Indian Journal of Pure & Applied Physics
Vol. 57, September 2019, pp. 664-667

Equation of state of a PNJL model with chemically equilibrium QGP

Anju Dahiya & S Somorendro Singh*

Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India

Received 3 July 2019

We study the equation of state (EOS) of PNJL model with chemically equilibrium QGP. By the effect of the chemical potential, the phase structure of QGP appears at lower critical temperature Tc. The result with the chemical potential found the phase structure of QCD is in agreement with the earlier result and almost matching with lattice data.

Keywords: QCD, Quark-Gluon Plasma

1 Introduction

Quantum-chromodynamics (QCD) is the theory of the strong interactions[1]. The theory predicts that there is a phase transition at high temperature and at very high chemical potential which exhibit a separation of these two matters[2]. In this transition phenomenon, there is a complete separation between these two matters, *viz.* the confined phase of hadronic matter and the deconfined phase of QGP matter at much lower temperature and there is mixed phase where the temperatures is around (150 - 170) MeV. It is also believed that the transition took place during the early phase of universe evolution and the evolution of universe can also be reproduced in contemporary heavy-ion collision experiments as colliding heavy-ions of Pb-Pb/Au-Au nuclei producing a state of matter as mini universe called quark-gluon plasma. So there are a number of experiments around the globe that these experiments are trying to prove the existence of quark-gluon-plasma formation. The study has proved the existence of QGP at RHIC at BNL and SPS at CERN. From these proofs the curiosity on the study of QGP formation has been increased from the last one decade and the proof can be signified from the different angles of signatures *viz.* photon/dilepton production, strange enhancements, hydrodynamical studies and QCD phase structure like equation of state (EOS). It indicates that the hot strongly interacting matter behave much like a nearly ideal relativistic fluid and this phenomenon of transition can be described by the models of relativistic hydrodynamics[3]. To describe such a complicated and crucial aspect is found as a description relating between local thermodynamic quantities, the equation of state (EoS). The calculation of the EoS is based on the parameters such as the temperature T and the chemical potential μ of the phenomenologically created matter of free quarks and gluons[4]. Most studies of the finite chemical potential EOSs were performed using monte carlo simulation work and through the high computer facility like lattice QCD[5,6]. In this paper, we follow the thermodynamic potential through the grand canonical ensemble and the potential is carried out to the leading order expansion of thermodynamic observables[7–11]. Then, we focus to find the equation of state using thermal quark mass in the PNJL model at finite temperature and chemical potential[12,13].

2 Gibbs Free Energy of PNJL Model

In this section we briefly describe the formalism of PNJL model with the introduction of thermal quark mass incorporating in the potential with the finite chemical potential. By the presence of thermal quark mass and chemical potential the early characteristic features of PNJL model is re-examined. The thermo dynamical potential for the two flavor quark in presence of chemical potential and thermal mass term in PNJL model[14] is written as:

$$\Omega(\varphi^*,\varphi,m;T) = U(\varphi,\varphi^*,T) + \frac{\sigma^2}{2G} - 2N_f qB\int d^3\,pE_p\frac{2}{\pi^3}$$
$$- 2N_f qB\int d^3\,p\frac{2}{\pi^3}\left(Trln\left(1+L^{\dagger}e^{\frac{E_p-\mu}{T}}\right)\right.$$
$$\left.+\,Trln\left(1+Le^{\frac{-E_p-\mu}{T}}\right)\right)$$

Where, $U(\varphi,\varphi^*,T) = \frac{-1}{2}b_2(T)\varphi^*\varphi + b_4(T)ln\big(1-$ *6φ*φ−3φ*φ2+4φ3+φ*3*

with

$$b_2(T)=\mathrm{a}_0+\mathrm{a}_1\left(\frac{T_0}{T}\right)+\mathrm{a}_2\left(\frac{T_0}{T}\right)^2 \ \ \& \ \ b_4(T)=\mathrm{b}_4\left(\frac{T_0}{T}\right)^3$$

---

[*] Corresponding author (Email: sssingh@physics.du.ac.in)

Where, $a_0= 3.51$, $a_1 = -2.41$, $a_2 = 15.22$, $b_4= -1.75$

In the above formula $E_p = \sqrt{p^2 + m^2(T)}$ is the energy calculated with the thermal quark mass. In field's φ and φ∗ are different at non-zero quark chemical potential which is already indicated in literature[15]. It also explains the primarily consequence as quantum fluctuations of the fields around their mean field values. In this present work we use mean field limit which implies φ = φ∗ in which we take μ = finite.

3 Thermodynamic Parameters of PNJL Model

We briefly discuss the standard thermodynamic properties like energy density, pressure, entropy and specific heat which can be derived from the above Gibbs potential incorporating the thermal mass and a chemical potential in PNJL model. These thermodynamically properties can describe the phase structure of QCD and the order of phase transition. From the theoretical development it is also believed that the order of transition is conjectured as first order, second order and crossover phase. In the meanwhile, the theoretical calculations related to the pressure are defined as a function of temperature T and chemical potential[16] μ. So the pressure as a function of temperature T and baryon chemical potential is given by the standard thermodynamic relation[17]:

$$P(T,\mu) = -\Omega(T,\mu)$$

Defining the energy density of the system, it is obtained by calculating the first derivative of Gibbs potential which is given as:

$$\epsilon(T,\mu) = -T^2 \frac{\partial\left(\frac{\Omega}{T}\right)}{\partial(T)}$$

Similarly, in continuation of the thermodynamic parameters we obtain entropy density associated in the system given as:

$$s(T,\mu) = \frac{-4}{3}\epsilon(T,\mu)$$

The entropy calculation is especially associated with the disordered condensate and it is used to measure in determining the equilibration of the system.

We also calculate the specific heat of the system, which is defined as the rate of change of energy density with temperature at constant volume.

$$C_v = \frac{\partial\epsilon}{\partial T}$$

It is also suggested that the temperature fluctuation in the heavy-ion collision experiments can be used to measure Cv. The measurement of Cv also directly test the relevance of conformal symmetry to finite temperature QCD.

4 Results

To discuss the QGP phase structure, we look forward the thermodynamic parameters at various temperatures with the inclusion of the thermal mass and chemical potential in PNJL model. Figures 1-4 show energy density, pressure, entropy and specific heat with respect to temperature at different chemical potentials. From Fig. 1 we know that the energy density at critical temperature is higher for all the starting point of the energy density for all the case of chemical potential[18]. It means the contribution of energy density is mainly dominated by the deconfined matter of free quarks and gluons. The value of energy density is found suddenly increase with the temperature reaching around 2.5 Tc and constancy the value of energy density is found beyond this temperature. The result is similarly behaving for all the values of chemical potentials beyond 2.5 Tc . In Fig. 2, we plot pressure with

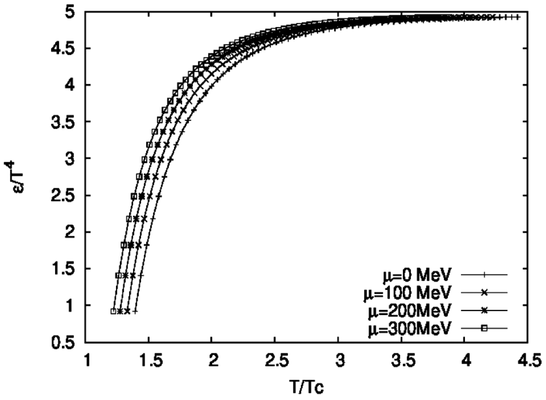

Fig. 1 — Energy density as a function of temperature at μ = (0, 100, 200, 300) MeV.

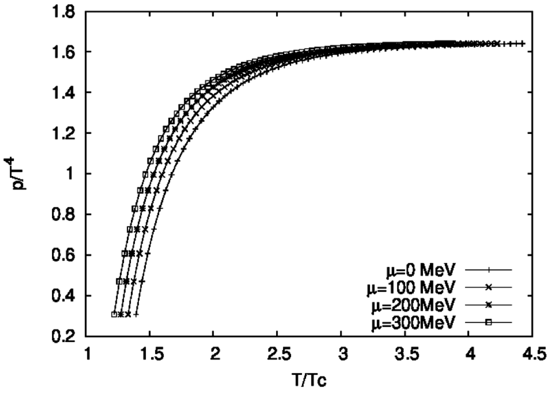

Fig. 2 — Pressure as the function of temperature at μ = (0; 100; 200; 300) MeV.

temperature at different values of chemical potential. It shows the behavior of the pressure is same with the energy density up to the temperature around 2.5 Tc . Pressure is equivalent to the energy density as $P = \frac{\epsilon}{3}$ in terms of magnitude

Fig. 3 we plot the entropy density with the variation of temperature and the result shows enhancement in the entropy density in comparison to the lattice data. The result is almost similar with the lattice when the temperature reach beyond 2.5 Tc . This indicates that the effect of thermal mass and chemical potential is negligible beyond this 2.5 Tc temperature. As shown in Fig. 4, Cv grows with increasing temperature and reaches a peak at 2.0 Tc . Then it slightly decreases sharply for a short range of temperature. These features are distinctly behaved with the increasing chemical potential and it is observed up to the range of temperature 2.0 Tc. As the temperatures increase beyond 2.0 Tc the effect of chemical potential disappear in its characteristic feature for all the chemical potentials. It implies the effect of chemical potential exist around 2.0 Tc and disappear the effect of chemical potential in the specific heat of QGP in this high temperature region.

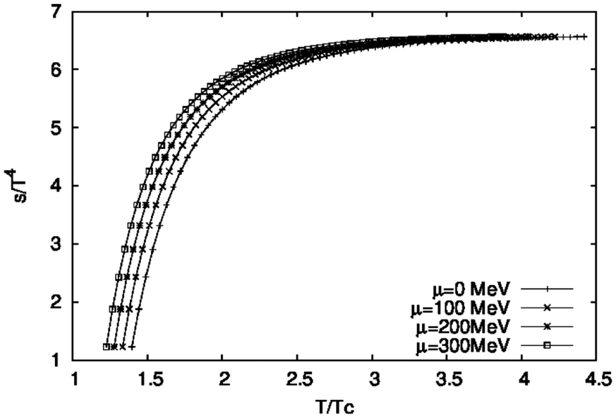

Fig. 3 — Entropy as a function of temperature at μ = (0, 100, 200, 300) MeV.

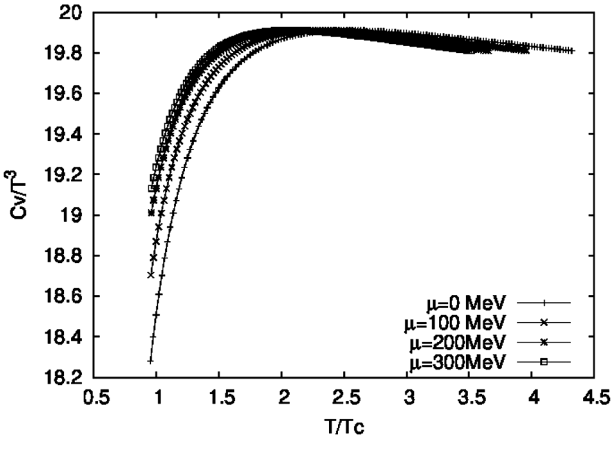

Fig. 4 — Specific heat as a function of temperature at μ = (0, 100, 200, 300) MeV.

5 Conclusions

It concludes that the QCD matter has paramagnetic properties which monotonically depend on the temperature and are not affected by the hadron-quark phase transition[19]. So the desirable properties can be achieved by adjusting the transition temperature. Again thoroughly studying the thermo dynamical properties of PNJL model it helps us to understand the PNJL model thoroughly with thermal mass and chemical potential and polyakov loop can be observed quite accurately. The effective model of QCD has taken into account in the features of both chiral symmetry breaking and deconfinement by using an effective potential obtained in pure gauge sector[20]. We explore the phase diagram and critical behavior of PNJL model with thermal mass and chemical potential. Through the effect of chemical potential and thermal mass, the picture of QCD model improved a lot in the features of QCD. It indicates that the study of QCD phase structure through the model of thermal mass and chemical potential is good for the exploration of QGP existence at the early phase of universe evolution.

Acknowledgement

Author (Anju Dahiya) likes to thank R Ramanathan for his comments and useful discussions in preparing the manuscript. She thanks the University and CSIR, Delhi for providing financial supports in terms of JRF for this work successful.

References

1 Meisinger P N & Ogilvie M C, *Phys Lett B*, 379 (1996) 163.
2 Fukushima K, *Phys Lett B*, 591 (2004) 277.
3 Karch F, *Phys Lett B*, 571 (2003) 67.
4 Allton C R, Doring M, Ejiri S, Hands S J, Kaczmarek O, Karsch F, Laermann E & Redlich K, *Phys Rev D,* 71 (2005) 054508.
5 Jacobs P & Wang X N, *Nucl Phys*, 54 (2005) 443.
6 Csikor F, Egiri G, Fodor Z, Katz S D, Szabo K K & Toth A I, *JHEP*, 0405 (2004) 046.
7 Fodor Z, Katz S & Szab K, *Phys Lett B*, 568 (2003) 7377.
8 Allton C R, Ejiri S, Hands S J, Kaczmarek O, Karsch F, Laermann E & Schmidt C, *Phys Rev D*, 68 (2003) 014507.
9 Allton C R, Döring M, Ejiri S, Hands S J, Kaczmarek O, Karsch F, Laermann E & Redlich K, *Phys Rev D,* 71 (2005) 054508.

10 Bernard C, DeTar C, Levkova L, Gottlieb S, Heller U M, Hetrick J E, Sugar R & Toussaint D, *Phys Rev D*, 77 (2008) 014503.

11 K Fukushima, Phys Lett B, 591 (2004) 277.

12 Y Nambu & G Jona-Lasinio, *Phys Rev*, 122 (1961) 345.

13 Klevansky S P, *Phys Rev Mod*, 64 (1992) 649.

14 Wang P, Thomas A W & Williams A G, *Phys Rev C*, 75 (2007) 045202.

15 Fukushima K, *Phys Lett B*, 591 (2004) 277.

16 Nambu Y & Jona-Lasinio G, *Phys Rev*, 122 (1961) 345.

17 Klevansky S P, *Phys Rev Mod*, 64 (1992) 649.

18 Wang P, Thomas A W & Williams A G, *Phys Rev C*, 75(2007) 045202.

19 Polyakov A M, *Phys Lett B*, 72 (1978) 477.

20 Gocksch A & Ogilvie M, *Phys Rev D*, 31 (1985) 877.