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Absence of Radiative Mass Shifts for Composite Goldstone Supermultiplets

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Abstract

A supersymmetric Dashen type formula is used to prove that there are no radiative mass corrections for Goldstone pion supermultiplets due to gauge interactions. In particular, for unbroken supersymmetry the charged to neutral pion mass difference is shown to vanish to all orders in electromagnetism.

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When the strong dynamics of a theory results in the spontaneous breakdown of its global symmetry group G to a subgroup H, Goldstone boson pions arise in one to one correspondence with the broken generators. If, in addition, the underlying Lagrangian contains explicit symmetry breaking terms, these pions will, in general, acquire a non-zero mass. Using the currents, j^{μ}_{5i}, associated with the spontaneously broken symmetries as interpolating fields for the Goldstone bosons, π_i, the current field identity

$$\partial_{\mu} j^{\mu}_{5i} = -f_{\pi_i} m^2_{\pi_i} \pi_i \tag{1}$$

can be used in conjunction with the broken Ward identities (WI) to yield information about the pion masses and interactions. In particular, the Dashen formula [1]

$$\begin{aligned} &(f_{\pi_i} m^2_{\pi_i})^2 \int d^4x \quad \langle 0 | T\, \pi_i(x) \pi_j(0) | 0 \rangle \\ &\quad = - \langle 0 | [Q_{5_j}, [Q_{5_i}, \mathcal{L}]] | 0 \rangle \quad , \end{aligned} \tag{2}$$

where Q_{5_i} is the charge obtained from the current $j^{\mu}_{5_i}$, relates the pion masses, m_{π_i}, and decay constants, f_{π_i}, to the symmetry breaking terms in the Lagrangian. If the breaking terms in $\mathcal{L}$ are invariant under H, then the number of independent masses and decay constants is the same as the number of H irreducible representations formed by the pions. If, however, terms in $\mathcal{L}$ also explicitly break H, then there will be mass splittings within the various irreducible representations which depend upon the form of the explicit breaking.

For the case of QCD with two massless flavors, the global chiral symmetry group SU(2) x SU(2) in spontaneously broken by quark condensate formation down to the vector subgroup $SU(2)_V$. Adding $SU(2)_V$ invariant up and down quark masses to the Lagrangian results in the $\pi_\pm$, π_o acquiring a common mass given via Dashen's formula in terms of the current quark mass, m_q, and quark condensates as

$$f_\pi^2 m_\pi^2 = -m_q [\langle \bar{u}u \rangle_o + \langle \bar{d}d \rangle_o] \quad . \tag{3}$$

When the electromagnetic interactions are introduced, however, the charged pions, $\pi_\pm$, will acquire additional mass corrections which will split them from the π_o mass [2]. Once again, Dashen's formula can be used to obtain this electromagnetic mass shift as

$$\begin{aligned} \delta m_{\pi^+}^2 &\equiv m_{\pi^+}^2 - m_{\pi^o}^2 \\ &= -\frac{1}{f_\pi^2} \{\langle 0|[Q_{5_-} [Q_{5_+}, L]]|0\rangle \\ &\quad - \langle 0|[Q_{5_o}, [Q_{5_o}, L]]|0\rangle\} , \end{aligned} \tag{4}$$

where $Q_{5_\pm} = \frac{1}{\sqrt{2}} (Q_{5_1} \pm iQ_{5_2})$ and $Q_{5_o} = Q_{5_3}$. This mass splitting can be calculated to lowest order in the QED fine structure constant α by using an effective one photon exchange electromagnetic Lagrangian which is proportional to the T product of two electromagnetic currents. The charge commutators in eq. (4) then transform this Lagrangian into a difference between the third component of the vector and axial vector two point functions. Finally, by using the current propagators spectral decomposition [3], the mass splitting can be obtained in terms of α and the ρ- meson mass as

$$\delta m_{\pi^+}^2 \simeq \frac{3\alpha}{2\pi} \ell n \; 2 \; m_\rho^2 \quad . \tag{5}$$

The purpose of the present note is to demonstrate that in supersymmetric (SUSY) theories, explicit global symmetry breaking due to gauge interactions yields vanishing radiative mass corrections for the Goldstone multiplets. Hence there is no pion electromagnetic mass splitting in SUSY QCD as long as SUSY remains unbroken. The vanishing of $\delta m^2_{\pi^+}$ in SUSY QCD has been established to lowest order in α by Lerche, Peccei, and Visnjic [4]. Using the superspace techniques of the present paper , this result is shown to be true to all orders in α in a more economical fashion.

As is the case for ordinary QCD, we assume that the strong dynamics of SUSY QCD results in the spontaneous breakdown of the global chiral symmetry to a vector subgroup, while leaving the supersymmetry unbroken [F1]. It follows from the SUSY Noether theorem [6] that for each symmetry generator of G (labelled by the index A) there corresponds a vector superfield current $J_A(x,\theta,\bar{\theta})$. For each of the broken generators (labelled by the index i), SUSY dictates the existence of a Goldstone chiral multiplet π_i consisting of the Goldstone boson and its supersymmetric partners. The currents can be used as interpolating superfields for the π_i with the current-field identity taking the form [6]

$$-\frac{1}{4}\bar{D}\bar{D}J_i = \frac{1}{2}m_{\pi_i}f_{\pi_i}\pi_i, \quad \text{(no sum on i)}. \tag{6}$$

In analogy to the ordinary case, we desire a SUSY Dashen type formula which expresses the pion superfield masses, m_{π_i} , as the double variation of a SUSY chiral Lagrangian. When the explicit global symmetry breaking term is a SUSY chiral mass term, such a formula, linear in m_{π_i} , was previously obtained independently by Veneziano [7] and the present authors [6]. In this paper, we shall extend the formula to also allow for global symmetry breaking arising from gauge interactions. We first construct the Noether currents.

A general superfield action can be written as

$$I = \int dV\, L_V(\bar{\phi},\phi,V) + \int dS\, L_S(\phi) + \int d\bar{S}\bar{L}_S(\bar{\phi}) \ , \tag{7}$$

with $\bar{D}_{\dot{\alpha}} L_S = 0 = D_\alpha \bar{L}_S$ and $dV = d^4x\, d^2\theta d^2\bar{\theta}$, $dS = d^4xd^2\theta$, $d\bar{S} = d^4xd^2\bar{\theta}$. Here $(\bar{\phi})\phi$ are underlying (anti-)chiral matter superfields and V are vector superfields associated with the gauged symmetries of the theory. Alternatively, defining the (anti-) chiral Lagrangians as

$$\begin{aligned} L &= -\frac{1}{8}\bar{D}\bar{D}\, L_V + L_S \ , \quad \bar{D}_{\dot{\alpha}} L = 0 \\ \bar{L} &= -\frac{1}{8} DD\, L_V + \bar{L}_S \ , \quad D_\alpha \bar{L} = 0 \ , \end{aligned} \tag{8}$$

it follows that

$$I = \int dS\, L + \int d\bar{S}\,\bar{L} \ . \tag{9}$$

Varying the (anti-) chiral Lagrangian leads to the (anti-) chiral Noether theorems

$$\begin{aligned} \delta_A \bar{L} &= -\frac{i}{4} DD\, J_A + \delta_A\bar{\phi}\ \frac{\delta I}{\delta\bar{\phi}} - \frac{1}{8} DD\, [\delta_A V \frac{\delta I}{\delta V}] \ , \\ \delta\ L &= \frac{i}{4}\bar{D}\bar{D}\, J_A + \frac{\delta I}{\delta\phi}\,\delta_A\phi - \frac{1}{8}\bar{D}\bar{D}\, [\delta_A V \frac{\delta I}{\delta V}] \ , \end{aligned} \tag{10}$$

where the current superfield is given by

$$\begin{aligned} J_A = \frac{i}{2} [\delta_A\bar{\phi}\ \frac{\partial L_V}{\partial\bar{\phi}} - \frac{\partial L_V}{\partial\phi}\,\delta_A\phi \\ + \sum_{(\alpha)} [(\mathcal{D}_{(\alpha)}\delta_A V) - (-)^{|\alpha| + \mathrm{perm}(\alpha)}\ \delta_A V(\mathcal{D}^T_{(\alpha)}\ \frac{\partial L_V}{\partial\mathcal{D}_{(\alpha)} V})]] \end{aligned} \tag{11}$$

The symbol $\mathcal{D}_{(\alpha)}$ represents a product of $|\alpha|$ of the SUSY covariant derivatives D_α or $\bar{D}_{\dot{\alpha}}$,

$$\mathcal{D}_{(\alpha)} = D_{\alpha_1} \ldots \bar{D}_{\dot{\alpha}_n} \ , \quad |\alpha| = n \ , \tag{12}$$

while $\mathcal{D}^T_{(\alpha)}$ corresponds to the product in transposed order, with perm(α) being the number of permutations required to transpose order the original sequence. Finally $\sum_{(\alpha)}$ dictates a summation over all possible number and configurations of such products.

By combining the current-field identity eq. (6) with the zero momentum WI for the two superspace point function of $\bar{D}\bar{D}\ J_A$, we arrive at the desired SUSY Dashen formula. To obtain this WI, we use the chiral Noether theorem to write

$$
\begin{aligned}
&-\left\langle 0\right|T(-\tfrac{1}{4}\,\bar{D}\bar{D}\ J_A(1))(-\tfrac{1}{4}\,\bar{D}\bar{D}\ J_B(2)\left|0\right\rangle \\
&= \left\langle 0\right|T(\delta_A L(1)\ \delta_B L(2)\left|0\right\rangle \\
&- \left\langle 0\right|T\ \delta_A L(1)\ \frac{\delta I}{\delta\phi(2)}\ \ \delta_B\phi(2)\left|0\right\rangle \qquad (13)\\
&+ \frac{1}{8}\ \left\langle 0\right|T\ \delta_A L(1)\bar{D}\bar{D}\ [\delta_B V\ \frac{\partial L_V}{\partial V}](2)\left|0\right\rangle \\
&+ i\ \left\langle 0\right|T\ \frac{\delta I}{\delta\phi(1)}\ \delta_A\phi(1)(-\tfrac{1}{4}\ \bar{D}\bar{D}\ J_B(2))\left|0\right\rangle \\
&- \frac{i}{8}\left\langle 0\right|T(\bar{D}\bar{D}[\delta_A V\ \frac{\partial L_V}{\partial V}](1))(-\tfrac{1}{4}\ \bar{D}\bar{D}\ J_B(2))\left|0\right\rangle \quad .
\end{aligned}
$$

Unbroken SUSY, however, dictates that the $(\theta,\bar{\theta})$ dependence of two superspace point functions take a specific form. Letting M and N be generic superfields, SUSY requires that

$$
\begin{aligned}
&\left\langle 0\right|T\ M(1)N(2)\left|0\right\rangle \qquad (14)\\
&\quad = \exp[i(\theta_1\sigma^\mu\bar{\theta}_2 - \theta_2\sigma^\mu\bar{\theta}_1\ )\partial_{2_\mu}]\ \ \left\langle 0\right|T\ M(0)N(2-1)\left|0\right\rangle \ .
\end{aligned}
$$

Moreover, if $(\bar{N})$ N is (anti-) chiral so that $(D_\alpha\bar{N} = 0)\ \bar{D}_{\dot{\alpha}}N = 0$, it follows that

$$\bar{N}(x,\theta,\bar{\theta}) = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}\bar{N}(x,0,\bar{\theta}) \qquad (15)$$

$$N(x,\theta,\bar{\theta}) = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}\ N(x,\theta,0) \quad , \qquad (16)$$

which further restricts the $(\theta,\bar{\theta})$ structure of two superspace point functions containing (anti-) chiral superfields. Thus integrating eq. (13) over x_2 and using eqs. (14 - 16), it follows, upon application of the action principle,

$$\langle 0|T\ \delta\phi\ \frac{\delta I}{\delta\phi}\ X|0\rangle = i\ \langle 0|T\ \delta\phi\ \frac{\delta X}{\delta\phi}\ |0\rangle \quad , \tag{17}$$

that the zero momentum WI takes the form

$$\begin{aligned}\int\ d^4x_2\ \ \langle 0|T\ \delta_A\ L(1)\ \delta_B\ L(2)|0\rangle \\ = i\ \int\ d^4x_2\ \ \langle 0|T\ \delta_B\phi(2)\ \frac{\delta}{\delta\phi(2)}\ \ \delta_A L(1)|0\rangle\ \ .\end{aligned} \tag{18}$$

However, using Noether's theorem and the current-field identity we recognize the left hand side of eq. (18) as the zero momentum chiral Goldstone pion propagator so that

$$\begin{aligned}\int\ d^4x_2\ \ \langle 0|T\ \delta_i L(1)\ \delta_j L(2)|0\rangle \\ = \frac{i}{4}\ (m_{\pi_i}\ f_{\pi_i})^2\ \ \frac{\delta(\theta_1 - \theta_2)}{m_{\pi_i}}\ \ \delta_{ij}\quad .\end{aligned} \tag{19}$$

Moreover, since the SUSY WI eqs. (14 - 16) imply that

$$\begin{aligned}\int\ d\bar{S}_2\ \ \langle 0|T\ \delta_B\bar{\phi}(2)\frac{\delta}{\delta\bar{\phi}(2)}\ \delta_A L(1)|0\rangle = 0 \\ \int\ dV_2\ \ \langle 0|T\ \delta_B V(2)\ \frac{\delta}{\delta V(2)}\ \delta_A L(1)|0\rangle = 0 \quad ,\end{aligned} \tag{20}$$

it follows that $-\frac{1}{4}\ D_2D_2$ on the right hand side of eq. (18) is simply the double variation of the chiral Lagrangian. We thus arrive at the SUSY Dashen type formula

$$\delta_{ij}\ m_{\pi_i}\ f_{\pi_i}^2\ = 4\langle 0|\delta_i\delta_j\ L(0)|0\rangle \quad . \tag{21}$$

Recalling that $L = -\frac{1}{8}\ \bar{D}\bar{D}\ L_V + L_S$ and noting that unbroken SUSY dictates that $\langle 0|L_V|0\rangle$ be superspace $(x,\theta,\bar{\theta})$ independent so that $\bar{D}\bar{D}\ \langle 0|L_V|0\rangle$ vanishes, it follows that

$$\langle 0|\delta_i\delta_j\ L(0)|0\rangle = \langle 0|\delta_i\delta_j L_S(0)|0\rangle \quad . \tag{22}$$

Thus only those breaking terms arising from pure chiral fields can contribute to a nonvanishing pion mass in the unbroken SUSY limit. Those explicit symmetry breaking terms arising from gauge interactions do not lead to any radiative non-trivial pion mass shifts. Thus for the particular example of SUSY QCD, it follows that after the inclusion of electromagnetic interactions the charged to neutral pion mass difference vanishes to all orders in the electromagnetic coupling.

The absence of perturbative mass shifts for zero tree mass fundamental chiral superfields can be understood in terms of the remarkable nonrenormalization properties of SUSY theories. We have seen here that even when the chiral superfield arises as a massless bound state of some complicated strong interaction dynamics, all radiative corrections to its mass again vanish order by order in the explicit symmetry breaking gauge coupling constant. It thus appears that a nonrenormalization theorem is also operating at the bound state level [4]. If the Goldstone multiplet is to develop any nonvanishing mass in the absence of SUSY breaking, it can do so only through some nonperturbative mechanism giving rise to pure chiral condensates of the form considered in refs. [6] and [7].

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Footnote:

[F1] The question of whether chiral symmetry breaking condensates form in SUSY theories is still an open one [5].

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