Overview of gravitational waves: Formation, polarization and detection

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Abstract. Gravitational waves (GWs) are reliable for proving general relativities. Also, they can be used in detecting physics phenomena in the universe. Motions of bodies make spacetime curved, and then gravitational waves are formed. Additionally, GWs can be produced in the early universe. A mathematical description of gravitational waves derived using linearized theory and transverse-traceless gauge shows GWs polarizations and the relationship between perturbation and strain. GW signal was first detected in the binary system PSR1913+16. Data presented an orbital decay, which followed general relativity's prediction. Direct detections today, designed according to GWs properties, are carried out by resonant detectors, interferometers, and pulsar timing arrays. Noise reduction is a key to accurate detection. Furthermore, analyzing the B-mode cosmic microwave background can be used in detecting primordial gravitational waves.

Keywords: Formation Of GW, GW Polarization, PSR 1913+16, Detectors, CMB Polarization.

1. Introduction

Gravitational waves are associated with the curvature of spacetime. They predict general relativity and any other relativistic theories of gravity. They provide a new method to examine general relativity and its alternatives in the high-speed, robust field regime [1]. By studying gravitational waves, we can get a picture of the coalescence of compact binaries.

We are looking forward to finding new physics corresponding to the connecting part of two inspiral bodies. Also, we can get information about the Big Bang and the early universe from gravitational waves formed around inflation.

This paper introduces general ideas of gravitational waves. Section 2 is about the formation of gravitational waves. GWs can be produced around the time of inflation. They can also be generated from the motions of bodies. Section 3 shows the polarization of gravitational waves. Assumptions and mathematical derivation of polarization are present. Also, strain, a property of gravitational waves related to their polarization, is mentioned in section 3. Section 4 is about indirect detection. The signal from PSR 1913+16 shows orbital period decays, which is consistent with general relativity's prediction. Section 5 presents three gravitational wave detectors: resonant, interferometer, and pulsar timing arrays. Operating principles and suitable frequency for each sensor are shown in this section. What's more, it will contain an error analysis. Section 6 briefly introduces using CMB (cosmic microwave background) polarization to detect primordial gravitational waves.

2. Formation of Gravitational Waves

2.1. Motion of Bodies
The motion of massive bodies like compact binaries can produce gravitational waves. Activities of bodies make spacetime curved. Gravitational waves are "ripples" in the curvature of spacetime and propagate at the speed of light. Most GWs caused by motions have frequencies ranging from $10^{-4}Hz$ to $10^{4}Hz$, depending on the masses of bodies.

2.1.1. Coalescence of Compact Binaries. Bodies in compact binary systems are neutron stars or black holes. The coalescence of binaries can be divided into 3 phases: inspiral, merger, and ring-down. Gravitational waves are produced in all three stages. During inspiral, bodies in a binary system orbit around each other. Distance between two bodies shortens, and relative velocity increases. The orbital period is reduced, and the system emits orbital energy through gravitational waves. Two bodies merge into a single one the merger, forming more intensive gravitational waves. Then, the strength of gravitational waves decays, and the system goes into the ring-down phase. During this stage, the new body emits gravitational waves to achieve equilibrium [2].

As for binary black holes, strain (Strain can reflect the strength of gravitational waves. More details are shown in section 3.) of gravitational waves is mildly increased during the inspiral. And then, strain changes rapidly in the merger phase. During ring-down, strain oscillates even milder and rests in the end. FIG.2, in reference, shows the strain's change of coalescing binary black holes. Themes of GWs from other compact binaries change in the same tendency with different values [3].

2.1.2. Stellar Core Collapse. The core collapse in massive stars is a promising source of gravitational waves. It contains necessities for generating gravitational waves: a mass of $1 - 100\mathrm{M}_{\odot}$ flows in a compact region at relativistic speed. Instabilities caused by highly rapid rotation (Most massive-star models indicate that such a process is unlikely to exist.) in core or asymmetric dynamics of core-collapse can lead to the emission of gravitational waves [4].

2.1.3. Rotating Neutron Stars. A rotating neutron star or GW pulsar is a prototypical source for continuous gravitational waves. One mechanism concerns the toroidal magnetic field $\mathrm{B_t}$, which distorts neutron stars into a prolate shape. An elastic neutron star is unstable under a large $\mathrm{B_t}$, and the star's magnetic axis mismatch with the star's angular momentum. Gravitational waves are more likely to be emitted when the magnetic axis and angular momentum are orthogonal [5].

2.2. In Early Universe
One model of generating gravitational waves is related to fluctuations. During inflation, initial quantum fluctuations were amplified into classical perturbation outside the visible universe. This process formed tensor modes, which can be viewed as gravitational waves. Gravitational waves generated in this way have an extensive frequency range [6].

Another model of gravity-wave production is associated with inhomogeneities. In many inflation models, accelerated expansion ends with preheating. During preheating, energy was extracted from the inflation field non-perturbatively and non-thermally, leading to a non-thermal distribution. Preheating introduces large and transient density inhomogeneities, which can be a source of stochastic gravitational waves [7]. In this model, the frequencies of gravitational waves are in the Hz to GHz range [8].

3. Polarization
As a ripple in the curvature of spacetime, the passage of gravitational waves leads to changes in the distance of particles in different directions, which present polarizations of gravitational waves. There are two types of polarizations: "plus" and "cross". If a gravitational wave propagates along z direction, there are six possible polarization modes. However, only 2 of them are allowed in general relativity. Fig.5.8 in reference shows the polarization modes of GWs [9].

3.1. Approximation in General Relativity

There is no apparent difference between the metric representing waves and other metrics. Hence, three approximations-linearized theory, short-wave approximation, and post-Newtonian theory-are applied to make a distinction.

Linearized theory approximates under a weak field, where motions happen in nearly flat space-time. In this case, the static part and perturbation part can be separated. Metric tensor(Metric tensor is used in coordinate transformation) then can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3.1}$$

Where $\eta_{\mu\nu}$ is the Minkowski metric (Minkowski metric is a metric tensor of 4-dimentional spacetime, whose spatial coordinate is Cartesian.) diag(-1,1,1,1), representing a static part, and $h_{\mu\nu}$ is a small perturbation of smooth background simulating waves. The amplitude of waves is small, so $h_{\mu\nu} \ll 1$.

In short-wave approximation, the background metric $h_{\mu\nu}$ has a larger curvature radius than the wavelength. Moreover, the metric is time-dependent and changes slowly. The most useful approximation is the post-Newtonian approximation, where waves arise when corrected in high order. The correction sets general relativity away from its Newtonian limit [10].

3.2. Transverse-Traceless (TT) gauge and GWs' Polarization

In linearized theory, a specific gauge is chosen to make metric perturbation purely spatial, namely

$$h_{tt} = h_{ti} = 0 \tag{3.2}$$

Transverse-traceless meter can be introduced:

$$\partial_i h_{ij} = 0 \tag{3.3}$$

A metric perturbation can be written as $\mathrm{h}_{\mathrm{ab}}^{\mathrm{TT}}$ when satisfying TT gauge. "i" and "j" here are related to space coordinates.

In vacuum, the linearized Einstein field equation is

$$h_{ij}^{TT} = 0 \tag{3.4}$$

A detailed derivation of (3.4) is shown in reference [11]. Assume a GW traveling along z-axis. Perturbation depending on z and time t can be a solution for (3.4). In this case, the metric perturbation can be $\mathrm{h}_{\mathrm{ij}}^{\mathrm{TT}}(\mathrm{t}-\mathrm{z})$. From TT gauge, we have

$$\partial_z h_{zj}^{TT} = 0 \tag{3.5}$$

(3.5) tells that $\mathrm{h}_{\mathrm{zj}}^{\mathrm{TT}}$ is irrelevant to z. "t" and "z" are associated in $\mathrm{h}_{\mathrm{ij}}^{\mathrm{TT}}(\mathrm{t}-\mathrm{z})$, so $\mathrm{h}_{\mathrm{zj}}^{\mathrm{TT}}(\mathrm{t}-\mathrm{z})$ is also unrelated to t and must be a constant. To meet the condition that $\mathrm{h}_{\mathrm{ij}} \longrightarrow 0$ as $\mathrm{r} \to \infty$, $\mathrm{h}_{\mathrm{zj}}^{\mathrm{TT}}(\mathrm{t}-\mathrm{z}) = 0$ is accepted. Given this and (3.2), the non-zero components of h^{TT} are $\mathrm{h}_{\mathrm{xx}}^{\mathrm{TT}}$,$\mathrm{h}_{\mathrm{xy}}^{\mathrm{TT}}$,$\mathrm{h}_{\mathrm{yx}}^{\mathrm{TT}}$ and $\mathrm{h}_{\mathrm{yy}}^{\mathrm{TT}}$. From symmetry, we have

$$h_{xx}^{TT} = -h_{yy}^{TT} \equiv h_+(t-z) \tag{3.6}$$

$$h_{xy}^{TT} = h_{yx}^{TT} \equiv h_\times(t-z) \tag{3.7}$$

From TT gauge and Einstein field equation, we can get (3.6) and (3.7), representing the two modes of GWs polarization.

3.3. Perturbation and Strain

Consider two particles separated on x-axis. One is located at the origin, and another is located at $x = L_c$. The line element is

$$\mathrm{d}l^2 = g_{ij} dx^i dx^j \tag{3.8}$$

The distance between the two particles can be written as

$$\mathrm{L} = \int_0^{Lc} \sqrt{g_{xx}(dx)^2} = \int_0^{Lc} \sqrt{g_{xx}} dx \tag{3.9}$$

When a GW propagates along z-axis, perturbation happens, and metric tensor g_{ij} has the form (3.1). (3.9) becomes

$$\mathrm{L} = \int_0^{Lc} \sqrt{1 + h_{xx}^{TT}(t-z)dx} \approx \int_0^{Lc} \left[1 + \frac{1}{2}h_+(t-z)\right] dx = L_c \left[1 + \frac{1}{2}h_+(t-z)\right] \tag{3.10}$$

Because of the perturbation, particle distance changes in one direction of GW's polarization. The proportion of change is

$$\frac{\delta L_x}{L} \approx \frac{1}{2}h_+(t-z) \tag{3.11}$$

The same is true for the situation that two particles are separated on y-axis and a gravitational wave travels along z-axis:

$$\frac{\delta L_y}{L} \approx -\frac{1}{2}h_+(t-z) \tag{3.12}$$

Strain of GW can be defined as

$$\mathrm{h} = \frac{\Delta L}{L} \tag{3.13}$$

And it is associated with the amplitude of the gravitational wave. (3.13) and (3.11) have similar form. It is natural to think that perturbation $\mathrm{h}_{\mathrm{xx}}^{\mathrm{TT}}$ works as a strain [11].

4. Indirect Detection–PSR 1913+16

The first evidence of gravitational waves is detected in the binary pulsar PSR 1913+16. In this binary system, there are 2 neutron stars, one of which is a pulsar. Pulsar makes the system perfect for testing general relativity. By fitting arrival times of pulse, component masses and size of orbit can be calculated. Combining them with orbital period P_b and eccentricity e, we can get the orbital period decay $\dot{P}_b$ rate. J. H. Taylor and J. M. Weisberg measured that $\dot{P}_\mathrm{b} = (-2.30 \pm 0.22) \times 10^{-12}$[12] (more detailed calculations are contained in this reference). This measurement is consistent with the prediction of gravitational waves from general relativity.

Today, a program named TEMPO can analyze TOAs (time of arrival) data and output orbital parameters: projected semimajor axis $a_p sini$, e, epoch of periastron passage T_0, P_b, and longitude of periastron ω_0. It can also output post-Keplerian parameters, including orbital period derivative $\dot{P}_\mathrm{b}$ [13].

5. GW Detectors

5.1. Resonant Detector

A resonant detector consists of a bar, suspensions, a resonant transducer, and an amplifier. Due to polarization, gravitational wave forces the bar to change in length and oscillate. Assume a bar with length $L{\sim}1m$ and mass $M = 1000kg$. Using (3.13), we can calculate the amplitude of bar's vibration caused by a short burst gravitational wave with $\mathrm{h} \sim 10^{-21}$:

$$\delta l_{gw} \sim hl \sim 10^{-21} m \tag{5.1}$$

where h is the strain of GW.

Resonant detector experiences three noises: thermal noise, sensor noise, and quantum limit.

Thermal noise is the most serious source of the noise. Place the bar in the environment with temperature T. When the kinetic energy of the bar equals to the thermal energy of a single degree of freedom, amplitude caused by thermal noise can be derived

$$\frac{M(\delta i)^2}{2}=\frac{kT}{2}$$

$$\langle\delta l^2\rangle_{th}^{\frac{1}{2}}=\left(\frac{kT}{4\pi^2 Mf^2}\right)^{\frac{1}{2}} \tag{5.2}$$

If $T=100mK$, then $\delta l_{th}\sim 6.10^{-18}m$. δl_{th} is much larger than δl_{gw}. GW cannot be measured accurately under large noise. To detect GW, we need to increase Q, defined as $Q=f\cdot\tau$. f is the resonant frequency, and τ is vibrations' decay time. Due to random walks, the amplitude is $Q^{\frac{1}{2}}$ times smaller than the expected amplitude from (5.2):

$$\langle\delta l^2\rangle_{th}^{\frac{1}{2}}=\left(\frac{kT}{4\pi^2 Mf^2 Q}\right)^{\frac{1}{2}}\sim 6\cdot 10^{-21}m \tag{5.3}$$

It has the same order as the GW's amplitude. Bars nowadays can be operated at $\delta l=\delta l_{gw}+\delta l_{th}\sim 10^{-20}m$ [10].

The resonant transducer transforms vibration into an electric signal, and the amplifier enlarges the call to a recorded level. The amplifier then introduces **sensor noise**. The amplitude of vibration is the largest near the resonant frequency so that sensor noise can be ignored. Sensor noise limits the detector's sensitivity to the resonant frequency.

Quantum limit is caused due to uncertainty principle. It causes the vibration with an amplitude $\langle\delta l^2\rangle_{quantum}^{\frac{1}{2}}\sim 4.10^{-21}m$. Quantum limit is dominant when thermal noise is solved.

Bars are designed to detect bursts and can observe frequencies higher than 500Hz. Usually, the resonant detector is a cylinder, which is only sensitive to one direction along the principal axis. There is also a spherical detector, which is sensitive to all orders and can observe all sky.

5.2. *Interferometer*

Interferometers are based on interference, the same principle as the Michelson-Morley Experiment. There are two types. One is ground-based, and another is space-based.

5.2.1. Ground-Based Observatories. LIGO and Virgo are ground-based observatories. They detect gravitational waves in the frequency range between a few Hz and a few kHz [14,15]. When the laser meets the beam splitter in the center of the detectors, it is divided into two perpendicular beams of light. The original lengths of the two light paths are the same. When the gravitational waves come, the size of one way will be changed. Two beams of light have different phases when they meet again. At the photodetector, interference happens.

A GW with $h\sim 10^{-21}$ will change the length of one arm in interferometer

$$\delta l_{gw}\sim hl \tag{5.4}$$

Set $l=4m$, then $\delta l_{gw}\sim 4.10^{-18}m$. The typical period of gravitational waves of interest is much more extended than when light travels one round in arm. Hence, keeping light in arms and making light run 100 games is possible. Length change becomes $\delta l_{gw}\sim 4.10^{-16}m$, which makes GW easier to detect. Reflection is a way to keep light in your arms. Fabry-Pérot cavities, optical cavities with low-transmissivity mirrors, can achieve it.

Ground-based interferometer experiences five types of noise: ground vibration, thermal noise, shot noise, quantum effect, and gravity gradient noise [16].

Vibration caused by ground motion is much larger than that caused by a GW. To avoid this noise, one can set mirrors on pendulums, which make ground noise small at high frequency and offer isolation.

The pendulum introduces **thermal noise** at a few Hz. Light bounces between mirrors, causing internal vibrations of mirrors with natural frequencies at a few kHz. These can be limited to a narrow frequency band. Hence they can have a small impact on GWs measurement. Besides, some mirrors are partly transmissive. During transmission, mirrors absorb light's energy and increase temperature. As a result, the index of refraction changes, affecting interference. This effect limits the laser power used in detection.

Photons are quantized, so they arrive at the photodetector randomly and make random changes in light intensity, similar to that caused by a GW. This mechanism causes deviation (**shot noise**) in length measurement.

$$\delta l_{shot} \sim \frac{\lambda}{2\pi\sqrt{N}} \tag{5.5}$$

where N is the number of photons and λ is the light wavelength. The detected frequency is f, then the frequency of measurement is $2f$.

$$\mathrm{N} = \frac{total\ energy}{single\ photon\ energy} = \frac{P \cdot \frac{1}{2f}}{\frac{hc}{\lambda}} \tag{5.6}$$

P is light power. Associating (5.5) and (5.6), a suitable method to reduce shot noise can be obtained: raising laser power. By setting a power-recycling mirror in front of the laser, photons' utilization can be increased, which has the same effect as improving strength.

According to the uncertainty principle, when length measurement is improved (shot noise is reduced), uncertainty in momentum measurement increases. **Quantum effects** are introduced. However, we don't need to know everything in the system. We only need to improve the measurements of quantities associated with GWs.

When GWs come, the gravitational field changes, leading to some natural sources like seismic waves and oscillations in air pressure. Detectors will respond to these sources and introduce **gravity gradient noise**. The noise has a huge impact when the frequency is low, so ground-based sensors are unsuitable for detecting frequencies below 1Hz.

5.2.2. Space-Based Observatories. One can launch the interferometer into space to avoid gravity gradient noise and detect GWs with a frequency below 1Hz [16].

LISA, TianQin, Taiji, and DECIGO are space-based observatories along Earth's orbit. (Diagrams of them are shown in references [17-20]. They share the same principle as ground-based observatories. In LISA, TianQin, Taiji, and spacecraft form triangles. DECIGO has four clusters, and each group has three satellites. The lengths of triangles are much longer than paths on the ground, so δl is more significant (according to (3.13)), which makes low-strain gravitational waves easier to observe. The minimum frequency that space-based observatories can detect is 0.1mHz.

5.3. Pulsar Timing Arrays

The arrival times of a pulse can be predicted by combining the pulsar's astrometric and orbital parameters, pulse propagation conditions, and spin. Comparing the prediction with observed TOAs, we can get the pulsar 'timing residuals', which represent space-time disturbances and is introduced by gravitational waves [21].

Pulsar timing arrays (PTAs) is a well-timed millisecond pulsars system that is perfect for detecting TOAs. The accuracy of its observation can reach 30ns. Therefore, PTAs can be used to measure timing residuals and, further, see gravitational waves. In order to improve the sensitivity of PTAs, (1) continuous observation of pulsars is required; (2) white and red noise needs to be reduced; (3) more pulsars need to be added.

Pulsar timing arrays are expected to detect ultra-low frequency $(\sim 10^{-9}-10^{-8}Hz)$ GWs. The primary sources of it are massive black hole binaries in stable orbits with periods of few or tens of years, stochastic background gravitational radiation, and GW bursts [22].

6. CMB Polarization

CMB (cosmic microwave background), left after the Big Bang, has 2 modes of polarizations: E mode and B mode, caused by scalar and tensor perturbations (gravitational waves) in the early universe separately. Hence, observing B-mode polarization is a possible way to detect primordial gravitational waves.

Three devices can be used in B-mode polarization detection: ground-based observatories, high-altitude balloons, and satellites. Ground-based observatories ([23] gives an example of ground-based observatories.) are allowed to equip large-diameter telescopes, which lead to high resolution. Balloon-borne experiments avoid interfering emissions from the atmosphere and can be improved quickly. Like balloons, satellites are also free from atmospheric emissions. They can detect large angular-scale fluctuations but have limited resolution due to small telescopes.

To make observation more convincing, three issues need to be handled [24]. The first is improving sensitivity. Detectors are suffering thermal emissions from telescopes and the environment. Only satellites in space are cold enough to prevent this emission. Fundamental photon noise is the prominent noise of the best detectors. Therefore, the sensitivity of a single detector cannot be improved much. One can increase instrument complexity and the number of sensors to get higher sensitivity. The second is to identify CMB from diffuse emission from Milky Way. One can do the measurement at multiple frequencies. Then, the emission can be removed with the knowledge that galactic emission has a different spectrum than CMB. The third is to control instrument systematic uncertainties. With the improvement of equipment, a systematic errors can be reduced.

7. Conclusion

One way to generate gravitational waves is through the motions of massive bodies. It can produce GWs with frequencies from $10^{-4}Hz$ to $10^{4}Hz$. One joint motion is a coalescence of compact binaries containing 3 phases: inspiral, merger, and ring-down. What's more, due to instabilities and asymmetry, stellar core collapse can lead to the emission of GWs. A mismatch of magnetic axis and angular momentum makes neutron stars a source for continuous GWs. Gravitational waves can also be produced around inflation in the early universe due to fluctuations and inhomogeneities. These primordial GWs have an extensive frequency range.

Three assumptions are made in general relativity to get a mathematical description of GWs: linearized theory, short-wave approximation, and post-Newtonian theory. Introducing small perturbation $h_{\mu\nu}$ in spacetime and combining approximations with TT gauge, 2 modes of polarizations, “plus” and “cross”, can be derived. Perturbations express polarization and act like strains.

A signal from PSR 1913+16 first proved the existence of gravitational waves. By fitting the data of pulse arrival time, we got binary's orbital period decay, which showed that the system emitted energy in the forms of GWs. The decay rate is consistent with general relativity. Today, TEMPO is designed to analyze TOAs.

The resonant detector is sensitive in a narrow band around its resonant frequency. It suffers thermal noise, sensor noise, and quantum limit. Sources with frequencies higher than 500Hz are suitable to detect using the deep detector. Interferometer is based on interference. Ground-based observatory detects GWs between a few Hz and a few kHz. It experiences ground vibration, thermal noise, shot noise, quantum effect, and gravity gradient noise. The last one limits its detectable frequency. To solve this problem, one can launch the observatory into space. Space-Based observatories can detect frequency below 1Hz; its minimum appreciable frequency is 0.1mHz. Pulsar timing arrays consist of well-timed millisecond pulsars. By analyzing their timing residuals, one can get information on GWs. PTAs can detect ultra-low frequency GWs.

Primordial GWs produced B-mode CMB polarization. Ground-based observatories, high-altitude balloons, and satellites detect B-mode polarization. Three issues need to be settled: (1) sensitivity, (2) identification of CMB, (3) systematic uncertainties.

With equipment and analysis methods improved, more detailed information in GWs signal can be explored. Through GWs, We look forward to discovering new theories corresponding to black hole properties, the early universe, and so on.

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