

Physical effects of gravitational waves: pedagogical examples

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Abstract

General relativity describes gravitation in terms of the geometry of spacetime. It predicts the existence of gravitational waves (GWs) that stretch and compress spacetime and were detected recently by state-of-the-art interferometer observations. Yet, for those who are not familiar with general relativity, it may be difficult to understand how the GWs actually stretch and compress spacetime. In this paper, after reviewing the fact that the effects of GWs can be understood as a force in Newtonian mechanics, we consider extreme and readily perceivable situations where large-amplitude GWs pass through the human body and the Earth, and demonstrate that GWs cause phenomena that commonly occur in daily life by back-of-the-envelope calculations. Our analysis provides intellectual and pedagogical materials for understanding the nature of GWs for nonexperts in general relativity.

Keywords: gravitational waves, black holes, general relativity, Newtonian mechanics

(Some figures may appear in colour only in the online journal)

1. Introduction

On February 11, 2016, the LIGO Scientific Collaboration and Virgo Collaboration announced the first direct detection of gravitational waves (GWs) generated by the coalescence of binary black holes (BHs) [1, 2]. It was a historical milestone for mankind to finally prove the



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Figure 1. Artistic image of extremely exaggerated physical effects of GWs. Owing to the passage of GWs, the Earth is stretched and compressed in a direction perpendicular to the propagation of GWs. In addition to this effect, two effects on the light arriving at the observer, i.e. color change of the light due to the red-/blue-shift and the change in light trajectories, also distort the appearance of the Earth.

prediction of Einstein's general relativity proposed about 100 years ago. GWs are ripples in spacetime. Now, it is almost common knowledge not only among scientists but also the general public that spacetime is stretched and compressed upon the passage of GWs. Many people are interested in GWs and general relativity; hence, it is time to think about how to cultivate intuitive understanding [3]. For a good review on the history of GWs, see [4].

Compared with, for example, electromagnetic waves, there are several obstacles to people being aware of the effects of GWs in their daily lives. First of all, it is difficult to imagine what happens when GWs stretch and compress spacetime. In particular, a common misconception is as follows: 'Rulers should also be stretched and compressed similarly to space. Thus, are GWs not physically observable inherently?' Actually, this is not the case, and GWs do lead to physically observable effects. In principle, one can notice the passage of GWs if their amplitude is sufficiently large to cause visible distortions (see figure 1). Although this conceptual issue has already been addressed in depth in the literature (e.g. see [5–8]), in the next section, we briefly provide the physical explanation of the reason why GWs can yield observable effects, which is basically a restatement of what is already known. Nevertheless, we believe that it helps readers confronted with this conceptual question recognize the reality of GWs and easily understand the subsequent sections where we study how the GWs with extreme amplitudes cause visible effects on daily phenomena.

The second reason why people do not commonly think about GWs as much as they do about electromagnetic waves is their extremely small amplitude. For instance, in the case of GWs emitted from a typical binary BH merger at a cosmological distance, the change in distance between the Earth and the Sun is merely about one-tenth of the Bohr radius! It is this smallness of GW amplitude that had prevented direct detection for about 100 years and rendered GWs irrelevant to daily physical phenomena.

Although these peculiar circumstances have led to the general public being less aware of GWs, given their scientific significance, it is still regrettable if only the experts of general relativity can appreciate their physical properties³. Fortunately, the physical effects of GWs

³ There are several educational papers that aim to spread the basic concept and recent experimental achievements of general relativity and GWs [9–13].

can be understood through familiar Newtonian mechanics [5, 7]. The aim of this paper is to provide intellectual and pedagogical materials for understanding the nature of GWs within the framework of Newtonian mechanics and at the level of the back-of-the-envelope calculation, by which students who have learned Newtonian mechanics but are not familiar with relativity can obtain a clear view of the physical effects of GWs. To this end, we consider extreme and readily perceivable situations. As it is often the case when learning a particular subject in physics, ideal examples enable us to easily understand the essence of the physical effects of GWs without being concerned with non-GW effects that are unnecessary for the current purpose but may become dominant in more realistic situations. We perform a quantitative analysis of several parameter regions in the amplitude-frequency space in which GWs would visibly affect us. Additionally, considering a BH binary system as the source of GWs, we shall estimate the corresponding BH mass and distance from the Earth.

We stress that our results should not be regarded as a warning of the risk of damage to human beings posed by GWs in actual daily life. As we shall see below, GWs may readily and perceptibly exert their effects only at a sufficiently close distance from massive BHs.

2. Physical effects of GWs: general arguments

Before presenting our clear examples of the physical effects of GWs, in this section we provide a general argument on how we can, in principle, detect GWs by addressing the following conceptual question that people who are not familiar with general relativity might ask: ‘If the passage of GWs causes nothing but the stretching and the compression of space, do they also stretch and compress a ruler for measuring the change in the size of the body simultaneously, making the effects of GWs unobservable?’ Although the answer to this question is already provided in several reports (e.g. [5–8]), we consider it useful not to skip this issue but to recapitulate the essential point of the answer here in order for readers to understand the background and hence the subsequent results of our analyses.

In a nutshell, an essential point in any experiment to detect the physical effects of GWs is the existence of an ‘absolute ruler’. Here, the absolute ruler refers to the ruler (not any ordinary ruler) that does not change upon the passage of GWs. The effects of GWs can then be perceived as the change of the system measured using the absolute ruler.

The absolute ruler is a set of fundamental physical constants that are literally absolute and do not change in the presence of GWs. The relevant physical constants depend on the types of systems and measurements considered. Here, we focus on the following two cases of physical phenomena caused by GWs and clarify how GWs induce physical effects that can be detected.

1. Variation of light travel time.
2. Variation of the size of a deformable or semirigid body with time.

The relevant physical constants in each phenomenon are the speed of light c (case 1) and those used to determine the electromagnetic force at the microscopic level, such as the electric charge (case 2).

Case 1 is actually implemented in current laser interferometers (such as LIGO). In laser interferometers, the distance L between the beam splitter and the mirror changes upon the passage of GWs, and consequently the travel time of the laser light over that distance, which the detector actually measures, also changes. Although the propagation speed of the laser light is not affected by GWs, the wavelength λ is stretched and compressed by them; hence, the laser light is red-/blue-shifted. Thus, the ratio L/λ remains constant. Nevertheless, this does

not mean that the laser interferometer cannot detect GW signals since it can detect the change in light travel time [8, 10].

In case 2, the physical phenomena induced by the distortion of a deformable body or vibrations of a semirigid body are measured. To understand this qualitatively, let us model the material of a body as a collection of point masses connected by springs. This model provides a convenient alternative to the electromagnetic force acting between atoms. The properties of the springs, such as their natural length and restoring force, are determined by physical constants, such as the electric charge and electron mass, and are not affected by GWs. When GWs pass through, the GWs change the length of the springs from their natural length. For the atoms and springs, this action is perceived as a *physical* force because natural length is absolute. As a result, the atoms oscillate around the equilibrium position, which, at the macroscopic level, can be seen as vibrations of the body.

For case 2, we can derive the effective force induced by GWs without resorting to general relativity. Consider a system consisting of two point masses floating in space, each with mass m and separated by a distance L . When a gravitational wave with amplitude $h(t)$ passes through the system, the distance between the masses (as measured by an absolute ruler) becomes $\sqrt{1+h(t)}L$ according to general relativity. If we assume that $h(t) \ll 1$, which is valid in most cases explored in this paper, we can use the binomial approximation $(1+\epsilon)^{1/2} \approx 1 + \frac{1}{2}\epsilon$ to say that the distance is approximately $\left[1 + \frac{1}{2}h(t)\right]L$, meaning that an observer with an absolute ruler would say that each mass is oscillating toward or away from the systems center of mass with an acceleration of $a = \frac{1}{2}\ddot{h}\left(\frac{1}{2}L\right)$ (since each mass is $\frac{1}{2}L$ from the center) $= \frac{1}{4}\ddot{h}L$. However we might interpret this in general relativity, from a Newtonian perspective, this acceleration must be caused by a force, meaning that each mass behaves as if it were experiencing a force of magnitude

$$F = \frac{1}{4}m\ddot{h}(t)L, \quad (1)$$

toward or away from each the systems center of mass. If the two masses happened to be connected by a spring, this effective force would measurably compress or expand the spring: indeed, the spring would behave as if it were being compressed by a total force of $F = \frac{1}{2}m\ddot{h}(t)L$.

The above discussion demonstrates that the stretching and compression of space by GWs do yield observable physical effects. In the following, we focus on case 2 and provide three extreme examples (destruction of organs, earthquakes, and tides) with which readers unfamiliar with general relativity can imagine the physical effects of GWs intuitively in terms of Newtonian mechanics. As the waveform of a GW, we consider a monochromatic wave given by

$$h(t) = h_0 \cos(2\pi f_{\text{gw}} t). \quad (2)$$

3. Example 1: Organ destruction

The first example we study is the potential destruction of the human body caused by passing GWs. With the typical realistic strain, the effect of GWs on the human body is totally negligible. However, as we have discussed in section 2, for a sufficiently large-amplitude of GWs, in principle, GWs could destroy organs or tissues inside our body. Let us provide an order-of-magnitude estimation of the stretching of GWs at which organs are destroyed by

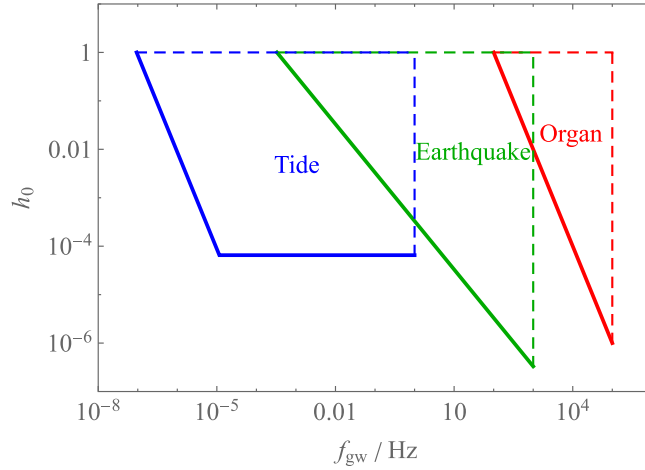


Figure 2. GW amplitudes h_0 sufficiently strong to cause large effects in terms of frequency f_{gw} . The solid lines indicate the threshold beyond which human lives would be threatened, whereas the dashed lines indicate the possible boundaries of the validity of the approximations we employed. The red, green, and blue lines are, respectively, the thresholds, beyond which the GWs cause organ destruction, major earthquakes, and large tides.

determining the force exerted by GWs of various amplitudes and frequencies on the parts of an organ and comparing against the maximum force the organ can tolerate without being destroyed.

In the order-of-magnitude estimation, we may replace an organ with two massive objects separated by a distance equivalent to the size of the organ and solve equation (1) as a force acting on the organ. Taking $L = 10$ cm and $m = 0.1$ kg as representative values for an organ (e.g. heart) and scaling frequency f_{gw} relative to 100 Hz, which is the typical frequency for an event measured by LIGO, Virgo, or KAGRA, we find that the amplitude of the total compressive force on the organ is approximately given by

$$F \approx 10^3 \text{ N} \left(\frac{f_{\text{gw}}}{100 \text{ Hz}} \right)^2 h_0. \quad (3)$$

Roughly speaking, an organ will collapse if more than $\sim 10^3$ N is exerted on it [14]. By adopting this criterion, we find that an organ is destroyed if the GW amplitude h_0 satisfies

$$h_0 \gtrsim \left(\frac{f_{\text{gw}}}{100 \text{ Hz}} \right)^{-2}. \quad (4)$$

The red solid line in figure 2 shows the boundary of this equality as a function of f_{gw} . The lower limit of f_{gw} around 10^2 Hz originates from the criterion $h_0 < 1$ for which our linear approximation is valid. Cases where $h_0 > 1$ are beyond the scope of this paper. The upper limit of f_{gw} is the inverse of the time that a sound wave travels across an organ:

$$f_{\text{gw}} < \frac{c_0}{L} = 10^5 \text{ Hz} \left(\frac{c_0}{10^3 \text{ m s}^{-1}} \right) \left(\frac{L}{10 \text{ cm}} \right)^{-1}, \quad (5)$$

where c_0 is the speed of sound passing across the organ. Above this frequency, the force from GWs changes its direction before it affects the whole organ, and it is not clear whether the

organ is destroyed even if the force given by equation (3) is applied. Hence, in our analysis, we consider the frequency range that satisfies this condition. In figure 2, the boundaries $h_0 = 1$ and $f_{\text{gw}} = 10^5$ Hz are shown as red dashed lines.

4. Example 2: Earthquakes

Next, we consider an earthquake that would be induced by the passage of GWs. A pioneering work was performed by Dyson, who calculated the seismic response of the Earth to the passage of 1 Hz GWs [15]. Constraints using potential seismic signatures have been extensively studied [16]. Dyson's computation heavily relies on tensorial equations and general relativity, which are beyond the scope of this paper. Instead, here, we consider a back-of-the-envelope calculation. Quite remarkably, results of our crude calculation reproduce those of Dyson's computation within a factor of $\mathcal{O}(1)$.

An earthquake is simply the vibration of the ground propagating as a sound wave. When a GW of frequency f_{gw} is passing through the Earth, a sound wave with the same frequency is induced and propagates. Let us model the motion of a segment of the Earth medium with a harmonic oscillator in which a point mass is attached to a spring of length L . As we have discussed in section 2, GWs induce the acceleration of the harmonic oscillator of the order of $\pi^2 f_{\text{gw}}^2 L h_0$. From the shape of the plane wave $e^{2\pi i x / \lambda_e}$, where λ_e is the wavelength of a transverse seismic wave passing through the Earth, the phases of the ground motion at different locations are nearly coherent (i.e. the phase difference is $< 2\pi$) if the separation is less than λ_e . In other words, modeling the vibration of the ground as oscillations of the harmonic oscillator is valid for the segment of the Earth medium with a (typical) size $\lesssim \lambda_e$. This means that we should take L to be λ_e . Then, the acceleration of the ground is estimated as

$$a = \pi^2 c_e f_{\text{gw}} h_0, \quad (6)$$

where $c_e = \lambda_e f_{\text{gw}}$ is the speed of a transverse seismic wave passing through the Earth, which is typically 3×10^3 m s⁻¹. Note also that since $f_{\text{gw}} = c / \lambda_{\text{gw}} = c_e / \lambda_e$ it holds that $\lambda_e \ll \lambda_{\text{gw}}$. The acceleration (6) is the same as that obtained by Dyson up to a factor of $\mathcal{O}(1)$ [15].

The induced earthquake will be catastrophic if the acceleration is greater than the gravitational acceleration $g \approx 9.8$ m s⁻². Thus, by equating acceleration (6) to g , we obtain the critical GW amplitude h_0 that yields a serious earthquake:

$$h_0 = \frac{g}{\pi^2 c_e f_{\text{gw}}}. \quad (7)$$

The green solid line in figure 2 shows h_0 that causes an earthquake with ground acceleration comparable to the gravitational acceleration. As in [15], we focus on the GW frequency range $1 \text{ mHz} < f_{\text{gw}} < 1 \text{ kHz}$. In this range, it is a good approximation to treat the surface of the Earth as a flat plane since the wavelength of the induced seismic waves is much smaller than the size of the Earth and larger than the surface irregularities.

5. Example 3: Tides

The third effect we consider is tides, i.e. the rise and fall of sea levels, which are usually generated by the gravitational forces from the Moon and Sun. Similarly, when GWs pass through the Earth, they can generate tides. Tides involve very complicated processes and it is difficult to analyze those generated by the passage of GWs. As in the previous two examples,

we will evaluate tides by a crude approximation. To this end, we treat the Earth as a rock sphere completely encompassed by a water shell of uniform depth H . In this model, we assume that the rock sphere is so rigid that it is not deformed by GWs and only the water shell is freely deformed (since it is liquid) by the force given by equation (1).

For this model, the first task is to determine the length scale L . We make a rough estimate; if the period of GWs is longer than one day, the ocean on the global scale will coherently respond to GWs similarly to the case in response to the tidal force generated by the Moon's gravitational force. In this case, the natural scale of L will be the Earth radius R_\oplus . Assuming $L = R_\oplus$, the force from the GWs exerted on the volume element ΔV is given by

$$F \simeq \pi^2 f_{\text{gw}}^2 \rho_w R_\oplus h_0 \Delta V, \quad (8)$$

where ρ_w is the density of water. Conversely, if the period of GWs is much shorter than one day but longer than the time scale of the sound wave traveling across the depth H , the response of the sea level will not be the same as that in the low-frequency case.

To understand this situation, let us consider the motion of a harmonic oscillator with a periodic external force as a simple model: $m\ddot{x} = -kx + m\omega^2 h_0 L \sin(\omega t)$. Here, x and the external force are assumed to represent the change in sea level and the effect by GWs, respectively. Note that we have taken the coefficient of the external force to be $m\omega^2 h_0 L$; with this expression, $h_0 L$ does not depend on ω and is simply proportional to h_0 (see equation (1)). Assuming $x = x_0 \sin(\omega t)$, x_0 is given by

$$x_0 = \frac{h_0 L \omega^2}{\omega_0^2 - \omega^2}, \quad (9)$$

where $\omega_0 = \sqrt{k/m}$. In the low-frequency limit $\omega \ll \omega_0$, we have $x_0 = h_0 L \omega^2 / \omega_0^2$, namely, x_0 is determined by the balance between the term $-kx$ and the external force. On the other hand, in the high-frequency limit $\omega \gg \omega_0$, we have $x_0 = -h_0 L$. Apparently, this might look inconsistent with the standard result of the forced harmonic oscillator, for which the amplitude goes to zero in the high-frequency limit. The point is that, as stressed above, the GW driving force actually increases with frequency for a given GW amplitude h_0 , and this leads to the conclusion that in the high-frequency limit x_0 becomes a constant value $-h_0 L$ independent of the frequency of the external force ω . Given these observations, it would be reasonable to consider that the tide induced by GWs becomes independent of f_{gw} for $f_{\text{gw}} \gg f_{\text{day}} = 1 \text{ d}^{-1}$.

The next nontrivial task is to estimate the magnitude of the tide induced by the force given in equation (8), which, in principle, can be done by solving the shallow-water equation. However, even at the level of the order-of-magnitude estimate, how this can be achieved is nontrivial. To overcome this issue, we simply compare force (8) with the tidal force induced by the Moon. For $f_{\text{gw}} < f_{\text{day}}$, we assume that the level of the tide induced by GWs is the same as that of an ordinary tide if the two forces have the same magnitude. The tidal force from the Moon acting on the volume element ΔV is estimated as

$$F_L \simeq \frac{GM_L \rho_w \Delta V}{D^3} R_\oplus. \quad (10)$$

Here, M_L is the lunar mass, and D is the lunar distance. By equating equations (8) to (10), we obtain the minimum h_0 that yields a tide comparable to the ordinary one as

$$h_0 = \frac{GM_L}{\pi^2 D^3 f_{\text{gw}}^2}. \quad (11)$$

On the other hand, for $f_{\text{gw}} \gg f_{\text{day}}$, on the basis of the above argument on the forced oscillator, we assume that the critical h_0 is given by the right-hand side of equation (11) with f_{gw} being replaced by f_{day} .

The blue solid line in figure 2 shows h_0 that causes a tide comparable to that caused by the Moon. The increase in sea level to this amount will threaten the lives of many people residing in coastal regions. The upper limit for $f_{\text{gw}} \lesssim 1$ Hz is imposed by the condition for the validity of the shallow-water approximation that the time for sound to pass across the depth of the ocean is shorter than the GW frequency.

6. GWs from BH binaries

In the previous sections, we considered extreme situations where the GWs cause visible effects on humans by causing stretching and shrinking of space, and we obtained the parameter regions in the amplitude–frequency space, as shown in figure 2. Now it is interesting to consider converting the parameter regions for the GWs to those for BH binaries. The strongest GWs are emitted during the last moment of the merging of BHs in a binary system. Therefore, it is natural to try to relate GWs to BH binary systems in such a way.

We assume that a BH binary system consists of two equal-mass BHs each of which has mass M and is located at the distance r from the Earth. Precise modeling of the GW waveform from such a phase requires numerous computations based on general relativity. Since we are working at the level of the order-of-magnitude estimate, we do not use the results obtained from full computations but make a rough estimate of the GW amplitude and frequency as follows.

At the last inspiral phase, the distance between BHs is comparable to the size of the BHs, i.e. the Schwarzschild radius $r_s = 2GM/c^2$, and the BHs are orbiting at a relativistic speed. In this extreme situation, the disturbance of the gravitational field caused by the motion of the BHs, which propagates outward as GWs, would be comparable to the gravitational field generated by the BHs when they are not moving. Thus, it is natural to think that the GW amplitude around the BHs is also $\mathcal{O}(1)$. The GW amplitude attenuates inversely proportionally to the propagation distance. Thus, by denoting r as the distance to the GW source, we find that the GW amplitude h_0 on the Earth would be on the order of

$$h_0 \simeq \frac{2GM}{c^2 r}. \quad (12)$$

The period of GWs is given by the time scale of the change of the GW source, which is r_s/c . Therefore, the frequency of the GWs would be on the order of

$$f_{\text{gw}} \simeq \frac{c}{r_s} = \frac{c^3}{2GM}. \quad (13)$$

This clearly shows that there is a one-to-one correspondence between f_{gw} and M .

In the previous sections, we estimated the critical values of h_0 above which phenomena catastrophic to humans occur at various values of f_{gw} . By using equation (13) to replace f_{gw} with M , we can draw curves corresponding to the critical values of h_0 in the M – r plane, as shown in figure 3. As expected, the results in figure 3 suggest that the GWs would cause serious damage only if too-heavy BHs merged at too-close distances from the Earth. It would be a good exercise to think about the farthest and/or lightest parameter value for each case. For instance, let us consider the farthest parameter value $(r, M) \sim (10^{15} \text{ km}, 10^{10} M_\odot)$ for the tide. It means that a large tide would be induced by GWs if two supermassive BHs each with a mass of $\sim 10^{10} M_\odot$ merged at about 100 light years from the Earth. To get a sense of this

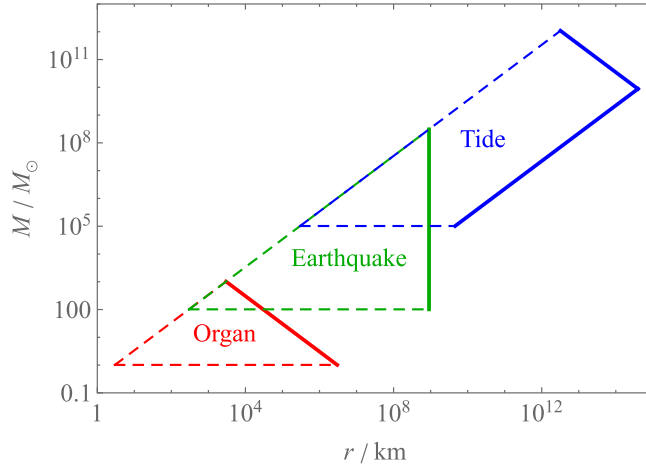


Figure 3. Parameter region corresponding to figure 2 in terms of the BH mass M and the distance r from the Earth assuming that GWs are emitted at the last inspiral stage of two BHs of equal mass. The BH mass M is normalized by the solar mass $M_{\odot} \approx 1.99 \times 10^{30}$ kg.

situation, let us compare the distance and the mass $(r, M) \sim (10^{15} \text{ km}, 10^{10} M_{\odot})$ of a BH with those of Sgr A*, which is the supermassive BH at the center of the Milky Way galaxy. The mass of Sgr A* is $4.2 \times 10^6 M_{\odot}$ and the distance is 2.7×10^4 light years from the Earth [17]. Therefore, even the farthest parameter value means that too-heavy BHs merged at too-close distances from the Earth. If such supermassive BHs existed, the Solar System would be destroyed by their gravitational force far before they merge. Therefore, the earthquakes and tides generated by GWs are totally irrelevant to reality. However, when space travel to BH binary systems becomes popular in the future, the effect of GWs on organs may be relevant during such travel. In that case, the red boundary in figure 3 must be included as a caution in the flight plan for astronauts.

7. Conclusion

GWs are ripples of spacetime traveling across the Universe. Not only spacetime but also everything in nature are stretched and compressed by GWs. Therefore, in principle, they could cause some large effects. After clarifying the conceptual issue about the detectability of GWs and translating the effects of GWs into the language of Newtonian mechanics, we performed the order-of-magnitude estimation for visible phenomena, such as organ destruction, earthquakes, and tides, that would be caused by large-amplitude GWs. We obtained the amplitude–frequency space in which GWs would cause some large effects. As a reference, we converted the parameter regions to those in BH mass–distance space. These are interesting examples highlighting the nature of GWs and are useful for gaining some understanding of the theory of relativity.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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