Attempts to understand the image reconstruction algorithms used in medical X-ray computed tomography

Davinder Siwal[1*] and G. Anil Kumar[2]
[1]*Department of Radiation Oncology, Guru Gobind Singh Medical College and Hospital, Baba Farid University of Health Sciences, Faridkot, Punjab 151203, India and*
[2]*Department of Physics, Indian Institute of Technology, Roorkee, Roorkee 247667, Uttarakhand, India*

Introduction

From our personal perspective, medical diagnostic and treatment therapy technology owes very much from the advancement of nuclear physics, thereby impacting every living species on earth. For instance, the versatile NaI(Tl) scintillation detector used in spectroscopic study and photon yield measurements in nuclear physics has routine usage in the thyroid gland uptake imaging − a deciding step for cancer detection (often cured by β^- particles emitted by ^{131}I based medicine accompanied by γ-rays, acquired by the NaI(Tl) detector probe), semiconductor based charge particle detectors employed in a typical reaction cross section measurement has potential applications in radiotherapy, proton and ion beam therapy, similarly monolithic scintillation detector array employed for the nuclear lifetime measurements, may be coupled with position sensitive sensors and are immensely used in the X/γ-ray tomographic reconstruction (CT, PET, SPECT) to reveal the human/animal anatomical and physiological situation, and the list goes on.

The position sensitive scintillation detectors employed in a typical X-ray CT machine are arranged diametrically opposite to the X-ray tube, receives the attenuated X-ray profiles slice-by-slice. These profiles carry the details of the intercepted tissue and the bone structure of the exposed region. The detector head captures all the profiles up to the rotation angle of 360 degrees. Profiles received during scanning are fused together to reveal the anatomical details of the patient in real time. The objective of this contribution is to illustrate the efforts made to understand the basic image reconstruction algorithms employed in X-ray computed tomography modalities.

*Electronic address: dsiwal@ggsmch.org

Shepp-Logan Phantom

A mathematical phantom of human head known as : “Shepp-Logan”, is available in the literature [1] to understand the performance of various image reconstruction algorithms. A two dimensional cross section of such phantom is depicted in Fig. 1 (a) which displays ; tumors, blood clots, ventricles, skull and brain [2]. This can be obtained by the superposition of ten ellipse of varying intensity. The general equation of ellipse is given as :

$$\frac{\Big((x-x_1)cos\theta+(y-y_1)sin\theta\Big)^2}{a^2}+\frac{\Big((x-x_1)sin\theta+(y-y_1)cos\theta\Big)^2}{b^2}=1 \qquad (1)$$

where (x_1,y_1) is the central co-ordinate of ellipse, θ is the angle of rotation with respect to major axis, and “a”, “b” are the length of major and minor axis respectively. The ellipses are drawn in space according to the parameters mentioned in the reference [3], such that the maximum bin content of an image become unity, representing the skull region.

Radon Transformation

The simplest acquisition in X-ray tomography is the parallel beam of photons traversing the tissues such that the remaining intensities are collected by the position sensitive scintillation/semiconductor detector assembly. If the

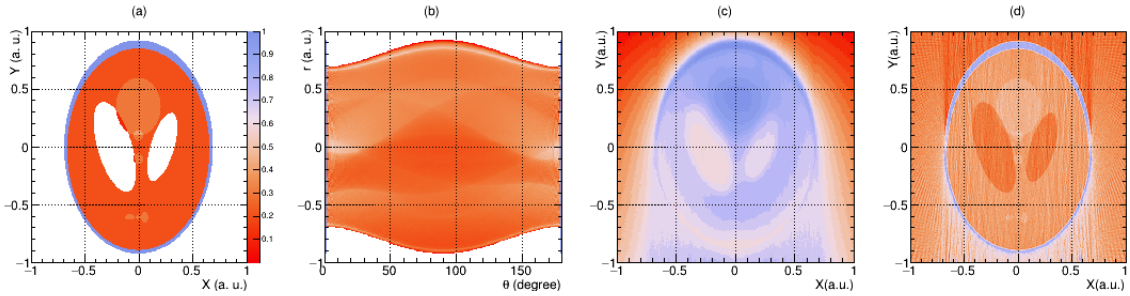

FIG. 1: Panel (a): Mathematical phantom, mimicking the cross section of human head, panel (b) : density distribution of a radon transform matrix, panel (c): Raw reconstructed image from back-projection method, panel (d) : Image obtained with "Ram-Lak" filter [3] in the Fourier domain.

photons are shot at a certain angle with varying radial position, detector records the radial profile which can be considered as a ray integral of the photon intensity, can be written as :

$$p(r,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(xcos\theta + ysin\theta - s)dxdy \qquad (2)$$

where $f(x,y)$ is the cross section of the patient body being intercepted by the photons, "s" is the perpendicular distance from the origin of the function $f(x,y)$, and θ is the angle at which the photons are collected by the detectors. The above integral is known as "Radon Transform". Such transformation is shown in Fig. 1 (b) for Shepp-Logan phantom which matches with the literature [3]. It can be shown that the ensemble of Fourier transform of the radial profile at the given angle $(FFT(p(r,\theta_1)) = F(\omega cos\theta, \omega sin\theta) = F(\omega_x,\omega_y))$ makes the transformed image in the frequency space. Using "Fourier Slice Theorem", the spatial response can be obtained from inverse Fourier transformation [3], given as :

$$f(x,y) = \int_0^{2\pi}\int_0^{\infty} F(\omega_x,\omega_y)H(\omega)\times e^{j2\pi\omega(xcos\theta+ysin\theta)}d\omega d\theta \qquad (3)$$

where $H(\omega)$ is a filter kernel.

Results and Discussion

In a real scenario, only the scanned profiles are available at various angle which forms the radon transform matrix. The profile resolution is governed by detector scintillation yield, photo-multiplier tube characteristics, front end signal processing electronics, and the position reconstruction algorithms. One has to infer about the actual image (in this case Fig. 1 (a)) obtained from the different image reconstruction algorithms. The simplest way is to backproject the profiles uniformly in the image space bin-by-bin. We can repeat this procedure for all the collected profiles such that their intersection reveals the image as shown in Fig. 1 (c). Because of inherent blurring introduced by the convolution with $\frac{1}{|\vec{r}|}$ term, it leads to poor contrast, therefore becomes difficult for medical practitioner to identify different regions in the image such as tumor, blood clotting etc. In order to improve the contrast, various filter functions are attempted in frequency space. For instance, with "Ram-Lak" filter [3], contrast improves significantly, as shown in Fig. 1 (d). Results obtained with other type of filters would be presented in the symposium.

References

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