

# v-USPhydro: Bulk Viscosity Effects on Event-by-Event Relativistic Hydrodynamics

**Jacquelyn Noronha-Hostler**

Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

E-mail: jakinoronhahostler@gmail.com

**Gabriel S. Denicol**

Department of Physics, McGill University, 3600 University Street, Montreal, Quebec, H3A 2T8, Canada

**Jorge Noronha**

Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

**Rone P. G. Andrade**

Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

**Frédérique Grassi**

Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

## Abstract.

v-USPhydro is a new relativistic 2+1 Lagrangian hydrodynamic code that incorporates the effects of bulk viscous hydrodynamics using Smoothed Particle Hydrodynamics (SPH) and is applicable to heavy ion collisions. Within this framework the bulk viscosity effects on collective flow harmonics i.e.  $v_2 - v_5$  are studied on an event-by-event basis. We discuss which corrections to the Cooper Frye model are most appropriate when bulk viscosity is considered. An enhancement of all the Fourier harmonics is seen when bulk viscosity correction to the Cooper Frye is considered even when the bulk viscosity to entropy density ratio,  $\zeta/s$ , is significantly smaller than  $1/(4\pi)$ .

## 1. Introduction

The Quark Gluon Plasma (QGP) found within heavy ion collisions is said to be a nearly perfect fluid. The large degree of collectivity evidenced by the Fourier harmonics of the azimuthal momentum distribution of charged hadrons are compatible [1] with viscous hydrodynamic calculations in which the shear viscosity to entropy density ratio,  $\eta/s$ , nears the uncertainty principle estimate  $\sim 1/(4\pi)$  [2, 3]. Recent studies on  $\eta/s$  suggest that there may be a minimum close to the critical region [4, 5]. Then  $\eta/s$  is larger in the high temperature perturbative regime [6, 7], is low close to the critical temperature in the hadron gap phase due to Hagedorn States [8], and increases again at low temperatures [9]. Thus far, the relativistic hydrodynamical models



used to study the collective flow effects have primarily only considered shear viscosity [10, 11] with the exception of a few that considered a nonzero  $\zeta/s$  within averaged initial conditions [12, 13, 14, 15, 16, 17] while some aspects involving event-by-event simulations have been studied in [18]. There is no a priori reason to neglect bulk viscosity, especially on an event-by-event basis so the purpose of this proceedings is to explore the effects of bulk viscosity within the framework of a 2+1 relativistic hydrodynamical code on an event-by-event basis. The hydrodynamical code is called viscous Ultrarelativistic Smoothed Particle hydrodynamics (v-USPhydro) [19], which is written in C++ using the Lagrangian method of Smoothed Particle Hydrodynamics (SPH).

## 2. Setup

Within v-USPhydro we assume a vanishing baryon chemical potential and the conservation of energy and momentum is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}T^{\mu\nu}) + \Gamma_{\lambda\mu}^\nu T^{\lambda\mu} = 0 \quad (1)$$

where  $\sqrt{-g} = \tau$  and the Christoffel symbol is

$$\Gamma_{\lambda\mu}^\nu = \frac{1}{2}g^{\nu\sigma}(\partial_\mu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\mu} - \partial_\sigma g_{\mu\lambda}). \quad (2)$$

The most general expression for the energy-momentum tensor (in the absence of shear viscosity effects) is

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu}, \quad (3)$$

where  $\Pi$  is the bulk viscous pressure, the spatial projector is  $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ . We employ the simplest second order formulation of the fluid dynamical equations of motion that can be causal and stable [15]

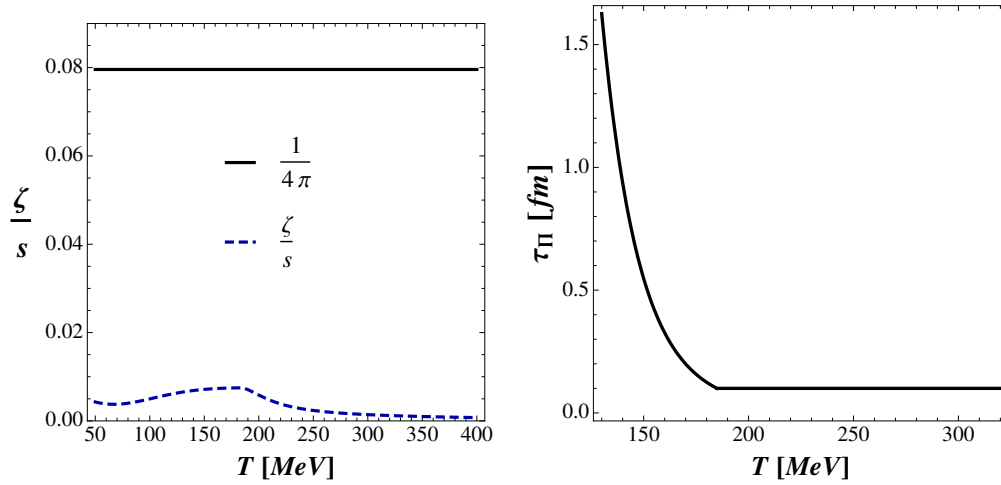
$$\tau_\Pi (D\Pi + \Pi\theta) + \Pi + \zeta\theta = 0, \quad (4)$$

where  $D = u^\mu \partial_\mu$  is the comoving covariant derivative,  $\theta = \tau^{-1} \partial_\mu (\tau u^\mu)$  is the fluid expansion rate,  $\zeta$  is the bulk viscosity, and  $\tau_\Pi$  is the relaxation time coefficient required to preserve causality [20].

This leaves us with two transport coefficients  $\zeta/s$  and  $\tau_\pi$  which are chosen in this work to be

$$\begin{aligned} \frac{\zeta}{s} &= \frac{1}{8\pi} \left( \frac{1}{3} - c_s^2 \right) \\ \tau_\Pi &= 9 \frac{\zeta}{\varepsilon - 3p} \end{aligned} \quad (5)$$

which are inspired by the calculations in [21] and [22], respectively. The coefficients are shown in Fig. 1. Here  $\varepsilon$  is the energy density and  $p$  is the pressure. We choose the time step  $dt = 0.1$  fm in our calculations. Note that, in order to solve the relaxation-type equations correctly, we must ensure that the relaxation time is larger than the time step used in the simulation. Since  $\tau_{\pi_i}$  goes to zero at large temperatures, this makes the simulation very time consuming. In our calculations, we fixed the time step to be  $dt = 0.1$  and use this as a lower bound for  $\tau_{\pi_i}$ , hence, the flattening in Fig. 1. Because the time step is small, such a lower bound does not affect the observables computed in this work. The initial conditions are taken from a Monte Carlo Glauber code [23]. Our initial time is  $t_0 = 1 fm/c$ , our freeze-out temperature is taken to be  $T = 150$  MeV, and we use the Lattice based equation of state (S95n-v1) in [24]. At this point no decays are included and all graphs in the Results Section only consider direct  $\pi^+$  results for the flow harmonics. In future work we will include hadronic decays and it would be especially interesting to include the effects of heavier resonances known as Hagedorn states [25].



**Figure 1.** The temperature dependence of the transport coefficients  $\zeta/s$  and  $\tau_{\Pi}$ , see Eq. (5).

As stated previously, we employ SPH, which is a Lagrangian method for solving hydrodynamics. In the SPH approach one introduces a conserved reference density current  $J^\mu = \sigma u^\mu$  where  $\sigma$  is the local density of a fluid element in its rest frame and  $h$  is known as the SPH length scale. As the fluid flows, the cell is deformed but its density obeys  $D\sigma + \sigma\theta = 0$ , which in hyperbolic coordinates is equivalent to  $\partial_\mu(\tau\sigma u^\mu) = 0$ . In terms of this reference density, the equations of motion defined above can be written as

$$\gamma \frac{d}{d\tau} \left[ \frac{(\varepsilon + p + \Pi)}{\sigma} u^\mu \right] = \frac{1}{\sigma} \partial^\mu (p + \Pi) \quad (6)$$

$$\gamma \frac{d}{d\tau} \left( \frac{s}{\sigma} \right) + \left( \frac{\Pi}{\sigma} \right) \frac{\theta}{T} = 0 \quad (7)$$

$$\tau_{\Pi} \gamma \frac{d}{d\tau} \left( \frac{\Pi}{\sigma} \right) + \frac{\Pi}{\sigma} + \left( \frac{\zeta}{\sigma} \right) \theta = 0. \quad (8)$$

These equations are completely equivalent to the above equations of motion but they are more suitable for the Lagrangian implementation via SPH.

The effects of the bulk viscosity must also be applied to the Cooper Frye method of freezeout. The distribution function can be split into an ideal term with a bulk correction for  $\pi^+$

$$f_{\mathbf{k}}^{(\pi)} = f_{0\mathbf{k}}^{(\pi)} + \delta f_{\mathbf{k}}^{(\pi)} \quad (9)$$

where the ideal term  $f_{0\mathbf{k}}^{(\pi)}$  is described by

$$f_{0\mathbf{k}}^{(\pi)} = \left( e^{E_{(\pi)}/T} + a_{(\pi)} \right)^{-1} \quad (10)$$

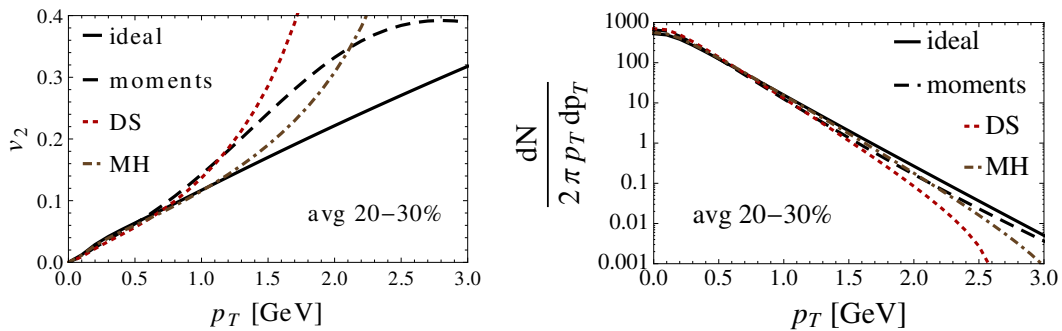
where  $E_{(\pi)}$  is the energy of the pion,  $T$  is the freeze out temperature, and  $a_{(\pi)}$  is -1 since  $\pi^+$  is a boson. Using the method of moments described in Refs. [26, 27], we compute the non-equilibrium contribution  $\delta f_{\mathbf{k}}^{(\pi)}$  associated with bulk viscosity effects to the momentum distribution function of a hadronic mixture. We obtain the following  $\delta f$  description

$$\delta f_{\mathbf{k}}^{(\pi)} = f_{0\mathbf{k}}^{(\pi)} \Pi \left[ B_0^{(\pi)} + D_0^{(\pi)} u \cdot k_\pi + E_0^{(\pi)} (u \cdot k_\pi)^2 \right]. \quad (11)$$

where the coefficients  $B_0$ ,  $D_0$ , and  $E_0$  for pions with freeze-out temperature  $T_{FO} = 150$  MeV are

$$\begin{aligned} B_0^{(\pi)} &= -65.85 \text{ fm}^4, \\ D_0^{(\pi)} &= 171.27 \text{ fm}^4/\text{GeV}, \\ E_0^{(\pi)} &= -63.05 \text{ fm}^4/\text{GeV}^2, \end{aligned} \quad (12)$$

see details in [19]. For freeze-out temperatures lower than 150 MeV, the  $\delta f$  contribution to the distribution function can become comparable to the ideal distribution  $f_0$ , which makes a perturbative analysis of the viscous effects at freeze-out unreliable. Thus, we used  $T_{FO} = 150$  MeV for the calculations in this paper. We also consider the two other derivations of  $\delta f$  described in [12] (MH) and in [17] (DS). A comparison of the three methods can be seen in Fig. 2 in which only the moments method has a well-behaved description all the way up to  $p_T = 3$  GeV for the elliptic flow and, thus, will be used for the rest of this paper. The results are for mid-rapidity RHIC's  $\sqrt{200}$  GeV 20 – 30% most central collisions in the case where the initial condition corresponds to a single average Glauber initial condition averaged over 150 events. The ideal case is the solid black line, our result for  $v_2$  computed using the  $\delta f$  obtained via the Moments method is the long dashed black line, results for the  $\delta f$  described in [17] is the short dashed red curve, while the short and long dashed brown curve is the result computed using the  $\delta f$  described in [12]. One can see also for the spectra that the results from [12] and [17] are significantly steeper.

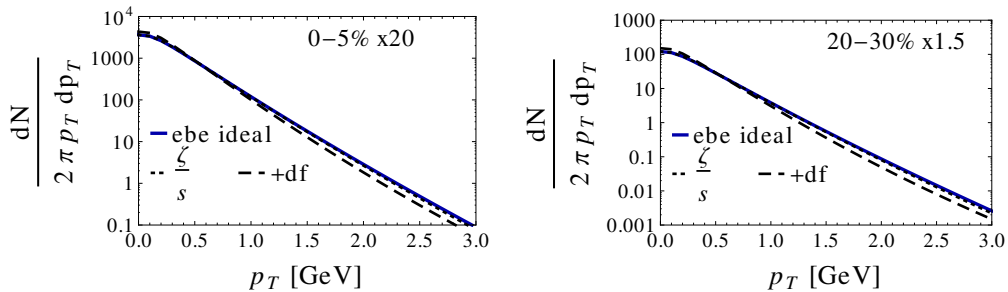


**Figure 2.** Dependence of the direct  $\pi^+$  differential elliptic flow (left) and spectrum (right) on the specific formula for the viscous  $\delta f$  contribution from the bulk viscosity that enters in the Cooper-Frye freeze-out. The results are for RHIC's  $\sqrt{200}$  GeV 20 – 30% most central collisions in the case where the initial condition corresponds to a single average Glauber initial condition. The ideal case is the solid black line, our result for  $v_2$  computed using the  $\delta f$  obtained via the Moments method is the long dashed black line, results for the  $\delta f$  described in [17] is the short dashed red curve, while the short and long dashed brown curve is the result computed using the  $\delta f$  described in [12].

Note that our moments method takes certain assumptions into account. We use only 100 of the known hadrons (up to mass  $M = 1.2$  GeV) due to the complexity of the calculations involved when further hadrons are included, we assume a constant cross-section, and we assume Navier Stokes scaling, which implies that  $\tau_\pi$  is small at freeze out [19]. All of these assumptions we plan to further test in an upcoming paper.

### 3. Results

All results below are shown for direct, thermal  $\pi^+$  on an event-by-event basis (for 150 events) using the event plane method [28]. We consider only  $p_T = 0 - 3$  GeV for RHIC's  $\sqrt{s} = 200$  GeV most central collisions (0 – 5%) and peripheral collisions (20 – 30%). The direct, thermal  $\pi^+$  spectra,  $dN/(dy p_T dp_T)$ , are shown for 0 – 5% and 20 – 30% centrality classes in Fig. 3. Because we fit the integrated  $\pi^+$  yields to 123 pions for most central collisions (roughly 41% of pions are direct pions at our freeze out temperature of  $T = 150$  MeV and RHIC measures 300 total  $\pi^+$  for most central collisions [29]) then the only difference between curves is the slope of the graphs themselves. The effect of bulk viscosity steepens the curve compared to the ideal fluid. However, the only noticeable differences comes from the correction to the Cooper Frye method, not from the hydrodynamical evolution itself. The difference is consistent across both centrality classes. As we will see in the following graphs the smaller number of high  $p_T$  pions contribute to an enhancement in the flow harmonics at high  $p_T$ .

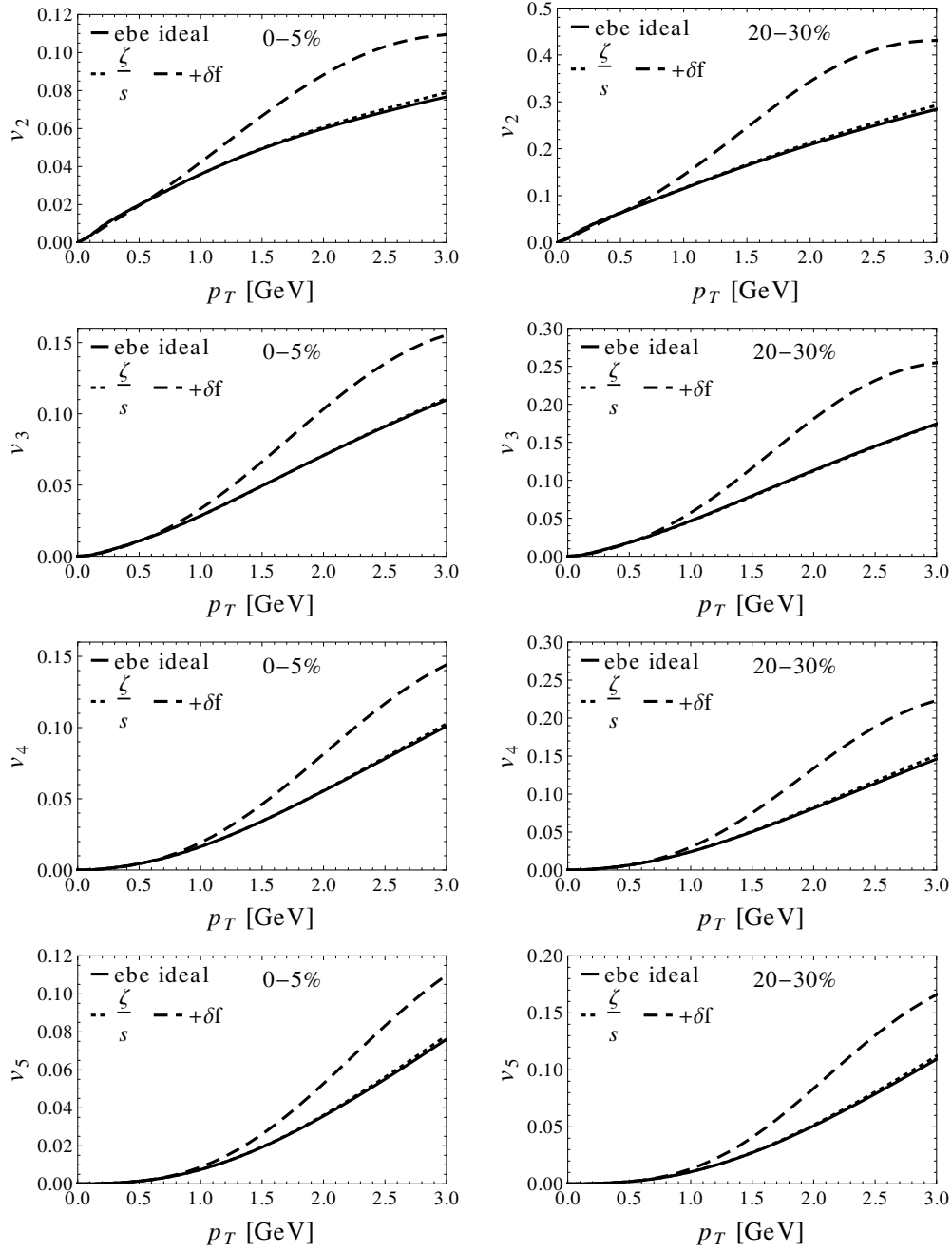


**Figure 3.** The  $\pi^+$  spectra  $dN/(2\pi p_T dp_T)$  for 0 – 5% and 20 – 30% centrality classes. The ideal fluid case is shown in a solid blue line, the result in the case where effects of bulk viscosity are included only on the hydrodynamical evolution but not on the freeze-out is shown by the short dashed black line while the long dashed black curve includes bulk effects on both the hydro evolution and freeze-out.

Results for the flow harmonics  $v_2$  to  $v_5$  for both most central collisions (0 – 5%) and peripheral collisions (20 – 30%) are shown in Fig. 4 for event-by-event Glauber initial conditions. The black solid lines indicate the ideal fluid whereas the short dashed lines include bulk viscosity corrections within the hydrodynamical evolution but at freezeout and the long dashed lines include bulk viscosity corrections in both the hydrodynamical evolution and freezeout. One can clearly see that the effects from the bulk viscosity solely in the hydrodynamical evolution are minimal. However, once the corrections to the freezeout are included the bulk viscosity is enhanced in the region of  $p_T = 1 - 3$  GeV. Both centrality classes experience this enhancement and it occurs for all  $v_n$ 's. When one compares these results to previous studies with shear viscosity, one is not surprised that the effects come primarily through the Cooper Frye correction term because the same effect was observed with shear viscosity. However, while shear viscosity depresses the flow harmonics for the higher  $p_T$  region, bulk viscosity has the opposite effect. One could expect that in the case where both shear and bulk viscosity are included in event by event simulations there could be some competition between the two effects.

### 4. Conclusions

Within the framework of our 2+1 viscous, Lagrangian relativistic hydrodynamical code we were able to study the effects of bulk viscosity on an event-by-event basis and compare them directly to an ideal fluid. We found that all flow harmonics  $v_2(p_T)$ - $v_5(p_T)$  there was an enhancement



**Figure 4.** Results for the bulk viscosity  $\zeta/s$  shown for most central collisions (0 – 5%) and non-central collisions (20 – 30%) computed using event-by-event simulations. The solid lines corresponds to the ideal fluid result, the short dashed lines include bulk viscosity only on the hydrodynamical evolution but not at freeze-out while the long dashed lines include bulk viscosity effects both on the hydro evolution and at freeze-out.

due to bulk viscosity in the momentum range above  $p_T > 1$  GeV for both central and non-central collisions. Even though our value of  $\zeta/s$  was almost an order of magnitude smaller than the commonly used value of shear viscosity to entropy density ratio  $\eta/s = 1/(4\pi)$ , we found significant deviations in our calculations compared to the ideal fluid flow harmonics.

Furthermore, this deviation shows up almost entirely through the non-equilibrium viscous correction to the Cooper Frye freeze out and not through the hydrodynamical process itself, as was expected from previous studies with shear viscosity.

Incidentally, the bulk viscosity has the opposite effect of the shear viscosity [11] when it comes to the  $p_T$  dependence of the flow harmonics. Previous studies have found that shear viscosity universally depresses the flow harmonics (for higher  $p_T$ 's in the range of  $p_T = 1 - 3$  GeV) whereas we found that the bulk viscosity does the opposite and enhances the flow harmonics in the same range. Because of this, it could be that much of the suppression of the flow harmonics due to shear viscosity is counteracted by the bulk viscosity. Thus, it is vital that studies that include shear viscosity also include the effect of bulk viscosity on an event-by-event basis in order to get a more accurate description and take this compensation into account. Additionally, because the effect is primarily seen through the non-equilibrium correction to the Cooper Frye it is vital that we have an accurate description and continue to invest in more accurate models.

#### 4.1. Acknowledgments

J. Noronha-Hostler, R. P. G. Andrade, J. Noronha, and F. Grassi acknowledge Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for financial support. G. S. Denicol is supported by the Natural Sciences and Engineering Research Council of Canada.

## 5. References

- [1] For a recent review, see U. W. Heinz, R. Snellings, arXiv:1301.2826 [nucl-th].
- [2] P. Danielewicz, M. Gyulassy, Phys. Rev. D **31**, 53 (1985).
- [3] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005) [hep-th/0405231].
- [4] T. Hirano and M. Gyulassy, Nucl. Phys. A **769**, 71 (2006) [nucl-th/0506049].
- [5] L. P. Csernai, J. I. Kapusta, L. D. McLerran, Phys. Rev. Lett. **97**, 152303 (2006) [nucl-th/0604032].
- [6] P. B. Arnold, G. D. Moore, L. G. Yaffe and , JHEP **0011**, 001 (2000) [hep-ph/0010177].
- [7] P. B. Arnold, G. D. Moore, L. G. Yaffe and , JHEP **0305**, 051 (2003) [hep-ph/0302165].
- [8] J. Noronha-Hostler, J. Noronha and C. Greiner, Phys. Rev. Lett. **103**, 172302 (2009) [arXiv:0811.1571 [nucl-th]].
- [9] M. Prakash, M. Prakash, R. Venugopalan and G. Welke, Phys. Rep. **227**, 321 (1993).
- [10] B. Schenke, S. Jeon, C. Gale, Phys. Rev. Lett. **106**, 042301 (2011) [arXiv:1009.3244 [hep-ph]].
- [11] B. Schenke, S. Jeon, C. Gale, Phys. Rev. C **85**, 024901 (2012) [arXiv:1109.6289 [hep-ph]].
- [12] A. Monnai, T. Hirano, Phys. Rev. C **80**, 054906 (2009) [arXiv:0903.4436 [nucl-th]].
- [13] H. Song, U. W. Heinz, Phys. Rev. C **81**, 024905 (2010) [arXiv:0909.1549 [nucl-th]].
- [14] P. Bozek, Phys. Rev. C **81**, 034909 (2010) [arXiv:0911.2397 [nucl-th]].
- [15] G. S. Denicol, T. Kodama, T. Koide and P. Mota, Phys. Rev. C **80**, 064901 (2009) [arXiv:0903.3595 [hep-ph]]; G. S. Denicol, T. Kodama and T. Koide, J. Phys. G **37**, 094040 (2010) [arXiv:1002.2394 [nucl-th]].
- [16] V. Roy, A. K. Chaudhuri, Phys. Rev. C **85**, 024909 (2012) [Erratum-ibid. C **85**, 049902 (2012)] [arXiv:1109.1630 [nucl-th]].
- [17] K. Dusling, T. Schfer, Phys. Rev. C **85**, 044909 (2012) [arXiv:1109.5181 [hep-ph]].
- [18] P. Bozek and W. Broniowski, Phys. Rev. C **85**, 044910 (2012) [arXiv:1203.1810 [nucl-th]].
- [19] J. Noronha-Hostler, J. Noronha, G. S. Denicol, R. P. G. Andrade, F. Grassi and C. Greiner, "v-USPhydro: Bulk Viscosity Effects in Event-by-Event Hydrodynamics" To appear soon.
- [20] G. S. Denicol, T. Kodama, T. Koide and P. Mota, J. Phys. G **35**, 115102 (2008) [arXiv:0807.3120 [hep-ph]].
- [21] A. Buchel, Phys. Lett. B **663**, 286 (2008) [arXiv:0708.3459 [hep-th]].
- [22] X. -G. Huang, T. Kodama, T. Koide and D. H. Rischke, Phys. Rev. C **83**, 024906 (2011) [arXiv:1010.4359 [nucl-th]].
- [23] H. -J. Drescher and Y. Nara, Phys. Rev. C **75**, 034905 (2007); Phys. Rev. C **76**, 041903 (2007).
- [24] P. Huovinen and P. Petreczky, Nucl. Phys. A **837**, 26 (2010) [arXiv:0912.2541 [hep-ph]].
- [25] J. Noronha-Hostler, J. Noronha, G. S. Denicol, R. P. G. Andrade, F. Grassi, C. Greiner, arXiv:1302.7038 [nucl-th].
- [26] G. S. Denicol, H. Niemi, E. Molnar and D. H. Rischke, Phys. Rev. D **85**, 114047 (2012) [arXiv:1202.4551 [nucl-th]].
- [27] G. S. Denicol, H. Niemi, arXiv:1212.1473 [nucl-th].

- [28] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998) [nucl-ex/9805001].
- [29] I. Arsene *et al.* [BRAHMS Collaboration], Nucl. Phys. A **757**, 1 (2005) [nucl-ex/0410020].