a total of $E + m$ qudits in a mixed state. Bob then performs a measurement on all $E + m$ qudits, yielding a random variable Y.

Theorem 1 *Alice applies a unitary transformation* U_x *(where* $x \in \{0, 1\}^n$*) to her part of an entangled state* $|\Phi_{d,E}^{AB}\rangle$. *She then send* m *of the* E *qudits to Bob. The resulting* $E + m$ *qudits mixed state* ρ_x *that Bob holds can be expressed as:*

$$\rho_x = (I_r \otimes U_x^T)\rho(I_r \otimes U_x^*), \tag{2}$$

where ρ *is the mixed state obtained by tracing out* m *qudits from the initial entangled state and* I_r *is the identity operator on the Hilbert space of the* m *qudits sended by Alice. The state* ρ *is given by:*

$$\rho = \sum_{l \in \{0,\cdots,d-1\}^{E-m}} \lambda_l |\psi_l\rangle\langle\psi_l|, \tag{3}$$

with $|\psi_l\rangle$ *forming an orthonormal basis for the subspace for the subspace spanned by the remaining* $E - m$ *qudits, and* $\lambda_l = \sum_{r \in \{0,\cdots,d-1\}^m} \lambda_{lr}$, $lr = a$, *where* r *is the index of the rightmost* m *qudits, and* l *is the index of the remain leftmost* $E - m$ *qudits.*

Proof If Alice applies the transformation U_x to her E qudits, the resulting entangled state is that

$$\begin{aligned}
&(U_x \otimes I) \sum_{a\in\{0,\cdots,d-1\}^E} \sqrt{\lambda_a}|a\rangle_A|a\rangle_B \\
&= \sum_{a\in\{0,\cdots,d-1\}^E} \sqrt{\lambda_a} U_x|a\rangle_A|a\rangle_B \\
&= \sum_{a\in\{0,\cdots,d-1\}^E} \sqrt{\lambda_a}|a\rangle_A U_x^T|a\rangle_B. \qquad (4)
\end{aligned}$$

Without loss of generality, Alice divides E qudits $|a\rangle$ into two groups, the left group of $E-m$ qudits $|l\rangle$ and the right group of m qudits $|r\rangle$. Alice sends the rightmost m qudits to Bob. After the communication, Alice and Bob's joint state may be written as

$$\sum_{l\in\{0,\cdots,d-1\}^{E-m}} \sqrt{\lambda_l}|l\rangle_A \frac{1}{\sqrt{\lambda_l}} \sum_{r\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_{lr}}|r\rangle_B U_x^T|lr\rangle_B, \qquad (5)$$

where $\lambda_l = \sum_{r\in\{0,\cdots,d-1\}^m}^m \lambda_{lr}$. Considering the mixed state obtained on Bobs side, if Alice measures her qudits in the standard basis, the probability of Alice observing any given l is λ_l. When Alice gets outcome l, the state of Bobs $E+m$ qudits is

$$\begin{aligned}
|\psi_l^x\rangle &= \frac{1}{\sqrt{\lambda_l}} \sum_{r\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_{lr}}|r\rangle_B U_x^T|lr\rangle_B \qquad (6)\\
&= (I_r \otimes U_x^T)\frac{1}{\sqrt{\lambda_l}} \sum_{r\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_{lr}}|r\rangle_B|lr\rangle_B \qquad (7)\\
&= (I_r \otimes U_x^T)|\psi_l\rangle. \qquad (8)
\end{aligned}$$

We may easily verify that these are orthonormal for different l as follows.

$$\begin{aligned}
\langle\psi_{l'}|\psi_l\rangle &= \frac{1}{\sqrt{\lambda_{l'}\lambda_l}} \sum_{r',r\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_{l'r'}\lambda_{lr}}\langle r'|r\rangle\langle l'r'|lr\rangle_B \\
&= \frac{1}{\sqrt{\lambda_{l'}\lambda_l}} \sum_{r\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_{l'r}\lambda_{lr}}\langle l'r|lr\rangle_B \\
&= \frac{1}{\lambda_l} \sum_{r\in\{0,\cdots,d-1\}^m} \lambda_{lr}\delta_{l'l} \\
&= \delta_{l'l}. \qquad (9)
\end{aligned}$$

Then, we have that

$$\begin{aligned}\rho_x &= \sum_{l\in\{0,\cdots,d-1\}^{E-m}} \lambda_l |\psi_l^x\rangle\langle\psi_l^x| \\ &= \sum_{l\in\{0,\cdots,d-1\}^{E-m}} \lambda_l (I_r \otimes U_x^T)|\psi_l\rangle\langle\psi_l|(I_r \otimes U_x^*) \\ &= (I_r \otimes U_x^T)(\sum_{l\in\{0,\cdots,d-1\}^{E-m}} \lambda_l |\psi_l\rangle\langle\psi_l|)(I_r \otimes U_x^*) \\ &= (I_r \otimes U_x^T)\rho(I_r \otimes U_x^*). \end{aligned} \tag{10}$$

□

Throughout this paper, notation will be clarified whenever a new symbol is introduced. Initially, let us establish a notation that will facilitate the subsequent discussions. Given any two linear operators T_1 and T_2 acting on the same Hilbert space $\mathcal{H}$, we utilize the symbol $T_1 \perp T_2$ to signify that the supports of T_1 and T_2 are orthogonal. By 'support of a linear operator T,' we refer to the subspace spanned by all eigenvectors corresponding to the nonzero eigenvalues of T. This support is denoted as Supp(T). Furthermore, Ker(T) signifies the supplementary space of Supp(T).

Now, let us proceed to present our primary findings.

3 Limitation on the capacity of arbitrary bipartite pure entangled states for the uambiguous transmission of classical information

In this section, we employ an unambiguous discrimination strategy to decode the quantum state ρ_x within the Hilbert space $\mathcal{H}$ spanned by $E + m$ qudits, where x is an element of the set $\{0, 1\}^n$.

Firstly, let us revisit the notation pertinent to unambiguous discrimination. For the unambiguous discrimination of quantum states ρ_x (where x belongs to the set $\{0, 1\}^n$), we must devise a measurement comprising $2^n + 1$ positive semidefinite operators, denoted as Π_x (for $x \in \{0, 1\}^n$) and Π_0, operating within the Hilbert space $\mathcal{H}$ spanned by $E + m$ qudits. These operators adhere to the following conditions:

$$Tr(\Pi_x \rho_{x'}) = \delta_{xx'} q_x, \tag{11}$$

where $x, x' \in \{0, 1\}^n$. Here, $\delta_{xx'}$ equals 1 if $x = x'$ and 0 otherwise, $0 < q_x \leq 1$, and

$$\Pi_0 + \sum_{x\in\{0,1\}^n} \Pi_x = I, \tag{12}$$

in which I signifies the identity operator on the Hilbert space $\mathcal{H}$ of $E + m$ qudits.

Condition (11) can also be expressed as:

$$\Pi_x \rho_{x'} = 0, \tag{13}$$

for all $x \neq x^{'}$, with $x, x^{'} \in \{0, 1\}^n$. Utilizing the measurement Π_x, $x \in \{0, 1\}^n$, if the system is prepared in the state ρ_x, then $Tr(\rho_x \Pi_x)$ $x \in \{0, 1\}^n$ represents the probability of successfully deducing the system to be in state ρ_x, while $Tr(\rho_x \Pi_0)$ signifies the inconclusive probability. Consequently, the average success probability, denoted as $Pr[u]$, is given by:

$$Pr[u] = \sum_{x \in \{0,1\}^n} p_x Tr(\rho_x \Pi_x). \tag{14}$$

A primary goal is to devise an optimal measurement strategy that maximizes this average success probability.

Subsequently, we employ an unambiguous discrimination strategy for decoding and proceed to derive an upper bound on the success probability, as outlined below.

Theorem 2 *Assume that Alice and Bob share arbitrary bipartite pure entangled states represented by*

$$|\Phi_{d,E}^{AB}\rangle = \sum_{a \in \{0,\cdots,d-1\}^E} \sqrt{\lambda_a} |a\rangle_A |a\rangle_B, \tag{15}$$

where each party holds E qudits. If Alice encodes messages $x \in \{0, 1\}^n$ with prior probabilities p_x into these entangled states without ancillary qudits, and transmits m qudits to Bob, then the success probability $Pr[u]$ of unambiguous decoding is bounded as follows:

$$Pr[u] \leq Pr\left(\{p_x \lambda_l\}, \frac{2^n(d^{E+m} - d^{E-m})}{2^n - 1}\right), \tag{16}$$

where $\lambda_l = \sum_{r \in \{0,\cdots,d-1\}^m} \lambda_{lr}$ (with $lr = a$ denoting the combined index, r denoting the index of the rightmost m qudits, and l denoting the index of the remain leftmost $E - m$ qudits), and $Pr(\{p_x \lambda_l\}, k)$ represents the sum of the k largest probabilities in the set $\{p_x \lambda_l\}$.

Proof Bob employs a POVM with elements Π_0 and $\{\Pi_x : x \in \{0, 1\}^n\}$ to unambiguously decode the states ρ_x. According to Theorem 1, the average success probability is given by:

$$\begin{aligned} Pr[u] &= \sum_{x \in \{0,1\}^n} p_x tr(\rho_x \Pi_x) \\ &= \sum_{x \in \{0,1\}^n} \sum_{l \in \{0,\cdots,d-1\}^{E-m}} p_x \lambda_l tr((I_r \otimes U_x^T)|\psi_l\rangle\langle\psi_l|(I_r \otimes U_x^*)\Pi_x), \end{aligned} \tag{17}$$

where $0 < p_x\lambda_l < 1$ and $\sum_{x,l} p_x\lambda_l = 1$. Furthermore, since the $|\psi_l\rangle$ are orthonormal and $\sum_{l\in\{0,\cdots,d-1\}^{E-m}} |\psi_l\rangle\langle\psi_l| \leq I$, we deduce that:

$$\begin{aligned}
&\sum_{x\in\{0,1\}^n} \sum_{l\in\{0,\cdots,d-1\}^{E-m}} tr((I_r \otimes U_x^T)|\psi_l\rangle\langle\psi_l|(I_r \otimes U_x^*)\Pi_x) \\
&\leq \sum_{x\in\{0,1\}^n} tr(\Pi_x) \\
&= tr(I - \Pi_0) \\
&= d^{E+m} - tr(\Pi_0).
\end{aligned} \tag{18}$$

Now, for each density operator ρ_x, we define the space Y_x as follows:

$$Y_x = Supp(\rho_x) \bigcap \sum_{x' \neq x} Supp(\rho_{x'}), \tag{19}$$

where the sum of subspaces denotes the space spanned by all possible linear combinations of vectors from those subspaces. Explicitly, $\sum_i H_i = \{\sum_k |\psi_k\rangle : |\psi_k\rangle \in H_k\}$. It is evident that Y_x is a subspace of $Supp(\rho_x)$.

Furthermore, we introduce $\bar{Y}_x$ such that $\bar{Y}_x \oplus Y_x = Supp(\rho_x)$ and define $Y = \sum_{x\in\{0,1\}^n} Y_x$ as the 'public space' shared among all the supports. With these definitions, we can express the whole Hilbert space H as:

$$H = \sum_{x\in\{0,1\}^n} Supp(\rho_x) = \bigoplus_{x\in\{0,1\}^n} \bar{Y}_x \bigoplus Y. \tag{20}$$

Consequently, the dimensions of these subspaces satisfy:

$$\sum_{x\in\{0,1\}^n} dim(\bar{Y}_x) + dim(Y) = d^{E+m}. \tag{21}$$

Additionally, based on the definitions of Y_x and $\bar{Y}_x$, and denoting $r = \sum_{x\in\{0,1\}^n} rank(\rho_x)$, we derive the following inequality:

$$r = \sum_{x\in\{0,1\}^n} [dim(\bar{Y}_x) + dim(Y_x)] \tag{22}$$

$$\leq \sum_{x\in\{0,1\}^n} [dim(\bar{Y}_x) + dim(Y)] \tag{23}$$

$$= \sum_{x\in\{0,1\}^n} dim(\bar{Y}_x) + 2^n \times dim(Y). \tag{24}$$

By combining the equality stated in (21), we arrive at the following inequality:

$$(2^n - 1)dim(Y) \geq r - d^{E+m}. \tag{25}$$

Given the condition that $\Pi_x \rho_{x'} = 0$, for $x \neq x'$ and the definition of Y, we deduce that Π_x is a positive semidefinite operator whose support lies entirely within the subspace $Y^\perp$. Here, $Y^\perp$ represents the orthogonal complement of the subspace Y. Consequently, we can infer that the summation $\sum_{x\in\{0,1\}^n} \Pi_i$ is also a positive semidefinite operator, and its support belongs to the subspace $Y^\perp$ as well.

Based on (12), we can express Π_0 as follows: $\Pi_0 = I - \sum_{x\in\{0,1\}^n} \Pi_x = P_y + (P_{y^\perp} - \sum_{x\in\{0,1\}^n} \Pi_x)$, where P_y represents the projector onto the subspace Y and $P_{y^\perp}$ represents the projector onto the subspace $Y^\perp$. Since $P_{y^\perp} - \sum_{x\in\{0,1\}^n} \Pi_x \geq 0$ (otherwise Π_0 would not be a positive semidefinite operator), we can deduce that

$$tr(\Pi_0) \geq tr(P_y) = dim(Y). \tag{26}$$

In other words, by combining the inequality above with the equality stated in (25), we arrive at

$$tr(\Pi_0) \geq dim(Y) \geq \frac{r - d^{E+m}}{2^n - 1}. \tag{27}$$

By combining $r = \sum_{x\in\{0,1\}^n} rank(\rho_x) \leq 2^n d^{E-m}$ with equations (18) and (27), we derive the following inequality:

$$\begin{aligned} & d^{E+m} - tr(\Pi_0) \\ & \leq d^{E+m} - \frac{d^{E-m}2^n - d^{E+m}}{2^n - 1} \\ & = \frac{d^{E+m}2^n - d^{E-m}2^n}{2^n - 1}. \end{aligned} \tag{28}$$

Hence, by combining the inequality above with the equality stated in (17), (18) and based on Lemma 1, we can conclude that

$$Pr[u] \leq Pr\left(\{p_x \lambda_l\}, \frac{2^n(d^{E+m} - d^{E-m})}{2^n - 1}\right). \tag{29}$$

This completes the proof. □

4 Discussion

Now let's give an example to illustrate the application of this theorem. For example, Alice and Bob share an bipartite pure entangled state with 2 qutrits each. This state can be expressed as

$$|\Phi_{3,2}^{AB}\rangle = \frac{1}{\sqrt{6}}|00\rangle_A|00\rangle_B + \frac{1}{\sqrt{6}}|01\rangle_A|01\rangle_B + \frac{1}{\sqrt{6}}|02\rangle_A|02\rangle_B \tag{30}$$

$$+\frac{1}{3}|10\rangle_A|10\rangle_B + \frac{1}{3}|11\rangle_A|11\rangle_B + \frac{1}{3}|12\rangle_A|12\rangle_B \tag{31}$$

$$+\quad \frac{1}{\sqrt{18}}|20\rangle_A|20\rangle_B+\frac{1}{\sqrt{18}}|21\rangle_A|21\rangle_B+\frac{1}{\sqrt{18}}|22\rangle_A|22\rangle_B. \tag{32}$$

Alice aims to transmit 4 bits of classical information with equal probability $\frac{1}{16}$ to Bob, using 1 qutrits without ancilla system, under the unambiguous discrimination decoding strategy. That is to say, $p_x = \frac{1}{16}, x \in \{0,1\}^4$, $\lambda_0 = \frac{1}{2}, \lambda_1 = \frac{1}{3}, \lambda_2 = \frac{1}{6}$, $n = 4, m = 1, E = 2, d = 3$. Based on previous findings, we are unable to estimate the upper bound of the success probability. However, according to the above theorem, we can present the probability bounded as

$$Pr[u] \leq Pr\left(\left\{\frac{1}{2}p_x, \frac{1}{3}p_x, \frac{1}{6}p_x\right\}_{x\in\{0,1\}^4}, \frac{2^4\times(3^{2+1}-3^{2-1})}{2^4-1}\right) = 0.7. \tag{33}$$

If Alice selects messages uniformly at random and transmits $m = E$ qudits to Bob, which means $\lambda_l = 1$, then according to the aforementioned theorem, we can straightforwardly deduce the following corollary.

Corollary 1 *Assume that Alice and Bob share bipartite pure entangled states*

$$|\Phi_{d,m}^{AB}\rangle = \sum_{a\in\{0,\cdots,d-1\}^m} \sqrt{\lambda_a}|a\rangle_A|a\rangle_B, \tag{34}$$

Alice encodes n bits of messages uniformly at random without using any ancilla qudits, and sends m qudits to Bob, then the success probability of unambiguous decoding is bounded by

$$Pr[u] \leq min\left(\frac{d^{2m}-1}{2^n-1}, 1\right). \tag{35}$$

Furthermore, in the special case where Alice chooses messages uniformly at random and sends $m = E$ qubits to Bob (implying $\lambda_l = 1$ and $d = 2$), we can immediately infer that the success probability $Pr[u]$ of unambiguous decoding is limited to $Pr[u] \leq \frac{2^{2m}-1}{2^n-1}$. This finding coincides precisely with the result presented in [22].

Consider an entangled state consisting of 2 qubits in the Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$. If Alice wants to transmit 2 bits of classical information uniformly at random using this state by sending 1 qubits, that is to say, $m = 1, n = 2$, the success probability for unambiguous transmission is bounded by $Pr[u] \leq \frac{2^2-1}{2^2-1} = 1$. As we know, in this case, Alice can transmit 2 classical bits with absolute fidelity using only 1 qubit in [23].

5 Concluding remarks

In this paper, we explore the limitations on the capacity of arbitrary bipartite pure entangled states for transmitting classical information. We consider the strategy of unambiguous discrimination for decoding in quantum communication. Specifically, we focus on a scenario where Alice endeavors to randomly transmit n bits of classical information to Bob by sending m qudits, leveraging bipartite pure entangled states as the shared resources. We derive an upper bound on the success probability of this transmission, thereby generalizing, to some extent, the findings presented in [22]. We anticipate that the results obtained in this paper will contribute to addressing the broader issue of quantum communication's relative information capability.

An intriguing open question remains. Is there an alternative, novel upper bound for unambiguous decoding that has the potential to surpass the current one? Furthermore, another compelling problem worthy of consideration is the impact on the established upper bound when, out of k bipartite entangled states, only $m < k$ EPR pairs can be distilled through local operations and classical communication (LOCC).

The entangled states examined in this paper are specifically bipartite pure entangled states, rather than general entangled states. If we broaden our scope to consider more general states, such as arbitrary mixed states, as shared resources between Alice and Bob, the question of the limit on unambiguously conveying classical information still warrants further consideration.

Author Contributions LL wrote the main manuscript text and reviewed the manuscript.

Funding This work is financially supported by the Guangdong Natural Science Foundation (Grant No. 2023A1515012450), the Guangdong Province Key Scientific Research Projects of Colleges and Universities (Grants No. 2022ZDZX1017), and the National Natural Science Foundation of China (Grants No. 11505042).

Data availability No datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare no conflict of interest.

References

1. Holevo, A.S.: Statistical decision theory for quantum systems. J. Multivariate Anal. **3**, 337 (1973)
2. Yuen, H.P., Kennedy, R.S., Lax, M.: Optimum testing of multiple hypotheses in quantum detection theory. IEEE Trans. Inf. Theory. **21**, 125 (1975)
3. Helstrom, C.W.: Quantum Detection and Estimation Theory. Academic Press, New York (1976)
4. Charbit, M., Bendjaballah, C., Helstrom, C.W.: Cut-off rate performance for phase shift keying modulated coherent states. IEEE Trans. Inf. Theory. **35**, 1131 (1989)

5. Eldar, Y.C., Megretski, A., Verghess, G.C.: Designing optimal quantum detectors via semidefinite programming. IEEE Trans. Inf. Theory. **49**, 1007 (2003)
6. Osaki, M., Ban, M., Hirota, O.: Derivation and physical interpretation of the optimal detection operators for several linearly independent signal sets. Phys. Rev. A. **54**, 1691 (1996)
7. Ban, M., Kurokawa, K., Momose, R., Hirota, O.: Optimum measurements for discrimination among symmetric quantum states and parameter estimation. Int. J. Theor. Phys. **36**, 1269 (1997)
8. Eldar, Y.C., Forney, G.D.: Optimal Detection of Symmetric Mixed Quantum States. quant-ph/0211111 (2002)
9. Barnett, S.M.: Minimum-error discrimination between multiply symmetric states. Phys. Rev. A. **64**, 030303(R) (2001)
10. Andersson, E., Barnett, S.M., Gilson, C.R., Hunter, K.: Minimum-error discrimination between three mirror-symmetric states. Phys. Rev. A. **65**, 052308 (2002)
11. Chou, C.L., Hsu, L.Y.: Minimum-error discrimination between symmetric mixed quantum states. Phys. Rev. A. **68**, 042305 (2003)
12. Herzog, U., Bergou, J.A.: Minimum-error discrimination between subsets of linearly dependent quantum states. Phys. Rev. A. **65**, 050305(R) (2002)
13. Ivanovic, I.D.: How to differentiate between non-orthogonal states. Phys. Lett. A. **123**, 257 (1987)
14. Chefles, A.: Unambiguous discrimination between linearly independent quantum states. Phys. Lett. A. **239**, 339 (1998)
15. Chefles, A., Barnett, S.M.: Optimum unambiguous discrimination between linearly independent symmetric states. Phys. Lett. A. **250**, 223 (1998)
16. Sun, Y., Bergou, J.A., Hillery, M.: Optimum unambiguous discrimination between subsets of nonorthogonal quantum states. Phys. Rev. A. **66**, 032315 (2002)
17. Rudolph, T., Spekkens, R.W., Turner, P.S.: Unambiguous discrimination of mixed states. Phys. Rev. A. **68**, 010301(R) (2003)
18. Feng, Y., Duan, R.Y., Ji, Z.F.: Condition and capability of quantum state separation. Phys. Rev. A. **72**, 012313 (2005)
19. Raynal, P., Lütkenhaus, N., van Enk, S.J.: Reduction theorems for optimal unambiguous state discrimination of density matrices. Phys. Rev. A. **68**, 022308 (2003)
20. Herzog, U., Bergou, J.A.: Optimum unambiguous discrimination of two mixed quantum states. Phys. Rev. A. **71**, 050301(R) (2005)
21. Bergou, J.A., Hillery, M.: Universal programmable quantum state discriminator that is optimal for unambiguously distinguishing between unknown states. Phys. Rev. Lett. **94**, 160501 (2005)
22. Li, L.J.: Limits on the ability of Einstein-Podolsky-Rosen pairs to unambiguously convey classical messages. Phys. Rev. A. **85**, 052304 (2012)
23. Bennett, C.H., Wiesner, S.J.: Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. Phys. Rev. Lett. **69**, 2881 (1992)
24. Holevo, A.S.: Bounds for the quantity of information transmitted by a quantum communication channel. Prob. Inf. Trans. **9**, 177 (1973)
25. Cleve, R., Van, D.W., Nielsen, M., Tapp, A.: Lecture notes in computing. Science **61**, 1509 (1998)
26. Nayak, A., Salzman, J.: Limits on the ability of quantum states to convey classical messages. ACM **53**, 184 (2006)