



# Low scale leptogenesis in a model with promising $CP$ structure

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**Abstract** We consider a simple extension of the standard model, which could give a solution to its  $CP$  issues through both the Peccei–Quinn mechanism and the Nelson–Barr mechanism. Its low energy effective model coincides with the scotogenic model in the leptonic sector. Although leptogenesis is known not to work well at lower reheating temperature than  $10^9$  GeV in simple seesaw and scotogenic frameworks, such low reheating temperature could be consistent with both neutrino mass generation and thermal leptogenesis via newly introduced fields without referring to the resonance effect. An alternative dark matter candidate to axion is prepared as an indispensable ingredient of the model.

## 1 Introduction

Strong  $CP$  problem [1,2] and origin of  $CP$  violating phases in the CKM matrix [3] are important issues unsolved in the standard model (SM). Recent neutrino oscillation experiments [4,5] suggest the existence of a  $CP$  violating phase in the PMNS matrix also [6–8]. The formers are known to be solved by introducing Peccei–Quinn (PQ) symmetry  $U(1)_{PQ}$  [9,10] and assuming complex Yukawa couplings for quarks [3]. Spontaneous breaking of the PQ symmetry brings about a pseudo Nambu–Goldstone boson called axion [11,12]. Since the PQ symmetry is explicitly broken to its discrete subgroup  $Z_N$  via nonperturbative QCD effects, axion potential is lifted to cause  $N$  degenerate vacua where the strong  $CP$  problem is dynamically solved. Unfortunately, the domain wall appears to separate  $N$  degenerate vacua [13]. Since it is topologically stable for a case with  $N \geq 2$ , its energy density overcloses the Universe. On the other hand, the domain wall can decay in the model with  $N = 1$  [14] and the problem is escapable. Such models can be realized for restricted field contents with special PQ charge assignment such as the KSVZ model [15,16]. Suitably constrained PQ

charge could be also relevant to mass hierarchy of quarks and leptons [17–21]. Astrophysical and cosmological analyses require the PQ symmetry breaking scale  $f_a$  to be within intermediate scales  $10^{9–11}$  GeV [22–24] as long as the PQ symmetry is broken after inflation. It comes from the fact that axion mass and strength of its interaction with the SM fields are inversely proportional to  $f_a$ .<sup>1</sup> An interesting feature of the model is that axion could be a dominant component of cold dark matter (DM) if  $f_a$  takes a value near the upper bound mentioned above [25,26].

Inflation is considered to determine an initial condition of the hot Big-Bang Universe through the decay of inflaton. If reheating temperature  $T_R$  is high enough to satisfy  $T_R > 10^9$  GeV, thermal leptogenesis based on out-of-equilibrium decay of a heavy right-handed neutrino [27,28] is expected to work well in the seesaw framework for the neutrino mass [29]. On the other hand, in the case  $T_R < 10^9$  GeV it cannot generate sufficient baryon number asymmetry through the thermal leptogenesis if we do not refer to resonance effects. As long as we consider such a low reheating temperature, we have to consider some extension of the model.

In this paper, we propose an extended model which gives a solution to the  $CP$  issues in the SM. The strong  $CP$  problem is solved by the PQ-mechanism and the  $CP$  violating phases in the CKM and PMNS matrices are spontaneously generated through Nelson–Barr mechanism for the strong  $CP$  problem. Extra fields introduced for it play a crucial role to solve the difficulty in the low scale leptogenesis. We show that the model can generate sufficient baryon number asymmetry through thermal leptogenesis in a consistent way with neutrino oscillation data for lower reheating temperature than  $10^9$  GeV. Although axion might not be a dominant component of DM, the model has another DM candidate as

<sup>1</sup> The vacuum expectation value  $v_{PQ}$  which breaks the PQ symmetry and the axion decay constant  $f_a$  are related each other by  $v_{PQ} = f_a N$  [1,2]. We use  $f_a$  as its breaking scale since we consider only a case with  $N = 1$  throughout the paper.

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an indispensable ingredient for the neutrino mass generation. The model is rather simple but we can treat several problems in the SM.

Remaining parts of the paper are organized as follows. In Sect. 2, we describe the model studied in this paper and discuss nature of the  $CP$  symmetry in the model. In Sect. 3, we first overview inflation and reheating considered in the model. After that, leptogenesis is discussed focusing on how sufficient lepton number asymmetry is generated at low reheating temperature less than  $10^9$  GeV. We summarize the paper in Sect. 4.

## 2 A model with spontaneous $CP$ violation

We consider an extension of the SM with a global  $U(1) \times Z_4$  symmetry and several additional fields. The SM contents are assumed to have no charge of this global symmetry. Added fermions are a pair of vector-like down-type quarks ( $D_L, D_R$ ), a pair of vector-like charged leptons ( $E_L, E_R$ ), and three right-handed singlet fermions  $N_k$  ( $k = 1, 2, 3$ ). Their representation and charge under  $[SU(3)_C \times SU(2)_L \times U(1)_Y] \times U(1) \times Z_4$  are given as<sup>2</sup>

$$\begin{aligned} D_L & \left( \mathbf{3}, \mathbf{1}, -\frac{1}{3} \right)_{0,2}, \quad D_R \left( \mathbf{3}, \mathbf{1}, -\frac{1}{3} \right)_{2,0}, \\ E_L & (\mathbf{1}, \mathbf{1}, -1)_{0,2}, \quad E_R (\mathbf{1}, \mathbf{1}, -1)_{2,0}, \\ N_k & (\mathbf{1}, \mathbf{1}, 0)_{1,1}. \end{aligned} \quad (1)$$

We also introduce an additional doublet scalar  $\eta$  and two singlet scalars  $\sigma$  and  $S$ , whose representation and charge under the above symmetry are given as

$$\eta \left( \mathbf{1}, \mathbf{2}, -\frac{1}{2} \right)_{-1,1}, \quad \sigma (\mathbf{1}, \mathbf{1}, 0)_{-2,2}, \quad S (\mathbf{1}, \mathbf{1}, 0)_{0,2}. \quad (2)$$

We note that this global  $U(1)$  has color anomaly as one in the KSVZ model [15, 16] and it can play a role of the PQ symmetry. Its charge assignment guarantees the domain wall number is one ( $N = 1$ ) so that the model can escape the domain wall problem. We assume that the model is  $CP$  invariant and then parameters contained in Lagrangian are all real.

The model is characterized by new Yukawa terms and scalar potential which are invariant under the imposed symmetry

$$\begin{aligned} -\mathcal{L}_Y = & y_D \sigma \bar{D}_L D_R + y_E \sigma \bar{E}_L E_R + \sum_{k=1}^3 \left[ \frac{y_{N_k}}{2} \sigma \bar{N}_k^c N_k \right. \\ & + y_{d_k} S \bar{D}_L d_{R_k} + \tilde{y}_{d_k} S^\dagger \bar{D}_L d_{R_k} \\ & \left. + y_{e_k} S \bar{E}_L e_{R_k} + \tilde{y}_{e_k} S^\dagger \bar{E}_L e_{R_k} + \sum_{\alpha=1}^3 h_{\alpha k}^* \eta \bar{\ell}_\alpha N_k \right] + \text{h.c.}, \end{aligned}$$

<sup>2</sup> Similar models with vector-like extra fermions have been considered under different symmetry structure in several contexts [30, 31].

$$\begin{aligned} V = & \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi)(\eta^\dagger \eta) \\ & + \lambda_4 (\phi^\dagger \eta)(\eta^\dagger \phi) + \frac{\lambda_5}{2M_*} [\sigma (\phi^\dagger \eta)^2 + \text{h.c.}] \\ & + \kappa_\sigma (\sigma^\dagger \sigma)^2 + \kappa_S (S^\dagger S)^2 + (\kappa_{\phi\sigma} \phi^\dagger \phi + \kappa_{\phi\sigma} \eta^\dagger \eta)(\sigma^\dagger \sigma) \\ & + (\kappa_{\phi S} \phi^\dagger \phi + \kappa_{\eta S} \eta^\dagger \eta)(S^\dagger S) \\ & + \kappa_{\sigma S} \sigma^\dagger \sigma S^\dagger S + m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta \\ & + m_\sigma^2 \sigma^\dagger \sigma + m_S^2 S^\dagger S + V_b, \end{aligned} \quad (3)$$

where  $\ell_\alpha$  is a doublet lepton and  $\phi$  is an ordinary doublet Higgs scalar.  $d_{R_k}$  and  $e_{R_k}$  are the SM down-type quarks and charged leptons, respectively. In Eq. (3), we list dominant terms up to dimension five and  $M_*$  is a cut-off scale of the model.  $V_b$  contains terms which are invariant under the global symmetry but violate the  $S$  number such as  $S^4$  and  $S^{\dagger 4}$ . If the singlet scalars  $\sigma$  and  $S$  get vacuum expectation values (VEVs) much larger than the weak scale

$$\langle \sigma \rangle = w e^{i\chi}, \quad \langle S \rangle = u e^{i\rho}, \quad (4)$$

we could have a low energy effective model with spontaneous  $CP$  violation. The global symmetry  $U(1) \times Z_4$  is broken to its diagonal subgroup  $Z_2$  by these VEVs. This  $Z_2$  guarantees the stability of a DM candidate as discussed later.

Although we find  $\chi = 0$  due to the global  $U(1)$  symmetry [32],  $V_b$  can cause a non-zero  $CP$  phase  $\rho$  at a potential minimum through  $\frac{\partial V_b}{\partial \rho} = 0$ . In fact, if we assume as an example,

$$V_b = \alpha (S^4 + S^{\dagger 4}) + \beta \sigma^\dagger \sigma (S^2 + S^{\dagger 2}), \quad (5)$$

$\cos 2\rho = -\frac{\beta}{4\alpha} \frac{w^2}{u^2}$  is obtained and its stability is found to require  $\cos^2 2\rho < \frac{3}{4}$ . Since the global  $U(1)$  symmetry works as the PQ symmetry, the VEV  $w$  should be assumed to satisfy  $10^9$  GeV  $\lesssim w \lesssim 10^{11}$  GeV. <sup>6</sup>

The axion in this model is characterized by a coupling with photon  $g_{a\gamma\gamma} = 1.51 \frac{10^{-10}}{\text{GeV}} \frac{(m_a)}{\text{eV}}$  [33]. If we redefine each radial component of singlet fields around the vacuum as  $\sigma = w + \frac{1}{\sqrt{2}} \tilde{\sigma}$  and  $S = u + \frac{1}{\sqrt{2}} \tilde{S}$ , the mass of  $\tilde{\sigma}$  and  $\tilde{S}$  is found to be  $m_{\tilde{\sigma}}^2 = 4\kappa_\sigma w^2$  and  $m_{\tilde{S}}^2 = 4\kappa_S u^2$ , respectively. In the example given by Eq. (5), mass of an orthogonal component to  $\tilde{S}$  is found to be  $12\alpha u^2 (1 - \frac{4}{3} \cos^2 2\rho)$ . Its detail depends on the assumed  $V_b$ .

Here we note that the effective model after the symmetry breaking can give an explanation for origin of  $CP$  phases in the CKM and PMNS matrices and the generation of neutrino masses. The former is based on a mass matrix which has been discussed by BBP [34] as a simple realization of Nelson–Barr mechanism [35–37] for the strong  $CP$  problem. The latter is based on the fact that the leptonic sector coincides with the scotogenic model [38]. In the next part, we discuss them in some detail.

## 2.1 $CP$ violating phases in CKM and PMNS matrices

The Yukawa couplings of down-type quarks and charged leptons shown in Eq. (3) derive mass terms such as

$$(\bar{f}_{Li}, \bar{F}_L) \mathcal{M}_f \begin{pmatrix} f_{Rj} \\ F_R \end{pmatrix} + \text{h.c.}, \quad \mathcal{M}_f = \begin{pmatrix} m_{fij} & 0 \\ \mathcal{F}_{fj} & \mu_F \end{pmatrix}, \quad (7)$$

where  $\mathcal{M}_f$  is a  $4 \times 4$  matrix. Characters  $f$  and  $F$  represent  $f = d, e$  and  $F = D, E$  for down-type quarks and charged leptons, respectively. Each component of  $\mathcal{M}_f$  is expressed as  $m_{fij} = y_{fij} \langle \phi \rangle$ ,  $\mathcal{F}_{fj} = (y_{fj} u e^{i\rho} + \tilde{y}_{fj} u e^{-i\rho})$  and  $\mu_F = y_F w$ . This mass matrix is found to have the same form proposed by BBP [34]. Since the global  $U(1)$  symmetry works as the PQ symmetry and all parameters in the model are assumed to be real,  $\bar{\theta} = \theta_{\text{QCD}} + \arg(\det \mathcal{M}_f) = 0$  is satisfied even if radiative effects are taken into account after the spontaneous breaking of the  $CP$  symmetry.

We consider diagonalization of a matrix  $\mathcal{M}_f \mathcal{M}_f^\dagger$  by a unitary matrix such as

$$\begin{pmatrix} A_f & B_f \\ C_f & D_f \end{pmatrix} \begin{pmatrix} m_f m_f^\dagger & m_f \mathcal{F}_f^\dagger \\ \mathcal{F}_f m_f^\dagger & \mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger \end{pmatrix} \begin{pmatrix} A_f^\dagger & C_f^\dagger \\ B_f^\dagger & D_f^\dagger \end{pmatrix} = \begin{pmatrix} \tilde{m}_f^2 & 0 \\ 0 & \tilde{M}_F^2 \end{pmatrix}, \quad (8)$$

where a  $3 \times 3$  matrix  $\tilde{m}_f^2$  is diagonal in which generation indices are abbreviated. Equation (8) requires

$$\begin{aligned} m_f m_f^\dagger &= A_f^\dagger \tilde{m}_f^2 A_f + C_f^\dagger \tilde{M}_F^2 C_f, \\ \mathcal{F}_f m_f^\dagger &= B_f^\dagger m_f^2 A_f + D_f^\dagger M^2 C_f, \\ \mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger &= B_f^\dagger \tilde{m}_f^2 B_f + D_f^\dagger \tilde{M}_F^2 D_f. \end{aligned} \quad (9)$$

If  $\mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger$  is much larger than each components of  $\mathcal{F}_f m_f^\dagger$ , which means  $u, w \gg \langle \phi \rangle$ , we find that  $B_f, C_f$  and  $D_f$  can be approximated as

$$B_f \simeq -\frac{A_f m_f \mathcal{F}_f^\dagger}{\mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger}, \quad C_f \simeq \frac{\mathcal{F}_f m_f^\dagger}{\mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger}, \quad D_f \simeq 1. \quad (10)$$

These guarantee the unitarity of the matrix  $A$  approximately. In such a case, it is easy to find

$$\begin{aligned} A_f^{-1} \tilde{m}_f^2 A_f &\simeq m_f m_f^\dagger - \frac{1}{\mu_F \mu_F^\dagger + \mathcal{F}_f \mathcal{F}_f^\dagger} (m_f \mathcal{F}_f^\dagger) (\mathcal{F}_f m_f^\dagger), \\ \tilde{M}_F^2 &\simeq \mu_F^2 + F_f F_f^\dagger. \end{aligned} \quad (11)$$

The right-hand side of the first equation is an effective mass matrix of the ordinary fermions, which is derived through the mixing with the extra heavy fermions. Since its second term can have complex phases in off-diagonal elements as long

as  $y_{fi} \neq \tilde{y}_{fi}$  is satisfied, the matrix  $A_f$  could be complex. Moreover, if the VEVs satisfy the condition

$$\langle \phi \rangle \ll w < u, \quad (12)$$

$\mu_F^2 \lesssim \mathcal{F}_f \mathcal{F}_f^\dagger$  could be realized for suitable Yukawa couplings  $y_{fj}, \tilde{y}_{fj}$  and  $y_F$ . In that case, the complex phase of  $A_f$  in Eq. (11) could have a substantial magnitude because the second term is comparable with the first one. The CKM matrix is determined as  $V_{CKM} = O_L^T A_d$  where  $O_L$  is an orthogonal matrix used for the diagonalization of an up-type quarks mass matrix. Thus, the  $CP$  phase of  $V_{CKM}$  is caused by the one of  $A_d$ . The same argument is applied to the leptonic sector and we have the PMNS matrix as  $V_{PMNS} = A_e^\dagger U$  where  $U$  is an orthogonal matrix used for the diagonalization of a neutrino mass matrix which is discussed in the next part. The Dirac  $CP$  phase in the CKM matrix and the PMNS matrix can be explained by the same origin. A concrete example of them can be found in Appendix of [31].

## 2.2 Neutrino mass and DM

The leptonic sector of the effective model is characterized by the  $Z_2$  invariant terms

$$\begin{aligned} -\mathcal{L}_{\text{scot}} = & \sum_{k=1}^3 \left[ \sum_{\alpha=1}^3 h_{ak}^* \bar{\ell}_\alpha \eta N_k + \frac{M_{N_k}}{2} \bar{N}_k^c N_k + \text{h.c.} \right] \\ & + \tilde{m}_\phi^2 \phi^\dagger \phi + \tilde{m}_\eta^2 \eta^\dagger \eta + \tilde{\lambda}_1 (\phi^\dagger \phi)^2 + \tilde{\lambda}_2 (\eta^\dagger \eta)^2 \\ & + \tilde{\lambda}_3 (\phi^\dagger \phi)(\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta)(\eta^\dagger \phi) \\ & + \frac{\tilde{\lambda}_5}{2} [(\phi^\dagger \eta)^2 + \text{h.c.}]. \end{aligned} \quad (13)$$

It is just the scotogenic model [38]. After the spontaneous breaking due to the VEVs of  $\sigma$  and  $S$ , parameters in Eq. (13) are determined through integrating out  $\tilde{\sigma}$  and  $\tilde{S}$  by using the ones in Eq. (3) as

$$\begin{aligned} \tilde{\lambda}_1 &= \lambda_1 - \frac{\kappa_{\phi\sigma}^2}{4\kappa_\sigma} - \frac{\kappa_{\phi S}^2}{4\kappa_S} + \frac{\kappa_{\sigma S} \kappa_{\phi\sigma} \kappa_{\phi S}}{4\kappa_\sigma \kappa_S} \\ \tilde{\lambda}_2 &= \lambda_2 - \frac{\kappa_{\eta\sigma}^2}{4\kappa_\sigma} - \frac{\kappa_{\eta S}^2}{4\kappa_S} + \frac{\kappa_{\sigma S} \kappa_{\eta\sigma} \kappa_{\eta S}}{4\kappa_\sigma \kappa_S} \\ \tilde{\lambda}_3 &= \lambda_3 - \frac{\kappa_{\phi\sigma} \kappa_{\eta\sigma}}{2\kappa_\sigma} - \frac{\kappa_{\phi S} \kappa_{\eta S}}{2\kappa_S} + \frac{\kappa_{\sigma S} \kappa_{\phi\sigma} \kappa_{\eta S} + \kappa_{\sigma S} \kappa_{\eta\sigma} \kappa_{\phi S}}{4\kappa_\sigma \kappa_S} \\ \tilde{\lambda}_5 &= \lambda_5 \frac{w}{M_*}, \quad M_{N_k} = y_k w, \end{aligned} \quad (14)$$

and

$$\begin{aligned} \tilde{m}_\phi^2 &= m_\phi^2 + \left( \kappa_{\phi\sigma} + \frac{\kappa_{\phi S} \kappa_{\sigma S}}{2\kappa_S} \right) w^2 + \left( \kappa_{\phi S} + \frac{\kappa_{\phi\sigma} \kappa_{\sigma S}}{2\kappa_\sigma} \right) u^2, \\ \tilde{m}_\eta^2 &= m_\eta^2 + \left( \kappa_{\eta\sigma} + \frac{\kappa_{\eta S} \kappa_{\sigma S}}{2\kappa_S} \right) w^2 + \left( \kappa_{\eta S} + \frac{\kappa_{\eta\sigma} \kappa_{\sigma S}}{2\kappa_\sigma} \right) u^2. \end{aligned} \quad (15)$$

Since  $\eta$  is supposed to have no VEV,  $Z_2$  is kept as an exact symmetry of the model. In this model, we assume both  $|\tilde{m}_\phi|$  and  $\tilde{m}_\eta$  have values of  $O(1)$  TeV although parameter tunings are required.

Neutrino mass is forbidden at tree level due to this  $Z_2$  symmetry but it could be generated through one-loop diagrams with  $\eta$  and  $N_k$  in internal lines. Its formula is given as

$$\mathcal{M}_{\alpha\beta}^v \simeq \sum_{k=1}^3 h_{\alpha k} h_{\beta k} \lambda_5 \Lambda_k, \quad \Lambda_k = \frac{\langle \phi \rangle^2}{8\pi^2} \frac{1}{M_{N_k}} \ln \frac{M_{N_k}^2}{M_\eta^2}, \quad (16)$$

since  $M_{N_k} \gg M_\eta$  is satisfied where  $M_\eta^2 = \tilde{m}_\eta^2 + (\lambda_3 + \lambda_4)\langle \phi \rangle^2$ . In order to make the point quantitatively clear, we assume a simple flavor structure for neutrino Yukawa couplings [39]

$$\begin{aligned} h_{ei} &= 0, \quad h_{\mu i} = h_{\tau i} \equiv h_i \quad (i = 1, 2); \\ h_{e3} &= h_{\mu 3} = -h_{\tau 3} \equiv h_3. \end{aligned} \quad (17)$$

This realizes the tri-bimaximal mixing which gives a simple and good 0-th order approximation for the analysis of neutrino oscillation data and leptogenesis. If we impose the mass eigenvalues obtained from Eq. (16) to satisfy the squared mass difference required by the neutrino oscillation data, we find

$$\begin{aligned} (h_1^2 \Lambda_1 + h_2^2 \Lambda_2) |\lambda_5| &= \frac{1}{2} \sqrt{\Delta m_{32}^2}, \\ h_3^2 \Lambda_3 |\lambda_5| &= \frac{1}{3} \sqrt{\Delta m_{21}^2}. \end{aligned} \quad (18)$$

If we assume  $M_{2,3} = O(10^7)$  GeV and  $M_\eta = 1$  TeV, we have  $\Lambda_{2,3} = O(1)$  eV from the neutrino oscillation data [40], and it can be satisfied by  $h_{2,3} = O(10^{-3})$  for  $|\tilde{\lambda}_5| = 10^{-3}$ . In that case,  $h_1$  can take a very small value compared with  $h_{2,3}$ .

The axion may be difficult to be a dominant component of DM for  $f_a < 10^{10}$  GeV although it depends on the contribution from the axion string decay [25, 26]. However, fortunately, the model has another DM candidate, the lightest neutral component of  $\eta$  with  $Z_2$  odd parity. It is known to be a good DM candidate which does not cause any contradiction with known experimental data as long as its mass is in the TeV range where the coannihilation can be effective [41–45]. In fact, if the couplings  $\tilde{\lambda}_3$  and  $|\lambda_4|$  take suitable values much larger than  $|\tilde{\lambda}_5|$ , both the DM abundance and the direct detection bound can be satisfied. Inelastic scattering between the neutral components mediated by  $Z^0$  is also constrained through direct search experiments. Its experimental bound can be satisfied for  $|\tilde{\lambda}_5| \lesssim 10^{-5}$  [46–50]. We should note that this constraint is closely related to the neutrino mass generation as seen above. In the next section we examine the thermal leptogenesis at low reheating temperature  $T_R < 10^9$  GeV taking account of the constraints from the neutrino oscillation data.

### 3 Leptogenesis at low reheating temperature

We should note that the singlet scalar  $S$  could play a role of inflaton in addition to give the origin of  $CP$  violation in both quark and lepton sectors. Since the  $Z_4$  symmetry constrains its coupling with the Ricci scalar, action relevant to the present inflation scenario is given in the Jordan frame as

$$\begin{aligned} S_J = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} M_{\text{pl}}^2 R - \frac{\xi_1}{2} S^\dagger S R - \frac{\xi_2}{4} (S^2 + S^{\dagger 2}) R \right. \\ \left. + \frac{1}{2} \partial^\mu S^\dagger \partial_\mu S - V_S \right], \end{aligned} \quad (19)$$

where  $M_{\text{pl}}$  is the reduced Planck mass and  $V_S$  stands for a corresponding part of the potential for  $S$  and  $S^\dagger$  in Eq. (3). If we assume  $\xi_1 = -\xi_2$  is satisfied, the coupling of  $S$  with the Ricci scalar reduces to  $\frac{1}{2} \xi S_I^2$  where  $S = \frac{1}{\sqrt{2}} (S_R + i S_I)$  and  $\xi = \xi_1 - \xi_2$ . It is well-known that a scalar field which couples non-minimally with the Ricci scalar in this way can cause inflation of the Universe [51, 52] and the idea has been applied to the Higgs scalar in the SM [53, 54] and its singlet scalar extensions [55, 56].<sup>3</sup>

If  $S$  is assumed to evolve along a constant  $\rho$  which is determined as a potential minimum  $\frac{\partial V_b}{\partial \rho} = 0$ , the radial component  $\tilde{S}$  can be identified with inflaton in this model. We suppose that  $\tilde{S}$  takes a large field value of  $O(M_{\text{pl}})$  as an inflaton and other scalars have much smaller values than it during the inflation. In that case,  $V_S$  can be expressed as  $V_S = \frac{\kappa_S}{4} \tilde{S}^4$ . After conformal transformation for a metric tensor in the Jordan frame, the corresponding potential  $U$  and the canonically normalised inflaton  $\chi$  in the Einstein frame is found to be written as

$$U = \frac{\frac{1}{4} \kappa_S \tilde{S}^4}{(1 + \frac{\xi \sin^2 \rho}{M_{\text{pl}}^2} \tilde{S}^2)^2}, \quad \frac{d\chi}{d\tilde{S}} = \sqrt{6 + \frac{1}{\xi \sin^2 \rho} \frac{M_{\text{pl}}}{\tilde{S}}}. \quad (20)$$

where we use  $S_I = \tilde{S} \sin \rho$ . If  $\tilde{S} \gg \frac{M_{\text{pl}}}{\sqrt{\xi \sin \rho}}$  is satisfied,  $U$  takes a constant value  $\frac{\kappa_S}{4\xi^2 \sin^4 \rho} M_{\text{pl}}^4$  to cause inflation. Since  $\rho$  is supposed to give the origin of  $CP$  violating phases in the CKM and PMNS matrices,  $\sin \rho = O(1)$  seems to be favored from a view point of low energy effective theory. If we use the e-foldings number  $N$ , slow-roll parameters in this inflation scenario is found to be expressed as  $\epsilon = \frac{3}{4N^2}$  and  $\eta = -\frac{1}{N}$  when  $6\xi \sin^2 \rho \gg 1$  is satisfied. If we suppose such a case, the scalar spectral index  $n_s$  and the tensor-to-scalar ratio  $r$  are given as  $n_s \sim 0.965$  and  $r \sim 3.3 \times 10^{-3}$  for  $N = 60$ , which coincide well with the ones suggested by the Planck data [62].

<sup>3</sup> Inflation in the scotogenic model extended with singlet scalars has been discussed from several motivations [57–61].

The spectrum of density perturbation predicted by the inflation is known to be expressed as

$$\mathcal{P}(k) = A_s \left( \frac{k}{k_*} \right)^{n_s-1}, \quad A_s = \frac{U}{24\pi^2 M_{\text{pl}}^4 \epsilon} \Big|_{k_*}. \quad (21)$$

If we use  $A_s = (2.101^{+0.031}_{-0.034}) \times 10^{-9}$  at  $k_* = 0.05 \text{ Mpc}^{-1}$  [62], we find the Hubble parameter during the inflation to be  $H_I = 1.4 \times 10^{13} \left( \frac{60}{N} \right) \text{ GeV}$  and the relation

$$\kappa_S \simeq 1.49 \times 10^{-6} \xi^2 \sin^4 \rho N^{-2}, \quad (22)$$

which should be satisfied at the horizon exit time of the scale  $k_*$ . Since the quartic coupling  $\kappa_S$  is a free parameter in this model, it allows  $\xi$  to take a much smaller value in comparison with the one of the usual Higgs inflation. For example,  $\xi \sin^2 \rho = O(10^2)$  realizes the observed value of  $A_s$  for  $N = 60$  if  $\kappa_S = O(10^{-6})$  is assumed.

After the end of inflation, the inflaton starts the oscillation around the vacuum  $\langle S \rangle$ . During this oscillation, inflaton is expected to decay to light fields. If we assume the mass pattern

$$2\tilde{m}_\eta < m_{\tilde{S}} < \tilde{M}_D, \tilde{M}_E, \quad (23)$$

the inflaton decay is expected to occur mainly through  $\tilde{S} \rightarrow \eta^\dagger \eta$  at tree level. The decay width could be estimated as

$$\Gamma \simeq \frac{\kappa_{\eta S}^2}{16\pi \kappa_S} m_{\tilde{S}} \sqrt{1 - \frac{4\tilde{m}_\eta^2}{m_{\tilde{S}}^2}}. \quad (24)$$

After the inflaton decays to  $\eta^\dagger \eta$ , the SM contents are thermalized through gauge interactions with  $\eta$  immediately. The reheating temperature can be estimated as

$$T_R \simeq 1.6 \times 10^8 \left( \frac{10^{-6}}{\kappa_S} \right)^{1/4} \left( \frac{\kappa_{\eta S}}{10^{-6.5}} \right) \left( \frac{u}{10^{10} \text{ GeV}} \right)^{1/2} \text{ GeV}. \quad (25)$$

As long as  $\tilde{M}_D, \tilde{M}_E < T_R$  is satisfied, these fermions are also expected to be thermalized immediately through gauge interactions. On the other hand, the right-handed neutrinos are considered to be thermalized through the neutrino Yukawa couplings. Here, we note that neutrino mass eigenvalues obtained from Eq. (16) require  $h_{2,3} = O(10^{-3})$  to explain the neutrino oscillation data if  $|\tilde{\lambda}_5| = O(10^{-3})$  and  $M_{N_{2,3}} = O(10^7) \text{ GeV}$  are assumed. Since the decay width  $\Gamma_{N_{2,3}}$  of  $N_{2,3}$  and  $T_R$  satisfy  $\Gamma_{N_{2,3}} > H(T_R)$  and  $T_R > M_{N_{2,3}}$  in such a case,  $N_{2,3}$  are also expected to be thermalized through the inverse decay simultaneously at the reheating period. However, it is not the case for  $N_1$  if its Yukawa coupling  $h_1$  is much smaller than them.

Baryon number asymmetry in the Universe [63–65] is expected to be generated through leptogenesis [27, 28] in this model. However, if reheating temperature is lower than

$10^9 \text{ GeV}$  which corresponds to the lower bound of the PQ symmetry breaking, leptogenesis might not work well since it coincides with the usually considered lower bound for successful leptogenesis in the seesaw framework [29]. Since both the production of the right-handed neutrinos and the generation of lepton number asymmetry through its out-of-equilibrium decay have to be caused only by neutrino Yukawa couplings there, it is difficult to yield a required lepton number asymmetry in a consistent way with the small neutrino mass generation. This situation does not change in the original scotogenic model either [44, 45].<sup>4</sup>

In the present model, the interaction between the right-handed neutrino  $N_1$  and extra vector-like fermions mediated by  $\tilde{\sigma}$  could change the situation in the similar way to the one discussed in [67, 68]. In fact, scattering  $\tilde{D}_L D_R, \tilde{E}_L E_R \rightarrow N_1 N_1$  mediated by  $\tilde{\sigma}$  could effectively produce the lightest right-handed neutrino  $N_1$  in the thermal bath since  $D_{L,R}$  and  $E_{L,R}$  are in the thermal equilibrium as mentioned above. We can examine this possibility briefly. We note first that the coupling constant  $y_{N_1}$  should take a value of  $O(10^{-2})$  at least to satisfy  $M_{N_1} < T_R$  since the VEV  $w$  should be larger than  $10^9 \text{ GeV}$ . Since the scattering becomes effective at the temperature  $T$  such that  $H(T) \simeq \Gamma_F$  where  $\Gamma_F$  is the reaction rate of this scattering,  $T > M_{N_1}$  should be satisfied to escape the Boltzmann suppression. Rough estimation of this condition gives

$$T \simeq 6 \times 10^8 \left( \frac{y_F}{10^{-1.2}} \right)^2 \left( \frac{y_{N_1}}{10^{-2}} \right)^2 \text{ GeV}, \quad (26)$$

and we find that the present scenario could work for suitable  $y_F$  since  $T > M_{N_1}$  could be satisfied. We should note that this does not depend on the magnitude of the neutrino Yukawa coupling  $h_1$  of  $N_1$ . It allows us to have successful leptogenesis even under  $T_R < 10^9 \text{ GeV}$ . Although this scattering process is expected to expand the allowed parameter space in the case  $T_R > 10^9 \text{ GeV}$ , we focus our present study only on the low scale leptogenesis at  $T_R < 10^9 \text{ GeV}$ .

If  $N_1$  is produced in the thermal bath successfully through the above mentioned extra fermions scattering mediated by  $\tilde{\sigma}$ , it decays to  $\ell_\alpha \eta^\dagger$  in out-of-equilibrium through a strongly suppressed Yukawa coupling  $h_1$ . Since the decay is delayed until a period where the washout process caused by the inverse decay could be freezed-out and the generated lepton number asymmetry can be effectively converted to baryon number asymmetry through sphaleron processes. We can check this scenario by solving Boltzmann equations for  $Y_{N_1}$  and  $Y_L (\equiv Y_\ell - Y_{\bar{\ell}})$ , which are defined by using  $\psi$  number

<sup>4</sup> Low scale leptogenesis in the scotogenic model has been studied intensively in [66]. However, the right-handed neutrino  $N_1$  is assumed to be in the thermal equilibrium initially. Some additional interactions are required to generate its thermal abundance. It is noticeable that such an interaction is included as an indispensable ingredient in the present model.

density  $n_\psi$  and entropy density  $s$  as  $Y_\psi = \frac{n_\psi}{s}$ . An equilibrium value of  $Y_\psi$  is represented by  $Y_\psi^{\text{eq}}$ . The Boltzmann equations analyzed here are given as

$$\begin{aligned} \frac{dY_{N_1}}{dz} &= -\frac{z}{sH(M_{N_1})} \left( \frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \\ &\times \left[ \gamma_D^{N_1} + \left( \frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} + 1 \right) \sum_{F=D,E} \gamma_F \right], \\ \frac{dY_L}{dz} &= -\frac{z}{sH(M_{N_1})} \left[ \varepsilon \left( \frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \gamma_D^{N_1} \right. \\ &\left. - \frac{2Y_L}{Y_\ell^{\text{eq}}} \sum_{k=1,2,3} \left( \frac{\gamma_D^{N_k}}{4} + \gamma_{N_k} \right) \right], \end{aligned} \quad (27)$$

where  $z = \frac{M_{N_1}}{T}$  and  $H(T)$  is the Hubble parameter at temperature  $T$ .  $CP$  asymmetry for the decay of  $N_1$  is expressed as  $\varepsilon$ .  $\gamma_D^{N_k}$  is a reaction density for the decay  $N_k \rightarrow \ell_\alpha \eta^\dagger$ , and  $\gamma_{N_k}$  is reaction density for lepton number violating scattering mediated by  $N_k$  [44, 45].  $\gamma_F$  represents a reaction density for the scattering  $\bar{D}_L D_R, \bar{E}_L E_R \rightarrow N_1 N_1$ . As an initial condition at  $T_R$ , we assume  $Y_{N_1} = Y_L = 0$ , and also  $(D_L, D_R)$  and  $(E_L, E_R)$  are in the thermal equilibrium.

Now we fix the VEVs as  $w = 10^9$  GeV and  $u = 10^{10}$  GeV as a typical example. In numerical study of Eq. (27), we fix the relevant parameters at the intermediate scale  $w$  to guarantee the generation of  $CP$  phases in the CKM and PMNS matrices through the mechanism discussed in the previous part. We require  $\tilde{M}_F^2 = \mu_F^2 + \mathcal{F}_f \mathcal{F}_f^\dagger > \mu_F^2$  for it. If we define  $\gamma$  as  $\tilde{M}_F = \gamma \mu_F$ , it should satisfy  $\gamma > 1$ . For simplification, we consider a case  $y_f = (0, 0, y)$  and  $\tilde{y}_f = (0, \tilde{y}, 0)$ , which brings about to a relation  $y^2 + \tilde{y}^2 = (\gamma^2 - 1) \frac{w^2}{u^2} y_F^2$  among Yukawa couplings of extra fermions. If  $y_F$ ,  $\gamma$  and  $\tilde{y}/y$  are fixed,  $y$  and  $\tilde{y}$  can be determined through this relation. We take  $\tilde{y}/y = 0.5$  in this study. By using these parameters, we can check the condition  $\tilde{M}_D, \tilde{M}_E, M_{N_k} < T_R$ . The stability of the inflaton potential at the inflation period can be also examined by using the RGEs with contributions from the extra vector-like fermions. The  $\kappa_S$  at the inflation period also allows us to find a value of  $\xi$  through Eq. (22).

As a typical value of parameters which could satisfy these conditions, we adopt  $y_D = y_E = 10^{-1.2}$ ,  $\kappa_\sigma = 10^{-4.5}$  and  $\kappa_{S\eta} = 10^{-6.5}$  at the intermediate scale  $w$ . We also fix parameters relevant to neutrino mass generation as

$$\begin{aligned} y_{N_1} &= 10^{-2}, \quad y_{N_2} = 2 \times 10^{-2}, \quad y_{N_3} = 4 \times 10^{-2}, \\ h_1 &= 6 \times 10^{-7}, \quad |\tilde{\lambda}_5| = 10^{-3}, \end{aligned} \quad (28)$$

and  $M_\eta = 1$  TeV. Since the mass of  $N_k$  is of  $O(10^7)$  GeV for these parameters, neutrino Yukawa couplings  $h_{2,3}$  are determined to be of  $O(10^{-3})$  by using the neutrino mass formula (16) and the neutrino oscillation data [40]. If we

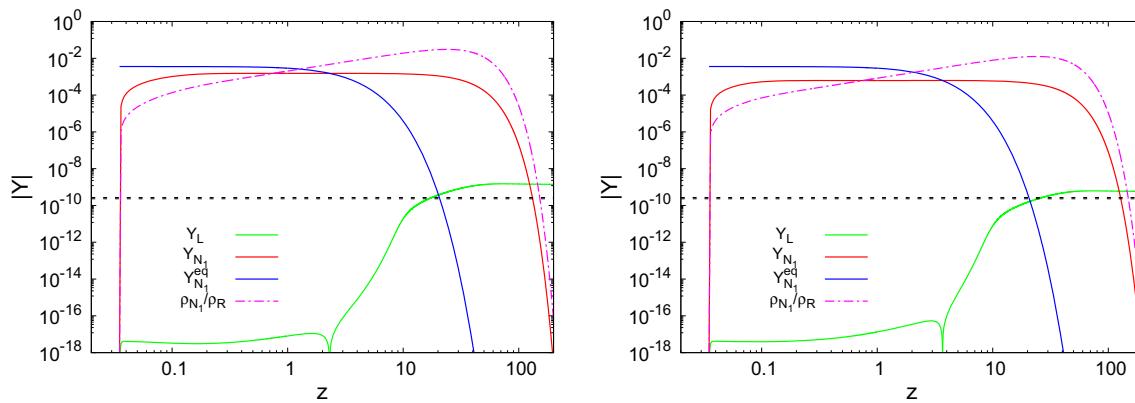
**Table 1** Expected values of nonminimal coupling, reheating temperature and baryon number asymmetry generated through leptogenesis.  $\kappa_S$  is a value at an intermediate scale  $w$  and  $\xi$  is estimated at an inflation period by using one-loop RGEs. The condition for the substantial generation of the  $CP$  phases in the CKM and PMNS matrices is satisfied for the used  $\gamma$

	$\kappa_S$	$\gamma$	$\xi \sin^2 \rho$	$T_R$ (GeV)	$Y_B$
(A)	$10^{-6}$	$\sqrt{2}$	49.1	$1.6 \times 10^8$	$3.3 \times 10^{-10}$
(B)	$10^{-6}$	2	48.9	$1.6 \times 10^8$	$1.6 \times 10^{-10}$
(C)	$10^{-6}$	$\sqrt{6}$	48.4	$1.6 \times 10^8$	$9.6 \times 10^{-11}$
(D)	$10^{-7}$	$\sqrt{2}$	15.4	$2.8 \times 10^8$	$4.9 \times 10^{-10}$
(E)	$10^{-7}$	2	14.6	$2.8 \times 10^8$	$2.9 \times 10^{-10}$
(F)	$10^{-7}$	$\sqrt{6}$	12.7	$2.8 \times 10^8$	$2.0 \times 10^{-10}$

assume a maximum  $CP$  phase in the  $CP$  asymmetry  $\varepsilon$  in the  $N_1$  decay,  $\varepsilon$  takes a value of  $O(10^{-7})$  for this parameter setting. Parameters  $\kappa_S$  and  $\gamma$  are fixed to the values given in Table 1 at the intermediate scale  $w$ .

Solutions for the Boltzmann equations for different  $\gamma$  values are shown in Fig. 1, which confirms the present scenario to work. The figures show that  $Y_{N_1}$  reaches a value near its equilibrium one  $Y_{N_1}^{\text{eq}}$  through the scattering of the extra fermions as expected. The out-of-equilibrium decay substantially occurs at  $z > 10$  to generate the lepton number asymmetry. This delay of the decay due to the small  $h_1$  could make the lepton number asymmetry possible to escape the effective washout-out. Sufficient lepton number asymmetry is found to be produced before the sphaleron decoupling at  $z_{EW} \sim \frac{M_{N_1}}{10^2 \text{ GeV}}$ . If  $\gamma$  becomes larger, the mass of extra fermions  $\tilde{M}_F$  becomes larger to suppress the reaction density  $\gamma_F$  due to the Boltzmann factor. As a result, the  $N_1$  number density generated through the scattering becomes smaller and the resulting lepton number asymmetry also becomes smaller as shown in the figures. If  $h_1$  is much smaller, entropy produced through the decay of relic  $N_1$  might dilute the generated lepton number asymmetry. If  $|\tilde{\lambda}_5|$  is taken to be smaller, Yukawa couplings  $h_{2,3}$  become larger and the washout effect could remain effective until a later stage to reduce the resulting baryon number asymmetry.

In the last column of Table 1, the baryon number asymmetry generated for the assumed parameters is presented. The difference of  $Y_B$  for the same  $\gamma$  can be also explained as a result of suppression of the reaction density  $\gamma_F$  by the Boltzmann factor for extra fermions, which becomes smaller in the case with lower reheating temperature. This result shows that the model with suitable parameters can generate a sufficient amount of baryon number asymmetry through leptogenesis although the reheating temperature is lower than  $10^9$  GeV. We should note that neutrino Yukawa couplings  $h_k$  change their values depending on  $w$  under the constraints of the neutrino oscillation data since the right-handed neutrino mass is



**Fig. 1** Evolution of  $Y_{N_1}$  and  $Y_L$  obtained as a solution of Boltzmann equations. The left panel is one for the case (D) in Table 1 and the right panel is one for the case (F). Initial values for them are fixed as

$Y_{N_1} = Y_L = 0$  at  $z = z_R$ .  $\rho_{N_1}/\rho_R$  represents a ratio of the energy density of  $N_1$  to the one of radiation

generated through  $M_{N_k} = y_k w$ . The extra fermion mass  $\tilde{M}_F$  is also fixed by  $w$  and  $\gamma$ . Since  $w$  is fixed as the lower bound of the  $PQ$  symmetry breaking and  $\gamma > 1$  is imposed for the realization of the  $CP$  phases in the CKM and PMNS matrices, we cannot expect that a scale of successful leptogenesis becomes much lower than the examples given here. If we do not impose these conditions, a sufficient  $Y_B$  could be generated even for much lower reheating temperature.

As mentioned already, it is difficult for the axion to be a dominant component of DM in the present parameter setting. However, the model has an alternative candidate, the lightest neutral component of  $\eta$ , as an indispensable component of the model. Thus, the model can give a simultaneous explanation for the strong  $CP$  problem, the origin of  $CP$  phases in the CKM and PMNS matrices, leptogenesis and DM relic abundance for the reheating temperature lower than  $10^9$  GeV.

#### 4 Summary

We have proposed a model which can give an explanation for the strong  $CP$  problem and the origin of the  $CP$  phases in both the CKM and PMNS matrices. It is a simple extension of the SM with vector-like extra fermions and several scalars. If the  $CP$  is spontaneously broken in a singlet scalar sector at an intermediate scale, it can be transformed to the CKM and PMNS matrices through mixings between the extra fermions and the ordinary quarks or the charged leptons. Their couplings are controlled by the imposed global symmetry. On the other hand, since the colored extra fermions play the same role as the ones in the KSVZ model for the strong  $CP$  problem, the strong  $CP$  problem can be solved through the  $PQ$  mechanism. After the symmetry breaking due to the singlet scalars, the leptonic sector of the model is reduced to the scotogenic model, which can explain the small neu-

trino mass and the DM abundance. We have shown that the model has an interesting feature in addition to these. The extra fermions could make the thermal leptogenesis possible to generate the sufficient baryon number asymmetry even if the right-handed neutrino mass is much lower than  $10^9$  GeV, which is well-known lower bound of the right-handed neutrino mass for successful leptogenesis in the ordinary seesaw scenario. Although the axion cannot be a dominant component of DM in that case, a neutral component of the inert doublet scalar can explain the DM abundance just as in the scotogenic model. It is remarkable that the model can explain various issues in the SM although the model is rather simple.

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