



# Sudakov effects in central-forward dijet production in high energy factorization

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## ABSTRACT

We discuss central-forward dijet production at LHC energies within the framework of high energy factorization. In our study, we profit from the recent progress on consistent merging of Sudakov resummation with small- $x$  effects, which allows us to compute two different gluon distributions which depend on longitudinal momentum, transverse momentum and the hard scale of the process: one for the quark channel and one for the gluon channel. The small- $x$  resummation is included by means of the BK equation supplemented with a kinematic constraint and subleading corrections. We test the new gluon distributions against existing CMS data for transverse momentum spectra in forward-central dijet production. We obtain results which are largely consistent with our earlier predictions based on model implementation of Sudakov form factors. In addition, we study dijet azimuthal decorrelations for the forward-central jets, which are known to be sensitive to the modeling of soft radiation.

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## 1. Introduction

Processes with jets remain one of the most important tools used to study Quantum Chromodynamics (QCD) at hadron colliders, in particular at the LHC [1,2] and future Electron Ion Collider (EIC) [3–6]. Amongst them, production of dijets proves particularly useful to address various questions concerning QCD dynamics. When both jets are produced in the central rapidity region, the energy fractions of the incoming partons are comparable and sizable. Theoretical predictions for such configuration can be safely calculated in the framework of collinear factorization. However, when one of the jets moves in the forward direction,  $y_{\text{jet}} \gg 0$ , one of the incoming hadrons is probed at relatively low momentum fraction  $x$ , and that leads to the appearance of large logarithms  $\ln x$ , which have to be resummed. The optimal description of this process is achieved within the hybrid factorization [7–10], where the matrix elements are evaluated with one of the incoming partons being off-shell. The momentum distribution of that parton obeys the BFKL equation [11–14], which depends not only on the longitudinal part of the momentum, but also on its transverse component. We will from now on refer to these as *transverse momentum dependent* parton distributions (TMDs). In addition, when both jets move forward, the value of  $x$  is even smaller and one starts being sensitive to saturation effects [15,16]. The corresponding evolution equation becomes nonlinear [17–21], as density of gluons at low  $x$  is very high.

While the small- $x$  effects can be taken into account by using one of the phenomenologically successful TMDs, there is another class of effects relevant for forward jet production which should also be accounted for, namely the resummation of Sudakov logarithms. They are important as the hard scale provided by jet transverse momentum opens phase space for logarithmically enhanced soft and collinear emissions [22–26]. See also recent Monte Carlo developments where one constructs TMD distributions that account for  $k_T$  and Sudakov effects [27].

As demonstrated in Refs. [28–32], small- $x$  and Sudakov resummations can be performed simultaneously in  $b_\perp$  space and can then be cast into transverse momentum dependent gluon distributions. Such TMDs have already been used in phenomenological calculations of di-hadron correlations at EIC [6] and in proton-nucleus collisions at RHIC [33,34]. While in [6,33] the Golec-Biernat-Wüsthof model [35] was employed to account for small- $x$  effects, in [34] the rcBK was used.

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In the present work, we focus on Sudakov effects in the process of central-forward dijet production in proton-proton collisions. Similarly to previous studies [45–47], we perform our calculations in the framework of *high energy factorization* (HEF) [7,15,36,37], where the cross section is calculated as a convolution of a hard sub-process [38,39] and nonperturbative parton densities, which take into account longitudinal and transverse degrees of freedom. At low  $x$ , gluons dominate over quarks, hence we consider only gluon TMDs.

In our earlier study of the central-forward dijet production [40], the Sudakov effects were introduced by means of a simplified procedure [40,41], which nevertheless turned out to be phenomenologically successful. We then used the same approach to study forward-forward dijet production in proton-proton and proton-lead collisions, focusing on the broadening of the dijet azimuthal correlation spectrum [42]. We found that a delicate interplay between the Sudakov effects and the saturation effects is needed to describe the LHC data. Although our phenomenological Sudakov model works well in that regard, a more systematic calculation of the Sudakov effects is necessary in order to solidify the predictions. One of the difficulties here comes from the proliferation of the small- $x$  TMD gluon distributions needed in the saturation regime [43,44]. The Sudakov resummation affects all these distributions in a rather complicated way. The following work is a first step towards a fully general approach and focuses on a single small- $x$  TMD gluon distribution, which appears in inclusive processes and in situations where saturation effects are mild.

In this work, we use the proper Sudakov factors derived within perturbative QCD [28–32] and profit from the recent progress on consistent merging of Sudakov resummation with small- $x$  effects [33]. These new elements allow us to significantly elevate theoretical status of our predictions for the discussed process of interest. We shall then compare the upgraded results to the ones which used simplistic models of including the Sudakov effects into the small- $x$  gluon, as well as to available experimental data. As in Ref. [40], the modeling of the small- $x$  effects in this work comes from the BK equation supplemented with a kinematic constraint and subleading corrections [47].

For the central-forward configuration of the final-state jets, one of the longitudinal fractions of the hadron momenta is much smaller than the other,  $x_B \ll x_A$ . This follows from simple kinematic relations

$$x_A = \frac{1}{\sqrt{s}} (|p_{1\perp}|e^{y_1} + |p_{2\perp}|e^{y_2}), \quad x_B = \frac{1}{\sqrt{s}} (|p_{1\perp}|e^{-y_1} + |p_{2\perp}|e^{-y_2}), \quad (1)$$

where  $\sqrt{s}$  is the center-of-mass energy of the proton-proton collision, while  $p_{i\perp}$  and  $y_i$  are the transverse momenta (Euclidean two-vectors) and rapidities of the produced jets. The formula for the *hybrid* high energy factorization reads [8,10]

$$d\sigma_{A+B \rightarrow j_1+j_2+X} = \int dx_A \int \frac{dx_B}{x_B} \int \frac{d^2k_{B\perp}}{\pi} \times \sum_{a,c,d} f_{a/A}(x_A, \mu) \mathcal{F}_{g^*/B}(x_B, k_{B\perp}, \mu) d\hat{\sigma}_{a+g^* \rightarrow c+d}(x_A, x_B, k_{B\perp}, \mu), \quad (2)$$

where  $\mathcal{F}_{g^*/B}$  is the so-called *unintegrated gluon density* or *transverse momentum dependent gluon distribution* (see [43,44,48,49] for more details on different gluon distributions),  $f_{a/A}$  are the collinear PDFs and  $d\hat{\sigma}_{a+g^* \rightarrow c+d}$  is built out of the off-shell gauge-invariant matrix elements. The indices  $a, c, d$  run over the gluon and all the quarks that can contribute to the inclusive dijet production. Notice that both  $f_{a/A}$  and  $\mathcal{F}_{g^*/B}$  depend on the hard scale  $\mu$ , and the latter depends also on the transverse momentum of the incoming gluon, whose value is linked to the final-state kinematics by the relation

$$|k_{\perp}|^2 = |p_{1\perp} + p_{2\perp}|^2 = |p_{1\perp}|^2 + |p_{2\perp}|^2 + 2|p_{1\perp}||p_{2\perp}|\cos\Delta\phi, \quad (3)$$

where  $\Delta\phi$  is the azimuthal distance between the jets. The hard scale dependence in the TMD is necessary to properly account for large Sudakov logarithms that appear predominantly in the back-to-back region, where  $k_{\perp}$  is small, but  $\mu$  remains large for relatively hard jets. As shown in Ref. [40], incorporating the hard scale dependence in the TMD is essential to successfully describe shapes of dijet spectra.

It is important to mention that, as discussed in Ref. [44], the high energy factorization formula (2) is valid only when  $Q_s \ll |k_{\perp}| \ll |p_{1\perp}|, |p_{2\perp}|$ , which corresponds to collisions of relatively dilute hadrons. The process of central-forward dijet production in  $p-p$  collision, which is the focus of our study, corresponds exactly to that situation. For processes which involve dense targets, like for example forward-forward dijet production in  $p-A$  collisions, Eq. (2) has to be replaced by a more general factorization formula with multiple transverse momentum dependent gluon distributions [43,44,50,51].

## 2. Dipole gluon with Sudakov form factor

The Sudakov effects are most conveniently included in position space. The resulting gluon TMD, which incorporates both small- $x$  and soft-collinear resummation, can be then transformed to momentum space as follows [33]

$$\mathcal{F}_{g^*/B}^{ag \rightarrow cd}(x, q_{\perp}, \mu) = \frac{-N_c S_{\perp}}{2\pi\alpha_s} \int_0^{\infty} \frac{b_{\perp} db_{\perp}}{2\pi} J_0(q_{\perp} b_{\perp}) e^{-S_{\text{Sud}}^{ag \rightarrow cd}(\mu, b_{\perp})} \nabla_{b_{\perp}}^2 S(x, b_{\perp}), \quad (4)$$

where  $S_{\perp}$  is the transverse area of the target and  $S(x, b_{\perp})$  is the so-called dipole scattering amplitude, which in the Color Glass Condensate (CGC) theory (see e.g. [52]) is related to the color average of the dipole operator, i.e. two infinite Wilson lines displaced in the transverse plane. (Notice the difference in the prefactor w.r.t. to Ref. [33], which comes from the fact that  $\mathcal{F}_{g^*/B} = \pi \mathcal{F}_{qg}^{(a)}$ .) The Sudakov factors come from the resummation of soft-collinear gluon radiation and they depend on the partonic channel. Hence, the gluon with the Sudakov acquires this dependence and, consequently, a single dipole gluon is replaced with a set of gluons  $\{\mathcal{F}_{g^*/B}^{ab \rightarrow cd}\}$ . In practice, the two channels that dominate in the central-forward productions are:  $qg \rightarrow qg$  and  $gg \rightarrow gg$ . Hence, we will need to determine two gluon TMDs:  $\mathcal{F}_{g^*/B}^{qg \rightarrow qg}$  and  $\mathcal{F}_{g^*/B}^{gg \rightarrow gg}$ .

It is appropriate to mention that in our study we resort to the so-called mean-field approximation (known to work very well, see e.g. [53]), which allows one to calculate quadrupole operators in terms of the dipole operators alone and thus to use the BK equation for evolution of the gluon density.

By taking the Fourier transform of Eq. (4), we can express the gluon with Sudakov resummation by the gluon without the Sudakov, all in momentum space

$$\mathcal{F}_{g^*/B}^{ab \rightarrow cd}(x, k_\perp, \mu) = \int db_\perp \int dk'_\perp b_\perp k'_\perp J_0(b_\perp k'_\perp) J_0(b_\perp k_\perp) \mathcal{F}_{g^*/B}(x, k'_\perp) e^{-S_{\text{Sud}}^{ab \rightarrow cd}(\mu, b_\perp)}. \quad (5)$$

For each channel, the Sudakov factors can be written as

$$S_{\text{Sud}}^{ab \rightarrow cd}(b_\perp) = \sum_{i=a,b,c,d} S_p^i(b_\perp) + \sum_{i=a,c,d} S_{np}^i(b_\perp), \quad (6)$$

where  $S_p^i(b_\perp)$  and  $S_{np}^i(b_\perp)$  are the perturbative and non-perturbative contributions. As argued in Ref. [33], as small- $x$  gluon TMDs for parton  $b$  may already contain some non-perturbative information at low- $x$ , the non-perturbative Sudakov factor associated with that incoming gluon  $b$  should not be included. In addition, according to the derivation in Ref. [28], the single logarithmic term in the perturbative part of the Sudakov factor – the so-called  $B$ -term – should also be absent for the incoming small- $x$  gluon. The perturbative Sudakov factors are given by [33]

$$S_p^{qg \rightarrow qg}(Q, b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ 2(C_F + C_A) \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) - \left(\frac{3}{2}C_F + C_A\beta_0\right) \frac{\alpha_s}{\pi} \right], \quad (7)$$

$$S_p^{gg \rightarrow gg}(Q, b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ 4C_A \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) - 3C_A\beta_0 \frac{\alpha_s}{\pi} \right], \quad (8)$$

where  $\beta_0 = (11 - 2n_f/3)/12$ ,  $\mu_b = 2e^{-\gamma_E}/b_*$ , and  $b_* = b_\perp/\sqrt{1 + b_\perp^2/b_{\text{max}}^2}$ . The  $gg \rightarrow q\bar{q}$  channel is negligible for the kinematics of this study. Following Ref. [33], for the non-perturbative Sudakov factor, we employ the parameterization [54,55]

$$S_{np}^{qg \rightarrow qg}(Q, b_\perp) = \left(2 + \frac{C_A}{C_F}\right) \frac{g_1}{2} b_\perp^2 + \left(2 + \frac{C_A}{C_F}\right) \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b_\perp}{b_*}, \quad (9)$$

$$S_{np}^{gg \rightarrow gg}(Q, b_\perp) = \frac{3C_A}{C_F} \frac{g_1}{2} b_\perp^2 + \frac{3C_A}{C_F} \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b_\perp}{b_*}, \quad (10)$$

with  $g_1 = 0.212$ ,  $g_2 = 0.84$ , and  $Q_0^2 = 2.4 \text{ GeV}^2$ .

As a basis for all calculations presented in this study, we use the nonlinear KS (Kutak-Sapeta) gluon TMD [47], which, for  $k_\perp^2 > 1 \text{ GeV}^2$ , comes from evolving the input distribution

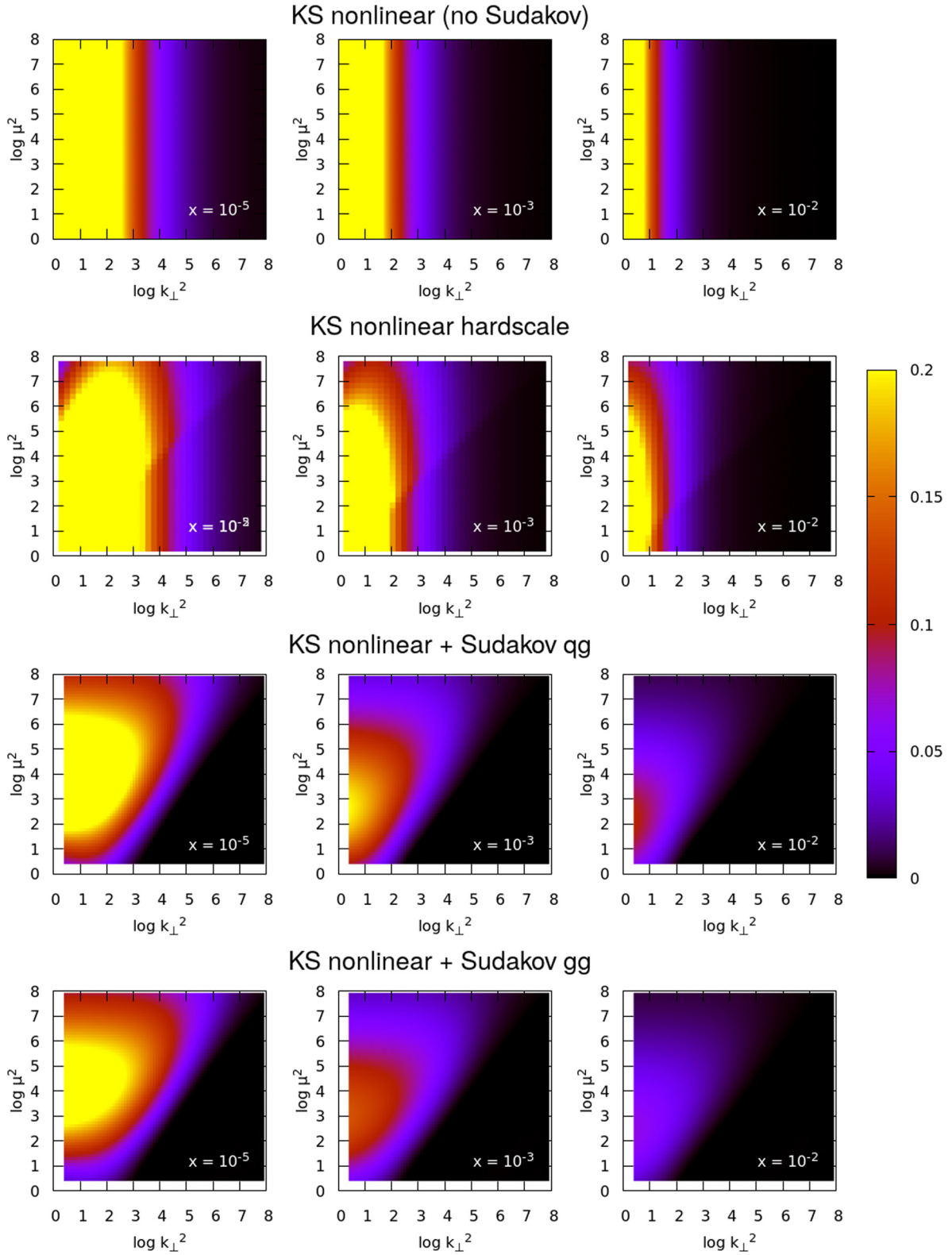
$$\mathcal{F}_{g^*/B}^{(0)}(x, k_\perp^2) = \frac{\alpha_s(k_\perp^2)}{2\pi k_\perp^2} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}\right), \quad \text{where} \quad xg(x) = N(1-x)^\beta(1-Dx), \quad (11)$$

with the extension of the BK (Balitsky-Kovchegov) equation [56], following the prescription of Ref. [57] to include kinematic constraint on the gluons in the chain, non-singular pieces of the splitting functions, as well as contributions from sea quarks. For  $k_\perp^2 \leq 1 \text{ GeV}^2$ , the gluon distribution is taken as  $\mathcal{F}_{g^*/B}(x, k_\perp^2) = k_\perp^2 \mathcal{F}_{g^*/B}(x, 1)$ , which is motivated by the shape obtained from the solution of the LO BK equation in the saturation regime [58].

The parameters of the gluon were set by a fit to the  $F_2$  data from HERA [59], which returned the values:  $N = 0.994$ ,  $\beta = 18.6$ ,  $D = -82.1$  and  $R = 2.40 \text{ GeV}^{-1}$ . The first three parameters correspond to the initial condition given in Eq. (11), while the last parameter is responsible for the strength of nonlinear effects in the evolution equation. The overall quality of the fit was good, with  $\chi^2/\text{ndof} = 1.73$ .

We emphasize that the gluon constrained by the above fit can be used in our study without any modifications. This comes from the fact that it corresponds to the small- $x$  kinematic regime and it is universal amongst DIS and central-forward jet production processes [44], where saturation effects are moderate. The same is true for the Sudakov factors used in our study. The perturbative part is parameter-free while the non-perturbative terms are universal in the kinematic domain of our study [54]. Moreover due to the high transverse momenta of the final state jets, non-perturbative effects in the Sudakov are less important than in the case of hadron production.

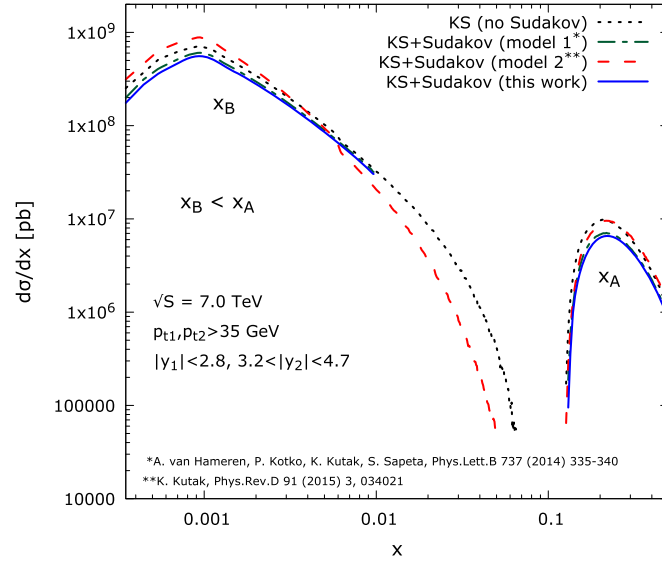
We introduce the Sudakov effects into the KS gluon distribution following the formalism described above. In addition, for reference, we use two methods employed in our earlier studies [40,41]. Those calculations used the Sudakov form factor, understood as the DGLAP evolution kernel, that has been applied on the top of the gluon TMD, together with constraints such as unitarity. Those methods should therefore be considered as models, in contrast to the proper resummation of Sudakov logarithms considered in this work. Nevertheless, the approaches used in Refs. [40,41] were phenomenologically successful (see also [42]), and one of the objectives of this study is to check how the predictions of those simplistic models compare with the proper way of including the Sudakov effects into the small- $x$  gluon. The reference models are:



**Fig. 1.** KS gluon distribution, Eq. (5) without and with the Sudakov form factors. The second row corresponds to the simple model-Sudakov given in Eq. (12), while the third and the fourth rows show results obtained with the Sudakov factors derived from QCD and given in Eqs. (7), (9), and (8), (10), respectively.

- Model 1: The survival probability model [40], where the Sudakov factor of the form [60]

$$T_s(\mu_F^2, k_\perp^2) = \exp \left( - \int_{k_\perp^2}^{\mu_F^2} \frac{dk_\perp'^2}{k_\perp'^2} \frac{\alpha_s(k_\perp'^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} dz' P_{a'a}(z') \right), \quad (12)$$



**Fig. 2.** Distributions of the longitudinal momentum fractions,  $x_A$ ,  $x_B$ , defined in Eq. (1) from calculations with various versions of the KS gluon distribution discussed in the article.

is imposed at the level of the cross section. This procedure corresponds to performing a DGLAP-type evolution from the scale  $\mu_0 \sim |k_\perp|$  to  $\mu$ , decoupled from the small- $x$  evolution.

- Model 2: The model with a hard scale introduced in Ref. [41]. The Sudakov form factor of the same form as in Eq. (12) is imposed on top of the KS gluon distribution in such a way that, after integration of the resulting hard scale dependent gluon TMD, one obtains the same result as by integrating the KS gluon distribution.

In Fig. 1 we show the KS gluon distributions, with and without Sudakov form factors, as functions of the transverse momentum  $k_\perp$  and the hard scale  $\mu$ . Three columns correspond to three different  $x$  values. The first row shows the original KS gluon distribution, which, as expected, does not depend on the value of  $\mu$ . In the second row, we show the KS hardscale gluon distribution of Ref. [41] (the other model [40] does not allow one to plot gluon distribution, as it applies Sudakov effects at the cross section level via a reweighting procedure). Here, the dependence on  $\mu$  is non-trivial and we see that the gluon develops a maximum in that variable. As shown in the figure, this maximum is rather broad. In the third and the fourth row of Fig. 1, we present our new KS gluon distribution with the Sudakov form factor described in this section. As explained earlier, this gluon exists in two versions, one for the  $qg$  and the other for the  $gg$  channel. The dependence on  $k_\perp$  and  $\mu$  is qualitatively similar between the new gluons and the naive KS hardscale gluon distribution. In the former case, however, the peak is significantly narrower in  $\mu$  as compare to the naive model of Ref. [41]. It is interesting to note that the  $qg$  gluon is broader than the  $gg$  gluon. This can be understood by comparing the color factors in the Sudakov functions (7) and (8). The color factor is bigger for the  $gg$  channel, hence, in that case, the Sudakov suppression is stronger along the  $\mu$  direction.

We have as well computed linear versions of the KS gluon distributions with the Sudakov, using the KS linear gluon distribution of Ref. [47]. We also used them to calculate differential distributions discussed in the following section. We observed that both sets of gluons (linear and nonlinear) give comparable results for the phenomenological observables. This is consistent with the expectation that saturation plays a limited role in central-forward dijet production in  $p-p$  collisions. Therefore, given that the nonlinear KS gluon distribution comes from a better fit to  $F_2$  than its linearized version [47], in the following, we present only the results obtained with the nonlinear gluon density.

The new gluons presented in this section are available publicly from the recent version of the KS package and can be downloaded from <http://nz42.ifj.edu.pl/~sapeta/KSgluon-2.0.tar.gz>.

### 3. Differential distributions

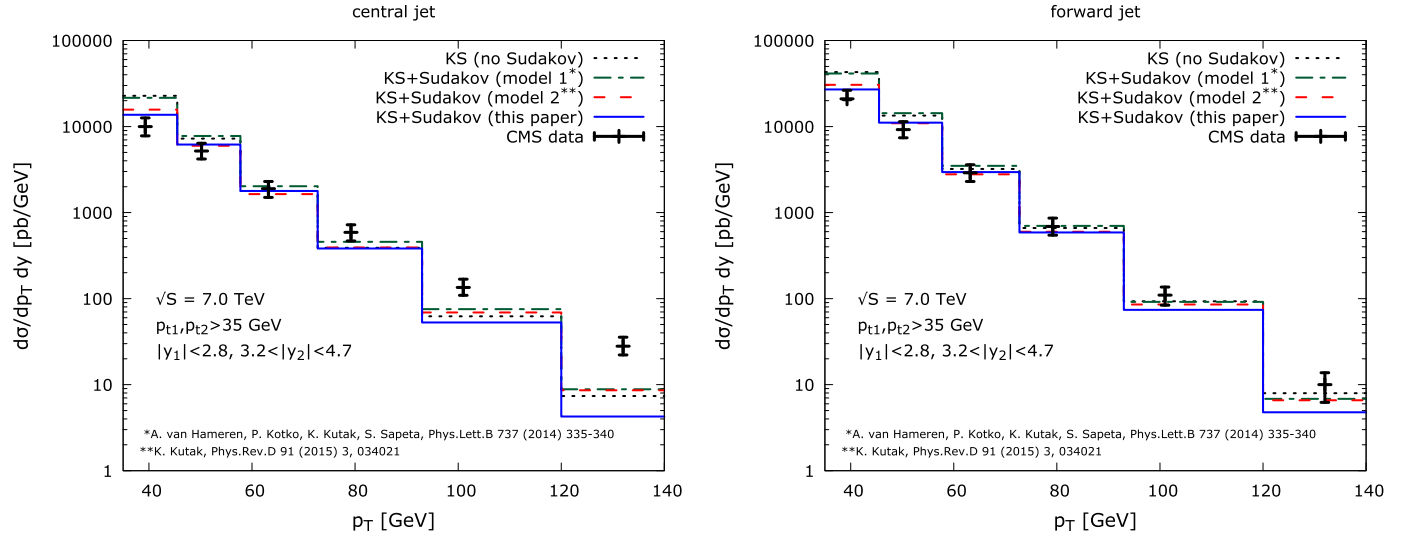
We now turn to the discussion of differential distributions in jets' transverse momenta calculated in the framework described in the preceding sections. We calculated the cross sections using the selection criteria of CMS [61]. The two leading jets were required to satisfy the cuts  $p_{1\perp}, p_{2\perp} > 35 \text{ GeV}$  and  $|y_1| < 2.8, 3.2 < |y_2| < 4.7$ . We used the CTEQ18 NLO PDF set [62] and LHAPDF [63] for the collinear PDFs and the KS gluon distributions with and without Sudakov for the gluon TMDs.

Our calculations have been performed and cross checked using two independent Monte Carlo programs [64,65] implementing the high energy factorization together with the off-shell matrix element calculated following the methods of Refs. [66–68]. We used the average transverse momentum of jets as both the renormalization and factorization hard scales.

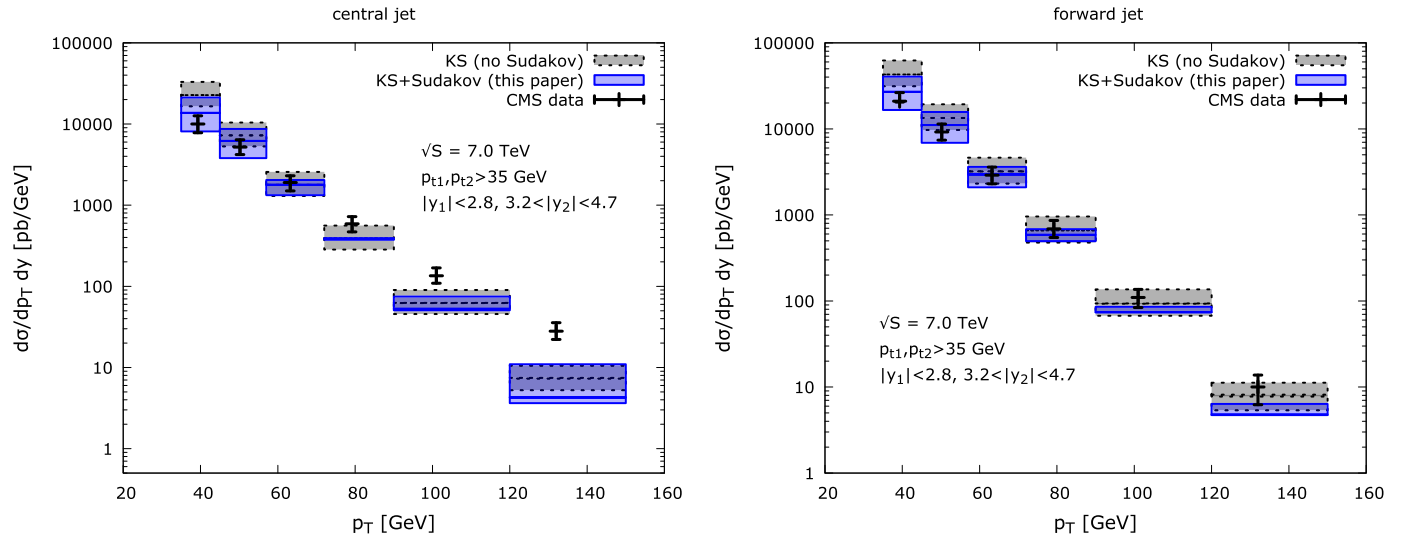
We start by showing in Fig. 2 distributions of the longitudinal momentum fractions probed by the central-forward dijet configurations. These results are consistent with the discussion of Section 1, in particular Eq. (1), and provide justification to the use of the hybrid factorization formula (2).

If Fig. 3 we show differential cross sections as function of the momenta of the forward and central jets. We compare central values of various predictions which differ by the gluon TMDs used in the HEF formula (2). The black dotted histograms correspond to the gluon without Sudakov, while the other three histograms use gluons with some form of Sudakov resummation. The main result of this paper is shown as a blue solid line, while the green and the red dashed curves correspond to the naive Sudakov modeling of Refs. [40,41].





**Fig. 3.** The transverse momentum spectra of the central (left) and the forward (right) jets obtained with the KS gluon distribution, with and without Sudakov effects, computed for the central value of the factorization and renormalization scale, compared to CMS data [61].



**Fig. 4.** As Fig. 3 but we only show predictions obtained with the original KS gluon distribution and the predictions with the KS gluon distribution with Sudakov from this work. The bands correspond to varying the renormalization and factorization scale by factors  $2^{\pm 1}$ .

We see that the predictions from this work, with the Sudakov effects included, tend to describe the data better than the predictions without the Sudakov, especially in the region of small  $p_{\perp}$ . However, the overall effect of the Sudakov form factor is not very strong for this particular observable.

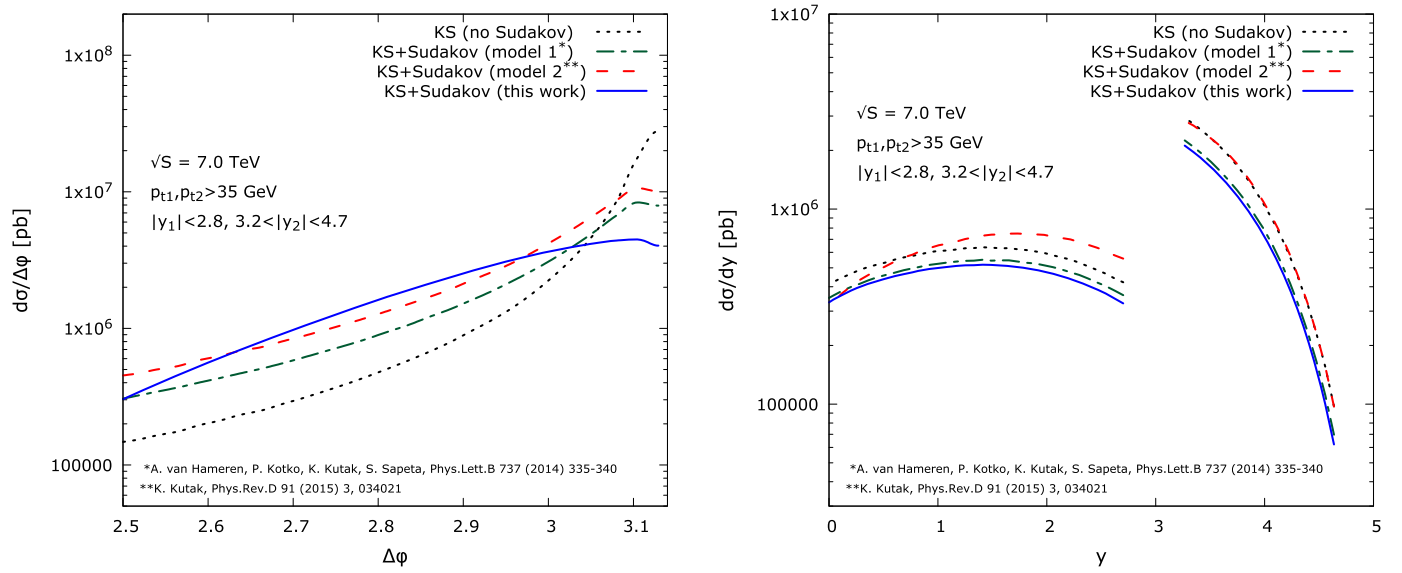
If Fig. 4, we show the same distributions of transverse momenta, but, here, we plot only two models (without Sudakov and with Sudakov from Section 2). This time, we show also the theoretical errors, estimated by the usual renormalization and factorization scale variation by the factors  $2^{\pm 1}$ .

We observe good agreement of our predictions with the CMS data [61], except the tail of the central-jet transverse momentum distribution. One has to remember however that, following Eq. (1), the tails of  $p_{\perp}$  distributions are sensitive to the region of large  $x$ , where, in principle, the gluon TMDs are not valid. Indeed, we have seen in our calculation that the KS gluon distribution with the Sudakov can sometimes get negative for larger  $x$  values. We interpret that as a sign of going outside of the validity region of the gluon distribution and, hence, in such situations, we set it to zero in the cross section calculation.

In Fig. 5 (left) we compare predictions for the distribution of the azimuthal angle between the two leading jets (aka azimuthal decorrelations). Again, we show results corresponding to calculations with and without the Sudakov. We observe that inclusion of Sudakov effects leads to qualitatively the same modification of  $\Delta\phi$  distributions. Namely, the region of large  $\Delta\phi$  is depopulated w.r.t. the result without Sudakov, while the opposite happens in the region of smaller  $\Delta\phi$ .

While qualitatively the predictions from KS gluon distribution + Sudakov from this work look similar to the earlier Sudakov models, quantitatively those cross sections differ to a certain degree, as seen in Fig. 5. In particular, the models 1 and 2, lead to convex functions for the azimuthal decorrelations, while the Sudakov of this study produces a concave curve.

We would also like to mention that the predictions using model 1 were shown to successfully reproduce the shapes of preliminary CMS data for the azimuthal decorrelations [40]. Since, as of today, these data are not published, we refrain from comparing them with the



**Fig. 5.** Differential cross sections as functions of the azimuthal distance between the jets  $\Delta\phi$  (left) and jet rapidities (right) obtained with the KS gluon distribution with and without Sudakov effects.

predictions of this work. We would only like to comment that, based on the comparison shown in Fig. 5, we expect the predictions from this study to be largely compatible with the earlier naive models, within theoretical errors.

Finally, in Fig. 5 (right) we show rapidity distributions resulting from the various versions of the KS gluon distribution, for the central and the forward jet. We see marked differences between predictions without and with Sudakov. Interestingly, inclusion of the Sudakov from this work suppresses both the central and the forward jet distribution, and this is largely consistent with the naive model 1. However, model 2 shows enhancement (central jet) or almost no effect (forward jet) in the rapidity differential cross sections.

The results presented in this section took advantage of the recent developments in the merging of the small- $x$  dynamics and the resummation of the Sudakov logarithms. Such calculations were not available at the time of our previous study [40], thus we had to resort to simple models of the Sudakov resummation. The calculations presented in this work are much more sound from the theory point of view. The new results show similar (or better) quality in the description of the transverse momentum spectra as compared to the methods of Refs. [40,41]. Likewise, in this work, we obtain predictions for azimuthal decorrelations, which are much more sensitive to the Sudakov resummation procedure. In particular, we see that the present approach gives a somewhat stronger suppression of the correlation peak.

Even though the predictions from this study are close to those from our earlier calculation, their theoretical status is much higher since, in this work, we used the proper Sudakov factor derived from first principles in QCD. And this was the main motivation behind the study presented in this paper.

We believe that our upgraded theoretical setup is useful in particular for the observables like the  $\Delta\phi$  distribution, where the Sudakov effects are strong, as shown in Fig. 5. The central-forward dijet production process provided us an excellent ground for validation of our framework. The latter can be used now to study small- $x$  dynamics, in particular the saturation effects, which are more pronounced in the production of the forward-forward dijet system, in particular in proton-lead collisions.

#### 4. Summary

We discussed Sudakov effects in central-forward dijet production at LHC energies within the framework of high energy factorization. Our study was triggered by recent progress on consistent merging of Sudakov resummation with the small- $x$  effects, which allowed us to compute hard-scale dependent gluon TMDs. As explained in Section 2, we were able to combine the phenomenologically successful KS gluon distribution [47] with the Sudakov factors directly in momentum space.

In our study, we used the Sudakov factors derived within perturbative QCD in Refs. [28–32]. For comparison, we also used simpler Sudakov models employed in our earlier studies [40,41].

We have calculated theoretical predictions for the differential cross sections as functions of  $p_\perp$  of the central and the forward jet, as well as azimuthal distance between the jets. The results are largely consistent with our earlier predictions based on simple phenomenological Sudakov models. We also achieved good description of CMS data for  $p_\perp$  distributions. Finally, we presented predictions for dijet azimuthal decorrelations.

It is worth emphasizing that our framework is relatively simple and all the parametrizations of non-perturbative physics were taken from external analyses, as explained in Section 2. Hence, no additional parameters were introduced in the calculation of the results presented in this work.

Overall, we conclude that the Sudakov resummation has a moderate effect on  $p_\perp$  spectra and a fairly sizable effect on the shapes of decorrelations. This is consistent with earlier phenomenological studies [40,42], which showed preference for gluons with Sudakov effects included.

Our future work will concern developing a full set of TMD gluon distributions exhibiting saturation effects and the Sudakov resummation, following the same perturbative calculations we used in the present paper. Such TMDs are necessary to confirm our previous calculations for forward-forward dijets [42] that show interplay of saturation effects and Sudakov effects consistent with the ATLAS data, where, however, the more naive Sudakov model was used.

Furthermore, in the future, we plan to address the dijet production in DIS, and a good understanding of the interplay of Sudakov effects and saturation is needed in order to provide robust predictions for the EIC [3] jet observables. We expect that by starting with central rapidities and going to more forward rapidities, one will be able to incrementally see the increasing importance of saturation effects and disentangle them from Sudakov effects.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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