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Compactification stability by global k-monopole in higher dimensional σ-model

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Abstract. We investigate compactification by k-monopole in nonlinear sigma model with kinetic terms $K(X) = -X - \beta^{-2}X^2$. We use both Einstein equations and effective potential from dimensional reduction to determine radius of compactification. The compactification channels can be found by introducing effective potential, and its derivatives shall determine the stability.

1. Introduction

Monopole is a noncontractible point defect due to its second fundamental group being nontrivial, $\pi_2(S^2) = \mathbb{Z}$ [1, 2]. Specifically due to the absence of gauge field, global monopole has divergent energy. It causes a peculiar feature when coupled to gravity: it gives no gravitational force on the surrounding matter, except near the core due to its tiny mass [3]. However, the space around global monopole suffers from deficit solid angle $\Delta \equiv 8\pi G\eta^2$, which makes it non-Euclidean. This deficit angle has a critical value $\eta < \sqrt{1/8\pi G}$. In [4], the authors suggest that when η is a little bigger than this upper bound, two angular dimensions of the space around the global monopole might be compactified into a cylinder. In [5, 6] global monopole was investigated in higher-dimensional spacetime when the staticity assumption is relaxed, i.e., the geometry can then be thought of as a "cigar". In other words, compactification of this type can be seen as nonstatic solution. Numerically, as discussed in [7], regular solutions in four-dimensional space could exist up to $\eta \lesssim \sqrt{3/8\pi G}$. Above that value, the singularity is interpreted in [8, 9] as the appearance of topological inflation.

Nonlinear terms in Lagrangian kinetic term, whose defects can arise from without providing a symmetry-breaking potential term [10], are named in [11] as k-defect. In the case of topological defect from monopoles, they are called in [12] as k-monopole.

In [13, 11] global k-monopole has been studied, and investigations of its gravitational field were done in [12, 14]. Qualitatively, they show that the gravitational property of Barriola-Vilenkin monopole still holds. The difference is their mass can be negative or positive, depending on the specific model of k-term considered. The region outside the monopole can be studied using the vacuum approximation where the Higgs field is approximation, where Higgs field is approximated as $|\phi| \simeq \eta$ so that the analytical solutions can be found. The authors of [15] show some gravitational solutions at the exterior of two different types of global k-monopoles [12].

Here we report our finding for a different model: a sigma model based on [16] with a cosmological constant and nonlinear term based on [11]. We investigate static compactification solutions and its stability by introducing an effective potential.

2. Global k-monopole in sigma model

Here we use $(+--...)$ metric g_{MN} $(M,N=0,...,p,\theta^1,...,\theta^{D-2})$ for static $(p+D)$-dimensional spacetime

$$ds^2 = g_{MN}dx^M dx^N = A^2(r)\eta_{\mu\nu}dx^\mu dx^\nu - B^2(r)dr^2 - C^2 d\Omega^2_{D-2}, \tag{1}$$

with $(p+1)$-dimensional flat space and $(D-2)$-sphere denoted by $g_{\mu\nu}dx^\mu dx^\nu = A(r)^2\eta_{\mu\nu}dx^\mu dx^\nu$ $(\mu,\nu=0,...,p)$ and $g_{ij}dx^i dx^j = -C^2 d\Omega^2_{D-2}$ $(i,j=\theta^1,...,\theta^{D-2})$ respectively, C is a real-valued constant and $D\geq 4$. The metric is induced in the action

$$\mathcal{S} = \int d^{p+D}x\,\sqrt{|g|}\,\left[\frac{R-2\Lambda}{2\kappa} + K(\mathfrak{X}) - \frac{\lambda}{4}\left(\vec{\Phi}^2-\eta^2\right)^2\right], \tag{2}$$

with $K(\mathfrak{X}) = -\mathfrak{X} - \beta^{-2}\mathfrak{X}^2$ and $\mathfrak{X} \equiv -(1/2)g^{MN}\partial_M\vec{\Phi}\cdot\partial_N\vec{\Phi}$. κ is Newton constant in $(p+D)$-dimensional spacetime. The nonlinear term containing β in the kinetic part goes linear when $\beta\to\infty$.

The action, being defined to be invariant under SO(n) transformation, has n degrees of freedom. Our object of interest is nonlinear sigma model with a constraint $\vec{\Phi}^2=\eta^2$, which means that the last term in (2) is just a term multiplied by Lagrange multiplier λ. The scalar field lives in n-dimensional manifold, but now it is constrained to a surface of $(n-1)$-sphere. By this we can introduce its internal basis coordinates in the S^{n-1} by $\vec{\Phi}\equiv\vec{\Phi}(\phi^i)$ with $i=1,...,n-1$. By these constructions (following [16]) we can write a nonlinear σ-model action

$$\mathcal{S} = \int d^{p+D}x\,\sqrt{|g|}\,\left[\frac{R-2\Lambda}{2\kappa} - X - \beta^{-2}X^2\right], \tag{3}$$

with $X = -(1/2)\eta^2 h_{ij}g^{MN}\partial_M\phi^i\partial_N\phi^j$. Its inner space metric $h_{ij}(\phi^k) = \frac{\partial\vec{\Phi}}{\partial\phi^i}\cdot\frac{\partial\vec{\Phi}}{\partial\phi^j}$ is defined to be dependent on $\phi^1,...,\phi^{n-1}$.

The nonlinear σ-model action then gives us the scalar-field's equation of motion and the energy-momentum tensor for the Einstein equation by the usual variational method

$$\frac{1}{\sqrt{|g|}}\partial_M\left(\sqrt{|g|}(1+2\beta^{-2}X)\eta^2 h_{ij}\partial^M\phi^j\right) = (1+2\beta^{-2}X)\frac{\eta^2}{2}\partial_M\phi^m\partial^M\phi^n\frac{\partial h_{mn}}{\partial\phi^i}, \tag{4}$$

$$T^M_N = \delta^M_N\left[\frac{\Lambda}{\kappa} + X + \beta^{-2}X^2\right] + (1+2\beta^{-2}X)\eta^2 h_{ij}\partial^M\phi^i\partial_N\phi^j. \tag{5}$$

Now it is crucial to choose an appropriate ansatz for the scalar field that satisfies the field equation of motion. This we follow the methods used in [16, 17]. The field's equation of motion is satisfied by choosing the scalar field to be dependent on the angular coordinates $\theta^1,...,\theta^{D-2}$ (the scalar field is now constrained in S^{D-2} with $D\geq 4$, following [6])

$$\phi^i(\theta^i) = \theta^i \tag{6}$$

if and only if we choose its inner metric $h_{ij}(\phi^k)$ to be related to $g_{ij}(C,\theta^k)$ by

$$h_{ij}(\phi^k) = -C^{-2}g_{ij}(C,\theta^k), \tag{7}$$

with C a real-valued constant which gives $X=(D-2)\eta^2/2C^2$. This then gives us the components of energy-momentum tensor

$$T^0_0 = T^r_r = \left[\frac{\Lambda}{\kappa} + X + \beta^{-2}X^2\right], \tag{8}$$

$$T^\theta_\theta = T^0_0 - (1 + 2\beta^{-2}X)\frac{\eta^2}{C^2}, \tag{9}$$

for some cosmological constant Λ of $(p+D)$-dimensional space. The Einstein tensor's components from the metric are

$$\begin{aligned} G^0_0 &= -\frac{p}{B^2}\frac{A''}{A} - \frac{p(p-1)}{2B^2}\left(\frac{A'}{A}\right)^2 + \frac{p}{B^2}\frac{A'B'}{AB} + \frac{(D-2)(D-3)}{2C^2}, &(10)\\ G^r_r &= -\frac{p(p+1)}{2B^2}\left(\frac{A'}{A}\right)^2 + \frac{(D-2)(D-3)}{2C^2}, &(11)\\ G^\theta_\theta &= -\frac{(p+1)}{B^2}\frac{A''}{A} - \frac{p(p+1)}{2B^2}\left(\frac{A'}{A}\right)^2 + \frac{(p+1)}{B^2}\frac{A'B'}{AB} + \frac{(D-4)(D-3)}{2C^2}. &(12)\end{aligned}$$

Combining the above equations we have a useful expression

$$\begin{aligned} pG^\theta_\theta - (p+1)G^0_0 - G^r_r &= -2\Lambda - \frac{(p+D-2)\kappa\eta^2}{C^2} - \frac{(2p+D-2)(D-2)\kappa\eta^4}{2C^4\beta^2} \\ &= -\frac{(p+D-2)(D-3)}{C^2}. \end{aligned} \tag{13}$$

Solving this when $\Lambda = 0$ we get

$$C^2 = \frac{(2p+D-2)(D-2)\kappa\eta^4}{2\beta^2(p+D-2)(D-3-\kappa\eta^2)}, \tag{14}$$

and when $\Lambda \neq 0$ we get

$$\begin{aligned} C^2_\pm &= \frac{(p+D-2)(D-3-\kappa\eta^2)}{4\Lambda} \\ &\quad \pm \frac{\sqrt{(p+D-2)^2(D-3-\kappa\eta^2)^2 - 4\Lambda(2p+D-2)(D-2)\kappa\eta^4\beta^{-2}}}{4\Lambda}. \end{aligned} \tag{15}$$

The dimension is constrained with $p + D > 2$ since $D \geq 4$. We can see that (14) satisfies

$$\eta^2 < \eta^2_{crit} \equiv (D-3)/\kappa \tag{16}$$

so that if (16) is applied to (14), $C^2_\pm$ is greater than zero. This condition also makes the first term in (15) greater than zero. As we limit (15) by $\beta \to \infty$, $C^2_- \to 0$ and $C^2_+ \to (p + D - 2)(D - 3 - \kappa\eta^2)/2\Lambda$. The latter is in agreement with [4].

Now we discuss metric solutions for the compactified space with manifold $Z_{p+2} \times S^{D-2}$. We obtain

$$B^{-2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) = \pm\omega^2, \tag{17}$$

using $G^\theta_\theta - G^r_r$, and we define

$$\pm\omega^2 \equiv \frac{D-3}{(p+1)C^2} - \frac{(1+2\beta^{-2}(D-2)\eta^2/2C^2)}{(p+1)}\frac{\eta^2}{C^2}. \tag{18}$$

This can be considered as the cosmological constant of Z_{p+2} space for the following reasons. We use positive-negative signs to ensure $\omega^2 \geq 0$; the positive (negative) sign is for $B^{-2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) > (<)\ 0$. Using $B = 1$, (17) gives us

$$ds^2 = \begin{cases} \frac{1}{\omega^2}(\sin^2\chi\ \eta_{\mu\nu}dx^\mu dx^\nu - d\chi^2) - C^2 d\Omega^2_{D-2}, & \text{for positive sign,} \\ \eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - C^2 d\Omega^2_{D-2}, & \text{for } \omega = 0, \\ \frac{1}{\omega^2}(\sinh^2\chi\ \eta_{\mu\nu}dx^\mu dx^\nu - d\chi^2) - C^2 d\Omega^2_{D-2}, & \text{for negative sign,} \end{cases} \tag{19}$$

with $\chi \equiv \omega r$. Using $B = A^{-1}$,

$$ds^2 = \begin{cases} (1-\omega^2 r^2)\ \eta_{\mu\nu} dx^\mu dx^\nu - \frac{dr^2}{(1-\omega^2 r^2)} - C^2 d\Omega^2_{D-2}, & \text{for positive sign,} \\ \eta_{\mu\nu} dx^\mu dx^\nu - dr^2 - C^2 d\Omega^2_{D-2}, & \text{for } \omega = 0, \\ (1+\omega^2 r^2)\ \eta_{\mu\nu} dx^\mu dx^\nu - \frac{dr^2}{(1+\omega^2 r^2)} - C^2 d\Omega^2_{D-2}, & \text{for negative sign.} \end{cases} \tag{20}$$

These give us the manifold of compactified space:

$$Z_{p+2} = \begin{cases} dS_{p+2}, & \text{for } B^{-2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) > 0, \\ M_{p+2}, & \text{for } B^{-2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) = 0, \\ AdS_{p+2}, & \text{for } B^{-2}\left(\frac{A'B'}{AB} - \frac{A''}{A}\right) < 0. \end{cases} \tag{21}$$

3. Compactification solutions stability

To study the stability of above metric solutions (19) and (20) with their radius (14) or (15) depending on their cosmological constant Λ, we look for the effective potential of the radion $V(\psi)$ by dimensional reduction. Considering a metric with signature $(+ - - \dots)$

$$ds^2 = G^{(p+D)}_{MN} dx^A dx^B, \tag{22}$$

with $G^{(p+D)}_{MN}$ metric on $(p+D)$-dimensional action

$$\mathcal{S} = \int d^{p+D}x \sqrt{|G|} \left[\frac{\mathcal{R}^{(p+D)}}{2\kappa} + \mathcal{L}_m \right]. \tag{23}$$

By defining the metric as

$$ds^2 = g^{(p+2)}_{\mu\nu}(x) dx^\mu dx^\nu - b^2(x) \gamma^{(D-2)}_{ij}(y) dy^i dy^j, \tag{24}$$

with $\gamma^{(D-2)}_{ij}$ a metric for a space with constant curvature with radius R_0, the action becomes

$$\mathcal{S} = \frac{V_{(D-2)}}{2\kappa} \int d^{p+2}x \sqrt{|g|} \left[b^{D-2} \mathcal{R}^{(p+D)} + 2\kappa b^{D-2} \mathcal{L}_m \right]. \tag{25}$$

Performing Weyl transformation to $\mathcal{R}^{(p+D)}$ by

$$g^{(p+2)}_{\mu\nu}(x) = b^{2a}(x)\ \tilde{g}^{(p+2)}_{\mu\nu}(x) \tag{26}$$

with $a = -(D-2)/p$, we obtain

$$\begin{aligned} \mathcal{S} \;=\; & \frac{V_{(D-2)}}{2\kappa} \int d^{p+2}x \sqrt{|\tilde{g}|} \left[\tilde{\mathcal{R}}^{(p+2)} + \frac{(D-2)(D-3)}{b^{2(p+D-2)/p} R_0^2} + \frac{2(D-2)}{p} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu (\tilde{\partial}_\nu \ln b) \right. \\ & \left. - \frac{(D-2)(p+D-2)}{p} \tilde{g}^{\mu\nu} (\tilde{\partial}_\mu \ln b)(\tilde{\partial}_\nu \ln b) + 2\kappa b^{-(D-2)/p} \tilde{\mathcal{L}}_m \right]. \end{aligned} \tag{27}$$

After we define the radion as

$$b \equiv \exp\left[\sqrt{\frac{p}{(D-2)(p+D-2)}} \frac{\psi}{M_P} \right], \tag{28}$$

with a $(p+2)$-dimensional Planck mass $M_P \equiv \sqrt{V_{(D-2)}/\kappa}$, we obtain an action in Einstein's frame

$$\mathcal{S} = \int d^{p+2}x\sqrt{|\tilde{g}|}\left[\frac{M_P^2\tilde{\mathcal{R}}^{(p+2)}}{2} - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\partial}_\mu\psi\tilde{\partial}_\nu\psi - V(\psi)\right], \tag{29}$$

whose effective potential is

$$V(\psi) = -e^{\sigma\psi/M_P}\frac{M_P^2(D-2)(D-3)}{2R_0^2} - e^{\chi\psi/M_P}M_P^2\kappa\tilde{\mathcal{L}}_m, \tag{30}$$

with $\sigma \equiv -2\sqrt{(p+D-2)/p(D-2)}$ and $\chi \equiv -2\sqrt{(D-2)/p(p+D-2)}$; its metric is given by

$$ds^2 = e^{\chi\psi/M_P}\tilde{g}_{\mu\nu}^{(p+2)}dx^\mu dx^\nu - e^{\xi\psi/M_P}\gamma_{ij}^{(D-2)}dy^i dy^j, \tag{31}$$

with $\xi \equiv 2\sqrt{p/(D-2)(p+D-2)}$. We restrict $p \geq 1$ so that $\sigma \neq \infty$, $\chi \neq \infty$ and $\xi \neq 0$.

Now we can write metric (31) by setting it as

$$ds^2 = e^{\chi\psi/M_P}\left[A^2(r)\eta_{\mu\nu}^{(p+1)}dx^\mu dx^\nu - B^2(r)dr^2\right] - e^{\xi\psi/M_P}\left[C^2 d\Omega^2_{(D-2)}\right], \tag{32}$$

such that it resemble (1) as $\psi = 0$. The radius R_0 now becomes C. By applying (32) into the matter terms in (3) we obtain the Lagrangian density $\tilde{\mathcal{L}}_m$ in (30) as

$$\tilde{\mathcal{L}}_m = -\frac{\Lambda}{\kappa} + K(X), \quad \text{with} \quad X = \frac{(D-2)\eta^2}{2C^2}e^{-\xi\psi/M_P}, \tag{33}$$

with $K(X) = -X - \beta^{-2}X^2$. This hence give us the effective potential for our model

$$\begin{aligned} V(\psi) &= -e^{\sigma\psi/M_P}\frac{M_P^2(D-2)(D-3)}{2C^2} + e^{\chi\psi/M_P}M_P^2\Lambda \\ &\quad + M_P^2\kappa\left[\frac{(D-2)\eta^2}{2C^2}e^{(\chi-\xi)\psi/M_P} + \frac{(D-2)^2\eta^4}{4\beta^2C^4}e^{(\chi-2\xi)\psi/M_P}\right]. \end{aligned} \tag{34}$$

Choosing the extrema to be at $\psi = 0$, we solve $V'(\psi = 0) = 0$ or

$$0 = \chi\Lambda + \frac{(D-2)}{2}\left[(\chi-\xi)\kappa\eta^2 - \sigma(D-3)\right]\frac{1}{C^2} + \left[\frac{(\chi-2\xi)(D-2)^2\kappa\eta^4}{4\beta^2}\right]\frac{1}{C^4}, \tag{35}$$

then we obtain

$$C^2 = \frac{(2p+D-2)(D-2)\kappa\eta^4}{2\beta^2(p+D-2)(D-3-\kappa\eta^2)}, \tag{36}$$

when $\Lambda = 0$, and

$$C^2 = \frac{(p+D-2)(D-3-\kappa\eta^2)}{4\Lambda}\left[1 \pm \sqrt{1 - \frac{(2p+D-2)(D-2)\kappa\eta^4\beta^{-2}}{4\Lambda(p+D-2)^2(D-3-\kappa\eta^2)^2}}\right], \tag{37}$$

when $\Lambda \neq 0$. Equation (37) needs to be $C^2 = (p+D-2)(D-3-\kappa\eta^2)/2\Lambda$ at $\beta \to \infty$, which is only satisfied with positive sign in (37), not the minus sign. Both are the same as (14) and (15), which we find using Einstein equations. By choosing the positive sign, it is clear that Λ cannot have negative value, otherwise it makes $C^2 < 0$. Both solutions have a stability condition

$$\begin{aligned} V''(\psi = 0) &= \frac{4(D-2)\Lambda}{p(p+D-2)} + \frac{2(\kappa\eta^2 - D + 3)(p+D-2)}{pC^2} + \frac{(2p+D-2)^2(D-2)\kappa\eta^4}{p(p+D-2)\beta^2C^4} \\ &> 0. \end{aligned} \tag{38}$$

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Equations (36) and (37) have a more compact expression with arbitrary Λ

$$\frac{1}{C_{\pm}^2} = \frac{(D-3-\kappa\eta^2)(p+D-2)\beta^2}{(2p+D-2)(D-2)\kappa\eta^4}\left(1 \pm \sqrt{1-\frac{4\Lambda(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2\beta^2}}\right), \tag{39}$$

which arises from (35). We use C_+ to ensure it becomes (36) when $\Lambda = 0$, but we use C_- for $\Lambda \neq 0$ due to $C_-^2 = (p+D-2)(D-3-\kappa\eta^2)/2\Lambda$ at $\beta \to \infty$ thus C_- corresponds to the positive sign in (37). The term under the square root needs to be positive, and thus we need $\beta^2 > 0$ when $\Lambda \leq 0$ or

$$\beta^2 \geq \beta_{crit}^2 \equiv \frac{4\Lambda(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2}, \tag{40}$$

when $\Lambda > 0$. It is clear that (16) must be satisfied. We set the minimum of the potential positioned at $\psi = 0$ for simplicity.

When the above condition is satisfied, we can set $V(\psi = 0, \Lambda = \Lambda_*) = 0$. Substituting (39) in to (35) to get

$$\begin{aligned} 0 &= \Lambda_* - \frac{(D-3-\kappa\eta^2)^2 p(p+D-2)\beta^2}{2(2p+D-2)^2\kappa\eta^4}\left(1 + \frac{2\Lambda_*(2p+D-2)(D-2)\kappa\eta^4}{p(p+D-2)(D-3-\kappa\eta^2)^2\beta^2}\right) \\ &\quad \mp \frac{(D-3-\kappa\eta^2)^2 p(p+D-2)\beta^2}{2(2p+D-2)^2\kappa\eta^4}\sqrt{1-\frac{4\Lambda_*(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2\beta^2}}. \end{aligned} \tag{41}$$

Since the left hand side is equal to zero, we can arrange it to get

$$\pm\sqrt{1-\frac{4\Lambda_*(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2\beta^2}} = \frac{2\Lambda_*(2p+D-2)(2p)\kappa\eta^4}{p(p+D-2)(D-3-\kappa\eta^2)^2\beta^2} - 1. \tag{42}$$

After squaring both sides we get $\Lambda_* = 0$ and

$$\Lambda_* = \frac{(D-3-\kappa\eta^2)^2\beta^2}{4\kappa\eta^4}. \tag{43}$$

We will use the latter expression and we restrict $\Lambda_* > 0$ so that (16) is satisfied. This treatment follows the method used in [16].

To see whether this compactification is stable we need to investigate the second derivative of the potential. After substituting (43) into (39), the inverse of radius squared becomes

$$\frac{1}{C_{\pm}^2} = \frac{4(p+D-2)\Lambda_*}{(2p+D-2)(D-2)(D-3-\kappa\eta^2)}\left(1 \pm \sqrt{1-\frac{\Lambda(2p+D-2)(D-2)}{\Lambda_*(p+D-2)^2}}\right). \tag{44}$$

This expression tells us that Λ cannot have higher value than $\Lambda = (p+D-2)^2\Lambda_*/(2p+D-2)(D-2)$ otherwise $1/C_{\pm}^2$ will be complex valued. Substituting (44) into (38) gives us

$$\begin{aligned} V''(\psi = 0, C_{\pm}^2) &= \frac{2(p+D-2)(D-3-\kappa\eta^2)^2\beta^2}{(2p+D-2)(D-2)\kappa\eta^4}\left(1 \pm \sqrt{1-\frac{4\Lambda(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2\beta^2}}\right) \\ &\quad - \frac{8\Lambda}{(p+D-2)} \\ &= -\frac{8\Lambda}{(p+D-2)} + \frac{8(p+D-2)\Lambda_*}{(2p+D-2)(D-2)}\left(1 \pm \sqrt{1-\frac{\Lambda(2p+D-2)(D-2)}{(p+D-2)^2\Lambda_*}}\right) \end{aligned} \tag{45}$$

where positive and negative signs correspond to C_+ and C_-, respectively. If $V'' \geq 0$ then the minimum of the potential at $\psi = 0$ can be determined by

$$\begin{aligned}\frac{V(\psi=0,C_\pm^2)}{M_P^2} &= \frac{2p\Lambda}{(2p+D-2)} \\ &\quad -\frac{p(p+D-2)(D-3-\kappa\eta^2)^2\beta^2}{(2p+D-2)^2 2\kappa\eta^4}\left(1\pm\sqrt{1-\frac{4\Lambda(2p+D-2)(D-2)\kappa\eta^4}{(p+D-2)^2(D-3-\kappa\eta^2)^2\beta^2}}\right) \\ &= \frac{2p\Lambda}{(2p+D-2)} - \frac{2p(p+D-2)\Lambda_*}{(2p+D-2)^2}\left(1\pm\sqrt{1-\frac{\Lambda(2p+D-2)(D-2)}{(p+D-2)^2\Lambda_*}}\right). \qquad (46)\end{aligned}$$

Now we can determine the compactified space and its stability. In the case of $\Lambda = 0$,

$$\frac{V(\psi=0,C_+,\Lambda=0)}{M_P^2} = -\frac{4p(p+D-2)\Lambda_*}{(2p+D-2)^2} < 0, \qquad (47)$$

$$V''(\psi=0,C_+,\Lambda=0) = \frac{16(p+D-2)\Lambda_*}{(2p+D-2)(D-2)} > 0, \qquad (48)$$

since $\Lambda_* > 0$ which imply that the compactification channel is $M_{p+D} \to AdS_{p+2} \times S^{D-2}$ and stable. In the case of $\Lambda > 0$,

$$\frac{V(\psi=0,C_-)}{M_P^2} = \frac{2p\Lambda}{(2p+D-2)} - \frac{2p(p+D-2)\Lambda_*}{(2p+D-2)^2}\left(1-\sqrt{1-\frac{\Lambda(2p+D-2)(D-2)}{(p+D-2)^2\Lambda_*}}\right), \qquad (49)$$

$$V''(\psi=0,C_-) = -\frac{8\Lambda}{(p+D-2)} + \frac{8(p+D-2)\Lambda_*}{(2p+D-2)(D-2)}\left(1-\sqrt{1-\frac{\Lambda(2p+D-2)(D-2)}{(p+D-2)^2\Lambda_*}}\right). \qquad (50)$$

Now we consider positive but nearly zero Λ

$$\frac{V(\psi=0,C_-)}{M_P^2} \simeq \frac{2p\Lambda}{(2p+D-2)} - \frac{\Lambda(D-2)p}{(2p+D-2)(p+D-2)} = \frac{2p\Lambda}{(p+D-2)} \simeq 0, \qquad (51)$$

$$V''(\psi=0,C_-) \simeq -\frac{8\Lambda}{(p+D-2)} + \frac{8\Lambda}{(p+D-2)} = 0, \qquad (52)$$

and at $\Lambda = (p+D-2)^2\Lambda_*/(2p+D-2)(D-2)$

$$\frac{V(\psi=0,C_-)}{M_P^2} = \frac{2p(p+D-2)^2\Lambda_*}{(2p+D-2)^2(D-2)} - \frac{2p(p+D-2)\Lambda_*}{(2p+D-2)^2} = \frac{2p^2(p+D-2)\Lambda_*}{(2p+D-2)^2(D-2)}, \qquad (53)$$

$$V''(\psi=0,C_-) = -\frac{8(p+D-2)\Lambda_*}{(2p+D-2)(D-2)} + \frac{8(p+D-2)\Lambda_*}{(2p+D-2)(D-2)} = 0. \qquad (54)$$

Both cases need the third derivative of V, which is

$$V'''(\psi=0,C_-) \simeq 8\sqrt{\frac{p}{(D-2)(p+D-2)}}\frac{\Lambda_*}{M_P}\frac{\left[(p+D-2)^3-(D-2)^2\right]}{p^2(p+D-2)} > 0 \qquad (55)$$

for $\Lambda \ll \Lambda_*$ case, and

$$V'''(\psi=0,C_-) = \frac{\left[(p+D-2)^3-(D-2)^2\right]}{(D-2)p^2}\frac{\Lambda_*}{M_P}\sqrt{\frac{p}{(D-2)(p+D-2)}} > 0 \qquad (56)$$

for $\Lambda = (p+D-2)^2\Lambda_*/(2p+D-2)(D-2)$ case, respectively. These imply that the compactification channels $dS_{p+D} \to (dS, M)_{p+2} \times S^{D-2}$ are unstable since the potential at $\psi = 0$ is a strictly increasing point of inflection. Combining these results we get tunneling channels $Y_{p+D} \longrightarrow Z_{p+2} \times S^{D-2}$ (with Λ and $V(\psi = 0)$ denoting Y_{p+D} and Z_{p+2} respectively)

$$dS_{p+D} \longrightarrow \begin{cases} dS_{p+2} \times S^{D-2}, \\ M_{p+2} \times S^{D-2}, \end{cases} \tag{57}$$

$$M_{p+D} \longrightarrow AdS_{p+2} \times S^{D-2}. \tag{58}$$

4. Conclusions

Here we investigate compactification by k-monopole in nonlinear sigma model with kinetic terms $K(X) = -X - \beta^{-2}X^2$. We can use both Einstein equations or effective potential from dimensional reduction to determine radius of compactification. The compactification channels can be found by introducing effective potential and its derivatives will determine the stability. Here we report that the compactification channels from M_{p+D} and dS_{p+D} to $Z_{p+2} \times S^{D-2}$, where Z is some topological manifold, are stable and unstable, respectively.

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