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κ-Exponential Inflation

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Abstract. We investigate a possible inflationary scenario where the expansion of the universe is driven by a slow evolution of a homogeneous single scalar field, whose potential $V(\varphi)$ is given by an unusual κ-generalized power law. Within the *slow-roll* approximation we obtain several of the main predictions of the model, as the scalar spectral index, the tensor-to-scalar ratio, the number of *e-folds*, and the local non-Gaussianity. We also show that this model admits a much wider set of solutions than the usual exponential approach, and that their theoretical predictions are contemplated by the observational data.

1. Introduction

Although strong concurrents have arisen in the last years, the inflationary cosmology is still the favorite paradigm to explain the origin of the anisotropies in the *Cosmic Microwave Background* (CMB), as well as to support the process of structures formation in the Universe. Arisen from the interface between the modern particle physics and standard Hot Big Bang cosmology, the cosmic inflation [1, 2] presuppose an abrupt expanding initial phase of the scalar factor, essentially induced by the negative pressure from the so-called *inflaton* scalar field, whose energy density dominates briefly the primordial cosmological scenario. Another aspect that makes this paradigm worthy of attention is its robust collection of predictions determining the observational features of the recent universe, such as: absence of magnetic monopoles, a strongly flattened spatial section, and a high degree of isotropy on scales greater than one degree of angular separation in the CMB sky as measured by [3, 4].

It must be emphasized that the microphysics origin of the inflation is still a mystery, so that any description of this period requires an appropriate extrapolation of the known physical laws. Consequently, the standard approach is strictly phenomenological, with a considerable freedom in modeling the potential energy of the field, which is responsible by the primordial inflationary period. In this sense, several forms of potential function have been widely explored in the literature, from the simplest models, such as the quartic $V(\varphi) \sim \lambda\varphi^4$ and quadratic $V(\varphi) \sim m^2\varphi^2$ potentials in Linde's Chaotic Inflation [5], to more elaborate ones, such as the tachyon [6], Higgs-dilaton [7], braneworlds [8] and those involving generalized Einstein's theories [9]. The final judgment for this impasse is given by the nature, where the results of the latest data release of the Planck satellite have shown that trivial models, such as simple power laws, are privileged by data [10].

In this paper, we study the main consequences and predictions of a inflationary scenario driven by the potential energy from *inflaton*

$$V(\varphi) \propto \exp_\kappa(-\lambda\varphi), \tag{1}$$

where the above generalized exponential function is defined by the following power law,

$$\exp_\kappa(-\lambda\varphi) \equiv \left(\sqrt{1+\kappa^2\lambda^2\varphi^2} - \kappa\lambda\varphi\right)^{\frac{1}{\kappa}}, \tag{2}$$

with the parameter κ defined in the interval $-1 \leq \kappa \leq 1$ and λ is the model parameter (associated with the energy scale of the inflation), both real. It can be easily verified that in the limit $\kappa \to 0$, we obtain $\exp_\kappa(-\lambda\varphi) \to \exp(-\lambda\varphi)$, reducing our model to the usual exponential inflation (see [11, 12, 13]). Since the *square-root* factor will occur often throughout this paper, we will simply take $\gamma(\varphi) \equiv \sqrt{1+\kappa^2\lambda^2\varphi^2}$, as originally treated in [14, 15, 16].

2. Slow-Roll Aproximation

In the classical approach of the inflationary paradigm, we assume that the φ-field evolution is dominated by the *drag* of the cosmological expansion, so that the potential energy of the *inflaton* dominates over the kinetic energy and the second derivative with respect to the physical time is negligible. In this *background*, the relation expressing the energy conservation is

$$3H\dot{\varphi} + V'(\varphi) \simeq 0, \tag{3}$$

where the Hubble isotropic expansion is given by

$$H \simeq \left[\frac{V(\varphi)}{3}\right]^{\frac{1}{2}}. \tag{4}$$

Throughout this paper we adopted units in which the Planck mass $m_{Pl} \equiv (8\pi G)^{-1/2} = c = 1$. Also, we define the *dot* and *prime* derivates as $\dot{\varphi} \equiv d\varphi/dt$ and $V' \equiv dV/d\varphi$. In this context, the *slow-roll* regime can be expressed in terms of the *slow-roll* parameters, $\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2$ and $\eta \equiv \frac{V''}{V} - \frac{1}{2}\left(\frac{V'}{V}\right)^2$ [17, 18].

To compare our model with the observational data, firstly we will consider the scalar spectral index n_s and the tensor-to-scalar ratio r, defined by

$$n_s - 1 = 2\eta - 6\epsilon \tag{5}$$

and

$$r = 16\epsilon, \tag{6}$$

respectively. By substituting our potential into the above equations, we obtain

$$n_s - 1 = -\lambda^2\left\{\frac{1+2\kappa}{\gamma^2(\varphi)} - \frac{2\kappa}{\gamma^3(\varphi)\left[\exp_\kappa(-\lambda\varphi)\right]^\kappa}\right\} \tag{7}$$

and

$$r = 8\lambda^2\frac{1}{\gamma^2(\varphi)}, \tag{8}$$

whose direct relation is found to be

$$n_s - 1 = -\frac{r}{8}\left[1 + 2\kappa\left(1 - \frac{r}{8\lambda^2 - \sqrt{8\lambda^2}\sqrt{8\lambda^2 - r}}\right)\right]. \tag{9}$$

A very important parameter describing the amount (or duration) of the inflation needed to solve the Big Bang puzzle is called the number of e-folds, defined by

$$N(\varphi) \equiv \int_{\varphi_e}^{\varphi} \frac{d\varphi}{\sqrt{2\epsilon(\varphi)}} = \frac{1}{2\lambda}\left\{\gamma(\varphi)\varphi + \frac{1}{\kappa\lambda}\ln\left[\gamma(\varphi) + \kappa\lambda\varphi\right]\right\}, \tag{10}$$

where the field at the end of inflation is $\varphi_e = \frac{1}{\kappa}\left(\frac{1}{2} - \frac{1}{\lambda^2}\right)^{\frac{1}{2}}$. We emphasize that the number of e-folds at the end of inflation is a relatively well-constrained quantity by the observations. On the one hand, for the causal horizon of the structures, as well as the flattening of the spatial section, to make physical sense, is necessary $N \approx 60$ [17]. From the other side, the Planck's constraints about the background gravitational waves provides the observational range $50 \leq N \leq 60$ [10].

To complete our description, we analyse the picture of possible interactions from the *inflaton* as well, that is, the non-Gaussianity level from primordial fluctuations [19]. The important quantity here is the parameter of local non-linearity f_{NL}^{local}, expressed in terms of the *slow-roll* parameters by (for analyses of f_{NL}^{local} in CMB data, see, e.g., [20, 21])

$$f_{NL}^{local} = -\frac{5}{6}(\eta - 2\epsilon) = \frac{5\lambda^2}{12}\left\{\frac{1+2\kappa}{\gamma^2(\varphi)} - \frac{2\kappa}{\gamma^3(\varphi)\left[\exp_\kappa(-\lambda\varphi)\right]^\kappa}\right\}, \tag{11}$$

from where we get the straight relation

$$f_{NL}^{local} = -\frac{5}{12}(n_s - 1). \tag{12}$$

This simple expression show us that there is a linear relation between the scalar spectral index and the local non gaussianity, is called the consistency relationship [22]. An interesting work regarding the non-gaussianity features of the primordial universe have been performed in reference [23].

3. Results and Discussions

The inflationary paradigm represents one of the greatest successes of the fusion between the concepts of the Physics of elementary particles and Cosmology, bringing changes in the form how we understand the primordial Universe. It is the favorite hypothesis of most cosmologists to explain the origin of primordial fluctuations, being the *seeds* for the emergence of structures, as well as for the flatness, isotropy, and homogeneity of the observed universe.

In this paper, we investigate some cosmological consequences from a new inflationary scenario driven by the generalized potential $V \propto \exp_\kappa(-\lambda\varphi)$ (something like that was done in [24]). One important aspect found in our model is that it generalizes the conventional exponential models found in the literature, with $V \sim e^{-\lambda\varphi}$. Within the *slow-roll* approximation, we obtained the straightforward relation between r and n_s, expressed by the Eq. (9), containing consistent results for the magnitude of the primordial tensor modes ($r \simeq 0.08$ at $n_s \simeq 0.97$, considering $\kappa = -1$, for example). According to the *Planck Collaboration 2018*, this is $n_s = 0.965 \pm 0.004$ (with 68% C.L.) and $r_{0.002} < 0.1$ (with 95% C.L.) [10]. Furthermore, our approach also includes the Harrison-Zel'dovich spectrum [25, 26, 27], if $r = 0$, for all values of the parameter κ.

Finally, we obtained the direct relation between the non-Gaussianity of perturbations and its espectral index, given by the Eq. (12). As expected in the *slow-roll* regime (see [22]), we obtained $f_{NL}^{local} \ll 1$. On the other hand, our result is consistent with the Planck collaboration 2018 estimate, $f_{NL}^{local} = -0.9 \pm 5.1$ [28]. Also it's seen that $n_s = 1$ implies $f_{NL}^{local} = 0$.

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