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# Number of nucleon-nucleon collisions vs. energy in modified Glauber calculations

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**Abstract.** We modified the standard Glauber model calculations of number of participants ($N_w$), number of collisions ($N_{coll}$) and their dispertions using a simplified estimate of the energy loss in nucleon-nucleon collision. A fraction equal to 1-$k$ of the initial nucleon momentum is allowed to be lost in each inelastic interaction, so that two "nucleon-like structures", emerging after the nucleon-nucleon inelastic process, have lower momenta. Therefore, further "nucleon-nucleon" inelastic collisions take place at a lower energy $\sqrt{s}$ and with the experimentally known cross-section ($\sigma_{inel}^{pp}\left(\sqrt{s}\right)$). We obtain a considerable reduction in the number of collisions $N_{coll}$ and in the total mean number of charged particles $N_{ch}$, compared to the standard Glauber approach, while the number of participants $N_w$ is practically unchanged. The model incorporates the experimental values of the average charged particle multiplicity for $pp$ and $p\bar{p}$ collisions as a function of $\sqrt{s}$, so that the total mean number of charged particles ($N_{ch}$) could be calculated and compared with the heavy-ion collision data. This model has been used to describe the available data on the average multiplicity obtained for $PbPb$ and $AuAu$ collisions from AGS to RHIC energies. Possible applications of this model to the pA and dA multiplicity data analysis are also discussed.

## 1. Introduction

Standard Glauber model is extensively used to compute the number of participants, the number of collisions and their dispertions [1], as a base for the analysis of data coming from multiple hadron production in high energy nucleus-nucleus collisions from AGS to SPS and RHIC energies.

The experimental data [2]-[3] present a number of interesting features showing hint for some collectivity effects as compared to the pp collisions: a non-trivial growth of multiplicity with the atomic number of colliding nuclei, the enhanced strange particles yields, not easy to explain dependence from collision centrality and energy, etc.

It is a common practice, both in experiment and in theory, to normalize the charged particle data yields using the number of nucleons participating to the collision [1], [2]-[4]. The latter are called "participants" or "wounded nucleons" and their number for a given collision is denoted below as $N_w$.

In experiments with heavy ions this number $N_w$, relevant to the centrality of collision, is usually defined straightforwardly for any event (using for example a Zero Degree Calorimeter whose signal is assumed to be proportional to the number of the spectator nucleons).

In any theoretical model $N_w$ could be defined as the number of nucleons involved in at least one inelastic interaction, in the given heavy nuclei collision. In the Glauber formalism the heavy ion collision of the nucleus A with the nucleus B is assumed to be *the incoherent* sum of individual nucleon-nucleon interactions occurring at the given collision energy $\sqrt{s}$ per nucleon-nucleon pair [1]. It is possible to obtain the number of the wounded nucleons $N_w$, the total number of inelastic nucleon-nucleon collisions $N_{coll}$ and their dispertions at the given impact parameter $b$ using the known Woods-Saxon geometry of the colliding nuclei and the values of the inelastic nucleon-nucleon cross-sections ($\sigma^{pp}_{inel}(\sqrt{s})$). The latter are defined from [5] for the given initial energy $\sqrt{s}$ of colliding nucleons.

In the Glauber model the energy $\sqrt{s}$ and the relevant $\sigma^{pp}_{inel}(\sqrt{s})$ are fixed for all nucleon-nucleon interactions. Therefore, one can expect that the number of nucleon-nucleon collisions $N_{coll}$ is usually overestimated in a standard approach that neglects energy loss in inelastic elementary collisions.

The mean energy loss in each inelastic nucleon-nucleon collision could be evaluated knowing the mean multiplicity, mean $p_t$ and mass of the particles produced at the given nucleon-nucleon collision energy. It could be also parameterized using a factor linked to the mean loss of energy or to the mean nucleon momentum loss in the inelastic nucleon-nucleon collision. In the present work the Glauber formalism in the form described in [1] is modified taking into account the energy conservation for all elementary nucleon-nucleon collision processes and using a single factor $k$ for the nucleon momentum mean loss in any inelastic collision.

The paper is organized as follows: in the $1^{st}$ section we describe our modification of the general Glauber formalism for the determination of $N_w$ and $N_{coll}$, aimed at a simplified energy loss evaluation. Available experimental data provide the possibility to test our modified model, - this is done in the next section using the data on total charged particle multiplicity $N_{ch}$ from AGS to RHIC energies. Finally, we present some results of calculations of $N_{coll}$, $N_w$, $N_{ch}$ and the estimates of their dispertions for various centrality classes for $PbPb$ collisions. Some $pA$ and $dA$ predictions are also produced for SPS and RHIC energies. A projection to LHC energy is also presented for the charged particle multiplicity in $PbPb$ collisions.

## 2. Glauber model modifications

We consider the collision of two heavy nuclei A and B (composed of nucleons $a$ and $b$) as *the incoherent* superposition of nucleon-nucleon (n-n) collisions in line with the Glauber model presented in detail in [1].

We assume that the leading particle –"the nucleon-like structure" [6], emerging after the 1st nucleon-nucleon inelastic process, has lower momentum then the projectile particle (because some energy of the 1st elementary n-n collision is lost in the production of secondaries).

In the present study we allow the leading particle, in every collision, to take away a fraction $k$ of the initial momentum $p$ (assuming the same $k$ value for the whole energy interval).

Let us denote as $p$ the momentum (in the CM system of two colliding nuclei) of the nucleon $a$ before the collision and $p'$ the momentum of the "nucleon like structure" after the collision (or the leading nucleon) $a'$. Then the coefficient $k$ could be defined as:

$$p'_a = kp_a \tag{1}$$

So, after the $1^{st}$ collision the "wounded nucleon" has a lower momentum. It means, that the next inelastic collision of two nucleons (one of which is the wounded one $a'$ and the next one is either $b$ or $b'$ (anyhow it will be denoted as $b'$) will occur at a different value of energy per pair $\sqrt{s}$ with the probability defined by the relevant cross-section $\sigma^{pp}_{inel}(\sqrt{s})$ of inelastic nucleon-nucleon interaction.

Therefore the main difference in our method with respect to the standard Glauber calculations is that for each nucleon-nucleon collision we always use the value of $\sigma^{pp}_{inel}$ of the correspondent

CM nucleon-nucleon collision energy. This one is calculated for any elementary collision taking into account the energy loss of the previous collisions. Then the number of participants $N_w$, the number of collisions $N_{coll}$, the multiplicity $N_{ch}$ and their dispertions are evaluated according to the standard model.

We found that in order to provide a detailed description of all collisions which takes into account their past history, it is sufficient to use at each step of the the MC code the proper transition to the center of mass (CM) system of the corresponding nucleons. In the most complex case one can have a collision of two nucleons $a$ and $b$ with different momenta ($p_a \neq p_b$) in some laboratory system. (We choose the z-axis along the collision direction and $p_x = p_y = 0$ so $p = p_z$).

In the first step the momenta must be transformed to the colliding nucleons CM. This is done in two steps.

The collision energy is defined as

$$\sqrt{s} = 2E_a^{CM} = 2E_b^{CM} = \sqrt{2} * \sqrt{m^2 - p_a * p_b + E_a * E_b} \tag{2}$$

where $m$ is a nucleon mass, $E = \sqrt{p^2 + m^2}$. The momenta can then be derived as:

$$p_a^{CM} = p_b^{CM} = \sqrt{\frac{s}{4} - m^2} \tag{3}$$

At this point we can consider the process of elementary nucleon-nucleon collision at $\sqrt{s}$ with the corrisponding inelastic cross-section. The momenta of both particles after the collisions are:

$$p'^{CM}_a = p'^{CM}_b = kp_a^{CM} = kp_b^{CM} \tag{4}$$

These reduced values of momenta are converted back into the laboratory system (of A+B) and the collisions of other pairs of nucleons are then evaluated.

## 3. Average number of charged particles per inelastic nucleon-nucleon collision

Following the Glauber model [1] we obtain the total multiplicity $N_{ch}$ of charged particles produced in a collision of two nuclei as *the incoherent* superposition of mean multiplicities in elementary nucleon-nucleon collisions. The model incorporates the known experimental values of the average multiplicity of charged particles as a function of $\sqrt{s}$ produced in $pp$ and $p\bar{p}$ collisions. In our calculations we parameterised the experimental data [5] by the following formula:

$$N_{ch}^{pp}(s) = -4.2 + 4.69s^{0.155} \tag{5}$$

Thus the total multiplicity of charged particles $N_{ch}$ produced in a given collision of two heavy ions could be calculated as a sum of all charged particles produced in all inelastic nucleon-nucleon interactions. For minimum bias events we used the values of the impact parameter in the range from 0 to 15 fm. The final results on $N_{ch}$ could be compared to the heavy-ion experimental data. The above mentioned $k$ (see equation (1)) is the only free parameter in these calculations.

## 4. Calculations

The Monte Carlo (MC) code was written to calculate the mean numbers of participants $N_w$, the number of collisions $N_{coll}$, the charged particle multiplicity $N_{ch}$ and their dispertions using a simplified evaluation of the nucleon momentum loss for any nucleon-nucleon collision.

This Modified Glauber (MG) MC model is applied to describe the available experimental data on the average multiplicity obtained for PbPb and AuAu collisions from AGS to RHIC energies. A compilation of the experimental data for total multiplicities from [7] is used.

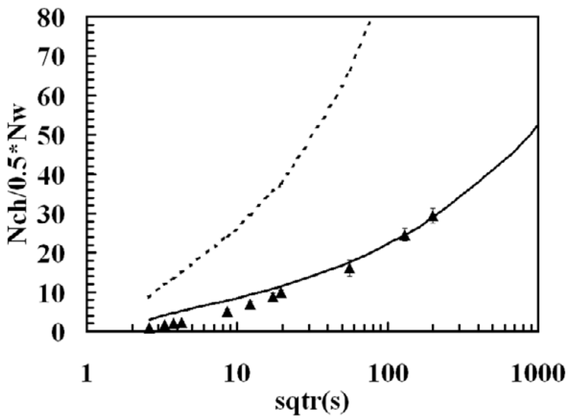


**Figure 1.** Mean total charged particles multiplicity $N_{ch}$ normalized to wounded nucleons vs. $\sqrt{s}$ (GeV): full triangles are PbPb and AuAu experimental data (compilation from [7]); full line is our modified Glauber calculations with the value $k = 0.21$ (see text); dashed line is standard Glauber calculations.

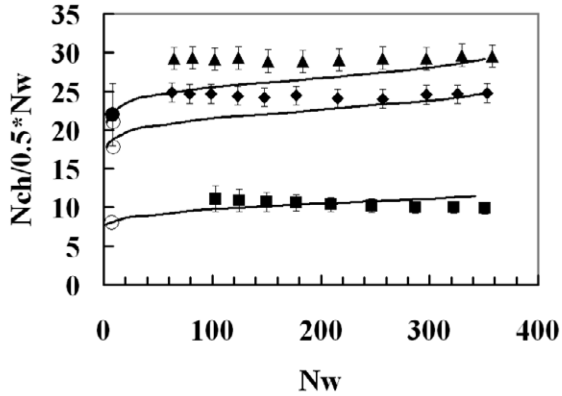


**Figure 2.** Centrality dependence of charged particles yields per participant in AuAu collisions. Experimental data for AuAu collisons are from [7]; full triangle: 200 GeV*A; full circle: 130 GeV*A; full square: 19.6 GeV*A; full circle: experimental data for dA collisions at 200 GeV*A from [8] (only for $N_w$=2); open circle: our modified Glauber calculations ($k = 0.21$) for dA collisions at 200, 130 and 19.6 GeV*A (only for $N_w$=2); full lines are our modified Glauber results ($k = 0.21$) for AuAu collisions at 200, 130 and 19.6 GeV*A.

As can be seen from figure 1 the modified Glauber calculations with the single parameter $k$=0.21 (solid line) are quite compatible with charged particle data yields in the whole energy range from AGS to RHIC energies. The standard model gives much larger values (see dotted line on the figure 1). The value of the parameter $k$=0.21 was defined by fitting the data at 200 GeV.

Some discrepancy between the MG predictions and the experiment is observed in charged particles yields at the SPS energies (figure 1). It can be explained due to our lack of knowledge of charged particle yields in pp collisions near the inelastic threshold.

The centrality dependence of charged particle yields per participant was also calculated in the MG model for AuAu collisions at 3 energies (19.6, 130 and 200 GeV*A) with the same parameter $k$=0.21 and then compared to the experimental data [7] (see figure 2). The general trend, i.e. the observed "stability" of the charged particle yields per participant for different centrality classes, is rather well reproduced in our simulation. This means that the experimentally observed scaling of the mean total charged particle yield with the number of participants [7] is rather well described by the modified Glauber model.

This scaling could be explained in the framework of the MG model by a balance of two factors, namely the values of $N_{coll}$ and the collision energies. In the case of central events (large $N_w$) we have a rather large number of $N_{coll}$, but the majority occurs at rather low nucleon-nucleon collision energies, therefore a relatively low mean amount of charged particles is produced per nucleon-nucleon collision. We have an opposite situation in peripheral collisions (see also the impact parameter dependence of $N_{coll}$ and $N_{coll}/N_w$) represented below in figure 7 and figure 9).

We found that the number of participants $N_w$ in AuAu and PbPb central collisions stays practically unchanged compared to the standard Glauber approach as expected (see figure 4, where the results of standard and modified models are shown along with some existing experimental data).

The main difference between standard and MG calculations is that energy conservation produces a considerable decrease in the number of $N_{coll}$ and in the corresponding $N_{ch}$, compared to the standard approach predictions (see figure 3). Some experimental estimates of $N_{coll}$ based on the standard Glauber model are also shown in figure 3. The standard Glauber calculations give about 800–1000 collisions in the domain of SPS-RHIC energies, while we obtain $N_{coll}$ values of about 500–600 in our modified Glauber calculations with the value k=0.21. It is worth mentioning the fact that our estimates of $N_{coll}$ using the Parton String Model [9], where energy conservation is taken into account in a natural way, give us values of $N_{coll}$ compatible with the MG calculations (see figure 3).

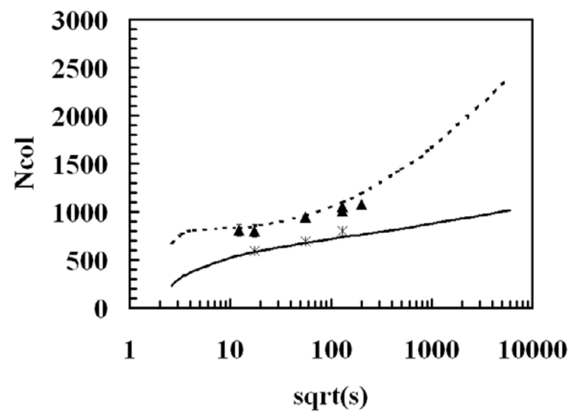


**Figure 3.** $N_{coll}$ vs $\sqrt{s}$ (GeV): various estimates for most central collisions. The full line is for our modified Glauber calculations with the value $k = 0.21$ (see text); the dashed line is for standard Glauber calculations; triangles: estimates [12], [10], [11], [2], [4] done at the corresponding energies; the stars are for Parton String Model results.

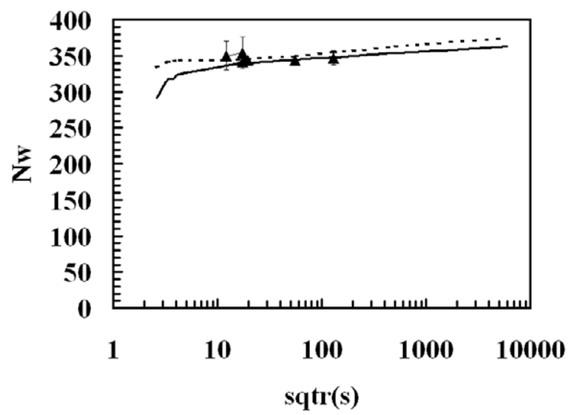


**Figure 4.** Energy dependence of number of nucleon-nucleon collisions $N_w$ for Pb-Pb and Au-Au in standard (full line) and modified Glauber (dashed line) calculations; triangles: defined in the experiments [12], [3], [10],[11], [2] at the corresponding energies.

## 5. Possible applications of the model

The fact that MG calculations seem to be efficient using only a single parameter for the description of global observable such as mean charged particle multiplicity in AA collisions for a wide energy range, points to the possibility that they can produce a better estimate of the relevant numbers $N_{ch}$, $N_w$, $N_{coll}$ and other quantities like dispertions with respect to the standard Glauber approach.

Some predictions of the total charged particles yields in AuAu collisions at the LHC energies are showm on the figure 5 where the total charged multiplicity $N_{ch}$ is shown vs. $\sqrt(s)$ for the standard and for the modified Glauber calculations. The total multiplicity produced by the MG model is about 10 times smaller then standard Glauber model (about 19 000 charged particles for PbPb collisions at 6 TeV*A for the MG vs. 170 000 charged particles for standard calculations).

We show some results of the calculations of $N_{coll}$, $N_{ch}$, $N_{ch}/N_w$, $N_{coll}/N_w$ and their dispertions for AuAu collisions at 19.6 GeV as an example of application for possible further analysis.

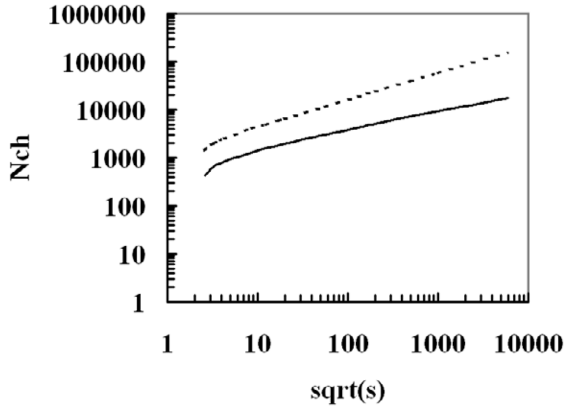


**Figure 5.** Total charged multiplicity $N_{ch}$ vs. $\sqrt{s}$ (GeV) standard (dashed line) and modified Glauber calculations with the value $k = 0.21$ (full line).

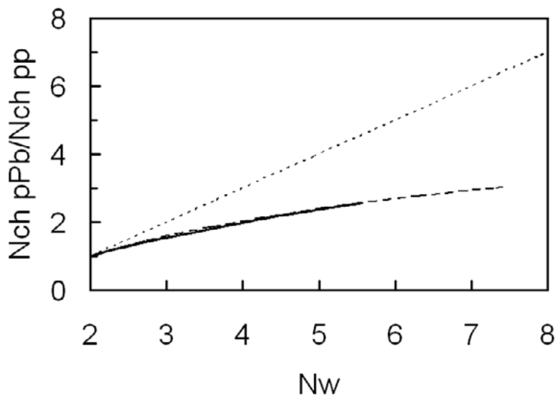


**Figure 6.** Relative yield of charged particles for $pPb$ with respect to $pp$ collisions vs. number of participants (it means the number of collisions here) in the standard and in the modified Glauber calculations with the value $k = 0.21$ at sqrt(s)=17.3 and 130 GeV: dotted line: standard Glauber calculations; full line: MG calculation with the value $k = 0.21$, 17.3 GeV; dashed line: MG calculation with the value $k = 0.21$, 130 GeV.

Calculations were done both for the centrality and for the energy dependence of $N_{ch}$ and $N_{coll}$, see figure 5 and see figure 11, 12, 3, 8. First of all, scaling with the number of participants is present in MG estimates (figure 11). Also, dispertions have maximal values for non-central events (see figure 8, figure 12) i.e. at impact parameter values of about 5–10 fm, and are very sensitive to the impact parameter window (that is relevant to the calorimeter resolution). The dispertion value are also found to be in general smaller then in case of standard Glauber predictions.

We also compared predictions from the MG model to some available experimental results on the mean charged particles yield per participant in the case of dA collisions [8]. In figure 2 we show our modified Glauber calculations ($k$=0.21) for the $dA$ collisions at 200, 130 and 19.6 GeV*A (open circles) for the charged particle multiplicity per participating nucleon, for $dA$ collisions at 200 GeV*A (data are taken from [8] for $N_w$=2, one experimental point (full circle) is shown).

Our MG estimates for the $AuAu$ collisions at the three given energies are also shown in full lines. One can see that the experimentally observed scaling with the number of participants of the mean total charged particles yield is followed rather closely by the modified Glauber model, even for $AuAu$ and $dA$ combined data.

A simpler case of $pA$ collisions was considered in the MG simulations, that is the $pPb$ case at 17.3 and 130 GeV CM collision energy. Results for charged particle relative yields ($pPb/pp$) are shown in figure 6. An important feature of MG calculations, in the case of proton-nucleus collisions, is the deviation from the Glauber model linear behavior: the relative yield of charged particles has a tendency for “saturation” with the growing number of inelastic collisions, giving the hint for a strong stopping power of the nuclear medium. This tendency was already observed experimentally by the NA49 and E910 collaborations ([15],[16] and references therein), the detailed comparison to the experiment is to follow.

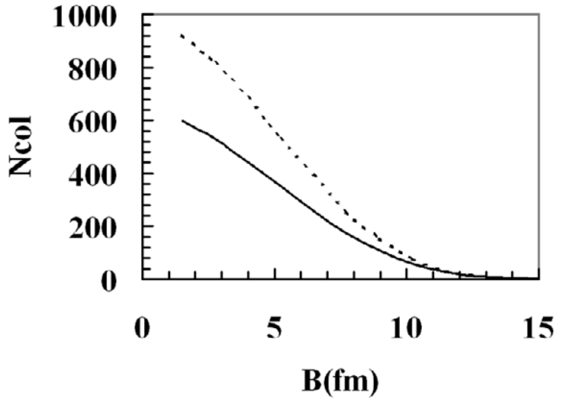


**Figure 7.** The mean number of collisions $N_{coll}$ vs. impact parameter $b$ calculated for AuAu collisions at $\sqrt{s} = 19.6$ GeV (results are averaged over 3 fm impact parameter window). Full line is for our modified Glauber fit with the value $k = 0.21$ (see text); dashed line is for the standard Glauber calculations.

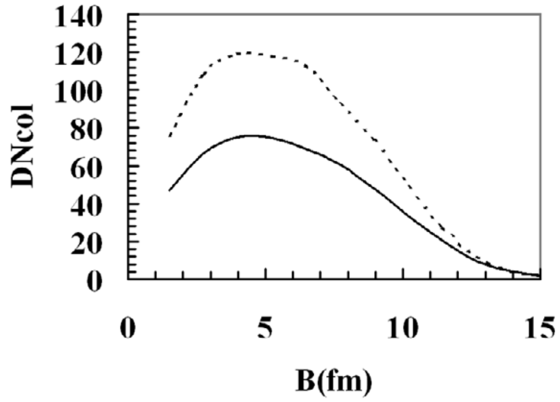


**Figure 8.** Dispertion of the mean number of collisions $N_{coll}$ vs. impact parameter $b$ calculated for AuAu collisions at $\sqrt{s} = 19.6$ GeV (see caption to figure 7).

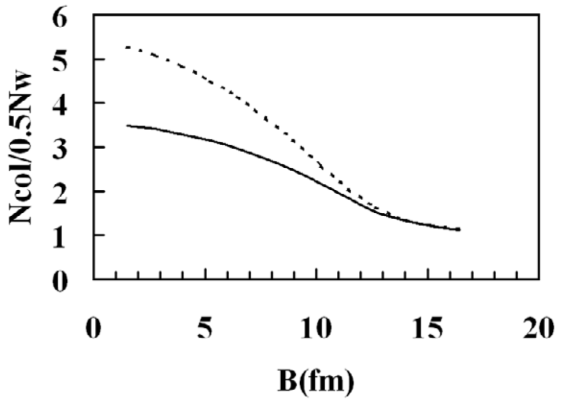


**Figure 9.** The mean number of collisions per wounded nucleon $N_{coll}/.5 * N_w$ vs. the impact parameter $b$ calculated for AuAu collisions at $\sqrt{s} = 19.6$ GeV (see caption to figure 7).

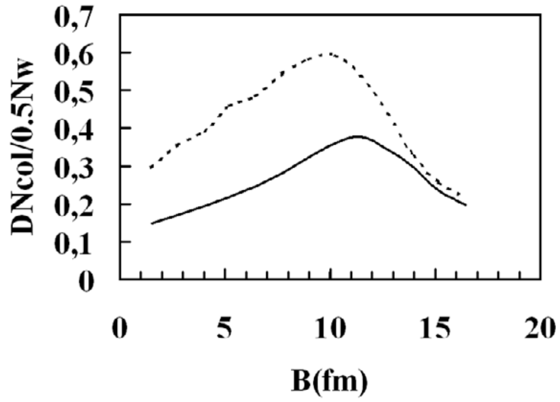


**Figure 10.** Dispertion of the mean number of collisions per wounded nucleon $dN_{coll}/.5 * N_w$ vs. impact parameter $b$ calculated for AuAu collisions at $\sqrt{s} = 19.6$ GeV (see caption to figure 7).

## 6. Conclusions

A simplified model, including energy loss calculation for heavy ion collisions has been produced in the framework of the modified Glauber model using a single parameter $k$=0.21 that accounts for the fraction of the initial nucleon momentum left after each inelastic nucleon-nucleon collision.

This approach seems to be efficient from AGS to RHIC energies in the description of global observables such as mean charged particle multiplicity. A tentative estimate of about 19000 charged particles for the mean total multiplicity was obtained for PbPb collisions at LHC energy.

A first application of the modified Glauber model to $pA$ and $dA$ experimental data analysis appears to describe reasonably well the general trends observed: scaling with the number of

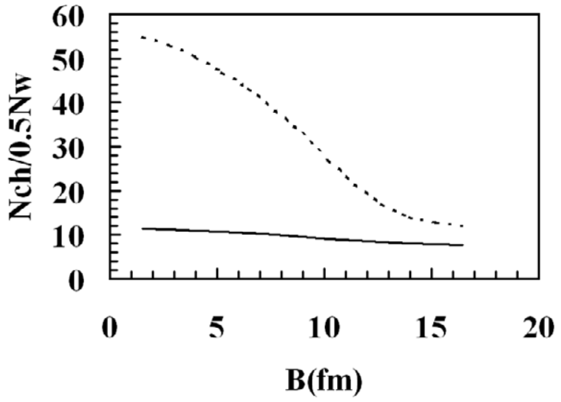


**Figure 11.** Mean total charged particles multiplicity $N_{ch}$ normalized per wounded nucleon calculated for $\sqrt{s} = 19.6$ GeV (see caption to figure 7).

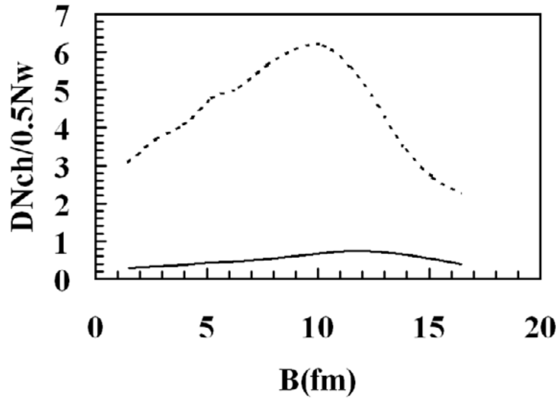


**Figure 12.** Dispertion of the mean total charged particles multiplicity $N_{ch}$ normalized per wounded nucleon calculated for AuAu collisions at $\sqrt{s} = 19.6$ GeV (see caption to figure 7).

participants is rather well reproduced for $dA$ and $AuAu$ collisions, and calculated relative charged particle yields ($pPb$ to $pp$ collisions) show a saturation trend with the growth of the number of nucleon-nucleon collisions and energy.

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