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The LHC higgsino discovery plane for present and future SUSY searches

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ABSTRACT

Considerations from electroweak naturalness and stringy naturalness imply a little hierarchy in supersymmetric models where the superpotential higgsino mass parameter μ is of order the weak scale whilst the soft SUSY breaking terms may be in the (multi-) TeV range. In such a case, discovery of SUSY at LHC may be most likely in the higgsino pair production channel. Indeed, ATLAS and CMS are performing searches in the higgsino mass discovery plane of $m_{\widetilde{\chi}_2^0}$ vs. $\Delta m^0 \equiv m_{\widetilde{\chi}_2^0} - m_{\widetilde{\chi}_1^0}$. We examine several theoretical aspects of this discovery plane in both the gravity-mediation NUHM2 model and the general mirage-mediation (GMM′) models. These include: the associated chargino mass $m_{\widetilde{\chi}_1^\pm}$, the expected regions of the bottom-up notion of electroweak naturalness Δ_{EW}, and the expected regions of stringy naturalness. While compatibility with electroweak naturalness allows for mass gaps $\Delta m^0 \sim$ 4-20 GeV, stringy naturalness exhibits a clear preference for yet smaller mass gaps of 4-10 GeV. For still smaller mass gaps, the plane becomes sharply unnatural since very large gaugino masses are required. This study informs the most promising SUSY search channels and parameter space regions for the upcoming HL-LHC runs and possible HE-LHC option.

The discovery of the Higgs boson with mass $m_h \simeq 125$ GeV at the LHC [1,2] is enigmatic within the context of the Standard Model (SM) in that m_h exhibits quadratic sensitivity to the highest mass scale (such as the Grand Unification scale) that the SM might couple to: radiative corrections would then drive its mass to far higher values. The introduction of softly broken supersymmetry (SUSY) results in cancellations leaving only logarithmic sensitivity to the scale of new physics, and the Higgs mass can be stabilized at its measured value [3]. Weak scale SUSY [4] finds a natural home within string theory, and is the oft-sought weak scale realization of string compactifications.[1] Weak scale SUSY is actually supported by a variety of *virtual* quantum effects, including 1. the measured values of the gauge couplings are consistent with unification within the context of the minimal supersymmetric standard model (MSSM) [6], 2. the measured value of the top mass is just right to radiatively drive electroweak symmetry breaking in the MSSM [7], 3. the measured value of m_h lies within the narrow window of predicted values in the MSSM [8] and 4. precision EW observables, especially within the m_W vs. m_t plane, slightly favor heavy SUSY over even the SM [9].

Even so, searches for SUSY at LHC Run 2 with 139 fb^{-1} of integrated luminosity [10] have led to limits (within the context of various simplified models) of $m_{\tilde{g}} \gtrsim 2.2$ TeV [11] and $m_{\tilde{t}_1} \gtrsim 1.1$ TeV [12]. In this case, one expects the corresponding soft SUSY breaking terms to lie in the (multi) TeV range. But if the soft SUSY breaking parameters are too large, then a Little Hierarchy (LH) emerges: one might expect the Higgs mass to be of order the soft breaking scale. This is exemplified by the fact that the LHC SUSY particle mass limits lie far beyond initial estimates from naturalness wherein values such as $m_{\tilde{g}}$, $m_{\tilde{t}_1} \lesssim 400$ GeV were expected [13–16]. In retrospect, it was pointed out that the log-derivative measure $\Delta_{BG} \equiv max_i|\frac{\partial \log m_Z^2}{\partial \log p_i}|$, where the p_i are fundamental parameters (usually the soft breaking terms are taken as the p_i) of the 4-d low energy effective SUSY theory, is highly model-dependent [17–20]. In a top-down approach within a more UV complete theory, such as string theory, then all soft terms are (in principle) calculable in terms of more fundamental parameters (such as the gravitino mass $m_{3/2}$ in gravity or anomaly–mediation), and the value of Δ_{BG} changes greatly from its effective theory value [20,21]. Alternatively, in the string theory landscape – wherein the soft terms may scan within the landscape – then

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[1] For recent discussion, see *e.g.* Ref. [5].

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selection effects may operate so that certain ranges of soft term values are statistically preferable to others [22–25].

A *model independent* bottom-up measure of electroweak naturalness emerges directly from minimizing the scalar potential of the MSSM in order to relate the Higgs field vevs to the MSSM Lagrangian parameters. The electroweak fine-tuning parameter [26,27], $\Delta_{\rm EW}$, is a measure of the degree of cancellation between various contributions on the right-hand-side (RHS) in the well-known expression for the Z mass:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2 \quad (1)$$

which results from the minimization of the Higgs potential in the MSSM. Here, $\tan\beta = v_u/v_d$ is the ratio of Higgs field vacuum-expectation-values and the Σ_u^u and Σ_d^d contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [27] and are included in the Isajet SUSY spectrum generator [28]. We also include leading two-loop terms from $m_{\tilde g}$ and $m_{\tilde t_{1,2}}$ as determined by Dedes and Slavich [29]. If the RHS terms in Eq. (1) are individually comparable to $m_Z^2/2$, then no unnatural fine-tunings are required to generate $m_Z = 91.2$ GeV. $\Delta_{\rm EW}$ is defined to be the largest of these terms, scaled by $m_Z^2/2$. Clearly, low electroweak fine-tuning requires that μ be close to m_Z and that $m_{H_u}^2$ – which sets the values of $m_{W,Z,h}$ – be radiatively driven to *small* negative values close to the weak scale. This scenario has been dubbed radiatively-driven natural supersymmetry or RNS [26,27] since it allows for large, seemingly unnatural GUT scale soft terms to be driven to natural values at the weak scale via RG running.

An advantage of Δ_{EW} is its model independence in that it depends only on weak scale Lagrangian parameters and sparticle masses. Thus, for a given mass spectrum, one obtains the same value of Δ_{EW} whether it was generated in some high scale model such as mSUGRA or else just within the pMSSM: *i.e.* it is both parameter independent and scale independent. If one moves to models with extra low-scale exotic matter beyond the MSSM, then additional terms may have to be added to the RHS of Eq. (1). It has been argued that $\Delta_{BG} \to \Delta_{EW}$ if appropriate underlying correlations between model parameters – that are usually assumed to be independent – are incorporated [17,18].

Under Δ_{EW}, the natural SUSY parameter space is found to be far larger than what is expected under Δ_{BG} [25]. Since top-squarks enter Eq. (1) at one-loop level, they can have masses into the several TeV regime while remaining natural, with $\Delta_{EW} \lesssim 30$.[2] Gluinos, which enter Eq. (1) at two-loop level [29], can range up to ~ 6 TeV at little cost to naturalness [30,31]. But since the SUSY conserving μ parameter enters Eq. (1) directly, then the lightest Higgs boson and the superpartner higgsinos must have mass not too far removed from $m_{weak} \sim m_{W,Z,h} \sim 100$ GeV. Thus, we expect from Δ_{EW} that higgsinos will be the lightest superpartners while other sparticles which gain mass from soft breaking terms may well be beyond the present LHC mass limits.

Such a scenario, it has been suggested [22,23], emerges naturally from the landscape of string theory vacua which also provides a solution to the cosmological constant problem. Rather general considerations of the string theory landscape lead to an expected distribution of soft terms for different pocket-universes within the multiverse which favors large values by a power law distribution: $m_{soft}^{2n_F+n_D-1}$ where n_F is the number of F-term fields and n_D is the number of D-term fields contributing to the overall SUSY breaking scale [32]. However, the overall SUSY breaking scale m_{soft} cannot be too large lest it lead to too large a value of m_{weak}^{PU} in different pocket universes (PU). The atomic principle [33] – that atoms as we know them ought to exist in a pocket-universe which gives rise to observers – requires that m_{weak}^{PU} be within a factor 2-5 of the measured value of m_{weak}^{OU} in our universe (OU). If the value of μ is determined by whatever solution to the SUSY μ problem is invoked [34], then μ is *unavailable* for (the usual) electroweak fine-tuning and the value of m_{weak}^{PU} is determined by Eq. (1). The requirement that $m_Z^{PU} \lesssim 4m_Z^{OU}$ is then equivalent to the above mentioned naturalness requirement that $\Delta_{EW} < 30$. Thus, the concept of *stringy naturalness* [25,35] favors soft terms as large as possible such that the weak scale remains not too far from its measured value in our universe. Under such conditions, superpartners are lifted beyond LHC search limits while the light Higgs mass m_h is pulled to a statistical peak at ~ 125 GeV [22,23]. In particular, the gluino mass is expected at $m_{\tilde g} \sim 4 \pm 2$ TeV while $m_{\tilde t_1} \sim 1.7 \pm 1$ TeV [23]. First/second generation matter scalars are pulled into the $m_{\tilde q,\tilde \ell} \sim 25 \pm 15$ TeV range leading to a mixed decoupling/quasi-degeneracy solution to the SUSY flavor and CP problems [36]. Under such (highly motivated) conditions, most sparticles may well lie beyond the reach of high-luminosity LHC (HL-LHC), with $\sqrt{s} \sim 14$ TeV and integrated luminosity ~ 3 ab^{-1}. The exception is the four higgsinos $\widetilde\chi_{1,2}^0$ and $\widetilde\chi_1^\pm$ which are expected to have mass $\sim 100 - 350$ GeV.

The search for higgsino pair production at the LHC is fraught with some difficulties. The lightest neutralino $\widetilde\chi_1^0$ is expected to form typically 10-20% of dark matter [37,38] with the remainder perhaps being composed of axions [39]. Indeed, such a scenario naturally emerges from the hybrid CCK/SPM solutions [40,41] to the SUSY μ problem [43], where both R-parity and the global $U(1)_{PQ}$ symmetry (needed for an axionic solution to the strong CP problem) emerge as accidental approximate remnant symmetries from a more fundamental $\mathbf{Z}_{24}^R$ symmetry (which itself is expected to emerge from compactification of a 10-d Lorentz string symmetry down to 4-d, $N = 1$ SUSY effective theory [42]). The small mass gaps $\Delta m^0 = m_{\widetilde\chi_2^0} - m_{\widetilde\chi_1^0}$ and $\Delta m^+ \equiv m_{\widetilde\chi_1^+} - m_{\widetilde\chi_1^0}$ (following Guidice & Pomarol notation, Ref. [44]) between the various higgsinos means that production of $\widetilde\chi_1^0\widetilde\chi_2^0$, $\widetilde\chi_1^\pm\widetilde\chi_2^0$ and $\widetilde\chi_1^+\widetilde\chi_1^-$ leads to very soft visible decay products, and where most of the energy goes into making the two lightest SUSY particles' (LSP) rest mass. In addition, $\widetilde\chi_1^0\widetilde\chi_1^0 j$ production provides a monojet at the level of 1/100 signal/background, where the dominant background comes from Zj production [45]. The reaction $pp \to \widetilde\chi_1^0\widetilde\chi_2^0$ with $\widetilde\chi_2^0 \to \mu^+\mu^-\widetilde\chi_1^0$ was proposed in Ref. [46] which would require a soft dimuon trigger to record the events. In Ref's [47,48], it was proposed to look at $\widetilde\chi_1^0\widetilde\chi_2^0 j$ production where an ISR jet radiation at high $p_T \gtrsim 100$ GeV could provide either a jet or MET trigger. Indeed, ATLAS [49] and CMS [50] have followed up on the opposite-sign dilepton plus jet(s) plus MET signature (OSDJMET), and have provided limits on such reactions in the $m_{\widetilde\chi_2^0}$ vs. Δm^0 plane. Due to its promising prospects for SUSY discovery, we will henceforth label this as the *LHC higgsino discovery plane*. Indeed, the latest ATLAS analysis from LHC Run 2 with 139 fb^{-1} finds some *excess of events* with low dilepton invariant mass $m(\ell^+\ell^-) \sim 5 - 10$ GeV in their SR-E-med analysis (see Fig. 11*a* of Ref. [49]). It will be exciting to see if this excess is confirmed in the forthcoming CMS 139 fb^{-1} analysis, or in future data from LHC Run 3 or HL runs.

The ATLAS and CMS searches within the higgsino discovery plane take place within simplified models which are appropriate for the OSDJMET search. Our goal in this paper is to place the higgsino discovery plane within the context of natural SUSY models and landscape SUSY models so as to provide theoretical context

[2] The onset of large finetuning for values of $\Delta_{EW} > 30$ is visually displayed in Fig. 1 of Ref. [30].

for the discovery plane. For instance, what features of the plane are model-dependent or model-independent, and which portions of the plane are favored by naturalness and by the string theory landscape? Identifying such regions should help focus OSDJMET searches onto the most promising portions of parameter space, and also help to prioritize searches in promising regions over searches within regions with implausible parameter choices.

To compare the simplified model of the higgsino discovery plane against expectations from theory, we work with two well-motivated models. The first is generic supergravity GUTs as portrayed in the two-extra-parameter non-universal Higgs model (NUHM2) [51]. This model takes similar parameters as the well-known mSUGRA/CMSSM model except that the two Higgs doublets acquire independent soft terms $m_{H_u}^2$ and $m_{H_d}^2$ whereas the three generations of matter scalars unify to m_0. This model is better motivated than mSUGRA/CMSSM since the Higgs multiplets necessarily live in different GUT multiplets from matter scalars, while the latter may unify in $SO(10)$ SUSY GUTs [52] or in stringy local GUTs [53]. In NUHM2, the gauginos still unify to $m_{1/2}$ at the GUT scale whilst trilinear soft terms unify to A_0. For convenience, the GUT values of $m_{H_u}^2$ and $m_{H_d}^2$ are traded for weak scale parameters μ and m_A. As usual, $\tan\beta$ is the ratio of Higgs vevs. Thus, the parameter space is given by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A \quad (NUHM2). \tag{2}$$

It is easy to generalize this to the NUHM3 or NUHM4 models where the third generation or each generation separately acquires an independent soft mass $m_0(i)$. But for illustration, we will take the generations as degenerate.[3]

A well-motivated alternative is the generalized mirage-mediation model [54] (GMM) which contains comparable moduli-mediated and anomaly-mediated contributions to soft terms. Their relative contributions are parametrized by $\alpha: 0 \to \infty$ where $\alpha \to 0$ gives the pure AMSB [55] soft terms and $\alpha \to \infty$ gives pure moduli (gravity) mediation. It is called mirage mediation because the gaugino mass universality is offset by AMSB contributions proportional to the corresponding gauge group beta functions. Then, evolution of gaugino masses from m_{GUT} to m_{weak} results in gaugino mass unification at the mirage scale [56,57]

$$\mu_{mir} = m_{GUT} \cdot e^{(-8\pi^2/\alpha)}. \tag{3}$$

The soft SUSY breaking terms in GMM are given by

$$M_a = M_s\left(\alpha + b_a g_a^2\right), \tag{4}$$

$$A_{ijk} = M_s\left(-a_{ijk}\alpha + \gamma_i + \gamma_j + \gamma_k\right), \tag{5}$$

$$m_i^2 = M_s^2\left(c_i\alpha^2 + 4\alpha\xi_i - \dot{\gamma}_i\right), \tag{6}$$

where $M_s \equiv \frac{m_{3/2}}{16\pi^2}$, b_a are the gauge β function coefficients for gauge group a and g_a are the corresponding gauge couplings. The coefficients that appear in (4)–(6) originally appeared as discrete quantities for particular orbifold compactifications where the n_i are modular weights. They are given by $c_i = 1 - n_i$, $a_{ijk} = 3 - n_i - n_j - n_k$ and $\xi_i = \sum_{j,k} a_{ijk}\frac{y_{ijk}^2}{4} - \sum_a l_a g_a^2 C_2^a(f_i)$. These coefficients are generalized in GMM to adopt continuous values to allow for more generic ways of moduli stabilization and potential uplifting [54]. The gaugino mass relations (4) are, however, much more robust [58]. Finally, y_{ijk} are the superpotential Yukawa couplings, C_2^a is the quadratic Casimir for the ath gauge group corresponding to the representation to which the sfermion $\tilde{f}_i$ belongs, γ_i is the anomalous dimension and $\dot{\gamma}_i = 8\pi^2\frac{\partial\gamma_i}{\partial\log\mu}$. Expressions for the last two quantities involving the anomalous dimensions can be found in the Appendices of Ref's. [57,59]. In the GMM model, the coefficients c_{H_u} and c_{H_d} can be traded for more convenient weak scale values μ and m_A as in the NUHM2 model, yielding the GMM′ model [54] with a parameter space given by

$$\alpha,\ m_{3/2},\ c_m,\ c_{m3},\ a_3,\ \tan\beta,\ \mu,\ m_A \quad (\mathrm{GMM}'). \tag{7}$$

Here, $m_{3/2}$ is the gravitino mass while c_m and c_{m3} vary the moduli-to-AMSB contributions for first/second versus third generation scalars and $a_3 \equiv a_{Q_3H_uU_3}$ performs the same task for trilinear soft terms. The GMM′ model is programmed into the spectrum generator of Isajet [28] which we use for our sparticle mass calculations. For simplicity, we take $c_m = c_{m3} = (5\ \mathrm{TeV}/\alpha M_s)^2$ so that matter scalar masses are ~ 5 TeV as in the NUHM2 case to be displayed in Figs. 1*a*), 2*a*) and 3*a*). We also take $a_3 = 1.6\sqrt{c_m}$.

One virtue of the LHC higgsino discovery plane is its relative model independence. Given some SUSY model, then for a given set of input parameters one can calculate the (loop corrected) values [44,60,61] of $m_{\tilde{\chi}_2^0}$ and Δm^0 and always locate a point on the discovery plane. Model dependence enters via the assumed value of $m_{\tilde{\chi}_1^\pm}$. The ATLAS and CMS groups assume $m_{\tilde{\chi}_1^\pm}^* \equiv (m_{\tilde{\chi}_2^0} + m_{\tilde{\chi}_1^0})/2$ which roughly holds at leading order in the deep higgsino region [44]. When higher order effects in the mass expansion or loop effects are included, then there are deviations to this ansatz. Since the details of the relative chargino mass hardly affect the OSDJMET signature, the effects are not so relevant, unless one begins leaving the nearly pure higgsino region where $|\mu| \ll m_{soft}$.

As an illustration, we plot contours of mass difference $\Delta m(\tilde{\chi}_1^\pm) \equiv m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^+}^*$ between the full one-loop corrected chargino mass from Isajet and the ATLAS/CMS ansatz $m_{\tilde{\chi}_1^+}^*$ in Fig. 1 for *a*) the NUHM2 model and *b*) the GMM′ model. The blue contour has mass difference zero so is an excellent fit to the ATLAS/CMS ansatz. However, as one proceeds to higher μ values then the mass differences becomes typically greater than zero with the chargino mass becoming larger than the average of the two light neutralinos. For very large μ, then one leaves the light higgsino region and the ansatz no longer obtains. The deviation of the chargino mass from the assumed simplified model value is not very relevant for the monojet plus soft dilepton searches considered below, but would be important for signals such as the golden trilepton signal for SUSY that originate from chargino-neutralino production [62].

In Fig. 2, we show some aspects of the higgsino discovery plane that are beyond the purview of the ATLAS/CMS simplified models and which depend on the entire SUSY particle mass spectrum. In Fig. 2*a*), we scan over the NUHM2 parameters $\mu: 50 - 1000$ GeV (which fixes the higgsino masses) and $m_{1/2}: 100 - 2000$ GeV (which for a given μ value varies the mass gap Δm^0). The remaining parameters are fixed at $m_0 = 5$ TeV, $A_0 = -1.6m_0$, $\tan\beta = 10$ and $m_A = 2$ TeV. Since the entire SUSY spectrum is calculated, now we can compute the corresponding value of Δ_{EW} for each point in the higgsino discovery plane. The green points have $\Delta_{EW} < 15$ while magenta points have $\Delta_{EW} < 30$ and hence qualify as natural. Yellow, blue and purple points have $\Delta_{EW} < 100$, 200 and 300 respectively. The grey-shaded region is already excluded by LEP2 searches for chargino pair production. From the plot, we see of course that the natural region is bounded by $m_{\tilde{\chi}_2^0} \lesssim 350$ GeV as expected. For small $m_{1/2}$ and $\mu > 350$ GeV, then the $\tilde{\chi}_2^0$ is actually wino-like and the model can become unnatural even for lower values of $m_{\tilde{\chi}_2^0} \sim 100 - 300$ GeV (which forms the upper edge of the

[3] Within the string theory landscape, first/second generation matter scalar masses are pulled to a (generation independent) upper bound in the 20 ± 10 TeV regime, offering a mixed decoupling/quasi-degeneracy solution to the SUSY flavor and CP problems [36].

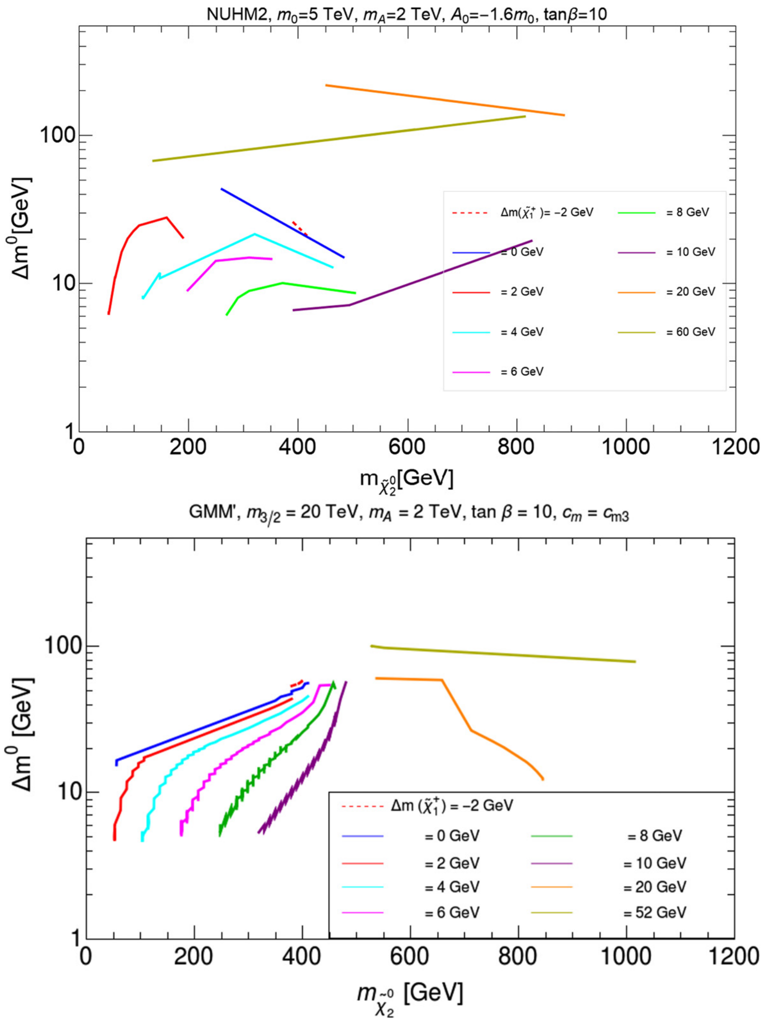

Fig. 1. Deviations in loop-corrected chargino mass as compared to simplified model value $\Delta m(\tilde\chi_1^\pm)$ *a*) in the NUHM2 model with varying μ and $m_{1/2}$ but with $m_0 = 5$ TeV, $A_0 = -1.6m_0$, $\tan\beta = 10$ and $m_A = 2$ TeV, and *b*) in the GMM′ model with varying μ and α but with $m_{3/2} = 20$ TeV and $c_m = c_{m3}$. Both models take $m_A = 2$ TeV and $\tan\beta = 10$.

naturalness envelope in Fig. 2*a*)). For fixed $\mu \sim 100 - 300$ GeV – but as $m_{1/2}$ increases – then the lightest electroweakinos become increasingly higgsino-like and the mass gap Δm^0 drops below ~ 8 GeV. The precise value of the mass gap where the model starts to become unnatural is somewhat sensitive to the assumptions of the NUHM2 model. Indeed somewhat lower values of the neutralino gap would have $\Delta_{EW} \lesssim 30$ if we allow generation-dependent matter scalar mass parameters, or if we give up the gaugino unification assumption. The point, however, is that for small mass gaps, the points become increasingly unnatural, in the NUHM2 case because large $m_{1/2}$ increases $m_{\tilde g}$ which feeds into the stop masses so that the $\Sigma_u^u(\tilde t_{1,2})$ become too large. Also, the two-loop contributions from $m_{\tilde g}$ and $m_{\tilde t_{1,2}}$ can become large [29]. This gives an important result: the region of higgsino discovery plane with mass gaps $\Delta m^0 \lesssim 5$ GeV becomes increasingly unnatural and hence less plausible. As mentioned above, the naturalness lower bound on Δm^0 is somewhat model-dependent and can reach as low as ~ 4 GeV in models like NUHM3 where first/second generation matter sfermions take values in the 20-40 TeV range. In that case, two-loop RGE effects suppress top squark soft term running [63], which allows larger $m_{1/2}$ values to be natural: these same large $m_{1/2}$ values lead to smaller neutralino mass gaps Δm^0. While searches in this unnatural region of very low Δm^0 are always warranted, spending an inordinate effort probing tiny mass gaps should be given a much lower priority in this rather implausible region.[4]

We also show in Fig. 2*a*) the corresponding contour of $m_{\tilde g} = 2.25$ TeV, the limit from ATLAS/CMS simplified model searches for gluino pair production. The region *above* the contour has $m_{\tilde g} < 2.25$ TeV and hence is largely excluded in the NUHM2 framework. We emphasize that this exclusion directly depends on our assumption of gaugino mass unification, and in more general models, the allowed natural region may be considerably larger. We also show the present ATLAS search contour for the OSDJMET channel as the black contour. The region to the left of the contour is thus excluded. Thus, the *allowed* NUHM2 natural search region has mass gap in the ∼7-20 GeV range, and this is the region where a SUSY signal may be expected. The lower bound depends on the specific parameter choices adopted and can range down to 4-5 GeV for other parameter choices. The current search results do cut well into the natural region of the NUHM2 model. We also remind the reader that the ATLAS search has yielded a slight excess in several invariant mass bins $m(\ell^+\ell^-) \sim 5 - 10$ GeV of this search channel. The projected reach of HL-LHC for CMS is shown by the red

[4] This is akin to the huge effort that went into placing limits on compressed stop-neutralino spectra in order to exclude natural SUSY, but under an overly-simplified measure of naturalness which emphasized (wrongly) that top squarks must be not too far removed from the weak scale.

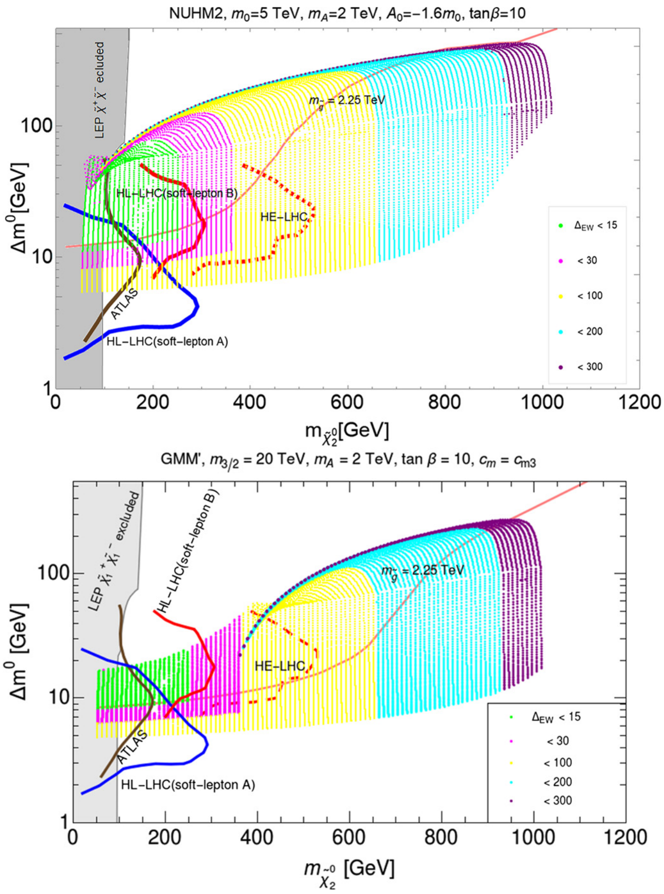

Fig. 2. Regions of naturalness Δ_{EW} in the higgsino discovery plane $m_{\tilde{\chi}^0_2}$ vs. $\Delta m^0 \equiv m_{\tilde{\chi}^0_2} - m_{\tilde{\chi}^0_1}$ from *a*) the NUHM2 model with varying μ and $m_{1/2}$ but with $m_0 = 5$ TeV, $A_0 = -1.6 m_0$ and *b*) the GMM′ model with varying μ and α but with $m_{3/2} = 20$ TeV and $c_m = c_{m3}$. For both models, we take $\tan\beta = 10$ and $m_A = 2$ TeV. We also show the present reach of the ATLAS experiment with 139 fb^{-1} and the ATLAS (soft lepton A) and CMS (soft lepton B) projected future reach at HL-LHC and also CMS at HE-LHC. The region above the $m_{\tilde{g}} = 2.25$ TeV contour is excluded by current LHC Run 2 gluino search analysis. The Higgs mass $m_h \sim 125$ GeV throughout the plane while $m_{\tilde{t}_1} > 1.1$ TeV everywhere.

contour, while the ATLAS HL-LHC projection is labeled by the blue contour. Some of the natural region of the higgsino discovery plane lies beyond the HL-LHC projected reaches. The ATLAS reach extends to lower mass gaps evidently due to the geometry of the ATLAS detector which allows for resolution of lower p_T leptons than CMS. The projected reach of HE-LHC with $\sqrt{s} = 27$ TeV for CMS is given by the dashed red contour [64]. The increased reach of HE-LHC is mainly due to the assumed increase in potential integrated luminosity when proceeding from HL- to HE-LHC: 3 ab^{-1} $\rightarrow$ 15 ab^{-1}. At face value, the projected HE-LHC reach apparently covers all the natural region of the NUHM2 model for the assumed set of parameters.

In Fig. 2*b*), we show the same higgsino discovery plane but for the GMM′ model where the mirage-mediation (MM) value $m_{1/2}^{MM} \equiv \alpha M_s$ again varies between 100-2000 GeV. For lower values of $m_{1/2}^{MM}$ we obtain tachyonic spectra (see Fig. 8 of Ref. [65]) so that no upper edge of unnaturalness ensues as it did in Fig. 2*a* until $m_{\tilde{\chi}^0_2} \gtrsim 400$ GeV. For GMM′, depending on α, we may have a compressed spectrum of gaugino masses as expected from mirage mediation. This means that for a given value of $m_{\tilde{g}}$, the wino and bino masses can be much larger than in the corresponding NUHM2 case with unified gaugino masses. The large wino/bino masses in GMM′ lead to smaller mass gaps and in fact here we find natural spectra with mass gaps down to $\Delta m^0 \sim 6$ GeV. In this case, more of the natural region is explored by the ATLAS rather than CMS cuts and indeed more of the natural SUSY parameter space appears to lie beyond HL-LHC reach. Even a tiny corner of magenta region seems to lie beyond projected HE-LHC reach. As in Fig. 2*a*), the region with mass gap $\Delta m^0 \lesssim 4-5$ GeV becomes increasingly unnatural.

From Fig. 2, it appears much if not most of the nature SUSY parameter space is now excluded, including the values with lowest Δ_{EW}. This is a reflection of the Δ_{EW} measure which is a bottom-up measure of *practical naturalness*: each of the independent contributions o_i to an observable $\mathcal{O}$ ought to be comparable to or less than its measured value. In contrast, from the successful application of the statistics of string theory vacua to the prediction of the cosmological constant (CC), the notion of *stringy naturalness* has arisen [25,35]: the value of an observable $\mathcal{O}_1$ is more natural than the value $\mathcal{O}_2$ if more phenomenologically viable vacua lead to $\mathcal{O}_1$ than to $\mathcal{O}_2$. For the case of the CC, for a uniform distribution of CC values Λ, then statistical selection of pocket universes within the multiverse favor a value of Λ nearly as large as possible such that galaxies condense, and structure forms in the universe. This reasoning allowed Weinberg to predict the value of Λ to within a factor of several well before its value was measured [66].

Applying similar reasoning to the SUSY breaking scale as expected from string theory, then with a number of hidden sectors available, the magnitude of the SUSY breaking scale is expected to scale as a *power law* [32]: $dN_{vac} \sim m_{soft}^n$ where $n = 2n_F + n_D - 1$. This is just a consequence of the fact that in string theory no particular SUSY breaking vev is favored, so all values are equally likely. Then the probability for the cumulative scale of SUSY breaking is just determined by the dimensionality of the space of SUSY breaking fields, which includes a factor of 2 for complex F-term breaking vevs and a factor 1 for real D-term breaking fields (as emphasized by Douglas and others [32,67,68]). Already for SUSY breaking by a single F-term field, there is a linear statistical draw towards large soft terms. However, phenomenological viability must also be addressed. In the case of 4-d SUSY theories containing the MSSM, the magnitude of the weak scale m_{weak} is determined by the values of soft breaking SUSY parameters and the superpotential μ term. Roughly, the larger the SUSY breaking scale, then the larger is the associated value of the pocket-universe weak scale m_{weak}^{PU}. Agrawal et al. [69] have used nuclear physics calculations to argue that in order for complex nuclei to form, and hence atoms as we know them, then the PU value of the weak scale must be within a factor 2-5 of the measured value of the weak scale in our universe: $m_{weak}^{PU} \lesssim (2-5) m_Z^{OU}$. For smoothly distributed values of the μ term and SUSY breaking scale, this amounts to $\Delta_{EW} < 8-50$. For simplicity, we adopt an intermediate value within this range: $\Delta_{EW} < 30$ to yield a phenomenologically viable weak scale.

In Fig. 3, we adopt a value of $n = 1$ for the gaugino masses since in a wide variety of string models the gaugino masses depend only on the dilaton field S gaining a vev, whereas the various moduli contribute subdominantly (the moduli and dilaton are expected to contribute comparably to other soft terms such as trilinears and scalar soft masses) [70]. We sample soft terms according to stringy naturalness with $n = 1$ for gaugino masses but with a uniform distribution in μ (since the μ parameter arises from whatever solution to the SUSY μ problem is assumed [34]) starting at $\mu > 100$ GeV. The resulting distribution of dots is displayed in Fig. 3. The density of dots is important in this case and higher density corresponds to greater stringy naturalness.

In the case of the NUHM2 model displayed in Fig. 3*a*), we see that the region of parameter space with small mass gap is favored by stringy naturalness over the regions with large mass gap. Thus, much of the stringy natural region still lies well beyond the present reach of LHC. This is consistent with the statistical predictions of stringy naturalness for the sparticle mass spectra:

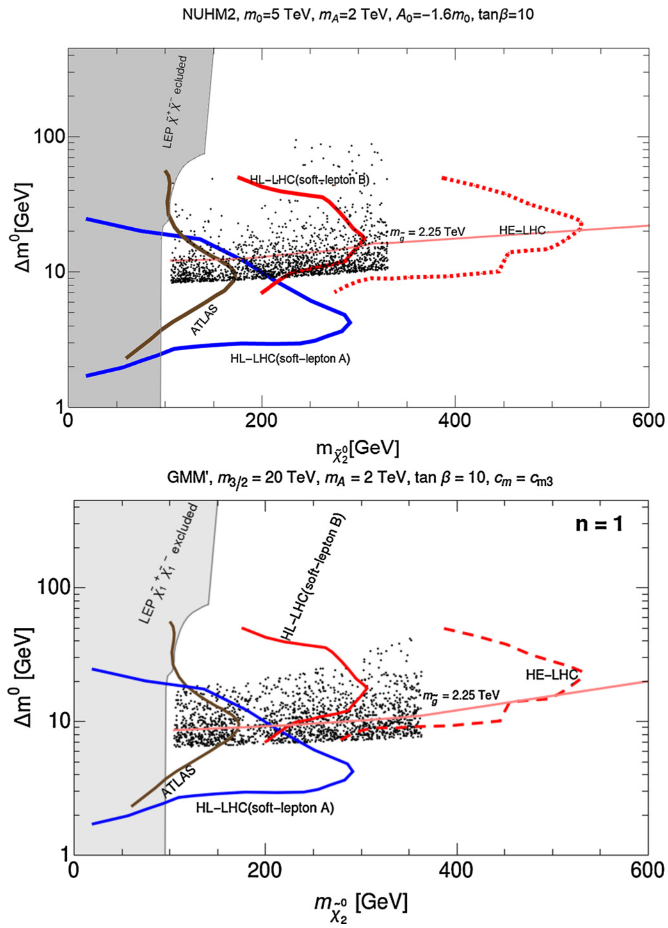

Fig. 3. Regions of stringy naturalness in the higgsino discovery plane $m_{\tilde{\chi}_2^0}$ vs. $\Delta m^0 \equiv m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ for *a*) the NUHM2 model with varying μ and $m_{1/2}$ but with $A_0 = -1.6m_0$ and *b*) in the GMM′ model with varying μ and α but with $m_{3/2} = 20$ TeV and $c_m = c_{m3}$. For both frames, we take $\tan\beta = 10$ and $m_A = 2$ TeV. We also show the present reach of the ATLAS experiment with 139 fb^{-1} and the ATLAS (soft lepton A) and CMS (soft lepton B) projected future reach at HL-LHC and CMS projected reach at HE-LHC. The region above the $m_{\tilde{g}} = 2.25$ TeV contour is excluded by current LHC Run2 gluino pair searches. The Higgs mass $m_h \sim 125$ GeV throughout the plane while $m_{\tilde{t}_1} > 1.1$ TeV everywhere.

stringy naturalness pulls the Higgs mass m_h to a peak around 125 GeV while gluinos are pulled up to $m_{\tilde{g}} \sim 4 \pm 2$ TeV and stops to $m_{\tilde{t}_1} \sim 1.5 \pm 0.5$ TeV [22,23,65]. Thus, stringy naturalness seems to explain what LHC is seeing: a Higgs of mass $m_h \simeq 125$ GeV with sparticles pulled beyond the present LHC reach. For a fixed value of μ, since stringy naturalness pulls the gaugino masses as large as possible – subject to maintaining m_{weak}^{PU} not too far removed from our measured value m_{weak}^{OU} – then we expect the $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ mass gap to be favored on the low allowed side: $\Delta m^0 \sim 5 - 10$ GeV. A similar calculation performed within the GMM′ model yields similar results in Fig. 3*b*): the low mass gap region is statistically favored within phenomenologically viable vacua within the multiverse.

Before concluding, it seems worthwhile to highlight the similarities and differences between the naturalness considerations in Fig. 2 and Fig. 3. The bottom-up measure Δ_{EW} is universal and applies independently of the details of UV physics. In contrast, the notion of stringy naturalness hinges on the existence of string vacua and their distribution as well as on the atomic principle that led to the cut-off, $m_{weak}^{PU} \lesssim (2-5)m_Z^{OU}$. These additional hypotheses about the nature of UV physics lead to a preference for lower values of Δm^0. We stress, however, that stringy naturalness together with the atomic principle is entirely compatible with electroweak naturalness. This is reflected in the fact that the envelope of points in Fig. 3 is essentially the same as that in Fig. 2. The reader who does not subscribe to the notion of stringy naturalness can simply disregard the preference for points with lower Δm^0 apparent in Fig. 3. However, the important conclusion that naturalness considerations require the neutralino mass gap to be not much below 4-5 GeV remains unaltered.[5]

Conclusions: Based on electroweak naturalness and even more on stringy naturalness, it may well be that gluinos and squarks, including top squarks, lie well beyond the reach of HL-LHC, and so may have to await an energy upgrade of LHC into the 27-50-100 TeV range for their discovery.[6] In contrast, the lightest electroweakinos are expected to be mainly higgsino-like with masses not too far removed from the measured value of the weak scale $m_{weak} \simeq m_{W,Z,h} \sim 100$ GeV. Thus, higgsino pair production is expected to occur at considerable rates at HL-LHC. The problem instead is one of visible energy: the small mass gaps $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and especially $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ are expected to be on the $5-10$ GeV range and most of the reaction energy goes into making the LSP masses $2m_{\tilde{\chi}_1^0}$. In such a case, it appears the soft opposite-sign dilepton plus jet plus MET signature OSDJMET is most promising, which depends on initial state radiation of a hard gluon or quark jet so that MET or $p_T(jet)$ can be used as a trigger. ATLAS and CMS have been analyzing these reactions and plotting excluded regions in the simplified model $m_{\tilde{\chi}_2^0}$ vs. Δm^0 plane and in fact ATLAS has a slight excess of events in this channel with $m(\ell^+\ell^-) \sim 4-12$ GeV from 139 fb^{-1} of data. From the theory perspective, not all parts of the higgsino discovery plane are equally plausible. In this paper we plotted out the natural portions of the discovery plane using the model-independent naturalness measure Δ_{EW}. Large portions of the natural region is already excluded by both gluino pair searches and by the OSDJMET search channel. However, considerable portions of the discovery plane remain unconstrained, especially those with low mass gaps $\Delta m^0 \sim 5-10$ GeV. Indeed these very portions are most favored by stringy naturalness, which also predicts $m_h \sim 125$ GeV with sparticles beyond the present LHC reach (along with the magnitude of the CC). Thus, experimental searches may wish to concentrate on these (stringy) natural regions, with perhaps lower priority efforts directed to mass gaps significantly below 4-5 GeV and certainly below 1 GeV. In those regions, huge gaugino masses are required which ultimately spoil the naturalness of the models.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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[5] A previous paper explored the compressed electroweakino mass spectrum from natural SUSY with an eye towards the possibility of long-lived charginos with sub-GeV mass gaps [72]. This work took place in the pMSSM11 model with upper limits on parameter choices arising from different notions of naturalness [73]. They also concluded that the mass gap should be larger than 5 GeV, and emphasized that metastable higgsinos would *not* be a signature of (that version of) natural SUSY.

[6] It is worth noting that in natural SUSY models with $\Delta_{EW} < 30$, a 27 TeV *pp* collider with an integrated luminosity of 15 ab^{-1} would discover at least one of the stop or the gluino, and possibly both; discovery of other squarks and sleptons may have to await yet higher energy colliders. [71].

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