THE INFLUENCE OF THE MEDIUM ON BREMSSTRAHLUNG IN HIGH-ENERGY ELECTRON-PHOTON SHOWERS †

A. A. VARFOLOMEEV, R. I. GERASIMOVA, I. I. GUREVICH, L. A. MAKAR'INA, A. S. ROMANTSEVA, I. A. SVETLOBOV and S. A. CHUEVA

Academy of Sciences, Moscow

As shown by Landau and Pomeranchuk [1)], the process of Bremsstrahlung is disturbed by multiple scattering as high-energy electrons go through a condensed medium. As a result, the probability of photon radiation, especially that of low energy photons, is decreased in comparison with the results of the usual Bethe - Heitler theory. If the condition

$$\omega/E \ll \left(\frac{E_s}{m}\right)^2 \cdot \frac{E}{m^2 L} = 10^{-8} \varrho E \qquad (1)$$

is fulfilled, in which ω is the frequency of the photon, E is the energy of the electron in MeV, ϱ is the density of the medium, $E_s = 21$ MeV is the scattering constant and L is the radiation length, then the radiation intensity according to Landau and Pomeranchuk is expressed by

$$dI_{L.P.} = \frac{e^2}{2\sqrt{6\pi}} \cdot \frac{E_s}{E} \cdot \frac{\sqrt{\omega}\, d\omega}{\sqrt{L}} \qquad (2)$$

instead of the Bethe - Heitler formula, whose classical analogue can be written as

$$dI_{B.H.} = \frac{e^2}{3\pi} \cdot \left(\frac{E_s}{m}\right)^2 \cdot \frac{d\omega}{L} \ . \qquad (3)$$

Ter-Mikaelyan [2)] has shown that the polarization of the medium leads to an additional weakening of the soft part of the Bremsstrahlung. This effect appears if

$$\frac{\omega}{E} \ll \frac{\omega_0}{m} = 10^{-5} \sqrt{\varrho}. \qquad (4)$$

In this region

$$dI_{T.M.} = \frac{e^2}{3\pi} \left(\frac{E_s}{E}\right)^2 \frac{\omega^2}{\omega_0^2} \cdot \frac{d\omega}{L},$$

where

$$\omega_0 = \sqrt{\frac{4\pi N e^2}{m}} \simeq 10^{16} \sqrt{\varrho}\, \sec^{-1} . \qquad (5)$$

† Appendix to Session 1. — Experimental.

Detailed formulae, taking into account both these effects and valid at $E \gg m$ in the whole domain of photon energy, have been derived by Migdal [3)].

An experimental study of these effects appears possible when investigating high-energy electromagnetic cascades in the cosmic radiation. The emulsion-stack method presents several advantages for such investigations. In big stacks the registration of electron-photon showers with energies up to 10^{12} eV is relatively efficient. Fig. 1 shows the curves of the Bremsstrahlung intensity $\frac{dI}{E} = \frac{\omega\, d\sigma}{E}$ in emulsions for $E = 10^{11}$ eV and 10^{12} eV according to Bethe - Heitler and Migdal (taking into account both effects of the medium). It will be seen that the effect of the medium will be noticeable at $E = 10^{12}$ eV in the radiation of photons of $\omega < 10^{9}$ eV. At small distances from the origin of the cascade it is possible to register with a great efficiency the electron pairs of the cascade starting from the energy of $10^{3} - 10^{7}$ eV and higher. Thus the method provides a way to determine experimentally the number of cascade pairs in a comparatively wide range of energy where the effects in question are due to appear. The use of the multiple scattering method gives the possibility of determining the energy spectrum of the pairs in the range up to 10^9 eV.

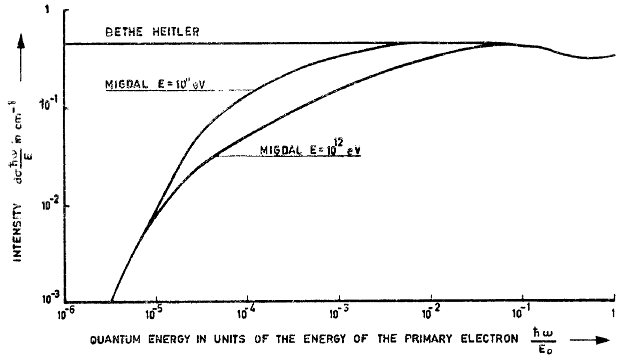

Fig. 1. *Bremsstrahlung spectrum in emulsion for electrons of energy of* 10^{11} *and* 10^{12} *eV according to Bethe - Heitler and Migdal.*

The effect produced by the medium shows all the more if the depth t, in which the measurement is carried out, is small. Too small a depth is undesirable as it decreases the number of results available for statistics. The optimum depth is found to be 1–1.5 t_0 where t_0 (= L) is the radiation length, equal to 2.9 cm in the emulsion.

Experimental spectra, gathered from such depths, cannot be immediately compared with the available results of the cascade theories, not even in the usual Bethe - Heitler variant, since the use of asymptotic formulae for cross-sections of elementary processes is not justified in the soft region of the spectrum at such shallow depths. For this reason we performed a calculation of electromagnetic cascades by the Monte-Carlo method, taking into account accurate (non-asymptotic) cross-sections of the elementary processes. The calculation was performed in two variants: according to the usual Bethe - Heitler theory disregarding the influence of the medium and with a recourse to the formulae given by Migdal [3)], accounting for the influence of multiple scattering and medium polarization on the Bremsstrahlung.

In this calculation the primaries were assumed to be electrons with energies 10^{11} eV and 10^{12} eV. All particles with energies greater than $E_{\min} = 1.5 \times 10^6$ eV were followed. The following elementary processes taking place in nuclear and electron fields of the emulsion matter were taken into account:

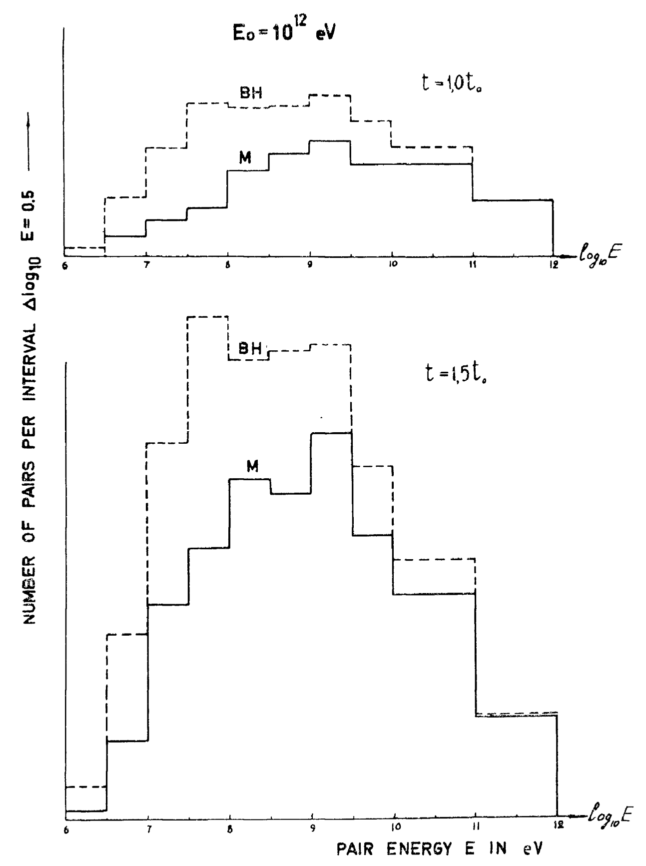

Fig. 2. *Spectrum of the pairs created by electrons of an energy of* 10^{12} *eV in emulsion up to depths of* $1.0\ t_0$ *and* $1.5\ t_0$*, calculated using the Monte-Carlo method.*

Bremsstrahlung, pair production by photons and electrons, Compton effect, photonuclear absorption of photons and ionization. Cross-sections for these processes were calculated taking into account the nuclear composition of the emulsion (which in this case was very similar to that of Ilford G-5).

Spatial distribution of the particles was disregarded, the problem being considered as unidimensional. The resulting data were referred to four depth values: $t_1 = 1.0\,t_0$, $t_2 = 1.5\ t_0$, $t_3 = 2.1\ t_0$ and $t_4 = 2.8\ t_0$. An electronic computer was used. About 100 cascades were computed for each depth and for each energy of primary electron. Fig. 2 shows the differential spectrum of pairs formed at depths up to $t_1 = 1.0\ t_0$ and $t_2 = 1.5\ t_0$ in a shower produced by a primary electron of energy $E = 10^{12}$ eV. Calculation results according to Bethe - Heitler formulae are given as well as those according to the Migdal formulae. The graph shows that the number of pairs with energies $<10^9$ eV is decreased by a factor 2 to 2.5 owing to the influence of the medium. Fig. 3 and 4 show the integral spectra of electrons which reach the depth $t = 1.5\ t_0$ according to both computations (B-H and M). Results by Arley [4)] and Jánossy [5)] for these spectra are shown for comparison purposes. Fig. 5 and 6 show the distributions of numbers of cascades as a function of the number of electrons with energies $> 1.5 \times 10^6$ eV at the depth $1.5\ t_0$. From these graphs fluctuations of the number of electrons in the cascade can be derived.

These calculations are valid for a definite medium — the emulsion. Our results can, however, be used for other media if certain assumptions are made.

In certain papers dealing with investigations of electromagnetic cascades in emulsions, a discrepancy between the observed spectra with the results of cascade theories could be noticed. The most detailed study of one cascade with an energy of $\sim 7.10^{11}$ eV was made by Męsowicz

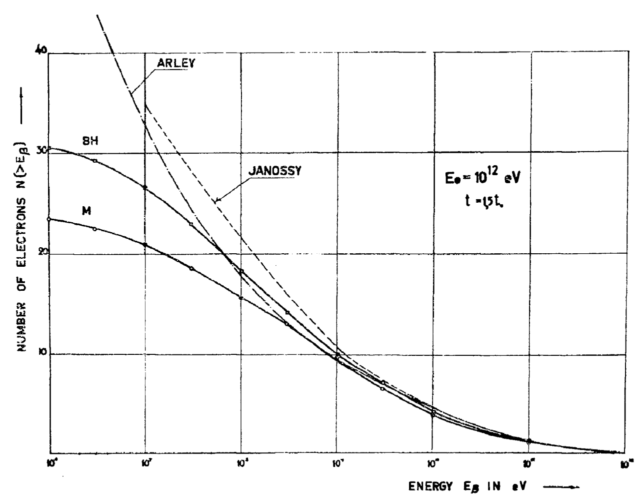

Fig. 3. *Integral spectrum of electrons reaching a depth of* $1.5\ t_0$ *in a shower initiated by a primary electron of energy* 10^{12} *eV calculated by a Monte Carlo method, using the Bethe - Heitler and the Migdal formula. For comparison the integral spectra according to Arley and Jánossy are shown.*

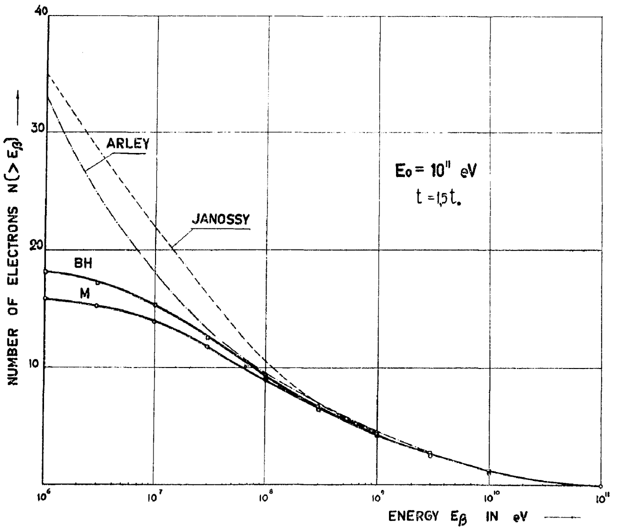

Fig. 4. *As Fig.* 3, *but for a primary electron energy of* 10^{11} *eV.*

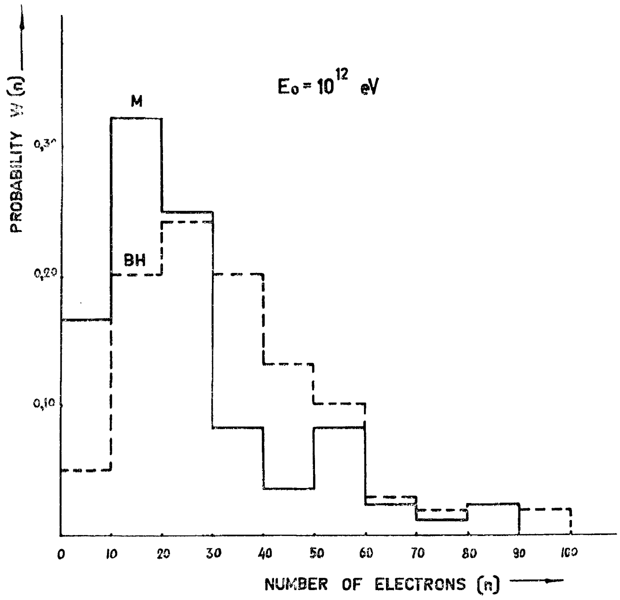

Fig. 5. *Calculated fluctuation of the total number of electrons above* 1.5 *MeV in a depth* 1.5 t_0 *for a primary electron energy of* 10^{12} *eV according to Bethe - Heitler and to Migdal.*

et al.[6]. However, the comparison made in this work is again based on the Jánossy curve (see Fig. 3 and 4).

In order to provide an experimental check concerning the influence of the medium on Bremsstrahlung we have performed an investigation of the energy spectra of pairs in electron-photon showers of high energy ($10^{11}-10^{12}$ eV). Table I gives the general data on the emulsion stacks in which the showers were registered. The exposure of the stacks was made at a high altitude in the stratosphere.

TABLE I

Symbol of shower	Range in one layer of emulsion mm	Stack	Volume of stack litres	Type of emusion
E-53	3.5	E	0.5	" R " NIKFI
O-209	12	O	3.0	,,
D-84	9	D	1.0	,,
D-44	6.5	D	1.0	,,
J-109	3	J	part of stack J	ILFORD G-5

In most stacks emulsions of the type " R " NIKFI were used, in which the grain density of relativistic tracks was equal to 27-30 grains/100 μ. Among the showers recorded in the experiment five were chosen, all of them produced by photons of energy $E_\gamma > 10^{11}$ eV.

The value E_γ was derived from the energy spectrum of electrons at the depth $t = 2.5-3.0\ t_0$. The method was similar to that used by Pinkau [7] and Międsowicz [6]. The electron spectrum was measured by the multiple scattering method at the above mentioned depth within the radius of 200-300 μ around the shower axis. From the number N of electrons with energies greater than $\varepsilon = 3\times10^8$ eV

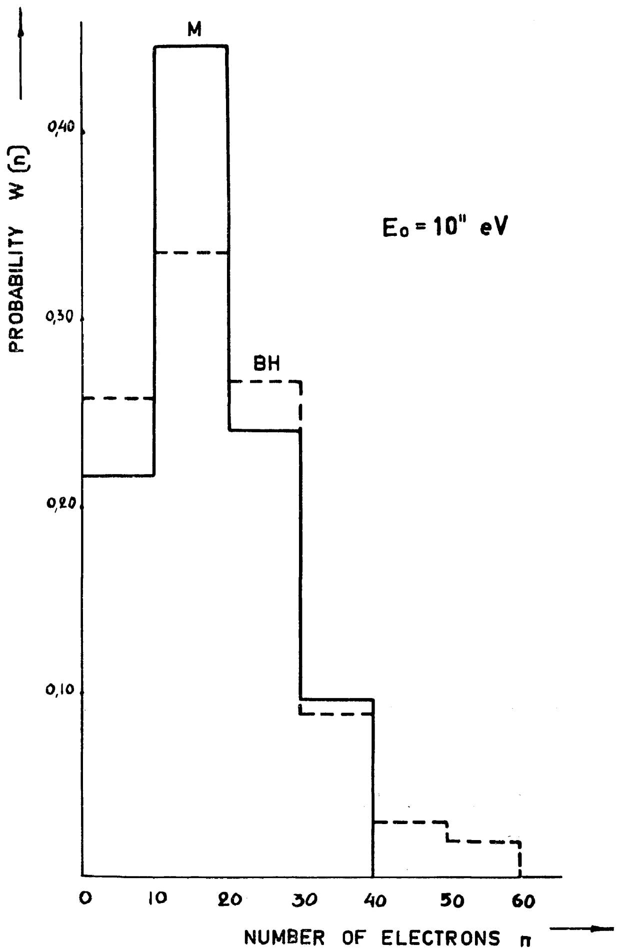

Fig 6. *As Fig.* 5 *for a primary electron energy of* 10^{11} *eV.*

(after corrections for the spatial distribution of electrons) the energy $E_0 = E_\gamma/2$ was determined using the cascade curves, first those of Jánossy [5] and then our own for the depth $t = 2.8\ t_0$. The results are listed in Table II:

TABLE II

Shower	Jánossy eV	Our calculation eV	Chudakov eV
E-53	$[8.4^{+7.2}_{-5.1}] \cdot 10^{11}$	$[1.60^{+3.4}_{-0.71}] \cdot 10^{12}$	$2 \cdot 10^{12}$
O-209	$[1.0^{+0.62}_{-0.45}] \cdot 10^{12}$	$[1.80^{+5.0}_{-0.92}] \cdot 10^{12}$	$2 \cdot 10^{12}$
D-84	$[1.8^{+1.44}_{-0.92}] \cdot 10^{12}$	$[3.24^{+7.76}_{-1.54}] \cdot 10^{12}$	$2 \cdot 10^{12}$
D-44	$[3.3^{+2.6}_{-1.3}] \cdot 10^{11}$	$[7.40^{+29.6}_{-4.5}] \cdot 10^{11}$	
J-109	$[2.6^{+1.24}_{-0.98}] \cdot 10^{11}$	$[4.44^{+17.56}_{-2.4}] \cdot 10^{11}$	

In the first case the errors were calculated as follows: from Jánossy's [8] fluctuation curves and the experimental numbers the possible $N_{max}(> \varepsilon)$ and $N_{min}(> \varepsilon)$ were determined, and from these and from cascade curves it was possible to determine $E_{0\,max}$ and $E_{0\,min}$. In the second case, starting from the computed data according to the Migdal formulae, cascade curves $\bar{N}(\varepsilon, E_0, t)$, $N_{max}(\varepsilon, E_0, t)$ and $N_{min}(\varepsilon, E_0, t)$ were plotted. $E_{0\,max}$ and $E_{0\,min}$ were determined from each curve successively. Values of $N_{max}(\varepsilon, E_0)$ and $N_{min}(\varepsilon, E_0)$ were determined from the curve giving the distribution of the number of cascades as a function of the number of electrons of energy $> \varepsilon$. The method of determination of $N_{max}(\varepsilon, E_0)$ and $N_{min}(\varepsilon, E_0)$ is schematized in Fig. 7.

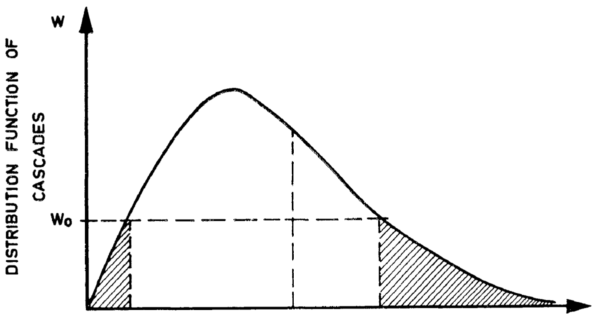

Fig. 7. *Method for finding the limits of error in the number of electrons. Energy limits are determined in an analogous way.*

On the curve showing the distribution $W(N)$ of the number of cascades against the number of electrons $N(> \varepsilon)$ in each cascade, a cut is made at the height W_0, corresponding to a shaded area equal to 30% of the whole area under the curve. The values of N_{max} and N_{min} are given by the abscissae of the intersections between the curve and the straight segments limiting the shaded area.

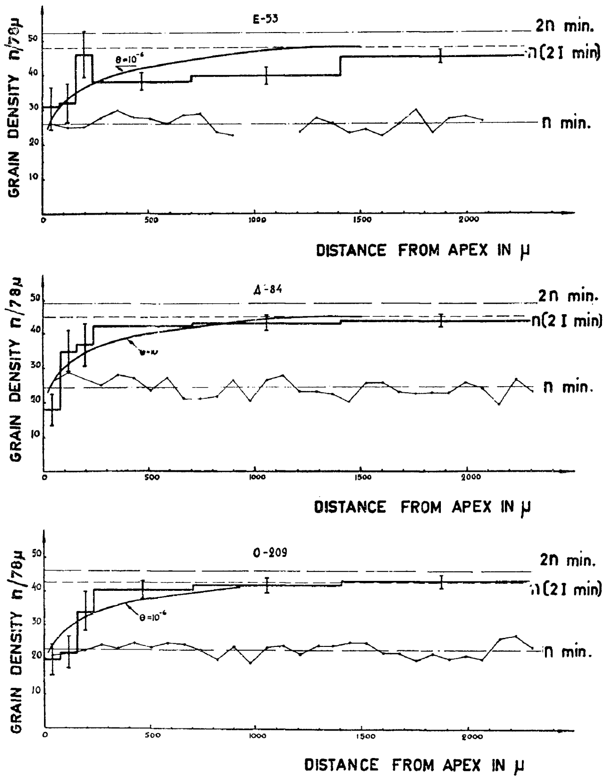

Fig. 8. *Grain density of the first pair of a shower of about* 10^{12} *eV energy as a function of distance from the apex. The broken curves show the grain density of extreme relativistic particles in the same region of the emulsion. The energy E is then determined via Chudakov's formula*

$$R = I/2\,I_0 = \{23.92 + \ln(\theta \times \sqrt{140x} + 1)\}/9.44$$

from $E = 4mc^2/\theta$. *(I is the grain density of the pair,* I_0 *is that of an extreme relativistic particle, x is the distance from the apex in cm and θ the opening angle of the pair in radians.)*

Density measurements on the tracks from the first pairs in the showers showed that in three showers of energy $\sim 10^{12}$ eV a decrease of ionization can be observed near the top, attributable to the mutual electron-positron screening (Chudakov-Perkins effect [9, 10]). Fig. 8 and 9 show the measured grain densities on the tracks from the first pairs in three showers. The broken line shows the measured densities of relativistic electron tracks near the measured segments of the pair track. The figures show a substantial diminution of the pair track density near the top compared with the track density of the doubly-ionizing particle $n(2\,I_{min})$.

The effect is observed along several hundreds of microns, which shows that the separation angle between the electron and the positron of the pair is small. Table II gives evaluations of energy of the first pairs according to the Chudakov [9] formula assuming equi-partition of energy among the components of the pair. It may be seen that these evaluations, made by differents methods, are in agreement for the pairs D-84 and O-209. In the shower E-53 one of the electrons from the first pair had a considerably lower energy than the other. In fact this shower is produced by one electron with a high energy: this is why

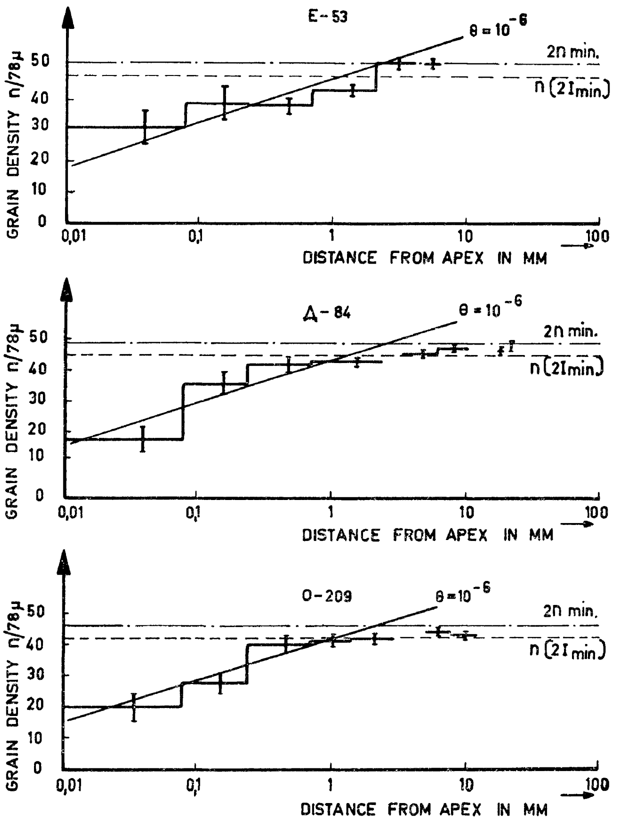

Fig. 9. *The same as in Fig.* 8, *but for greater distances from the apex.*

the energy derived from the separation angle was found to be higher than that derived from the development of the cascade.

Further treatment of the experimental data on these showers consisted first in a reconstruction of the space picture of each shower over a distance up to 1.5 t_0 from the apex of the first pair. A careful search was made for pair and electron tracks within a radius greater by 100-200 μ than the zone in which the cascade electrons were recorded. Next, the energy of each pair was determined from the angle of the multiple scattering of the electrons. Basically a 250 μ cell was used. The full noise of second differences was 0.13 μ per 250 μ. Measurements were made with about 20 cells. In these conditions the error of measurement of the electron energy was about 20% up to the energy (5—7) 10^8 eV. In some single cases the determination of the pair energy was made with a cell of 500 μ (noise 0.2 μ) or by measuring the relative multiple scattering. In a few cases owing to unfavourable conditions of multiple scattering measurements, the pair energy was estimated from the separation angle according to the Borsellino formula [11]. In any case it may be considered that the pair energy was measured with sufficient accuracy up to 10^9 eV.

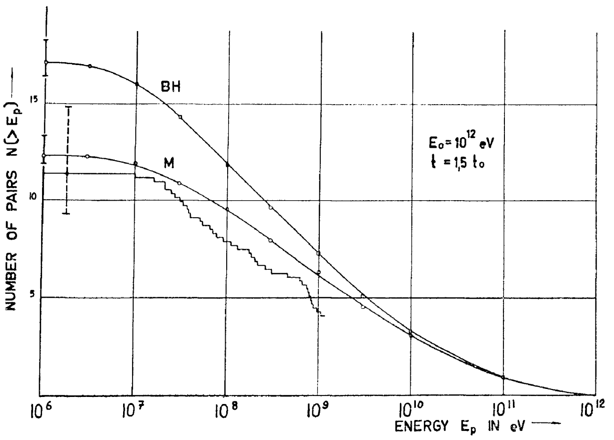

Fig. 10. *Integral energy spectrum of pairs in showers initiated by electrons by a primary energy of* 10^{12} *eV in photographic emulsion. The broken curve is the experimental one, obtained from 5 showers.*

Fig. 10 and 11 compare the experimental results and the theoretical data for the spectra of pairs formed at depths up to 1.5 t_0; they give integral theoretical spectra (smooth curves) and integral experimental spectra (broken curves) on the average for one primary electron. The histogram in Fig. 10 records the results from the showers O-209, D-84 and E-53 (five primary electrons of energy $\sim 10^{12}$ eV), that in Fig. 11 from showers D-44 and J-109 (four primary electrons with energy $\sim 10^{11}$ eV).

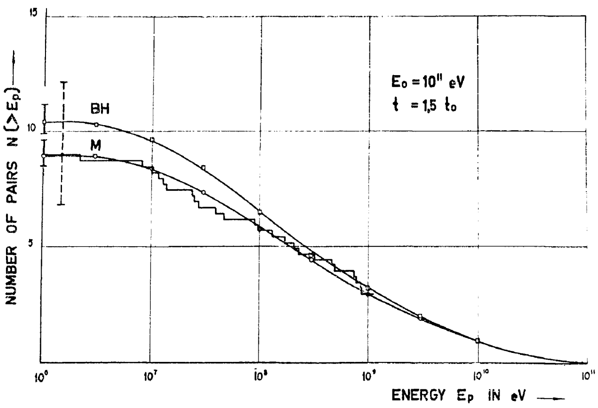

Fig. 11. *As Fig.* 10, *but for a primary electron energy of* 10^{11} *eV.*

Errors shown on Fig. 10 and 11 were determined in a way similar to that for the electron number $N(\varepsilon, E_0, t)$ (see above). The distribution curve of counted cascades against the number N_p at depths up to 1.5 t_0 determined the referential limits $N_{p_{\max}}$ and $N_{p_{\min}}$ corresponding to a 70% probability (see Fig. 7). For the number of pairs in a given shower we have $N_p = \bar{N}_p {}^{+\Delta_1}_{-\Delta_2}$, where $\bar{N}_p$ is the true mean number of pairs. Correspondingly for the mean number from s showers:

$$N_{p_{\text{mean}}} = \bar{N}_p {}^{+\delta_1}_{-\delta_2}, \quad \text{where } \delta_1 = \Delta_1/\sqrt{s}, \quad \delta_2 = \Delta_2/\sqrt{s}.$$

Consequently :

$$\overline{N}_p = N_{p_{\text{mean}}} {}^{+\,\delta_2}_{-\,\delta_1}\,.$$

This procedure was also used in the determination of the errors in the computed curve.

When comparing experimental and computed curves it is also necessary to take into account errors committed in the determination of shower energies. If the energy assigned to a shower is greater than its true energy there will be a discrepancy between the experimental spectrum and that calculated from the Bethe-Heitler formula, which will have nothing to do with the effects of the medium. Therefore, to prove the effects we have to be certain that the assumed shower energies are not too high. For the three showers with energy $\approx 10^{12}$ eV, this condition is fulfilled because the Chudakov effect is present.

It may be seen from Fig. 11 that the discrepency between the curves computed from the formulae of Bethe-Heitler and of Migdal for $E_0 = 10^{11}$ eV is smaller than the experimental errors. For showers with $E_0 = 10^{12}$ eV (Fig. 10) the observed discrepancy is somewhat higher than the errors. The sizeable experimental errors shown here may in fact be somewhat too large because of the selection methods of showers according to their energy. Fluctuations at the depths 1.5 t_0 and 2.8 t_0 are correlated; therefore, showers with electron numbers close to $\overline{N}$ at $t = 2.8\ t_0$ should not have big fluctuations in pair number at the depth $t = 1{,}5\ t_0$.

For these reasons we consider that our results confirm in a qualitative way the effect of the influence of the medium on Bremsstrahlung. Our results show that a quantitative verification of the medium effect at $E_0 \simeq 10^{12}$ eV requires an increase of the statistical material by a factor of about 4.

LIST OF REFERENCES

1. Landau, L. and Pomeranchuk, I. Ya. Doklady Akad. Nauk SSSR, *92*, p. 535 and p. 735, 1953.
2. Ter-Mikaelyan, M. L. Doklady Akad. Nauk SSSR, *94*, p. 1033, 1954.
3. Migdal, A. B. Phys. Rev., *103*, p. 1811, 1956; Zh. eksper. teor. Fiz., *32*, p. 633, 1957.
4. Arley, N. Proc. roy. Soc., A, *168*, p. 519, 1938.
5. Jánossy, L. and Messel, H. Proc. roy. Irish Acad., A, *54*, p. 217, 1951.
6. Mięsowicz, M., Stanisz, O. and Wolter, W. Bull. Acad. Polon. Sci., Cl. 3, *4*, p. 811, 1956.
7. Pinkau, K. Nuov. Cim., *3*, p. 1285, 1956.
8. Jánossy, L. Acta phys. Hungar., *2*, p. 289, 1952.
9. Chudakov, A. E. Izv. Akad. Nauk SSSR, Ser. Fiz., *19*, p. 651, 1955.
10. Perkins, D. H. Phil. Mag., *46*, p. 1146, 1955.
11. Borsellino, A. Phys. Rev., *89*, p. 1023, 1953.