

## PARTICLE SYSTEMATICS

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ABSTRACT

Several aspects of hadron spectroscopy are reviewed. For the baryons, the status of the even parity  $70$  multiplets is examined in some detail. For the mesons, a rapid survey of the state of the  $qq$  multiplets leads on to a discussion of the identification of the  $0^{++}$  mesons as  $(q^2\bar{q}^2)$  configurations. A brief account of Jaffe and Low's approach to this problem via the  $P$ -matrix is included. Throughout the review the interesting interplay between ideas from non-charmed and charmed hadron spectroscopy is underlined.

0. INTRODUCTION

In this talk I shall concentrate on the spectroscopy of the 'light' hadrons - those composed of  $u$ ,  $d$  and  $s$  quarks - but I will try to convince you that insights gained here may be of use for charmed hadron spectroscopy. Instead of attempting a comprehensive review, the discussion will be restricted to a few of the recent developments that I believe to be important and I will therefore make an attempt to set these in context. The selection of topics clearly reflects my personal opinions and omission does not necessarily constitute a value judgement.

A few words of warning. It is now fashionable to attach to everything the label, "predicted by QCD". In fact, despite favourable auguries, the confinement problem has not yet been solved and there are no rigorous results from QCD for hadron spectroscopy. Nevertheless, QCD has given some interesting clues and suggestions: I will attempt to separate these possible "QCD successes" from successes not specifically related to QCD. As always, good phenomenology must tread a delicate path between "random assumption models" on the one hand, and "rigorous theory" on the other.

1. THE BARYONS1.1 The Harmonic Oscillator Shell Model

To set the scene for our discussion of the low-lying baryon resonances it is helpful to review the expectations of the simplest quark model - the non-relativistic harmonic oscillator model. In this theory the dynamics of the three quarks in a baryon are described by a non-relativistic Hamiltonian with harmonic forces between each pair of quarks

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} + \frac{1}{2}k|\underline{r}_{12}|^2 + \frac{1}{2}k|\underline{r}_{23}|^2 + \frac{1}{2}k|\underline{r}_{31}|^2 \quad (1.1)$$

In the  $SU(3)$  (and  $SU(6)$ ) limit all quark masses are equal

$$m_1 = m_2 = m_3 = m \quad (1.2)$$

and the centre of mass motion may be separated off in the usual fashion. The relative

quark motion is then described by two independent harmonic oscillators whose coordinates are conventionally chosen to be

$$\underline{\rho} = \frac{1}{\sqrt{2}} (\underline{r}_1 - \underline{r}_2) \quad (1.3)$$

and

$$\underline{\lambda} = \frac{1}{\sqrt{6}} (\underline{r}_1 + \underline{r}_2 - 2\underline{r}_3) \quad (1.4)$$

Notice that the  $\rho$ -oscillator is antisymmetric, and the  $\lambda$ -oscillator symmetric, under exchange of quark labels 1 and 2. The spatial wavefunctions obtained by exciting these oscillators then have the corresponding permutation symmetry. Under the assumptions that quarks have spin  $\frac{1}{2}$  and are colour triplets, and that the low-lying hadrons are colour singlets and are composed of three active flavours of quarks (u, d, s), the allowed wavefunctions for baryons may be enumerated:-

$$| 3q \rangle = \underbrace{| \text{flavour} \rangle}_{\text{Antisymmetric}} \underbrace{| \text{spin} \rangle}_{\text{SU}(6) \times O(3)} \underbrace{| \text{space} \rangle}_{\text{Symmetric}} \underbrace{| \text{colour} \rangle}_{\frac{1}{c} \text{Antisymmetric}}$$

The spectrum of the lowest  $SU(6) \times O(3)$  multiplets in the harmonic oscillator shell model is shown in Figure 1. Clearly the equal spacing of the levels and the degeneracy structure are specific to the choice of harmonic interactions: introduction of a non-harmonic perturbation alters these splittings and lifts the degeneracies.

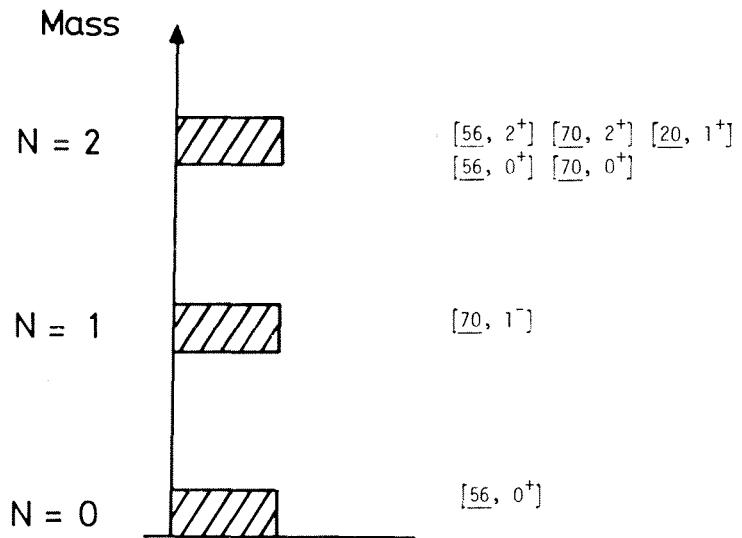


Figure 1 Spectrum of allowed  $SU(6) \times O(3)$  multiplets in the harmonic oscillator quark shell model. The mass is labelled by the principal quantum number  $N$  of the harmonic oscillator and multiplets are labelled by their  $SU(6)$  representation, and their orbital angular momentum and parity,  $L^P$ .

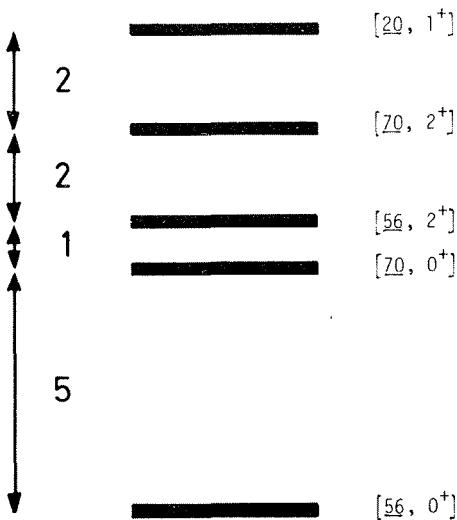


Figure 2 Pattern of splitting of  $N=2$  Band Multiplets caused by a non-harmonic perturbation.

In fact, there is an amusing general result<sup>1)</sup>: in first order perturbation theory, the pattern of splitting of the  $N=2$  multiplets is independent of the form of the perturbing potential  $U(r_{ij})$ . This pattern is shown in Figure 2; it has the attractive feature of lowering the radial excitation of the ground state - the  $[56, 0^+]$  multiplet - and raising the enigmatic  $[20, 1^+]$  multiplet, relative to the remaining 56 and 70's.

### 1.2 Algebraic SU(6) Models

Over the past decade or so there have been two main types of attempt to bring order to the enormous amount of experimental data available on baryon resonances. Both are algebraic rather than dynamical in character - in that they parametrize the data in terms of a small number of unknown SU(6) reduced matrix elements instead of using a specific quark model, embodying many assumptions about quark dynamics, to predict matrix elements. It is appropriate to briefly review the ingredients and achievements of both these approaches in order to set more recent developments in context:-

#### (1) SU(6) Mass Operator Analyses<sup>1-7)</sup>

The mass operator for baryon resonances is parametrized in terms of 2-body SU(6) tensor operators. To reduce the large number of possible operators, specific assumptions are made as to which operators have an important effect on the spectrum and which may be discarded. Detailed fits to the masses of all resonances within the low-lying negative and positive parity SU(6) multiplets have been performed, using harmonic oscillator wave-functions for the quarks. The unknown reduced matrix elements are determined by a careful choice of "well-known" input states and the model then predicts the masses and SU(6) composition of all the remaining states. In particular, specific predictions are made for the masses of (as yet) unobserved resonances.

(2) SU(6)<sub>W</sub> Decay Operator Analyses<sup>8-13)</sup>

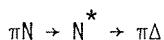
These models concentrate on decay systematics and assume that resonance decay takes place via meson emission from a single active quark. The single quark transition operator is parametrized by the most general allowed SU(6)<sub>W</sub> structure. No detailed forms for the spatial wavefunctions are used but an SU(6)  $\times$  O(3) classification of the resonant states is assumed. Again, the small number of reduced matrix elements are determined by a fit to data thus allowing the decay properties of hitherto unobserved states to be predicted.

It is clear that these two approaches are complementary to each other: to predict where best to look for a missing state one needs both the mass and the decay properties of the state. Most of the early mass fits took no account of the decay systematics of resonances while the decay fits made no attempt to explain the observed SU(6) mass splittings. It would clearly seem desirable to obtain a consistent picture of both aspects of an SU(6) multiplet - although it must be emphasized that the use of SU(6) symmetry for masses and of SU(6)<sub>W</sub> symmetry for decays involves different theoretical assumptions. What then is the status of such attempts?

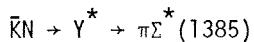
1.3 Status - Pre-1978Negative Parity States: [70, 1<sup>-</sup>]

(i) SU(6) mass fits<sup>7, 14)</sup> were able to obtain a good description of the observed negative parity resonances below about 2 GeV.

(ii) SU(6)<sub>W</sub> decay analyses<sup>11-13)</sup> enjoyed great success in correlating elastic and inelastic resonant amplitudes. In particular, the agreement with the inelastic amplitude signs for the reactions



and



determined by isobar model analyses was spectacular. A recent isobar analysis of the  $\pi^+ p$  channel<sup>15)</sup> confirms this good agreement and also confirms the  $\rho N$  amplitude signs predicted by such an SU(6) model<sup>12)</sup>.

(iii) Simultaneous SU(6) mass and decay fits were found to fail.<sup>4, 7, 14)</sup> The basic reason for the incompatibility of these mass and decay analyses may be illustrated by the following example. Physical states are found to be mixtures of pure SU(6) states. In particular, the  $\frac{1}{2}^+ N^*$  resonances are mixtures of quark spin  $\frac{1}{2}$  and  $\frac{3}{2}$  SU(6) 70 states. e.g.

$$| S11(1530) \rangle = - \sin \theta_S | ^48 \rangle + \cos \theta_S | ^28 \rangle \quad (1.5)$$

In order to accomodate the observed AK decay of the higher mass S11(1670), decay analyses immediately require substantial mixing

$$\theta_S \sim -30^\circ$$

(in the conventions of reference 11). On the other hand, the specific assumptions of the mass operator fits<sup>14)</sup> tied the amount of mixing to the mass difference of the S11(1530) and the D13(1520). Thus the S11 states were predicted to be essentially unmixed - in contradiction with the decay analysis result. The situation for the more complicated 3-way mixing of the  $\gamma^*$ 's is similar, but less clear cut.

#### Positive Parity States

The mass fits predicted<sup>7)</sup> that the S=0 and S=-1 states of the four N=2 band multiplets -  $[56, 0^+]_2$ ,  $[56, 2^+]$ ,  $[70, 0^+]$  and  $[70, 2^+]$  - should all lie below about 2.1 GeV in mass. The  $[20, 1^+]$  states are higher than this. Physical states are now mixtures of different SU(6) multiplets.

In contrast to this SU(6) picture of the positive parity states, decay analyses<sup>11-13)</sup> obtained good agreement with data by only invoking  $[56, 0^+]$  and  $[56, 2^+]$  multiplets. In fact, the mass analyses themselves found the particular states used in the decay analyses were predominantly 56 in character with only a small 70 contamination. Nevertheless, the existence of even parity 70 multiplets at low mass, or indeed at all, was brought into question.

To bring out the issues more clearly, Table 1 lists all two, three and four star, positive-parity resonances below 2.1 GeV, according to the 1978 edition of the Particle Data Group compilation<sup>16)</sup>.

Table 1

Positive-parity states below 2.1 GeV according to the Particle Data Group 1978 compilation. (One star resonances have been omitted.) States with controversial SU(6) assignments have been ringed.

N	$\Delta$	$\Sigma$	$\Lambda$
$\frac{1}{2}^+$ (1470)	$\frac{3}{2}^+$ (1690)	$\frac{1}{2}^+$ (1660)	$\frac{1}{2}^+$ (1600)
$\frac{5}{2}^+$ (1688)	$\frac{5}{2}^+$ (1890)	$\frac{1}{2}^+$ (1880)	$\frac{1}{2}^+$ (1800)
$\frac{1}{2}^+$ (1780)	$\frac{1}{2}^+$ (1910)	$\frac{5}{2}^+$ (1915)	$\frac{5}{2}^+$ (1815)
$\frac{3}{2}^+$ (1810)	$\frac{7}{2}^+$ (1950)	$\frac{7}{2}^+$ (2030)	$\frac{3}{2}^+$ (1860)
$\frac{7}{2}^+$ (1990)	-	$\frac{3}{2}^+$ (2080)	$\frac{5}{2}^+$ (2110)
$\frac{5}{2}^+$ (2000)			
Total Number of States	6	4	5

Four of these states - NP11(1780), NF17(1990), AP01(1800) and AF05(2110) - were assigned by Jones, Dalitz and Horgan<sup>7)</sup> to the  $[70, 0^+]$  and  $[70, 2^+]$  multiplets. Litchfield, Cashmore and Hey<sup>11, 12)</sup> on the other hand suggested that these four states may be more economically assigned to higher-lying 56 multiplets. The motivation for this suggestion was based on a count of missing states. Tables 2 and 3 list the S=0 and S=-1 states required for the 56 and 70 multiplets: the existence of even parity 70's requires twice as many  $\gamma^*$ 's than for 56 multiplets. The number of states required by the mass fits and the number of observed states is as follows:-

	$N^* \text{ and } \Delta^*$	$\Sigma^* \text{ and } \Lambda^*$
Predicted (Tables 2 and 3)	19	33
Observed (Table 1)	10	10

In view of the large discrepancy it seemed fair to question the evidence for positive parity 70's. Why have the missing states not been observed in partial wave analyses? - or at the very least, the missing states with high spin?

Table 2

Predicted S=0 and S=-1 states of  
the  $[56, 0^+]$  and  $[56, 2^+]$  multiplets.

	N	$\Delta$	$\Sigma$	$\Lambda$
$[56, 0^+]$ $^2_8$	$\frac{1}{2}^+$		$\frac{1}{2}^+$	$\frac{1}{2}^+$
		$\frac{3}{2}^+$	$\frac{3}{2}^+$	
$[56, 2^+]$ $^2_8$	$\frac{5}{2}^+$		$\frac{5}{2}^+$	$\frac{5}{2}^+$
	$\frac{3}{2}^+$		$\frac{3}{2}^+$	$\frac{3}{2}^+$
$^4_{10}$		$\frac{7}{2}^+$	$\frac{7}{2}^+$	
		$\frac{5}{2}^+$	$\frac{5}{2}^+$	
		$\frac{3}{2}^+$	$\frac{3}{2}^+$	
		$\frac{1}{2}^+$	$\frac{1}{2}^+$	
Total no. of states	3	5	8	3

Table 3

Predicted  $S=0$  and  $S=-1$  states of the  $[70, 0^+]$  and  $[70, 2^+]$  multiplets.

	N	$\Delta$	$\Sigma$	$\Lambda$
$[70, 0^+]$				
$2_8$	$\frac{1}{2}^+$		$\frac{1}{2}^+$	$\frac{1}{2}^+$
$4_8$	$\frac{3}{2}^+$		$\frac{3}{2}^+$	$\frac{3}{2}^+$
$2_{10}$		$\frac{1}{2}^+$	$\frac{1}{2}^+$	
$2_1$				$\frac{1}{2}^+$
$[70, 2^+]$				
$2_8$	$\frac{5}{2}^+$		$\frac{5}{2}^+$	$\frac{5}{2}^+$
	$\frac{3}{2}^+$		$\frac{3}{2}^+$	$\frac{3}{2}^+$
	$\frac{7}{2}^+$		$\frac{7}{2}^+$	$\frac{7}{2}^+$
$4_8$	$\frac{5}{2}^+$		$\frac{5}{2}^+$	$\frac{5}{2}^+$
	$\frac{3}{2}^+$		$\frac{3}{2}^+$	$\frac{3}{2}^+$
	$\frac{1}{2}^+$		$\frac{1}{2}^+$	$\frac{1}{2}^+$
$2_{10}$		$\frac{5}{2}^+$	$\frac{5}{2}^+$	
		$\frac{3}{2}^+$	$\frac{3}{2}^+$	
$2_1$				$\frac{5}{2}^+$
				$\frac{3}{2}^+$
Total no. of states	8	3	11	11

#### Summary of Pre-1978 Status

It is clear that the overall situation was far from satisfactory. Mass operator analyses - at least with their present choice of operators - failed to fit masses and decays simultaneously. They were thus unable to answer the question of why the missing  $70$  states had not been observed. Decay analyses were very successful but were more limited in ambition. In particular, no attempt was made to obtain an understanding of either the observed mass spectrum or the mixing matrices. Moreover, both approaches lack much intuitive appeal. We now turn to more recent developments which go some way toward clarifying these questions and answering these objections.

#### 1.4 Post-1978 Developments

Although, as stressed in the introduction, there are few, if any, areas of hadron spectroscopy which can be said to provide evidence for specific QCD effects, it is certainly true that the paper of de Rujula, Georgi and Glashow<sup>17)</sup> sparked off a revival of interest in non-relativistic potential models for the baryon spectrum. The new feature of this paper was the inclusion of a short-range potential arising from coloured gluon exchange between quarks, in addition to a long-range confining potential. Isgur and Karl<sup>18-21)</sup> and other authors<sup>22-30)</sup> have examined in detail the consequences of this gluon exchange potential for the excited baryon multiplets.

I will discuss in some detail the specific non-relativistic oscillator model of Isgur and Karl since, in addition to a treatment of the one-gluon corrections, it has another very interesting feature - although one which has nothing specific to QCD. This is the phenomenon of 'kinematic mixing' - SU(6) configuration mixing arising simply from the non-equality of non-strange and strange quark masses.

(a) Coloured Gluon Exchange

The non-relativistic reduction of the one-gluon exchange contribution to the Hamiltonian leads to the so-called Breit interaction. This contains a magnetic dipole-dipole interaction of the form (for two quarks  $i$  and  $j$ )

$$H_{\text{Hyperfine}}^{ij} = A \left\{ \frac{8\pi}{3} \underline{S}_i \cdot \underline{S}_j \delta^3(\underline{\rho}) + \frac{1}{\rho^3} (3 \underline{S}_i \cdot \hat{\underline{\rho}} \underline{S}_j \cdot \hat{\underline{\rho}} - \underline{S}_i \cdot \underline{S}_j) \right\} \quad (1.6)$$

where  $\underline{\rho} = \frac{1}{2}(\underline{r}_i - \underline{r}_j)$ . The first term is the Fermi contact term, which only operates when the pair  $(ij)$  have zero orbital angular momentum, and the second term is a spin tensor force which is operative only when the pair have non-zero orbital angular momentum.

For the ground state, only the contact term contributes and this term is responsible for the  $\Delta$ -N and  $\rho$ - $\pi$  splitting. The non-Abelian nature of coloured gluon exchange is reflected in the fact that the  $\Delta$ -N and  $\rho$ - $\pi$  splitting have the same sign, in agreement with experiment. This term is also responsible for the  $\Sigma$ - $\Lambda$  splitting of the ground state baryons.<sup>17)</sup>

For the  $[70, 1^-]$  both terms should be present and Isgur and Karl have made a quantitative study using harmonic oscillator quark wavefunctions. For the non-strange baryons, the contact term splits the  $^28$  states from the  $^48$  and  $^210$  states in good qualitative agreement with the observed states. The presence of the tensor force does not alter this qualitative success but does induce mixing between the pure  $^28$  and  $^48$  states since

$$\langle S = \frac{3}{2} | H_{\text{tensor}} | S = \frac{1}{2} \rangle \neq 0 \quad (1.7)$$

The detailed calculations of Isgur and Karl<sup>18, 20)</sup> predict little mixing for the D13 states but find substantial mixing for the S11 resonances; namely

$$\theta_S \sim -30^\circ$$

in good agreement with decay analyses<sup>11, 13)</sup>. Before euphoria sets in some comments are in order:

(1) The reason for the discrepancy between mass and decay analyses now seems to be identified: Tensor forces were explicitly rejected in the early mass operator analyses.

(2) The mass operator analyses explicitly retained spin-orbit forces: the model of Isgur and Karl ignores them entirely, despite the fact that the Breit interaction has a specific spin-orbit contribution. It certainly seems clear that the data does not require

strong spin-orbit forces and several authors<sup>20, 29, 31-33)</sup> have investigated the possibility that there is a substantial cancellation between the L.S force from vector gluon exchange and from a (presumed) scalar confining potential.

(3) From a theoretical point of view, the relevance of the non-relativistic treatment of single gluon exchange is not obvious. Indeed similar success for the ground state hadrons was obtained, at about the same time as de Rujula et al., in the framework of the MIT Bag<sup>34)</sup>, in which the quarks are highly relativistic. Even in the context of a 'non-relativistic' potential model, a recent re-analysis<sup>30)</sup> of the Hamiltonian of de Rujula et al. shows that the quarks are in fact moving relativistically and 'relativistic corrections' must be treated with caution.

(b) Kinematic Mixing

The basic mechanism for this type of configuration mixing was first noticed in the context of the MIT Bag.<sup>35)</sup> Isgur and Karl, however, chose instead the much more tractable non-relativistic harmonic oscillator Hamiltonian. The basis of the effect is merely the observation that SU(3) breaking via quark masses

$$m_s > m_u$$

requires that the frequency of the  $\lambda$ -oscillator is lower than that of the  $\rho$ -oscillator

$$\omega_\lambda < \omega_\rho$$

The  $\rho$ - and  $\lambda$ -modes have been defined by equations 1.3 and 1.4: the two types of oscillations may be visualized by the one-dimensional analogy shown in Figure 3.

A simple illustration of this mechanism of work is given by the  $\Sigma$ - $\Lambda$  splitting of the  $\frac{5}{2}^-$  states of the  $[70, 1^-]$ .<sup>19)</sup> For the ground state  $\Sigma$  and  $\Lambda$  we have

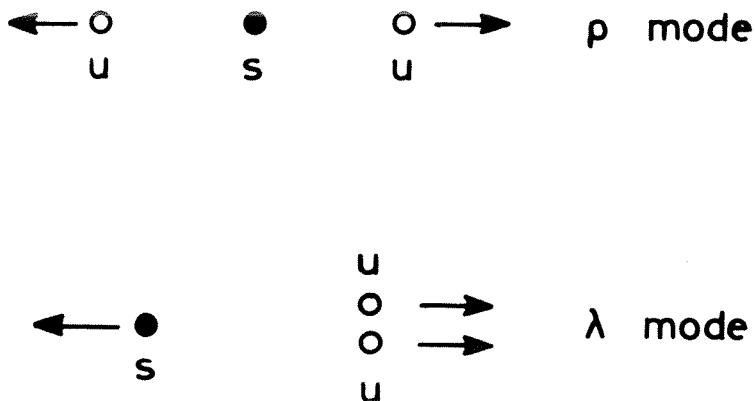


Figure 3 One-dimensional representation of  $\rho$  and  $\lambda$  oscillator modes

$$\Sigma \frac{1^+}{2} (1190) > \Lambda \frac{1^+}{2} (1115)$$

whereas we find

$$\Sigma \frac{5^-}{2} (1760) < \Lambda \frac{5^-}{2} (1830)$$

in the  $[70, 1^-]$ . This reversal may be understood by considering the symmetries of the non-strange quarks in these states. Both  $\Sigma$  and  $\Lambda$  have the non-strange quarks coupled to spin 1 and a colour  $\bar{3}$ : the product of the isospin and spatial wavefunctions must therefore be symmetric. Thus

$$\Sigma : (I = 1) \rightarrow (12)_S \lambda\text{-mode}$$

and

$$\Lambda : (I = 0) \rightarrow (12)_A \rho\text{-mode}$$

so that the  $\Sigma$  is predicted to be the lighter state. Inclusion of the gluon exchange contribution reduces the splitting somewhat, but does not alter the conclusion.<sup>19)</sup>

The most important consequence of this kinematic effect concerns the mixing of the  $Y^*$ 's. The pure SU(6) states are not diagonal in the  $\rho, \lambda$  - basis: on diagonalisation the highest mass  $Y^*$ 's correspond to pure  $\rho$ -states. Under the standard assumption that resonance decays are described by single-quark operators a fascinating selection rule emerges. Kaon emission from a  $Y^*$  must necessarily involve the strange quark: since for the  $\rho$ -mode it is the non-strange quarks that are orbitally excited we obtain the result:

$$(Y^*)_\rho \not\rightarrow \bar{K}N \quad (1.8)$$

This is illustrated schematically in Figure 4. It is interesting that this phenomenon - that the higher mass  $Y^*$ 's of the  $[70, 1^-]$  tend to couple only weakly to  $\bar{K}N$  - had been noticed empirically by Rosner and Petersen<sup>36)</sup>, and by Faiman<sup>37)</sup>, (who termed this "Ideal mixing for Baryons").

For the  $[70, 1^-]$ , the mixing generated by the simple SU(3) mass splitting provides qualitative agreement with the observed mixing of SU(6) Decay analyses.<sup>13)</sup> What of the positive-parity baryons? The oscillator states of the N=2 level may be characterized as  $\rho\rho$ ,  $\rho\lambda$  and  $\lambda\lambda$ : Of these, only the  $\lambda\lambda$  modes can couple to  $\bar{K}N$  via a single quark operator. Thus there is the immediate prediction that many  $Y^*$ 's will essentially decouple from the  $\bar{K}N$  formation channel. Isgur and Karl have investigated this in detail including a non-harmonic perturbing potential and gluon corrections.<sup>21)</sup> The qualitative expectation is borne out by their results: their most spectacular example concerns the  $\frac{3^+}{2} \Lambda^*$  states. At the N=2 level, including the  $70$ 's and the  $20$ , seven  $\frac{3^+}{2} \Lambda^*$  resonances are expected: the kinematic mixing decouples all but one of these states from  $\bar{K}N$ . According to the Particle Data Group, only one such resonance has been observed!

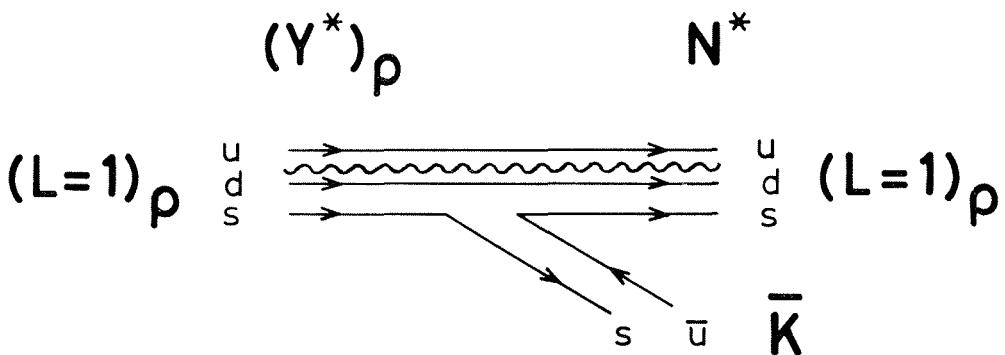


Figure 4 Quark line diagram illustrating the selection rule  $(Y^*)_p \not\rightarrow \bar{K}N$ .

Again, some additional comments are in order:

(1) The most spectacular decouplings occur for the  $\Lambda^*$  resonances:  $\Sigma^*$ 's do not in general decouple from  $\bar{K}N$ . There are still some missing  $\Sigma^*$  states which should couple to  $\bar{K}N$  but in the specific model of Isgur and Karl these are either above 2 GeV, or involve low-spin states. With the present state of  $Y^*$  phase-shift analyses, it is plausible that there may be no conflict with the existence of even parity 70's.

(2) It would therefore seem that the most severe constraints on this model arise from the well-explored non-strange sector, where no decoupling of  $N^*$ 's and  $\Delta^*$ 's is predicted. Even here after two new phase-shift analyses<sup>38, 39</sup> this year, the question is not resolved. An example will illustrate this point. Isgur and Karl predict two  $\Delta_{\frac{3}{2}}^{3+}$  states around 1950-2000 MeV. The analysis of Cutkosky et al. finds one state, the  $\Delta_{\frac{3}{2}}^{3+}(1910)$  but include the comment that "alternate fits containing additional resonances are possible".

### 1.5 Conclusions

(1) A plausible case can be made for the existence of even-parity 70's. The integrity of phase shift analyses in not finding hitherto "theoretically desirable states" is impressive.

(2) Given the extent of configuration mixing in the  $S=-1$  sector of the  $N=2$  Band multiplets, it is no longer clear to me than an  $SU(6)$  classification is useful. Perhaps it is preferable to work directly with a quark basis that distinguishes strange and non-strange quarks.

### 1.6 Implications for Charmed Baryons

After the beautiful semiquantitative success of the Isgur-Karl model for non-charmed baryons, it is of interest to investigate the predictions of an extension of this model to the charmed quark sector. For the  $[70, 1^-]$  the lowest-lying  $\Lambda^*$  is the  $\Lambda_{\frac{1}{2}}^1(1405)$ ,

corresponding to a  $\lambda$ -mode excitation. Using the  $\Lambda_c(\frac{1}{2}^+)$  state to determine the charmed quark mass, Copley et al.<sup>40)</sup> predict that the corresponding  $\Lambda_c(\frac{1}{2}^-)$  state is probably stable against strong decay! They suggest a search for the electromagnetic decay

$$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c(\frac{1}{2}^+) + \gamma$$

which is certainly of experimental interest.

## 2. THE MESONS

### 2.1 $(q\bar{q})$ Spectroscopy

Figure 5 shows the spectrum of  $q\bar{q}$  nonets predicted by a harmonic oscillator quark model. Again, deviations from this pattern are caused by the inclusion of non-harmonic forces, spin-spin and spin-orbit forces and so on. A recent investigation,<sup>41)</sup> for example, claims evidence for spin-tensor forces in the spectrum of observed mesons.

There is no time to review in detail all the experimental contributions on meson resonances that were presented in the parallel sessions of this conference. Instead, I shall give a very cursory survey of the current status of the  $N=1$  and  $N=2$  ( $q\bar{q}$ ) multiplets.

#### (1) $2^{++}, 3^{--}$

These leading trajectory nonets remain in good shape and little essential has changed since the Tokyo conference.<sup>42)</sup>

#### (2) $1^{++}, 1^{+-}$

For these multiplets there has been some clarification. I was particularly impressed by the enormous statistics of the data on  $3\pi$  and  $K\pi\pi$  channels presented at this conference

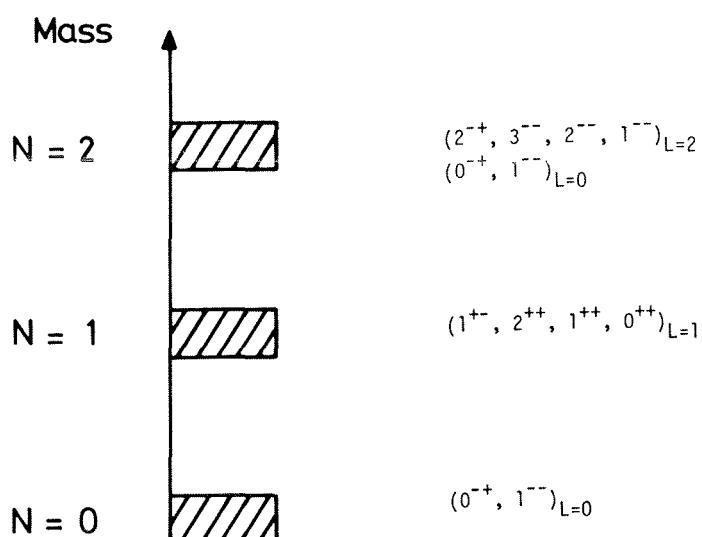


Figure 5 ( $q\bar{q}$ ) multiplets of the Harmonic Oscillator Model. Nonets are labelled according to the principal quantum number  $N$  and the orbital angular momentum  $L$ , as well as their spin, parity and charge conjugation  $JPC$ .

by members of the ACCMOR collaboration.<sup>43, 44)</sup> Their analysis confirms the existence of two resonant states in the Q region of  $K\pi\pi$ , and a convincing resonant amplitude is seen in the  $1^{++}$  ( $I=1$ )  $\pi\rho$  channel: They quote an  $A_1$  mass of around 1300 MeV. It is now a little embarrassing that after so many years living without an  $A_1$ , there are now claims for an  $A_1$  resonance at 1100<sup>45, 46)</sup>, 1300 in this experiment, and even<sup>47)</sup> at 1500 MeV! It must be said, however, that the statistics of the present (diffractive production) experiment are an order of magnitude better than previous ones. We must wait until the dust settles.

(3)  $2^{-+}, 2^{--}$

Only the  $A_3$   $I=1$   $2^{-+}$  state has been confirmed<sup>48)</sup>, but there should soon be a detailed isobar analysis of the L region of  $K\pi\pi$ , which should show two  $I=\frac{1}{2}$  states as in the Q region. I probably ought not to mention the hint of some 'extra' activity in the  $2^{-+}$   $I=1$  channel that may not be explicable by a single resonance .....

(4)  $0^{-+}, 1^{--}$  : Radial excitations

The isobar analyses<sup>43, 44)</sup> of  $K\pi\pi$  and  $3\pi$  show new indications of possible  $K'$  ( $\sim 1400$ ) and  $\pi'$  ( $\sim 1300$ ) states. In view of these masses, and in the absence of any confirmation of the  $\rho'$  (1250) from present  $e^+e^-$  experiments<sup>49)</sup> the  $\rho'$  (1600) looks a safer bet for theorists to model as the radial excitation of the  $\rho$ . Definite statements about possible  $\omega'$  and  $\phi'$  states are still awaited.

(5)  $N=4, L=3$  Band?

An  $I=1$   $5^{--}$  state has been deduced from a moments analysis of a  $K^+K^-n$  (by missing mass) experiment.<sup>50)</sup> The favoured mass and width are 2.30 GeV and 270 MeV, respectively. Figure 6 shows this state on an almost forgotten Chew-Frautschi plot.

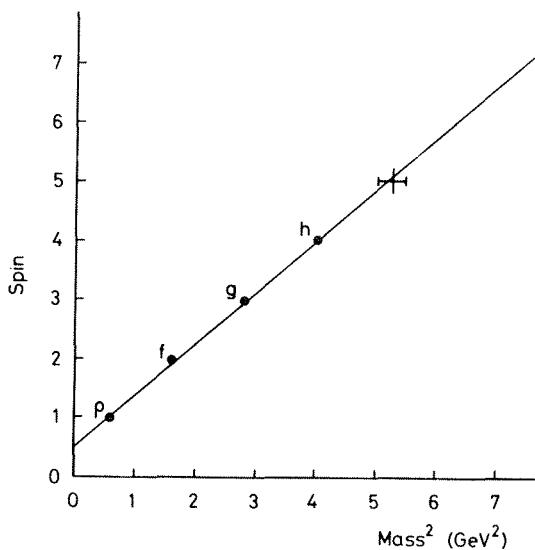


Figure 6 Chew-Frautschi plot including the new  $5^{--}$  state.

## 2.2 $(q^2\bar{q}^2)$ Spectroscopy and the $0^{++}$ Mesons

In 1976 Jaffe challenged the orthodox view of the low mass  $0^{++}$  mesons as  $(q\bar{q})_{L=1}$  states and proposed their assignment as "crypto-exotic"  $q^2\bar{q}^2$  states.<sup>51)</sup> In general, multi-quark  $q^2\bar{q}^2$  states are expected to be rather broad because of the possibility of super-allowed fall-apart decays (Figure 7). However, for the  $0^{++}$  states coloured gluon exchange is effective in lowering the zeroth order  $q^2\bar{q}^2$  mass significantly. In Jaffe's original calculations in the MIT Bag the following identification was possible

$$\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d})s\bar{s} \sim S^*(993)$$

$$u\bar{d}s\bar{s}, \text{ etc} \sim \delta(976)$$

$$u\bar{u}d\bar{d} \sim \epsilon(700)$$

The  $\epsilon$ -state was predicted to be very broad owing to its fall-apart decay to  $\pi\pi$ , but the  $S^*$  and  $\delta$  states must couple to  $K\bar{K}$  and are narrow because of the closeness of the  $K\bar{K}$  threshold. Despite the encouraging qualitative success of these assignments there was a problem with the prediction of the accompanying  $\kappa$  state at around 900 MeV: the  $K\pi$  phase shift is certainly not resonant below 1 GeV. Moreover, an exotic  $I=2$  S-wave  $\pi\pi$  state is predicted around 1100 MeV: the relevant phase shift is repulsive up to at least 1500 MeV. What are we to make of all these predictions?

## 2.3 The P-Matrix

In a recent paper, Jaffe and Low<sup>52)</sup> have made some interesting observations concerning "mass" predictions for  $q^2\bar{q}^2$  states. They argue that "masses" calculated for  $q^2\bar{q}^2$  in the MIT Bag do not in general correspond to physical resonances. The motivation for their

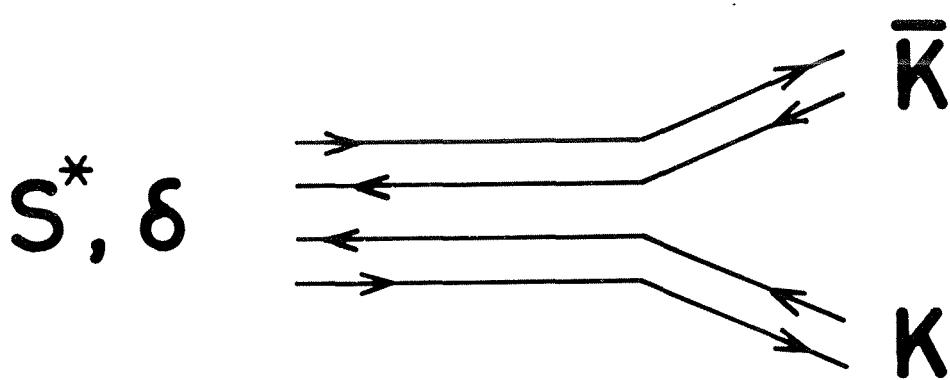


Figure 7 Quark line diagram for superallowed "fall-apart" decay of a  $q^2\bar{q}^2$  state.

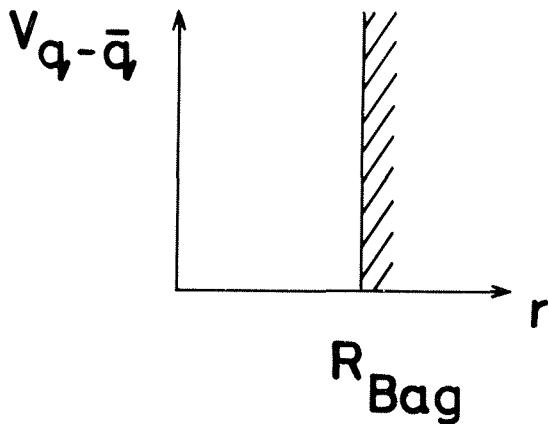


Figure 8 Bag approximation to the  $q\bar{q}$  confinement potential

analysis is sketched below.

For a  $(q\bar{q})$  system coupled to an overall colour singlet, the Bag may be considered as approximating the colour confining potential by an infinite square well (Figure 8). A colour singlet  $(q^2\bar{q}^2)$  system, on the other hand, has projections on to  $(q\bar{q})(q\bar{q})$  systems both with colour octets coupled to an overall singlet, and with colour singlets.

$$\begin{array}{c}
 \xrightarrow{\hspace{2cm}} [(q\bar{q})_{8c} - (q\bar{q})_{8c}]_{1c} \\
 [(q^2\bar{q}^2)_{1c}] \swarrow \searrow \\
 \xrightarrow{\hspace{2cm}} [(q\bar{q})_{1c} - (q\bar{q})_{1c}]_{1c}
 \end{array}$$

For the colour octet component a colour confining potential such as Figure 8 is appropriate: for the colour singlet projection, however, some weak non-confining potential is expected (e.g. Figure 9). In Bag calculations this component has been artificially confined.

A tacit premise of all quark model calculations of resonance masses is a narrow resonance approximation: this is clearly invalid for  $q^2\bar{q}^2$  states with their large fall-apart decay modes. In the absence of realistic calculations of fissioning Bags and so on, Jaffe and Low have proposed the "P-matrix" in an attempt to obtain a relation between observed phase-shifts and Bag model  $q^2\bar{q}^2$  "primitives". The term "primitive" is introduced to underline the fact that these are not necessarily physical states.

An idea of their approach may be gained by considering an analogy<sup>52)</sup> of S-wave scattering from a weak square well potential of radius  $b$  (Figure 10). For such a

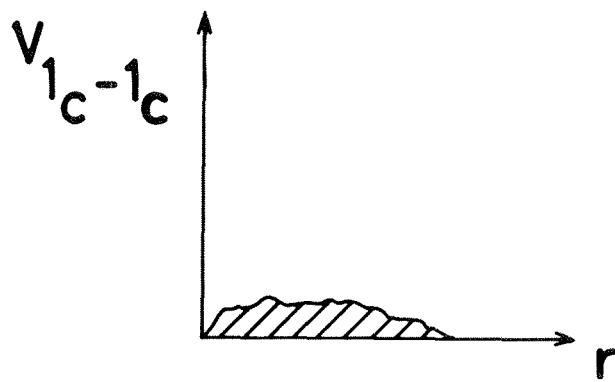


Figure 9 An artist's impression of a weak non-confining  $(q\bar{q})_{1c} - (q\bar{q})_{1c}$  potential.

potential there are no bound states or resonances and the phase shift is found by solving the Schrödinger equation for  $r < b$  and for  $r > b$  and equating  $\psi'(r)/\psi(r)$  at  $r=b$ . One obtains the condition

$$q \cot q b = k \cot(kb + \delta(k)) \quad (2.1)$$

where the three momenta are related by

$$q^2 = k^2 - 2mV$$

Now consider the following question. What is the connection, if any, between the phase shift  $\delta(k)$  of this problem, and the eigenvalues of an infinite square well of radius  $b$ ? (Figure 11). The eigenvalues of the infinite square well problem are obtained by imposing the boundary condition

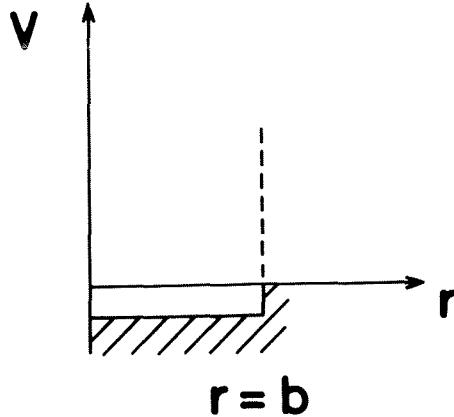


Figure 10 Weak square-well potential.

$$\psi(b) = 0 \quad (2.2)$$

corresponding to a pole in the logarithmic derivative

$$\left. \frac{\psi'(r)}{\psi(r)} \right|_{r=b} = q \cot qb \quad (2.3)$$

These occur for values of  $q$  such that

$$q_n b = n\pi$$

These are the infinite set of "primitives" for this problem. Is there a connection between these primitives  $q_n$  and the phase-shift  $\delta(k)$  of our original problem? The answer is yes: primitives correspond to values of  $q$  such that  $q \cot qb$  has a pole. By the definition of the phase-shift (equation 2.1) this occurs at values of  $k$  where the quantity

$$P(k) = k \cot(kb + \delta(k))$$

has a pole. Thus the primitives of the infinite square well correspond to values of  $k$  satisfying

$$\delta(k) = n\pi - kb$$

where  $\delta(k)$  is the phase-shift of our weak potential problem.

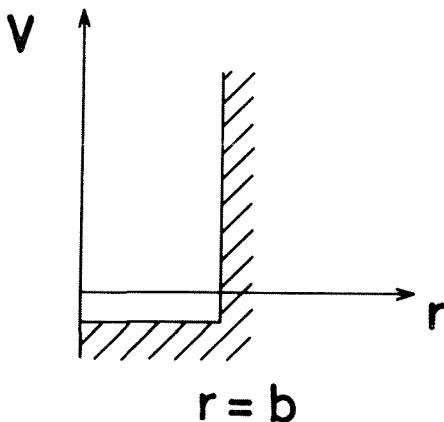


Figure 11 'Artificial' Infinite Square Well Potential.

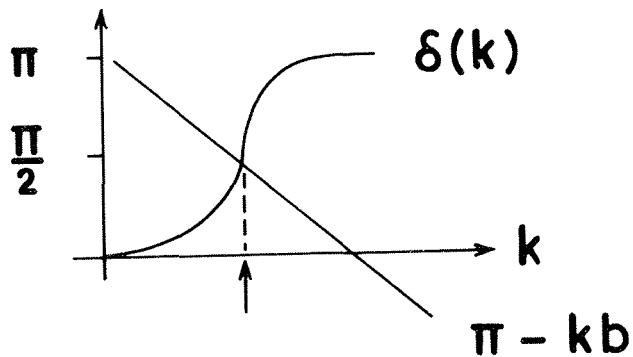


Figure 12 Position of P-matrix pole for narrow resonance.

Consider two examples:-

(1) Strong Forces, Narrow Resonance

This is analogous to the situation we expect for genuinely confined channels. The position of the P-matrix pole is most easily exhibited by a plot of  $\delta(k)$  and  $\pi - kb$  versus  $k$  (Figure 12). The phase-shift rises rapidly through  $\pi/2$  and the P-matrix pole is rather insensitive to the value of  $b$  and close to where  $\delta(k)$  passes through  $\pi/2$ .

(2) Weak Forces, No Resonances

This is analogous to the situation we expect for unconfined channels. Despite the absence of resonant states there will be P-matrix poles. Figure 13 shows three cases:

(a) No potential:  $\delta(k) \equiv 0$

The P-matrix pole is at  $k=\pi/b$ : Jaffe and Low call this the "compensation mass"  $M_{\text{comp}}$ . It is clearly very sensitive to the choice of  $b$ .

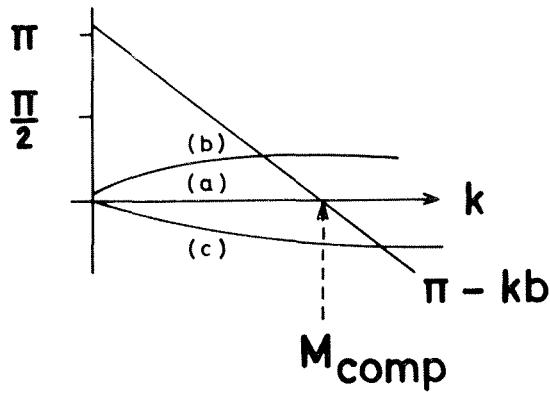


Figure 13 Position of P-matrix poles for (a) No interaction, (b) Weak attraction and (c) Weak repulsion. Case (a) defines the "compensation mass"  $M_{\text{comp}}$ .

(b) Weak attractive potential:  $\delta(k) > 0$

The pole in the P-matrix is lower than the compensation mass.

(c) Weak repulsive potential:  $\delta(k) < 0$

In this case the P-matrix pole is at a mass larger than  $M_{\text{comp}}$ .

This I hope illustrates the idea of Jaffe and Low. They have applied their formalism to S-wave  $\pi K$  and  $\pi\pi$  phase-shifts and find P-matrix poles corresponding to all the Bag model primitives. For example, they find poles in the  $K\pi$  channel at 960 MeV and in the  $I=2 \pi\pi$  channel at 1.04 GeV, in good agreement with the original Bag estimates.<sup>51)</sup> Jaffe and Low make a further strong claim: the relative splittings of the S-wave  $I=2$  and  $I=0 \pi\pi$  P-matrix poles with respect to the compensation mass correspond to "direct experimental evidence for the repulsive color magnetic interactions predicted in exotic channels by QCD".

It is clear that these results need careful examination. For example, one question is the sensitivity of the results to the choice of  $b$ . On physical grounds one expects

$$b \sim R_{\text{Bag}}$$

and Jaffe and Low have detailed arguments and consistency checks for their precise choice.

#### 2.4 Conclusions

(1) Predictions of resonant masses for multiquark states certainly need care: for such states the narrow resonance approximation is clearly invalid.

(2) If we accept the  $(q^2\bar{q}^2)$  identification for the  $0^{++}$  states below 1 GeV, there are presumably some additional  $(q\bar{q})_{L=1} 0^{++}$  states at higher masses. There are some hopeful signs of 'extra' activity in S-wave  $\pi\pi \rightarrow K\bar{K}$  around 1300 MeV. At the moment the theoretical situation is confused: Martin and Ozmutlu<sup>53)</sup> are unable to find any satisfactory multi-resonance description of the  $I=0$  S-wave.

#### 2.5 Implications

An obvious extrapolation of the idea of multiquark states is the inclusion of charmed quarks. Several authors<sup>54-56)</sup> have looked at such states and naive estimates put several  $c\bar{c}q\bar{q}$  states below the  $\psi'$ . One must re-examine these predictions in the light of Jaffe and Low's analysis of  $q^2\bar{q}^2$  mesons. If any narrow states survive they could lead to curious decays of the  $\psi'$  and the possibility of charged "x-like" states!

There have also been attempts to<sup>57-59)</sup> explain the structure in the  $e^+e^-$  total cross section around 4 GeV in terms of a high orbital angular momentum approximation for  $c\bar{c}q\bar{q}$  states.

3. CONCLUDING REMARKS

Before concluding I want to try to demonstrate that there are still interesting questions to ask of hadron spectroscopy.

The first question concerns multiquark hadrons. If we have granted acceptance to a  $q^2\bar{q}^2$  interpretation of the scalar mesons, then we must assume other multiquark hadrons exist - in particular  $q^4\bar{q}$  states.<sup>60-64)</sup> Estimates in the MIT Bag of the lowest-lying negative parity  $q^4\bar{q}$  states include many states in well-explored regions of phase-shift analyses. An extreme, and, in my opinion, a not very successful suggestion,<sup>64)</sup> challenges the usual assignment of resonances to the  $qqq[70, 1^-]$ . However, given our experience in the meson sector, Bag predictions must be regarded as 'primitives' and some sort of P-matrix analysis performed. A recent paper<sup>65)</sup> claims some success along these lines.

The second question concerns the  $I=0$   $1^{+-}$  and  $1^{++}$  states of the usual  $(q\bar{q})_{L=1}$  multiplets. No  $I=0$  partners of the B are known and the D and E mesons sit rather uneasily in the  $1^{++}$  nonet. Given the success of non-relativistic potential models for charmonium, it seems worthwhile to try to extrapolate such models down to strangeonium. One such attempt is by Barbieri et al.<sup>66)</sup> These authors determine the strange quark mass scale by fitting to the  $\phi(s\bar{s})1^{--}$  state and then predict the P-wave and radially excited S-wave ( $s\bar{s}$ ) states. They obtain reasonable agreement for the  $f'(s\bar{s})2^{++}$  around 1.6 GeV, and predict a  $\phi'(s\bar{s})1^{--}$  state at about 1.7 GeV. The situation with regard to the  $1^{++}$ ,  $1^{+-}$ , and  $0^{++}(s\bar{s})$  states is not so clear and probably warrants further attention.

Thirdly, I wish to bring to your attention some other curious states. Given Bjorken's beautiful argument<sup>67)</sup> for the existence of glueballs and constituent gluons, what about  $q\bar{q}g$ <sup>68)</sup> and  $qqqg$  states. Although it is difficult to make firm estimates for the properties of these states they deserve serious attention. One signal perhaps for mesons might be a  $J^{PC}$  exotic state: so far there is little encouragement from experiment.

To conclude:

- (1) Clear evidence for the existence of non  $q\bar{q}$  or  $qqq$  states is needed from experiment
- (2) There are still interesting questions for light quark spectroscopy
- (3) There is an interesting interplay between the charmed and non-charmed sectors.

ACKNOWLEDGEMENTS

Many people were helpful in my preparation of this talk: amongst the many I must thank are Hugh Burkhardt, Roger Cashmore, Jon Ellis, Bob Jaffe, Peter Litchfield and Alan Martin.

\* \* \*

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#### DISCUSSION

*Chairman:* Yu.D. Prokoshkin

*Sci. Secretaries:* W.M. Geist and J.P. Martin

*Chan Hong-Mo:* You divided baryon spectroscopy into a pre-QCD and a post-QCD era, the latter being characterized by the work of Isgur and Karl. I wish to point out that the relation to QCD of these new fits is almost purely temporal in that they were done after QCD became popular, but in rather little else. At the level we are considering, namely one-gluon-exchange, QCD enters at two places: i) colour dependence in terms of the colour charges  $\lambda_i$  of the quarks; ii) the vector nature of the gluon. To test (i), one must have systems with different colours. In the baryon, however, all subsystems, namely quarks  $q$  and diquarks  $qq$ , have colour 3 so that no colour dependence can be tested. The vector nature (ii) of the gluon implies certain relative strengths between the spin-spin, spin-orbit and tensor terms. Isgur and Karl kept the spin-spin and tensor terms but dropped the spin-orbit term, which, as you pointed out, is mainly an assumption. One has not therefore tested the vector nature of the gluon. It is not excluded that one can get equally good fits with other combinations of spin-dependent forces. For example, there is a new preprint by Goldstein and Maharana in which they claim to obtain an equally good fit with quadrupole forces based on a quark-diquark model.

*A.J.G. Hey:* It is clear that you are not testing much QCD. There are many assumptions in the Isgur and Karl analysis which you have to worry about before you can claim that it was derived from QCD. I agree that the relation of QCD to the spectrum, at the moment, is very tenuous, except if you believe the analysis of Jaffe and Low. They claim to see the direct effect of the colour magnetism term. That is a possibility. The analysis of Goldstein which gives evidence for quark-diquark structure of baryons was based upon the assumption that 70-plets do not exist at low mass. I am not so clear that this evidence is valid. Therefore I am very sceptical about analyses which make as a keystone of their starting point the fact that 70-plets are not observed.

*Chan Hong-Mo:* Yes, my remark applies only to the baryons. For multiquark states, one has subsystems of different colours so that the colour dependence can be tested. This is one good reason why they are interesting. Also, I did not mean to blame you for unwisely ascribing to QCD any success of the recent fits to the baryon spectrum -- I was just unhappy that the truth is not more clearly stated in the literature.

*A.J.G. Hey:* I agree on the baryons.

*P. Minkowski:* For the absence of L·S-force there is a clear case when you state what the non-strange  $\frac{5}{2}^-$ ,  $\frac{3}{2}^-$ ,  $\frac{1}{2}^-$  baryon states -- what you call  $^48$ , where the total spin is  $\frac{3}{2}$  -- there is no visible mass splitting.

*A.J.G. Hey:* I agree. But one has to understand for whatever reason L·S-forces are suppressed in the baryon spectrum.

*P. Minkowski:* Maybe we learn more by a discussion of the equations written down by Leutwyler, Stern et al., who have such a model for mesons: your oscillator model, relativistic and no L·S-forces whatsoever.

*A.J.G. Hey:* I looked into the paper about the model you are talking about. I am afraid the greatest of its successes was correctly predicting the position of the  $\eta_c(2.8)$ .

*H.J. Schnitzer:* I should emphasize that the baryon model of Isgur and Karl is the only one which agrees with similar models of meson spectroscopy.

*A.J.G. Hey:* In my opinion the models for meson spectroscopy are premature. There are not enough data on the  $q\bar{q}$  excitations of the mesons. I discussed fits similar to those of Isgur and Karl in the meson sector. They predict all these states at masses which we have looked for and they have no reason why we do not see them. And before we do not have some more experimental evidence I take all the spectroscopists' calculations with a pinch of salt.

*J. Rosner:* Are you able to place any theoretical bounds on the mass of the  $A_1$ ?

*A.J.G. Hey:* No comment.

*C.S. Kalman:* We have calculated the four-quark ground state using the formalism of Isgur and Karl in a paper to be considered in the Parallel Session on Hadron Spectroscopy. The hyperfine treatment is identical with Jaffe and the parameters are obtained *solely* from baryon data. The lowest mass is roughly 300 MeV higher than Jaffe's  $E(700)$  and is in fact bang on the  $\rho$  mass.

*A.J.G. Hey:* OK. There are no reliable models for predicting masses.