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Unified inflation and dark energy with accelerated particle holographic model

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Abstract In this paper and in the follows of the holographic approach to describe the primary acceleration and the late-time acceleration eras, we have considered and unified inflation and dark energy while accelerated particle holographic (APHM) density is used (Fazlollahi in Chin Phys C 47:035101, 2023). As discussed, establishing holographic model for constant roll inflation during the very early Universe leads one to explicit form of Hubble parameter which satisfies inflationary era. The validity of such holographic constant roll inflation with respect to the Planck data puts a certain bound on the infrared cut-off at the time of horizon crossing. Beside the mere inflation, the Hubble parameter gives explicit form for density of accelerated particle holographic model. It is shown such density due to inflationary bound condition on infrared cut-off could present late-time acceleration epoch. Consequently, inflation and late time expansion era unified in unique infrared cut-off model. Moreover, such corresponding fractional density gives more matter regimes during matter era. Nevertheless, studying matter structure formation during matter era reveals the model is not in conflict with ΛCDM model.

1 Introduction

Following observational data, the Universe expands with acceleration phase in two different eras, primary accelerated era called inflation [1,2], and late-time accelerated epoch debugged dark energy [3,4]. One of the most important puzzles in current cosmology is unifying these two eras in unique model. Several higher curvature gravity theories like $f(R)$ theory [5,6] and the Gauss–Bonnet gravity theory [7] have been used to resolve this issue. It suggests that the higher curvature theories originated from the diffeomorphism property of the gravitational action and or from string theory could unify these eras. In this context, one of the arenas of string theory is the holographic principle originates from black hole thermodynamics and establishes a connection of the infrared cutoff of a quantum field theory with the largest distance of this theory [8–10]. With eye to holographic principle dozen of density forms proposed for density of dark energy [11–14], proves to successfully describe the late time evolution of the Universe [15–19]. However, apart from the dark energy models, the holographic density is also found to be useful to realize and study the early (primary) steps of the Universe evolution like the inflationary era [20–22]. Actually, during the early Universe the holographic density which is inversely proportional to the squared infrared cut-off of the theory, becomes large enough to drive inflation. The first study in this context was done in [23] wherein the original model of Higgs inflation is studied. As a result, it was found when the Higgs field couples with the Ricci scalar, model does not satisfy the holographic bound. However, it is shown coupling Higgs field to Einstein tensor passes the holographic test [23]. Despite a considerable application of the holographic principle to describe evolution of the Universe in primary and late time accelerated expansion eras, unification of these two eras through holographic principle is done recently by using general form of holographic cut-off L_{IR} [24]:

$$L_{IR} = L_{IR}\left(L_p, \dot{L}_p, \ddot{L}_p, \ldots, L_f, \dot{L}_f, \ddot{L}_f, \ldots, a, H, \dot{H}, \ddot{H}, \ldots\right) \quad (1)$$

where L_p and L_f are particle horizon and the future horizon, respectively, a is the scale factor, H presents the Hubble parameter and over dot denotes derivative with respect to cosmic time. Although in [24], unified holographic models corresponding to $f(R)$ theory and Gauss–Bonnet gravity is studied, it seems that it is necessary to study unified version for explicit forms of infrared cut-off L_{IR} and check how holographic inflation restricts acceleration phase in late time.

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Therefore, in the present work, we are interested to provide a unified framework of holographic inflation with holographic dark energy through [25]

$$\rho_H = \frac{3\alpha}{\kappa^2}\left(\frac{\ddot{L}_p}{L_p}\right) = \frac{3\alpha}{\kappa^2}\left(\dot{H} + \beta H^2\right) \tag{2}$$

where $\kappa^2 = 8\pi$ (we set $G = c = 1$) and α and β are two arbitrary constants of the model. Comparing relation (1) with Eq. (2) through Friedmann equation shows cut-off L_{IR} recasts to

$$L_{IR} = L_{IR}\left(L_p, \ddot{L}_p, a, H, \dot{H}\right). \tag{3}$$

Hence, the unified holographic model for density (2) is not function of the future horizon L_f and its time derivatives. As result, unlike scenarios given in [24], our model developed in the absence of future horizon and high orders of time derivative of the Hubble parameter.

The plan of paper is as follows: after going through some basics of constant roll inflation in scalar–tensor theory in Sect. 2, the holographic connection of constant roll inflation and the unification of constant roll inflation with the dark energy era is examined in Sect. 3. In Sect. 4 due to inflation constraints on cut-off (3), the evolution of the Universe in current epoch will be considered. Furthermore, the corresponding parameters of matter structure formation during matter-dominated era is studied. The paper ends with conclusions and remarks in Sect. 5.

2 Brief on constant roll inflation

In this section, we revisit the main essence of constant roll inflation driven by a scalar field with action [26]

$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{4}$$

where ϕ is the scalar field with the potential $V(\phi)$. With aid of the flat Friedmann–Lemaître–Robertson–Walker metric, the Friedmann equations and the Klein–Gordon like equation for scalar field are given by,

$$\frac{3}{\kappa^2}H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{5}$$

$$-\frac{2}{\kappa^2}\dot{H} = \dot{\phi}^2 \tag{6}$$

$$0 = \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} \tag{7}$$

respectively. In the slow-roll regime the term $\ddot{\phi}$ is negligible with respect to the Hubble friction and the resorting force. Thus, in general scenario of the constant roll inflation the scalar field rolls as a constant rate, in particular

$$\ddot{\phi} = \gamma H\dot{\phi} \tag{8}$$

where γ is the positive constant known as the constant roll parameter [27]. Clearly, for $\gamma \approx 0$ the above condition obtains the standard slow-roll inflationary scenario. Differentiating Eq. (6) with respect to time and applying the constant roll condition (8), we have

$$\ddot{H} = 2\gamma H\dot{H}. \tag{9}$$

On integrating which gives a first-order differential equation of the Hubble parameter, one finds

$$\dot{H} = \gamma\left(H^2 - \zeta^2\right) \tag{10}$$

where ζ is an arbitrary integration constant. It is to be noted that this selection of constant ζ let us to put origin of time axis at $t = 0$, present Universe, and thus all epochs of the Universe from big bang to current time happened in past, $t \leq 0$. As result, inflation era occurred at $t \ll 0$ while current era onset at $t < 0$. Furthermore, dissolving above differential equation gives the explicit form of the Hubble parameter

$$H = -\zeta\, tanh\left(\gamma\zeta t\right) \equiv -\zeta\, tanh(\chi). \tag{11}$$

Using Eqs. (8) and (11) yields the scalar field

$$\phi = \frac{2}{\kappa}\sqrt{\frac{2}{\gamma}}\, tan^{-1}\left(e^{\gamma\zeta t}\right). \tag{12}$$

Consequently, with aid of Eqs. (6) and (12) immediately one finds

$$V = \frac{3\zeta^2}{\kappa^2}\left(\frac{3-\gamma}{6}\right)\left[1 + \left(\frac{3+\gamma}{3-\gamma}\right) cos\left(\sqrt{2\gamma}\kappa\phi\right)\right]. \tag{13}$$

To have a positive Hubble parameter, Eq. (11) depicts that the ζt ranges within $-\infty < \zeta t < 0$. So, during the early Universe, i.e., $\chi \ll -1$, the Hubble parameter from Eq. (11) behaves as $H \approx \zeta$, which illustrates a de-Sitter inflationary stage. Furthermore, one finds the first-roll parameter as,

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \gamma\left(\frac{\zeta^2}{H^2} - 1\right) = \frac{\gamma}{sinh^2\left(\gamma\zeta t\right)}. \tag{14}$$

This demonstrates that ϵ_1 is an increasing function with respect to the time only for positive values of γ. To finish inflationary stage, ϵ_1 must reach unity at some points in time depending on the value of γ. For instance, if one takes $\gamma = 0.01$, the ϵ_1 reaches to unity at

$$\zeta t \approx -9.98. \tag{15}$$

The relation (14) shows the magnitude of ζt is depending on positive value of γ. Keeping this in mind, we will study and establish the holographic unified model with positive γ.

3 Holographic constant roll inflation

According to the holographic principle, the holographic density is proportional to the inverse squared infrared cutoff L_{IR}, which could be related to the causality given by the cosmological horizon,

$$\rho_H = \frac{3\sigma^2}{\kappa^2 L_{IR}^2} \tag{16}$$

where σ is a free parameter. Consequently, the Friedmann equation gives,

$$H = \frac{\sigma}{L_{IR}}. \tag{17}$$

The infrared cutoff L_{IR} is usually assumed to be the particle horizon L_p

$$L_p \equiv a \int_0^t \frac{dt}{a}, \tag{18}$$

or the future event horizon L_f

$$L_f \equiv a \int_t^\infty \frac{dt}{a}. \tag{19}$$

The fundamental form of the cut-off is still a debatable topic, and along this direction studying the general form (1) maybe is useful. Nevertheless, we will study form (3), reveals infrared cut-off is not depending on the future event horizon and its time derivatives. Based on this restriction, we can establish the constant roll inflation built from infrared cut-off. In particular, plugging Eq. (11) in (17), we find

$$\frac{\sigma}{L_{IR}} = -\zeta tan(\chi). \tag{20}$$

This solution of infrared cut-off helps to calculate the observable quantities, by which we can consider validity of the holographic inflationary scenario. The slow-roll parameters in the holographic inflation model are given by

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{1}{\sigma}\frac{dL_{IR}}{dt}, \tag{21}$$

and

$$\epsilon_{n+1} = \frac{\dot{\epsilon}_n}{\epsilon_n H} = \frac{\dot{\epsilon}_n}{\sigma \epsilon_n} L_{IR} \tag{22}$$

respectively with $n \geq 1$. Thus, the scalar spectral index n_s and the tensor-to-scalar ratio r defined as,

$$n_s = 1 - 2\epsilon_1 - 2\epsilon_2 \tag{23}$$

$$r = 16\epsilon_1. \tag{24}$$

The L_{IR} (20) immediately gives ϵ_1 and ϵ_2 as follows:

$$\epsilon_1 = \gamma \left(\frac{\zeta^2 L_{IR}^2}{\sigma^2} - 1 \right) \tag{25}$$

$$\epsilon_2 = \frac{2\gamma\zeta^2}{\sigma^2} L_{IR}^2. \tag{26}$$

Plugging the above expressions of the slow-roll parameters into Eqs. (23) and (24), we find

$$n_s = 1 - 2\gamma \left(\frac{3\zeta^2 L_{IR}^2}{\sigma^2} - 1 \right) \tag{27}$$

$$r = 16\gamma \left(\frac{\zeta^2 L_{IR}^2}{\sigma^2} - 1 \right). \tag{28}$$

Equations (27) and (28) show n_s and r depend on the parameters γ and $\frac{\zeta^2 L_{IR}^2}{\sigma^2}$. Consequently, one can directly constrain these two parameters through observational data. The spectral index and the tensor-to-scalar ratio with the Planck 2018 results [28] are given by,

$$n_s = 0.9649 \pm 0.0042 \quad \text{and} \quad r < 0.064. \tag{29}$$

For the holographic model in hand, these two indices n_s and r prove to be simultaneously compatible with the Planck data for the following ranges of the parameters

$$\sqrt{\frac{3}{2}} < \frac{\zeta}{\sigma} L_{IR} < \sqrt{2} \quad \text{and} \quad 0.0038 < \gamma < 0.004. \tag{30}$$

Thus, the holographic model of Eq. (20) onsets the constant roll inflationary scenario which is compatible with the Planck constraints while both constants γ and ζ are positive.

4 Unified model for APHM

In this section, we will study how the infrared cut-off (20) satisfies late time observations. In this context and in follows of [25], the accelerated particle holographic density given by

$$\rho_H = \frac{3\alpha}{\kappa^2} \left(\dot{H} + \beta H^2 \right). \tag{31}$$

From Eqs. (16) and (17) this density can be rewritten as function of infrared cut-off (20) and or the Hubble parameter (11) and onsets inflation era for

$$\frac{\zeta}{\kappa\sqrt{6}} < \sqrt{\rho_H} < \frac{\zeta}{\kappa}\sqrt{2}. \tag{32}$$

Furthermore, plugging Eq. (10) into Eq. (32), one may find

$$\rho_H = \frac{3\alpha}{\kappa^2} \left((\beta + \gamma) H^2 - \gamma\zeta^2 \right) \tag{33}$$

where the cut-off density (16) and relation (20) imply that the density ρ_H is function of χ, namely,

$$\rho_H = \frac{3\gamma\alpha\zeta^2}{\kappa^2} \left(\left(1 + \frac{\beta}{\gamma} \right) tan^2(\chi) - 1 \right). \tag{34}$$

In particular, using e-folding number $x = ln(a)$ in Eq. (11), we should have

$$t = \frac{1}{\gamma\zeta} tan^{-1} \sqrt{e^{-2\gamma x} - 1}. \tag{35}$$

With this equation and using relation between e-folding number and redshift i.e., $e^{-x} = 1 + z$, one can explore density (34) as function of redshift. In fact. In this scenario late time density (31) constrained by constant roll condition (8) that satisfies relations (32) for early Universe (see i.e., relation (32)).

To consider density (34) in current time, ignoring effects of radiation component yields first Friedmann equation

$$\frac{3}{\kappa^2} H^2 = \rho_H + \rho_m \tag{36}$$

where ρ_m denotes density of whole matter includes baryonic and dark matter. Although ρ_H triggers and governs inflation era, relation (32), in the late time can be considered as dark energy. Thus, in follows of this study by setting $\rho_H = \rho_X$, we assume same inflationary density presents late time acceleration phase. Also, the continuity equation of such Friedmann equation given by:

$$\dot{\rho}_{tot} + 3H\left(\rho_{tot} + p_{tot}\right) = 0 \tag{37}$$

where we define $\rho_{tot} = \rho_X + \rho_m$ and pressure $p_{tot} = p_X + p_m$. Observations suggest that in large scale structure, the matter field evolves like pressureless fluid [29,30]. With this assumption and decoupling Eq. (37) into its components, one gets

$$\dot{\rho}_X + 3H\rho_X\left(1 + \omega_X\right) = -Q \tag{38}$$

$$\dot{\rho}_m + 3H\rho_m = Q \tag{39}$$

where $\omega_X = p_X \big/ \rho_X$ is the equation of state of dark energy and Q presents the interaction term. In the absence of mechanism of microscopic origin of the interaction, one can use the different forms of Q, phenomenologically [31]. However, in this study, as simplest scenario we assume these two fields do not interact to each other, $Q = 0$. As result, the equation of state ω_X and matter density are,

$$\omega_X = -1 - \frac{\dot{\rho}_X}{3H\rho_X} \tag{40}$$

$$\rho_m = \rho_{m0} a^{-3} \tag{41}$$

respectively. Defining fractional densities

$$\Omega_X = \frac{\kappa^2}{3H^2}\rho_X, \quad \Omega_m = \frac{\kappa^2}{3H^2}\rho_m \tag{42}$$

and constraining model for current Universe, $z = 0$, and plugging Eqs. (34) and (41) into the Friedmann equation (36) yields

$$\alpha = -\frac{\Omega_{X0} H_0^2}{\gamma \zeta^2} \tag{43}$$

which implies α is negative parameter. Substituting (11) and (34) into Eq. (40), the equation of state reads,

$$\omega_X = -1 - \frac{2}{3}\left(\frac{\gamma\left(\beta + \gamma\right)}{\left(\beta + 2\gamma\right) cos^2\left(\gamma\zeta t\right) - \beta - \gamma}\right). \tag{44}$$

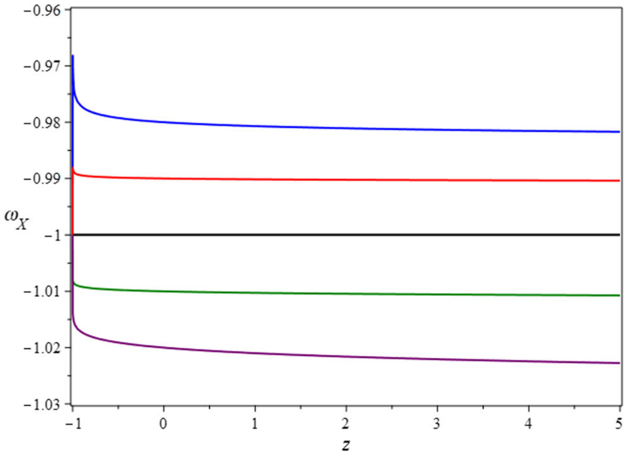

Fig. 1 The equation of state of dark energy versus redshift for different values of $\omega_{X0} = -0097, -0.98, -1, -1.01$ and -1.02

Setting $\omega_X\left(z = 0\right) = \omega_{X0}$ and using Eq. (44), we find

$$\beta = -\gamma - \frac{3}{2}\left(1 + \omega_{X0}\right). \tag{45}$$

Consequently, setting $\omega_{X0} = -1$, one finds $\beta = -\gamma$. The constant β can be positive only for $\omega_{X0} < -1.002$.

Also, deceleration parameter of the late-time evolution is defined as follows,

$$q \equiv \frac{1}{2}\left(1 + \frac{3p_{tot}}{\rho_{tot}}\right) \tag{46}$$

which it can be rewritten as function of equation of state (44), namely

$$q \equiv \frac{1}{2}\left(1 + 3\omega_X \Omega_X\right). \tag{47}$$

The joint analysis of SNe+CMB data with the ΛCDM model suggests transition point $z_T = 0.52 - 0.73$ [32]. With aid of this interval and setting $H_0 = 67.4$ and $\Omega_{m0} = 0.315$ [33], it indicates equation of state for $\omega_{X0} < -1$ gives phantom models in high redshift while in $z = -1$ it coincides with the ΛCDM model and thus model with initial condition $\omega_{X0} < -1$ only coincides with ΛCDM model at $z = -1$. On the other hand, for $\omega_{X0} \geq -1$, one finds same evolution when model behaves like the cosmological constant at $z = -1$. However, using initial condition $\omega_{X0} > -1$, one finds

$$\lim_{z \to \infty} \omega_X \approx -1 \tag{48}$$

which implies Eq. (44) coincides with ΛCDM model for high redshift. The evolution of equation of state (44) and deceleration parameter (47) are plotted in Figs. 1 and 2, respectively.

The evolution of fractional densities (42) compared with ΛCDM model is plotted in Fig. 3. As shown, when $\omega_{X0} = -0.99$ and $= -1.01$ dark energy is not in conflict with the standard cosmological model in current Universe when we only have small errors around $\sim 1\%$ with respect to ΛCDM

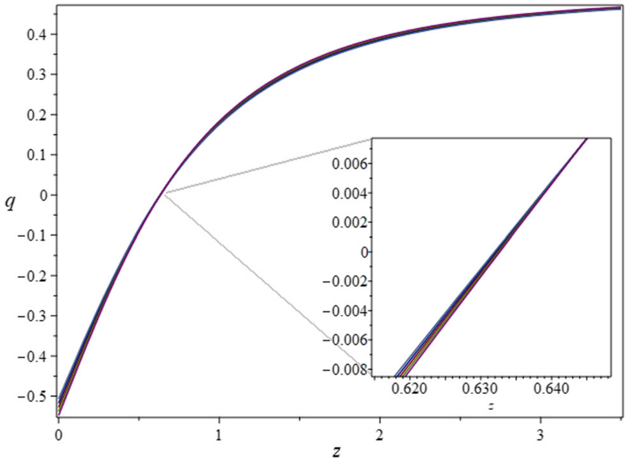

Fig. 2 Deceleration parameter as function of redshift for different values of $\omega_{X0} = -0097, -0.98, -1, -1.01$ and -1.02

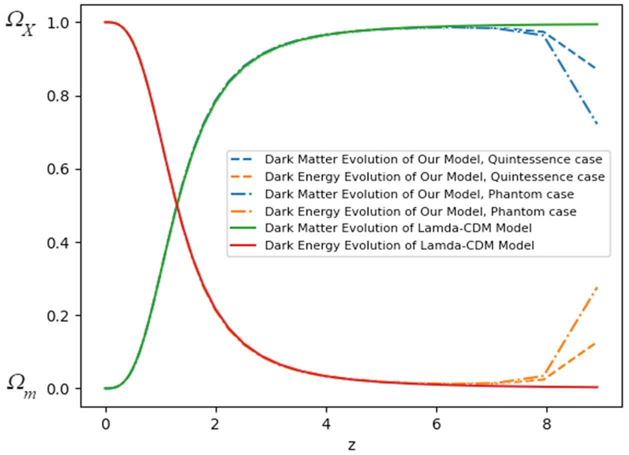

Fig. 3 Fractional densities of our model compared with ΛCDM theory for $\omega_{X0} = -0.99$ (dash curves) and $\omega_{X0} = -1.01$ (dash-dot curves)

theory. The important result traces back to high redshifts wherein fractional density Ω_X plays key role in first steps of the Universe as trigger of inflationary era.

Till now, with aid of constant roll condition (8), the Hubble parameter (11) is obtained which satisfies infrared cut-off inflationary scenario for condition (32). Using same Hubble parameter in late-time Universe demonstrates that density (31) triggers accelerated expansion in current time. As result, with eye to Eq. (16) and conditions (32), density (31) could explain both primary and late time acceleration eras as unique field (Fig. 3). However, as one plausible scenario, may some errors between fractional densities impose more matter regimes during matter dominated era compared with ΛCDM model. At this step, it is worthwhile to check whether evolution of density ρ_X during matter epoch affect on matter distributions or not. Thus, in follows we have explored structure formations like the matter power spectra and angular spectrum of CMB temperature anisotropy. To calculate these parameters numerically, we use the modified version of the Boltzmann code CAMB [34]. In Fig. 4, we have compared the matter power spectra in our model with that in the ΛCDM model at $z = 0$. As shown, our model deviates from ΛCDM model around $\sim 1.5\%$ for different k. These deviations trace back to extra matter regimes due to behavior of fractional density Ω_X during matter era.

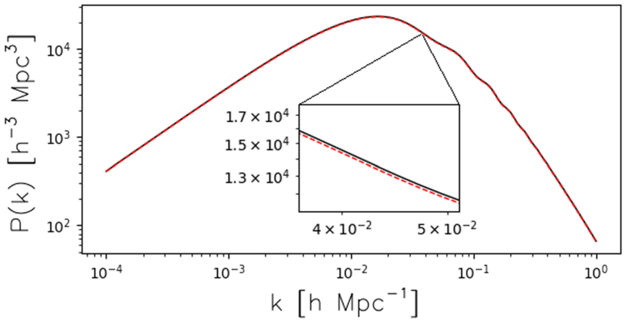

Fig. 4 The matter power spectrum versus wavenumber k at $z = 0$. The solid and dashed curves represent our model and ΛCDM theory, respectively

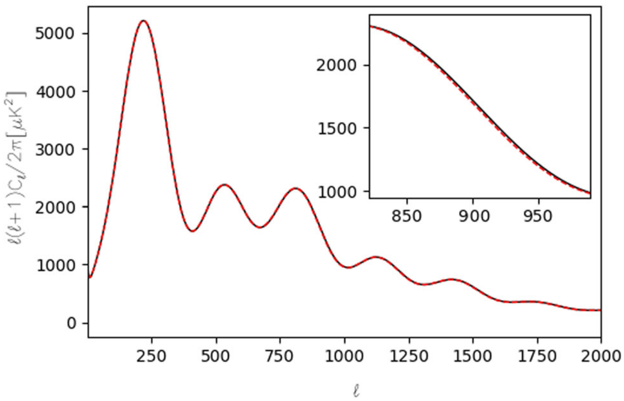

Fig. 5 The theoretical CMB TT spectrum of our model (solid curve) compared with ΛCDM model (dashed curve)

The angular spectrum CMB temperature anisotropy is plotted in Fig. 5. The largest difference between our model with that in ΛCDM model is around $\sim 1.8\%$ at $l = 905$. The evolution of theoretical CMB TT spectrum and matter power spectrum (Fig. 4) demonstrates that extra matter field due to behavior of dark energy in high redshifts gives no tangible effects on matter density during matter dominated era. Thus, the dark energy (31) satisfies observations.

5 Remarks

The holographic principle has earned a lot of attention due to its application in cosmology for explaining primary and late-time acceleration eras. However, unifying these two eras through this principle is considered, recently due to studying infrared cut-off relation (1). In this context, exploring and unifying inflationary era and dark energy epoch through specific form of infrared cut-off is worthwhile and can lead us to see whether proposed explicit cut-off models can illustrate inflation and dark energy as unique field or not.

In this study, we have attempted to explore one of the new infrared cut-off models in which $L_{IR}^2 \approx L_p/\ddot{L}_p$. Consequently, infrared cut-off is given by the particle horizon and its second derivative with respect to time. Investigating such cut-off scenario with Planck data yields strong conditions (34). Under these conditions, our cut-off model satisfies primary accelerated expansion and thus inflation governs with its corresponding Hubble parameter (11). Furthermore, as calculated, using this Hubble form for late time for same cut-off form gives density of dark energy, proves to successfully describe the late time evolution of the Universe. Consequently, cut-off (20) unifies very early and late time acceleration eras when density (31) is used.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and we only have used some modules such as CAMB.]

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