

# $\beta^+/\text{EC}$ - decay half-life study of $sd$ space nuclei using shell model

Surender<sup>1</sup>, P. C. Srivastava<sup>2</sup>, and Vikas Kumar<sup>1\*</sup>

<sup>1</sup>*Department of Physics, Banaras Hindu University Varanasi 221 005, INDIA*

<sup>2</sup>*Department of Physics, Indian Institute of Technology Roorkee 247 667, INDIA*

## Introduction

The nuclear shell model has been successful in the description of various aspects of nuclear structure and beta decay probabilities of exotic nuclei [1, 2]. The nuclear structure study of  $N = Z$  nuclei in the nuclear chart are very interesting because these nuclei show several phenomena such as shape coexistence along the  $N = Z$  line and the role of pairing correlation of neutron and proton [3]. Proton-rich nuclei hold a pivotal role in astrophysics, particularly in processes like nucleosynthesis within stars. Motivated with the recent experimental data [4] of  $\beta^+/\text{EC}$  - decay half-lives in  $sd$  space nuclei, we have reported a comprehensive  $\beta^+/\text{EC}$  - decay study using nuclear shell model (SM) in the present work. The USDB [5] and SDNN [6] effective interactions have been used for the shell model calculations in  $sd$  model space. The shell model code NuShellX@MSU [7] is used for the diagonalization of energy matrices.

The SM results of  $\beta^+/\text{EC}$  - decay half-lives, excitation energies,  $Q$ -values,  $\log ft$  values, and branching fractions are discussed and compared with the available experimental data. The calculated  $\beta^+/\text{EC}$  - decay half-lives, excitation energies,  $Q$ -values,  $\log ft$  values, and branching fractions are in a good agreement with the available experimental data.

## $\beta$ - decay formalism

In  $\beta$ -decay processes the atomic number  $Z$  changes by one unit while the mass number  $A$  of the parent nucleus remains unchanged.

The total decay half-life of a combined  $\beta^+$  and electron-capture (EC) transition is given by

$$f_0 t_{1/2} = \left[ f_0^{(+)} + f_0^{(EC)} \right] t_{1/2} = \frac{\kappa}{[g_A^2 * B(GT) + B(F)]} \quad (1)$$

where,  $g_A$  ( $= 1.270$ ) represents the axial-vector coupling constant of the weak interactions and  $f_0$

is the phase-space factor (or Fermi integral). The latest updated value of  $\kappa$  is

$$\kappa \equiv \frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 c^4 (G_F \cos \theta_C)^2} = 6289s, \quad (2)$$

where, the  $\theta_C$  is the Cabibbo angle. The Fermi reduced transition probability  $B(F)$  is given by

$$B(F) \equiv \frac{g_V^2}{2J_i + 1} |M_F|^2, \quad (3)$$

where,  $g_V$  ( $= 1.0$ ) represents the vector coupling constant of the weak interaction and  $M_F$  is the Fermi matrix element.

The Gamow-Teller reduced transition probability  $B(GT)$  is given by

$$B(GT) = \langle \sigma \tau \rangle^2. \quad (4)$$

In the above expression, the nuclear matrix element for the Gamow-Teller operator is given by

$$\langle \sigma \tau \rangle = \frac{\langle f | \sum_k \sigma^k \tau_{\pm}^k | i \rangle}{\sqrt{2J_i + 1}}, \quad (5)$$

where initial and final states are represented by the quantum numbers  $i$  and  $f$ , respectively.

For  $\beta^{\mp}$  decay, the phase-space factor is given by

$$f_0^{(+)} = \int_1^{E_0} F_0(-Z_f, \epsilon) p \epsilon (E_0 - \epsilon)^2 d\epsilon, \quad (6)$$

where,  $F_0$  is called Fermi function and

$$\epsilon \equiv \frac{E_e}{m_e c^2}, E_0 \equiv \frac{E_i - E_f}{m_e c^2}, p \equiv \sqrt{\epsilon^2 - 1}, \quad (7)$$

where  $E_e$  is the total energy of the emitted electron/positron and  $E_i$  and  $E_f$  are the energies of the initial and final nuclear state. The  $\log ft \equiv \log_{10}(f_0 t_{1/2} [s])$ . The total half-life related to partial half-life ( $t_i$ ) of the daughter state  $i$  using the following expression:

$$t_{1/2} = \left( \sum_i \frac{1}{t_i} \right)^{-1}. \quad (8)$$

\*Electronic address: [vikasphysics@bhu.ac.in](mailto:vikasphysics@bhu.ac.in)

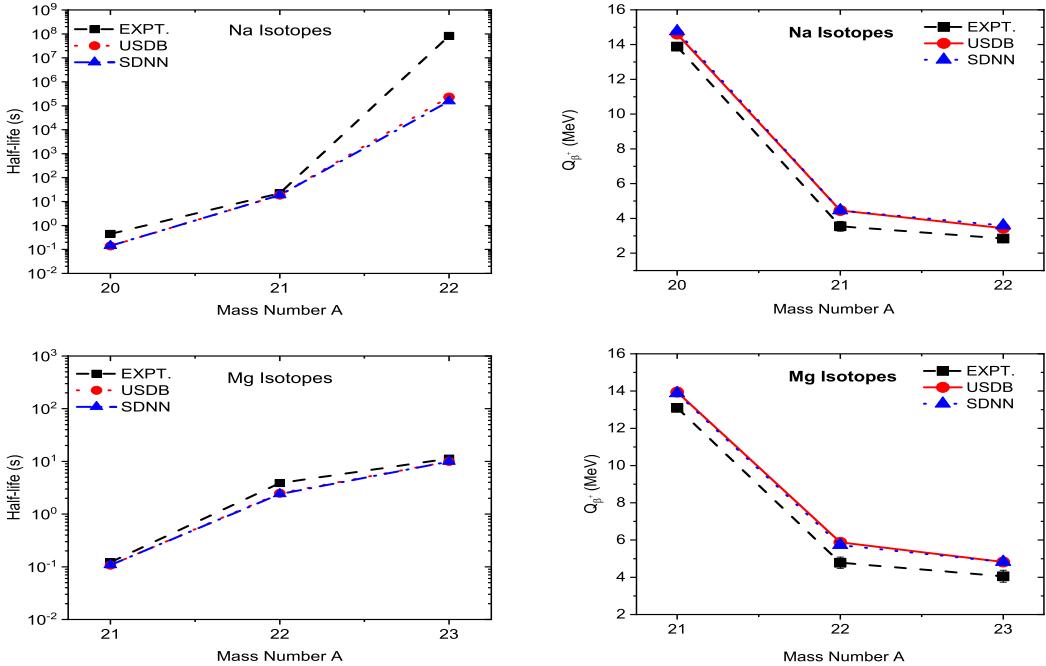


FIG. 1: The theoretical and experimental  $\beta^+$ /EC- decay half-life and  $Q$  values versus mass number A of the concerned nuclei for Na and Mg isotopes.

The branching ratio  $b_r$  is related to partial half-life  $t_i$  and the total half-life  $t_{1/2}$  of the allowed  $\beta$ -decay as

$$t_i = \frac{t_{1/2}}{b_r}. \quad (9)$$

## Results and Discussion

The experimental data with theoretical shell-model results of  $\beta^+$ /EC-decay half-lives are compared in left panel of the fig.1, while the  $Q$ -values are compared in right panel. As from fig.1, the half-life of Na and Mg isotopes increase with increase mass number while the  $Q$ -values decrease with increase mass number. The theoretical shell-model results of excitation energies and  $\beta^+$ -decay properties like  $\log ft$  values, branching percentages of Na and Mg nuclei of  $sd$  shell are compared between theoretical and experimental data. The shell model results for  $sd$  shell nuclei are in good agreement with the experimental data for excitation energies,  $\log ft$  values, and the branching ratios, except for few cases where the theoretical results deviate from

the experimental data. During the meeting, we will present the results of the remaining  $sd$  shell nuclei.

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