Holographic Quantum Foam

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Abstract.

Quantum fluctuations can endow spacetime with a foamy structure. The three different methods we have employed to calculate the magnitude of the foaminess yield the same result and all agree with what the holographic principle dictates and hence the name "holographic quantum foam" (HQF). HQF is shown to yield a cosmology (which we call holographic foam cosmology) with an effective dynamical cosmological constant and critical energy density the quanta of which obey an exotic statistics known as infinite statistics (or quantum Boltzmann statistics). We also show that, in the gravitational context, the properties of HQF are intimately related to the phenomenon of turbulence. And we interpret inflation in the early universe as due to the onset of turbulence with the graceful exit problem being naturally solved by the fact that the quanta of HQF obey infinite statistics. Finally we discuss methods to detect HQF, among which are the use of laser interferometers and the observation of high-energy photons from extragalactic sources. We also briefly discuss, in the context of HQF, dark matter the quanta of which also obey infinite statistics.

1 Introduction

According to John Wheeler [1], probed at small scales, spacetime appears to be very complicated – something akin in complexity to a turbulent froth which he dubbed "spacetime foam", with the implication that spacetime is "foamy" on very small scales. In his talk at the Chapel Hill Conference on gravity in 1957 he also illustrated the idea of spacetime foam using an analogy with the surface of the ocean: Imagine yourself flying a plane over an ocean. At high altitude the ocean appears smooth. But as you descend, it begins to show roughness. Close enough to the ocean surface, you see bubbles and foam. Analogously, spacetime appears smooth on large scales; but on sufficiently small scales, it will appear rough and foamy. Many physicists believe the foaminess is due to quantum fluctuations of spacetime, hence spacetime foam is also known as "quantum foam". [2]

Then it is natural to ask: "How foamy is spacetime? How large are the quantum fluctuations?" To be concrete, let us start with the basic question: "What is the uncertainty δl in the measurement of a distance l?" We expect

$$\delta l \gtrsim l^{1-\alpha} l_P^{\alpha} \tag{1}$$

on the average, with the Planck length $l_P \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33}$ cm providing the intrinsic length scale in quantum gravity, and the parameter $\alpha \sim 1$ characterizing the different spacetime foam models. We want our discussion to be as general as possible, depending only on the quantum characteristics of spacetime measurements, and not on any particular theory.

This talk is organized as follows: In section 2, we start the discussion on the quantum fluctuations of spacetime by using three different methods (viz., using a gedanken experiment, applying the holographic principle, and applying the method of mapping the geometry of spacetime respectively) to measure the quantum foam-induced uncertainties in distance measurements. Then, as an application, we make a "post-diction" on the existence of a dark sector in the universe. In section 3, we show that the physics of quantum foam naturally leads to a model of cosmology which we call "holographic foam cosmology" that automatically yields a critical energy density and the existence of dark energy/matter obeying a new kind of statistics known as infinite statistics in our universe. In section 4, we give a short summary of the properties of the dark matter model that is motivated by holographic quantum foam (which was discussed in some detail in my talk in IARD 2016.) In section 5, we show the deep similarities between turbulence and the quantum foam phase of strong gravity to arrive at a Kolmogorov-like scaling in quantum gravitational setting, and then apply the results to discuss inflation in the early universe (with a natural "graceful" exit scenario). Section 6 is devoted to a discussion of three possible ways to detect/test quantum foam. (This section also serves as an introduction to the talk by Dr. Eric Steinbring in this (IARD2024) conference.) We give a short summary in section 7, followed by four appendices on (A) metric, energy, momentum, and speed-of-light fluctuations for quantum foam models (to be used in section 6), (B) cumulative effects of spacetime fluctuations (also to be used in section 6), (C) the Margolus-Levitin theorem (used in section 2), and (D) infinite statistics (see section 3). For simplicity we will adopt the set of units (most of the time) in which $c = 1, \hbar = 1$, and the Boltzmann constant $k_B = 1$. The subscript "P" will denote Planck units.

2 Quantum Fluctuations of Spacetime

In this section we use three different methods to calculate δl, the accuracy with which a distance l can be measured, taking into account the uncertainties/ fluctuations in l.

2.1 Method 1: Gedankan Experiment

To measure the distance l between two points, following Salecker and Wigner [3], we [4] put a clock at one of the points and a mirror at the other. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance. But the uncertainties in the positions of the clock and the mirror introduce an inaccuracy δl in the measurement. Let us concentrate on the clock (of mass m and size d). If it has a linear spread δl when the light signal leaves the clock, then its position spread grows to $\delta l + \hbar l(mc\delta l)^{-1}$ when the light signal returns to the clock, with the minimum at $\delta l = (\hbar l/mc)^{1/2}$. Thus quantum mechanics dictates that $\delta l^2 \gtrsim \frac{\hbar l}{mc}$. Next, we note that, for the uncertainty in distance measurement not to be greater than δl, the clock must tick off time fast enough so that $d/c \lesssim \delta l/c$. On the other hand, general relativity demands that d must be larger than the Schwarzschild radius Gm/c^2 of the clock, for otherwise one cannot read the registered time. From these conditions imposed by quantum mechanics and general relativity, we conclude that [1]

$$\delta l \gtrsim (l l_P^2)^{1/3} = l_P \left(\frac{l}{l_P}\right)^{1/3}. \tag{2}$$

A gedanken experiment to measure a time interval T gives an analogous expression: $\delta T \gtrsim (T t_P^2)^{1/3}$, with t_P being the Planck time. (Fluctuations of this kind correspond to what statisticians call fractional Brownian motion with self-similarity parameter $\frac{1}{3}$.)

2.2 Method 2: Holographic Principle

According to the holographic principle, [6, 7, 8] the Universe which we perceive to have 3 spatial dimensions can actually be encoded on a 2-dimensional surface, like a hologram. Effectively this principle stipulates that the maximum amount of information in a region is bounded by its surface area, i.e., order-of-magnitude wise, its number of degrees of freedom is no more than its area in Planck (l_P^2) units.

Now consider partitioning a big cube (l^3) into cubes as small as physical laws allow (with average size $(\delta l)^3$), so intuitively one degree of freedom is associated with each small cube. Applying the holographic

[1]See also Ref. [5] which has some overlap with our work.

principle, we immediately find that $\left(\frac{l}{\delta l}\right)^3 \lesssim \frac{l^2}{l_P^2}$, yielding $\delta l \gtrsim l^{1/3} l_P^{2/3}$. Accordingly the spacetime foam model corresponding to $\delta l \gtrsim l^{1/3} l_P^{2/3}$ is known as the holographic model of spacetime foam, also known as the holographic quantum foam (HQF) model. [9]

2.3 Method 3: Mapping the Geometry of Spacetime

This method involves the use of a global positioning system for a volume of space-time with radius l over time l/c it takes light to cross the volume. [10] Fill the space with a swarm of clocks, exchanging signals with the other clocks and measuring the signals' time of arrival. An interesting question is: "How accurately can these clocks (of total mass M) map out this space-time volume?"

The process of mapping the geometry of spacetime is a kind of computational operation. Hence we can apply the Margolus-Levitin theorem [11] (see Appendix C) which stipulates that the rate of operations is bounded by the energy $E/\hbar$ that is available for the operation. In other words, the total number of operations is bounded by $< (E/\hbar)\times$ time $= \frac{Mc^2}{\hbar}\frac{l}{c}$. Next, to prevent black-hole formation, we need to impose the condition $M < \frac{lc^2}{G}$. Then we end up with the upper bound for the number of operations or events (which can be interpreted as the number of spacetime "cells" for the volume of spacetime under consideration) being $l^2 \frac{c^3}{\hbar G} = \frac{l^2}{l_P^2}$. For maximum spatial resolution, each clock ticks only once; therefore each "cell" occupies a spatial volume of $\frac{l^3}{l^2/l_P^2} = l l_P^2$ implying that the average spatial separation of the "cells" is $l^{1/3} l_P^{2/3}$. Again we obtain the result $\delta l \gtrsim l^{1/3} l_P^{2/3}$. Now it is timely to make the following observation: Maximum spatial resolution requires maximum energy density $\rho \sim \frac{3}{8\pi}(l l_P)^{-2}$, yielding the number of bits $\sim l^2/l_P^2$. [9] This observation will be useful for the discussion in the next section.

We end this section with the following corollary, viz., *a heuristic argument on why the Universe cannot contain ordinary matter only.* [12] Let us start by assuming that the Universe (of size $l = R_H$, the Hubble horizon) has only ordinary matter. According to the statistical mechanics for ordinary matter at temperature T, (and in volume l^3) energy scales as $E \sim l^3 T^4$ and entropy goes as $S \sim l^3 T^3$. Black hole physics can be invoked to require $E \lesssim \frac{l}{G} = \frac{l}{l_P^2}$, resulting in the bound on the entropy given by $S \lesssim (l/l_P)^{3/2}$.

We can repeat verbatim our earlier argument to conclude that, if only ordinary matter exists, $\delta l \gtrsim \left(\frac{l^3}{(l/l_P)^{3/2}}\right)^{1/3} = l^{1/2} l_P^{1/2}$ which is much greater than $l^{1/3} l_P^{2/3}$ for the case of the holographic quantum foam model. In other words, ordinary matter contains only an amount of information dense enough to map out spacetime at a level with much coarser spatial resolution. Thus, there must be other kinds of matter/energy with which the Universe can map out its spacetime geometry to a finer spatial accuracy than is possible with the use of conventional ordinary matter. We conclude that there must be a dark sector in the Universe! One can draw this conclusion, independent of recent observations of dark energy and dark matter. [2]

By the way, the quantum foam model corresponding to $\delta l \gtrsim l^{1/2} l_P^{1/2}$ is appropriately called the random walk (RW) foam model [13] (which would be the correct quantum foam model if there were only ordinary matter obeying Bose or Fermi statistics in our Universe. And in that case, the entropy bound is given by $(area)^{3/4}$.) For the different spacetime foam models, henceforth to be parametrized by $\alpha \sim 1$ so that $\delta l \gtrsim l^{1-\alpha} l_P^{\alpha}$ (as in Eq. (1)), then the values of $\alpha = 1/2, 2/3, 1$ correspond respectively to the RW model, the holographic model, and the minute Planck length fluctuation model. In this talk we concentrate on the holographic model.

There is another interesting fact: The ($\sim (R_H/l_P)^2$) bits/"particles" of the dark sector vastly outnumber the ($\sim (R_H/l_P)^{3/2}$) particles of ordinary matter (by an enormously huge factor of $(R_H/l_P)^{1/2} \sim 10^{31}$ for the present observable universe.)

3 Holographic Foam Cosmology

Let us now generalize the quantum foam discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by substituting l by $R_H = 1/H$ where H and R_H are respectively the Hubble parameter and the Hubble radius.[12] Then the resulting cosmology (to be called

[2] Recent cosmological observations (Supernovae, cosmic microwave background, galaxy clusters etc) show: The two main ingredients of the Universe are unconventional matter: dark energy $\sim 70\%$; dark matter: $\sim 25\%$.

Journal of Physics: Conference Series **2987** (2025) 012015 doi:10.1088/1742-6596/2987/1/012015

the holographic foam cosmology) has the following two features: (1) the cosmic energy has the critical value: $\rho = \frac{3}{8\pi}\left(\frac{H}{l_P}\right)^2 \sim (R_H l_P)^{-2}$; (2) the Universe contains $I \sim (R_H/l_P)^2$ bits of information. Thus the average energy carried by each bit is $\rho R_H^3/I \sim R_H^{-1}$. Such long-wavelength bits or "particles" carry negligible kinetic energy resulting in the pressure of the unconventional energy being roughly equal to minus its energy density, leading to accelerating cosmic expansion. (This scenario is very similar to that of quintessence [14].)

Furthermore, these long-wavelength "particles" constitute the dark energy which acts like a dynamical cosmological constant $\Lambda \sim 3H^2$. In the next section we will show that this cosmological constant is intimately connected to a crucial property of the dark matter model in holographic foam cosmology.

Let us dig deeper into the properties of the long-wavelength "particles" of dark energy. How different are these "particles"? Consider $N \sim (R_H/l_P)^2$ such "particles" obeying Boltzmann statistics in a volume $V \sim R_H^3$ at temperature $T \sim R_H^{-1}$. The corresponding partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$ with thermal wavelength λ yields an entropy $S = N[ln(V/N\lambda^3) + 5/2]$ for the system with $\lambda \sim T^{-1}$. But $V \sim \lambda^3$, so S becomes negative unless $N \sim 1$ which is equally nonsensical. Solution: The N inside the log in S, i.e, the Gibbs factor $(N!)^{-1}$ in Z_N, must be absent. But that means the N "particles" are distinguishable! Then $S = N[ln(V/\lambda^3) + 3/2] \sim N$, which is positive as required! [9]

Now the only known consistent statistics in greater than 2 space dimensions without the Gibbs factor is the quantum Boltzmann statistics, also known as infinite statistics. [15, 16, 17, 18] Thus we conclude that the "particles" constituting dark energy obey infinite statistics, rather than the familiar Fermi or Bose statistics! This is the overriding difference between dark energy and conventional matter. [9, 19] For completeness, a short discussion of infinite statistics is given in Appendix D.

4 Dark Matter

In his Modified Newtonian Dynamics (MoND), Milgrom [20] introduced a critical acceleration parameter to account for the observed flat galactic rotation curves (v independent of r) and to explain the observed Tully-Fisher relation (speed of stars correlated with galaxies' brightness: $v^4 \propto M$ at large r), which the cold dark matter (CDM) scheme fails to accomplish. On the other hand, there are problems with MoND at the cluster and cosmological scales, where CDM works much better.

An obvious question then is this: Can we reconcile the CDM approach with the MoND approach? [21] As I reported in IARD 2016, [3] my collaborators and I succeed in doing so by generalizing Jacobson's idea of gravitational thermodynamics [22] and E. Verlinde's idea of entropic gravity [23] for $\Lambda = 0$ to the case of de-Sitter space with positive Λ. We find that the dynamical cosmological constant (originated from quantum fluctuations of spacetime) found in Section 3 automatically gives rise to a critical acceleration parameter of the same (i.e.,correct) magnitude as the one Milgrom put in by hand in MoND. We call our dark matter model "MoNDian dark matter" (later renamed "Modified dark matter" (MDM) [24]).

Furthermore, in the MDM scenario, we find that, at the galactic scale, there is an interesting connection among dark matter (mass M'), ordinary matter (M) and dark energy ($\sim$ cosmological constant Λ) given by $M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$. We also succeed in fitting rotation curves for 30 local spiral galaxies [25]; and our test of MDM at cluster scales fares well too. Finally we show that "particles" constituting DM, like dark energy, obey infinite statistics! (Can that be why it is so difficult to experimentally detect dark matter "particles"?)

5 Quantum Foam, Turbulence, and Inflationary Universe

According to Wheeler [1], spacetime, when probed at very small scales, as is the case for the early universe, will appear very complicated like a chaotic turbulent froth. In this section we will examine how accurate this picture is and explore one of its important physical implications.

5.1 Quantum Foam and Turbulance

First let us show the deep similarities between the spacetime foam phase of strong quantum gravity and turbulence [26]. The connection between them can be traced to the role of diffeomorphism symmetry in classical gravity and the volume preserving diffeomorphisms of classical fluid dynamics. In the case of

[3] The discussion on dark matter in this section is very brief as this subject was already explored in some detail in an earlier IARD conference.

Journal of Physics: Conference Series **2987** (2025) 012015 doi:10.1088/1742-6596/2987/1/012015

irrotational fluids in three spatial dimensions, the equation for the fluctuations of the velocity potential can be written in a geometric form [27] of a harmonic Laplace–Beltrami equation: $\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}g^{ab}\partial_b\varphi) = 0$. Here the effective space time metric takes on the form $ds^2 = \frac{\rho_0}{c}[c^2dt^2 - \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)]$, with c being the sound velocity. Comparing this metric with the spacetime metric in the standard ADM form $ds^2 = N^2dt^2 - h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$, we find that, in the fluid dynamics context, v^i plays the role of the shift vector N^i. Thus a fluctuation of v^i plays the role of a fluctuation of the shift vector N^i. This is possible provided the metric of spacetime fluctuates, which is an intuitive definition of the spacetime foam.

Next consider fluid dynamics in the large limit of the so-called Reynolds number (to be defined shortly below). Kolmogorov [28] observed that (in the presence of a constant energy flux $\frac{v^2}{t} \sim \varepsilon$), the scaling of velocity with length scale $\ell \sim vt$ is $v \sim (\varepsilon\ell)^{1/3}$. The two-point function of velocity goes as $\langle v^i(\ell)v^j(0)\rangle \sim \ell^{2/3}\delta^{ij}$ (the famous "two-thirds law").

Let us now recall $\delta\ell \sim \ell^{1/3}\ell_P^{2/3}$ for quantum foam. If one defines the velocity as $v \sim \frac{\delta\ell}{t_c}$, (with natural characteristic time scale $t_c \sim \frac{\ell_P}{c}$), then $v \sim c\left(\frac{\ell}{\ell_P}\right)^{1/3}$ yielding Kolmogorov-like scaling; to wit, the velocity scales as $v \sim \ell^{1/3}$, and the two-point function has the two-thirds power law. We conclude that Wheeler is correct: quantum foam does act like a turbulent froth! But it is important to realize that, of all the quantum foam models (parametrized by α in $\delta l \sim l^{1-\alpha}l_P^{\alpha}$), Kolmogorov scaling is consistent only with the holographic model (with $\alpha = 2/3$).

5.2 Turbulance and Inflation in the Early Universe
So far we have only applied the holographic foam cosmology (HFC) to the present cosmic era, predicting an accelerating expansion. Whether HFC can also explain the inflation in the early universe [29] has yet to be studied. But there are reasons to be optimistic: For example, 1. One of the HFC's main features, that the cosmic energy is of critical density, is a hallmark of the inflationary universe paradigm. Thus the flatness problem is automatically solved. 2. It is quite possible that HFC provides sufficient density perturbation as the model contains the essence of a k-essence model. 3. Spacetime foam physics is, in a nutshell, black hole physics in a quantum setting, hence it is intimately related to wormhole physics. But it has been argued [30] that wormholes in a Friedmann-Robertson-Walker universe can be used to solve the horizon problem.

We will now argue that the brief inflation in the early universe was due to a natural onset of cosmic turbulence. First recall that the onset of turbulence corresponds to the case of very large Reynolds number defined by $Re = Lv/\nu$, where v is the velocity field, L is a characteristic scale and ν is the kinematic viscosity which is given by the product of the mean free path $\tilde{l}$ and an effective velocity factor $\tilde{v}$. Below, for illustration, we follow the folklore that inflation occurred at time, say, 10^{-35} sec.: In that case, we have (1) $\nu \sim cl_P$ since, in that regime, momentum transport could only be due to Planckian dynamics; (2) $v \sim 10^{-2}c$ and $L \sim 10^8 l_P$ (see, e.g., Ref. [31]); (3) $\tilde{l} = l_P \ll L \sim 10^8 l_P$. It follows that, in the early universe, the Reynolds number $Re \gg 1$ precipitating the onset of turbulence.

But is there a natural way to implement a "graceful" exit from inflation? That is, how can one get a small enough Re to transit to the laminar phase and to end inflation in the process? It is here that the nonlocality property enjoyed by the quanta of spacetime foam (due to the fact that they obey infinite statistics) came to the rescue since the length scale $\tilde{l}$ in ν would naturally and eventually extend to the order of L, yielding a small enough Re to suppress turbulence and to naturally end inflation [32] !

6 Possible Ways to Test/Detect Spacetime Foam
There have been numerous proposals to detect quantum foam. In this section we discuss three such proposals: 1. using gravity-wave interferometers, 2. looking for possible spreads in arrival times for high-energy gamma rays from distant gamma ray bursts, 3. using quantum foam-induced phase incoherence of light from distant galactic sources to probe Planck-scale physics.

6.1 Laser interferometry
To measure the minute fluctuations $\delta l \sim l^{1/3}l_P^{2/3}$, we need instruments capable of accurately measuring fluctuations in length over long distances. Modern gravitational-wave interferometers (such as LIGO/VIRGO, LISA with extraordinary sensitivity $\sim 10^{-18}$ meter) come to mind. The uncertainty δl

manifests itself as a displacement noise (in addition to noises from other sources such as thermal noise, seismic noise, photon shot noise etc.) that infests the interferometers. [33, 34]

The displacement noise due to spacetime fluctuations that involves a time interval t is given by $\delta l(t) \sim l_P^{2/3}(ct)^{1/3}$. Let f denote the frequency of the interferometer bandwidth. We can decompose the displacement noise in terms of the associated displacement amplitude spectral density $S(f)$ of frequency f to give $S(f) \sim c^{1/3} l_P^{2/3} f^{-5/6}$ (by applying $(\delta l(t))^2 = \int_{1/t}[S(f)]^2 df$). For the "advanced phase" of LIGO, it is claimed that $S(f = 100\text{Hz}) \sim 10^{-20}\text{mHz}^{-1/2}$ with the implication that one can probe l_P down to $\lesssim 10^{-31}$ cm! But, the above estimate assumes that spacetime foam affects the paths of all the photons in the laser beam coherently. In reality, incoherent contributions from different photons cut down sensitivity (by some sizable fractional powers of the number of photons.) On the other hand , one should remember a couple of useful features of this technique: 1. Correlation length of spacetime foam noise is extremely short, so this noise can be easily distinguished from other noises; 2. With $S(f) \propto f^{-5/6}$, one can optimize the technique at low frequencies.

We conclude this subsection by noting that there are other kinds of interferometers that we can make use of, including atom interferometers, optical interferometers, matter-wave interferometers (bosons or fermions), matter-wave interferometric gravitational-wave observatory (using atomic beams from supersonic atomic sources), and perhaps especially the twin table-top 3D interferometers [35] under construction at Cardiff University in Wales.

6.2 Energy-dependent spread in the arrival times of photons?

Due to quantum foam effects, the speed of light $v = \frac{\partial E}{\partial p}$ fluctuates by $\delta v \sim 2\epsilon c(E/E_P)^\alpha$, where $\epsilon \sim \pm 1$. (See Appendix A.) Thus, for photons emitted simultaneously from a distant source coming towards our detector, we would expect an energy E-dependent spread in their arrival times given by $\delta t \sim \delta v(\ell/c^2) \sim t(E/E_P)^\alpha$, where $t = \ell/c$ is the average overall time of travel from the photon source (distance ℓ away). But these results for the spread of arrival times of photons are not correct, because we have wrongly used $\ell/\lambda \sim Et/\hbar$ as the cumulative factor instead of the correct factor $(\ell/\lambda)^{1-\alpha} \sim (Et/\hbar)^{1-\alpha}$. Using the correct cumulative factor (as described in Appendix B), we get a much smaller $\delta t \sim t^{1-\alpha} t_P^\alpha \sim \delta\ell/c$ for the spread in arrival time of the photons, independent of energy E (or photon wavelength, λ), as expected, leading to a time spread too small to be detectable [36].

6.3 Phase incoherence of photons from distant galaxies

Another proposal to observationally constrain models of spacetime foam is to examine if the light wavefront from a distant quasar or GRB can be noticeably distorted by spacetime-foam-induced phase incoherence. [37, 38, 39, 40, 41, 42] First recall that the phase ϕ fluctuates by $\Delta\phi = 2\pi\delta l/\lambda \sim 2\pi l_P^\alpha l^{1-\alpha}/\lambda$. As the phase fluctuations are proportional to the distance to the source, but inversely proportional to the wavelength, ultra-high energy photons from distant sources are particularly useful.

The need to use large l means that cosmological effects must be taken into account. Since the angular broadening caused by spacetime foam does not explicitly depend on flux but merely the number of "scattering events" along the line of sight, the appropriate cosmological distance measure [40] is the line-of-sight co-moving distance $D_C(z) = D_H \int_0^z dz'/E(z')$ with $E(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$, where $D_H = c/H_0$ is the Hubble distance, Ω_M, Ω_k and Ω_Λ are the density parameter associated with matter, curvature and the cosmological constant respectively.

Now an exciting piece of news: There has been a recent development in using this method to probe quantum foam by Eric Steinbring.[42] We refer the readers to his contribution to the Proceedings of this (IARD2024) conference. What we have done in this short Subsection so far is merely intended to introduce his talk.

7 Summary and Conclusion

We summarize by collecting some of the salient points presented in this talk:

• Spacetime is foamy: it undergoes quantum fluctuations such that, e.g., distance fluctuations scale as the cube root of the distances: $\delta l \gtrsim (l l_P^2)^{1/3}$.

• Three different methods [4] have been used to derive the magnitude of quantum foaminess of spacetime. The discussion only depends on the quantum characteristics of spacetime measurements, and not on any particular theory; the results are consistent with the holographic principle, hence the term "holographic quantum foam (HQF)".

• HQF, in conjunction with thermodynamics, naturally demands the existence of a dark sector (different from ordinary matter).

• Applied to the cosmos, HQF yields a cosmology (called "holographic foam cosmology (HFC)") which has two main features: (1) the cosmic energy density has the critical value: $\rho = \frac{3}{8\pi}\left(\frac{H}{l_P}\right)^2 \sim (R_H l_P)^{-2}$; (2) the dark energy (formed by the quanta of its constituents in the current universe) acts like a dynamical cosmological constant Λ with $\sqrt{\Lambda/3} \sim H$.

• "Particles" constituting the dark energy obey an exotic statistics (called infinite statistics).

• HQF allows one to construct a dark matter model (called Modified Dark Matter (MDM) model) which acts like the Cold Dark Matter model at cluster and cosmological scales, but it behaves like the Modified Newtonian Dynamics (MoND) model at the galactic scale [5] (explaining the observed galactic flat rotation curves and the Tully-Fisher relation). The dark matter particles in MDM obey infinite statistics.

• In the gravitational context, the phenomenon of turbulence is intimately related to the properties of quantum foam.

• Inflation in the early universe is due to the onset of turbulence. The "graceful" exit problem is solved by the fact that the quanta of quantum foam obey infinite statistics. (So, in essence, we have a scenario in which both the late and the early cosmic acccelerations have a common origin and they can be traced to HQF.)

• To detect quantum foam, we can make use of laser interferometry techniques. Another way is to examine if the light wave-front from a distant quasar or GRB can be noticeably distorted by spacetime-foam-induced phase incoherence.

• Detecting the tiny effects of spacetime fluctuations is difficult, but not impossible. Read the fascinating talk by Eric Steinbring in these Proceedings!

Acknowledgments

I would like to dedicate this paper to the memory of James D. "BJ" Bjorken who kindly took an interest in my work on holographic quantum foam. I also thank him for his trust in my instinct and for his encouragement. The work reported here was supported in part by the US Department of Energy, the Bahnson Fund and the Kenan Professorship Research Fund of UNC-CH.

Appendix A. Metric, energy, momentum, and speed of light fluctuations

Just as there are uncertainties in spacetime measurements, there are also uncertainties in metric and energy-momentum measurements. The spacetime fluctuation parametrized by α translates into a metric fluctuation over a distance ℓ given by $\delta g_{\mu\nu} \gtrsim (\ell_P/\ell)^{\alpha}$. One can also show that the uncertainties in energy (E)-momentum (p) measurements are respectively given by $\delta E \sim E(E/E_P)^{\alpha}$ and $\delta p \sim p(p/(m_p c))^{\alpha}$, where m_P and $E_P = m_P c^2$ denote the Planck mass and Planck energy. These energy-momentum uncertainties modify the dispersion relation for the photons: $E^2 \simeq c^2p^2 + \epsilon E^2(E/E_P)^{\alpha}$, where $\epsilon \sim \pm 1$. Thus the speed of light $v = \partial E/\partial p$ fluctuates by $\delta v \sim 2\epsilon c(E/E_P)^{\alpha}$.

Appendix B. Cumulative Effects of Spacetime Fluctuations

Consider a large distance ℓ. Divide it into l/λ equal parts each of which has length λ. Given the fluctuation $\delta\lambda$ from each part, the following (subtle) question presents itself: what is the cumulative factor [39] $\mathcal{C}_\alpha$ in $\delta\ell = \mathcal{C}_\alpha\, \delta\lambda$ for the whole distance ℓ? Since $\delta\ell \sim \ell_P(l/\ell_P)^{1-\alpha}$ and $\delta\lambda \sim \ell_P(\lambda/\ell_P)^{1-\alpha}$, it follows that $\mathcal{C}_\alpha = (\ell/\lambda)^{1-\alpha}$; in particular, for the holographic quantum foam model: $\mathcal{C}_{\alpha=2/3} = (\ell/\lambda)^{1/3}$. Note that the $\delta\lambda$'s from the ℓ/λ parts in ℓ do not add coherently as $\delta\ell/\delta\lambda \neq \ell/\lambda$.

[4] For yet another method which makes use of unimodular gravity and causal-set theory, see my IARD 2016 talk.

[5] HQF can be used to successfully derive MoND with the correct magnitude of the critical galactic acceleration $a_c \sim \sqrt{\Lambda/3} \sim H$).

Appendix C. The Margolus-Levitin Theorem

The theorem states that the maximum speed of computation for any system is given by $4/h$ times the energy available to the system for computation. One way to prove this theorem is to consider the maximum speed of dynamical evolution for that physical system. [11]

Assume that the system has a discrete energy spectrum $E_n, n = 0, 1, 2...$ with the lowest energy chosen to be $E_0 = 0$. Write an arbitrary state $|\psi\rangle$ as a superposition of energy eigenstates, with coefficients c_n at time $t = 0$. Let $|\psi_0\rangle$ evolve for a time t to become $|\psi_t\rangle$. Consider the transition amplitude $S(t) = \langle\psi_0|\psi_t\rangle$. We want to find the smallest value of t such that $S(t) = 0$.

Noting that the real part of S $Re(S) = \sum_n |c_n|^2 cos(E_n t/\hbar)$ and using the inequality $cosx \geq 1 - (2/\pi)(x + sinx)$, valid for $x \geq 0$, we get $Re(S) \geq 1 - 2Et/(\pi\hbar) + 2Im(S)/\pi$, where E denotes the average energy of the system. But $S(t) = 0$ implies both $Re(S) = 0$ and $Im(S) = 0$. So this inequality becomes $0 \geq 1 - 4Et/h$, where $h = 2\pi\hbar$. Thus the earliest that $S(t)$ can possibly equal zero is when $t = h/4E$. Applied to a computer, this result implies that the maximum speed of computation is given by $4/h$ times the energy available for computation. QED.

Appendix D. Infinite Statistics

A Fock realization of infinite statistics [15, 16, 17, 18] is given by $a_k a_l^\dagger = \delta_{k,l}$. Any 2 states obtained by acting on the vacuum $|0>$ with creation operators in different order are orthogonal to each other: $<0|a_{i1}...a_{iN}a_{jN}^\dagger...a_{j1}^\dagger|0> = \delta_{i1,j1}...\delta_{iN,jN}$, implying that particles obeying infinite statistics are virtually distinguishable. The corresponding partition function is $Z = \Sigma e^{-\beta H}$ (NO Gibbs factor). In infinite statistics, all representations of the particle permutation group can occur. Furthermore, theories of particles obeying infinite statistics are non-local. (More precisely, the fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are point-like functionals of the fields.) In particular, the number operator $n_i = a_i^\dagger a_i + \sum_k a_k^\dagger a_i^\dagger a_i a_k + \sum_l \sum_k a_l^\dagger a_k^\dagger a_i^\dagger a_i a_k a_l + ...$, and Hamiltonian, etc., are both nonlocal and non-polynomial in the field operators. It is also known that the TCP theorem and cluster decomposition still hold and that quantum field theories with infinite statistics remain unitary. [18]

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