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C 11 Unifying AH Elementary-Particle Forces Including Gravity

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It is a final goal in physics to unify all the four basic forces, strong, weak, electromagnetic, and gravitational. I have found it hard to make one story out of many different models and diverse interests existing at this stage of the game in this field. The following is the best I can do.

In this talk, I would like to review some of the contributed papers allocated to this Session other than what Pati,[1] Palla,[2] and Freund[3] will give us talks on. Let me first discuss unified gauge theories of strong, weak, and electromagnetic interactions. There are two standard models: the model of Pati and Salam[4] in which leptons have the fourth color and the model of Georgi and Glashow[5] in which a simple group SU(5) is assumed for grand unification. Since Fritzsch[6] has given in Session CIO and Pati will give us here some features of these "orthodoxies," I shall pick up only new ideas contributed to this Session.

In the original Georgi-Glashow model,[5] leptons and quarks are assigned to a fundamental quintet and an antisymmetric decuplet of SU(5):

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ l^+ \\ \bar{\nu} \end{pmatrix}_R \quad \text{and} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1' & -d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2' & -d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3' & -d_3 \\ u_1' & u_2' & u_3' & 0 & -l^+ \\ d_1 & d_2 & d_3 & l^+ & 0 \end{pmatrix}_L.$$

It is then very tempting to enlarge the gauge group so that these fifteen leptons and quarks may be put in a same multiplet. Such an extension of the Georgi-Glashow model to a grand unified model of SU(6) gauge group has been made by Inoue, Kakuto, Nakano, Abud, Buccella, Ruegg, Savoy, Lee, Weinberg, and Yoshimura.[7] In the contributed paper, "Unified SU(6) Gauge Theory of the Strong, Weak, and Electromagnetic Interactions" by S. K. Yun,[8] the eight leptons, *(e", E~, v_e; E°)* and *(fjL~, M~, v^; M°)*, and the eight quarks, *(t_{i9}* ***Ui*** *g_u* ***Ci)*** with charge 2/3 and *(d_{i9} b_t\ s_{i9} h_t)* with charge —1/3, form two left-handed 15-plets and two left-handed singlets:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \bar{t}_3 & -\bar{t}_2 & -u_1 & -d_1 & -b_1 \\ -\bar{t}_3 & 0 & \bar{t}_1 & -u_2 & -d_2 & -b_2 \\ \bar{t}_2 & -\bar{t}_1 & 0 & -u_3 & -d_3 & -b_3 \\ u_1 & u_2 & u_3 & 0 & e^+ & -E^+ \\ d_1 & d_2 & d_3 & -e^+ & 0 & \bar{E}^0 \\ b_1 & b_2 & b_3 & E^+ & -\bar{E}^0 & 0 \end{pmatrix}_L, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \bar{g}_3 & -\bar{g}_2 & -c_1 & -s_1 & -h_1 \\ -\bar{g}_3 & 0 & \bar{g}_1 & -c_2 & -s_2 & -h_2 \\ \bar{g}_2 & -\bar{g}_1 & 0 & -c_3 & -s_3 & -h_3 \\ c_1 & c_2 & c_3 & 0 & \mu^+ & -M^+ \\ s_1 & s_2 & s_3 & -\mu^+ & 0 & \bar{M}^0 \\ h_1 & h_2 & h_3 & M^+ & -M^0 & 0 \end{pmatrix}_L,$$

$$(\bar{\nu}_e)_L, \text{ and } (\bar{\nu}_\mu)_L.$$

They also form two right-handed 15-plets and two right-handed singlets with suitable combinations of leptons and quarks. Assuming two 35-plet Higgs scalars, the one breaking SU(6) down to $SU(3)_0$ x SU(3V x U(l) and the other breaking 811(3)^, as well as a 15-plet Higgs scalar and also a singlet one, he has derived two mass relations for leptons and quarks:

$$\frac{m_d}{m_b} = \frac{m_s}{m_h} \quad \text{and} \quad m_d + m_s + m_b + m_h$$
$$= m_e + m_\mu + m_{E^-} + m_{M^-},$$

the latter of which is weaker than the original one of Georgi and Glashow, *m_d=m_e, m_s~m^* etc., in a model with a single quintet Higgs scalar. From these relations, he has predicted

$$m_b = 3.5^{+1.3}_{-0.4}\ \text{GeV and } m_h = 4.6^{+1.7}_{-0.5}\ \text{GeV}.$$

The quantization of electric charge of elementary particles is one of the most satisfactory features in grand unified gauge theories. In order for electric charge to be quantized, it is not, however, necessary that the strong gauge group $G_c=SU(3)_c$ and the weak and electromagnetic gauge group $G_w x G_A$ (where G_A is an Abelian factor) be unified into a simple or semisimple group. What is necessary is either that the group G_w X G_A is unified by a semisimple flavor gauge group G_F or that the group $G_A x G_c$ is unified by a semisimple strong gauge group G_s- In the contributed paper, "Embedding Weak-hypercharge in Strong Gauge Group" by Inoue, Kakuto, Komatsu, and Nakano,[9] the latter possibility has been investigated in great detail. The first example of their models is the case where $G_s=SU(4)$.[4] Since $SU(3)_c$ should be electrically neutral, the charge operator Q is written as

$$Q=Q_W+Q_S \quad \text{with} \quad Q_S=2\sqrt{6}\,xF_{15},$$

where F_{16} is a generator of SU(4) and x is a parameter to be determined. Due to the color constraint that fermion representations should contain only **1,** 3, and 3* of color $SU(3)_c$, allowed candidates are **1, 4,** 6, and **4*** of SU(4). They are decomposed into representations of subgroup SU(3)<7 xU(1) in the notation $(d_{Cy}\ Q)$ as

$$\mathbf{1}=(\mathbf{1},0),\ \mathbf{4}=(\mathbf{3},x)+(\mathbf{1},-3x),$$
$$\text{and } \mathbf{6}=(\mathbf{3},-2x)+(\mathbf{3}^*,2x).$$

The relation between the gauge coupling constants of SU(3)<7 and U(1) is given by

$$g_C=2\sqrt{6}\,xg'.$$

They take two choices: GV=SU(2) or $SU(2)_L$ X $SU(2)_R$. In the case of G^=SU(2),

$$Q_W=I_3$$

where 7_3 is a SU(2) generator. They have found that one of the most interesting models is given by

$$x=\frac{1}{6},\ I(f_1)=0,\ I(f_4)=\frac{1}{2},\ \text{and } I(f_6)=0.$$

The leptons and quarks are assigned as

$$(f_4^{I=1/2})_L=\binom{u}{d}_L+\binom{\nu_e}{e}_L,$$

$$(f_4^{I=1/2})_R=\binom{u}{b}_R+\binom{N}{e}_R,$$

$$(f_6^{I=0})_R=d_R+b_R^*,\ \text{and } (f_1^{I=0})_L=N_L.$$

Following the renormalization group analysis of Georgi, Quinn, and Weinberg,[5] they obtain

$$\ln\left(\frac{m_Y}{\mu}\right)=\frac{2\pi}{11}\left[\frac{3}{2}\alpha^{-1}(1-\sin^2\theta_W)-\alpha_G^{-1}\right].$$

This indicates that m_Y, the mass of superheavy gauge bosons, must be of order $10^{33}\sim10^{40}$ GeV, far above the Planck mass. In this way, they have studied every combination of SU(4), Sp(3), Sp(4), SO(7), and SO(8) for the strong gauge group, and SU(2) and $SU(2)_L$ x $SU(2)_R$, for the weak gauge group. One of their conclusions is that, in this approach, the constraint relations between the gauge couplings, the weak mixing angle and the mass scale of symmetry breaking owing to the renormalization effect are not so severe compared with those in the grand unified models. The mass scale, however, becomes far above the Planck mass in some cases, or becomes too small to ensure the proton stability in some other cases.

In grand unified gauge theories, there is only one gauge coupling constant. The Higgs potential and the Yukawa coupling between fermions and Higgs scalars are, however, arbitrary so that one can choose whatever hierarchy of spontaneous symmetry breakdown one wants by adjusting Higgs representations and their parameters. In the contributed paper, "Eigenvalue Conditions and Asymptotic Freedom of SO(7V) Gauge Theories" by Chang and Perez-Mercader,[10] it is demonstrated that this is not true if the theroy is required to be asymptotically free.[11] In order to make the theory asymptotically free, they have imposed "eigenvalue conditions"[12] on all the coupling constants other than the gauge coupling. The new features of their grand unified scheme are 1) The number of "carbon copies" of the basic fermion family (w, d, ν_e, e, • • •) is limited. 2) Each fermion multiplet of $SO(N)$ unifies the light fermions (w, d, ν_e,e, • • •) with superheavy fermions (I/, D, • • •). The superheavy fermions have masses which are of the order of the superheavy gauge boson masses. 3) Even though SO(12)[13] contains $SU(4)xSU(2)_L xSU(2)_R$, the structure of the vacuum indicates a breakdown of manifest left-right invariance, already at super-high energies. For SO(7V) gauge theories, the renormalization group equations[14] for the gauge coupling *g*, the Yukawa coupling *h*, and the quartic Higgs couplings *X* and *A* are given by

$$16\pi^2\frac{dg}{dt}=-g^3\left[\frac{22}{3}(N-2)-\frac{2n_F n_D}{3}-\frac{n_B}{3}(N-2)\right],$$

$$16\pi^2\frac{dh}{dt}=h^3\left[2n_F n_D+\frac{N(N-1)}{2}+(N-4)^2-N\right]-6g^2h(N-2)-\frac{3}{4}g^2h[(N-4)^2-N],$$

$$16\pi^2\frac{d\lambda}{dt}=\lambda^2[4N(N-1)+64]+\lambda\Lambda(16N-8)+12\Lambda^2-24(N-2)g^2\lambda+18g^4-6n_F n_D h^4+8n_F n_D\lambda h^2,$$

and

$$16\pi^2\frac{d\Lambda}{dt}=\Lambda^2(8N-4)+96\lambda\Lambda-24(N-2)g^2\Lambda+6(N-8)g^4-24n_F n_D h^4+8n_F n_D\Lambda h^2,$$

where n_F and n_D are the number of identical fermion and the dimension of spinor representation, respectively.

They assume a special solution of the type

$$h(t)=\bar{h}g(t),\ \lambda(t)=\bar{\lambda}g^2(t),\ \text{and}\ \Lambda(t)=\bar{\Lambda}g^2(t).$$

Then, the coupled set of differential equations is reduced to a coupled set of algebraic equations for the constants A, *X*, and *À*, which can be solved. For exmaple, in the SO(12) model with $n_F=7$, the solutions are

A^2-0.1171 *g\ X=-0M12g*

and 1-0.6166 *g*

Since the parameters of Higgs potential are fixed, the structure of the vacuum after spontaneous symmetry breakdown is also fixed. It is U(2) x SO(10) in this case. They emphasize that, in this kind of asymptotically free grand unified theory, the hierarchy of symmetry breaking is dictated by the theory. There is no longer room to declare a range of quartic self-couplings so as to choose one vacuum *vs* another.

The baryon number non-conservation is one of the most intriguing features common to grand unified gauge theories. In the contributed paper, "Unified Gauge Theories and the Baryon Number of the Universe," Yoshimura[15] has made a very interesting suggestion that the dominance of matter over antimatter in the present universe is a consequence of baryon number non-conserving reactions in the very early fireball. Grand unified gauge theories provide a basis for such a conjecture. He has made a computation in a specific SU(5) model of the Georgi-Glashow type[5] and within the standard big-bang cosmology,[16] and found a small ratio of baryon to photon number density in rough agreement with observation. Let me quickly sketch his idea in the following. The time development of the baryon number density $N_B(t)$ in a hot universe is given by

$$\frac{dN_B(t)}{dt}=-3\frac{\dot{R}}{R}N_B+\sum_{a,b}(\Delta n_B)\langle\sigma v\rangle N_a N_b$$

where *R* is the cosmic scale factor, **(<JV}** is the thermal average of the cross-section for *a+b-** anything times the relative velocity of *a* and *b*, An_B is the change of bay on number in this process, and N_a is the number density of *a*. In the universe (the temperature *T)* dominated by highly relativistic particles, the following relation holds:[16]

R/R=-T/T=($7cpG_N l3y^12

where ***p*** is the energy density

$$\rho=d_F\pi^2T^4/15.$$

The effective number of degrees of freedom d_F is 1/2 or 7/16 for each boson or fermion species, respectively. From these relations, he obtains the rate equation

dF_B— *(WG_N d_F\-v*(3\\,,*

d f - ~ v 4 5 ; \ *j)* F r d

where ***ô=X(Jn_BXov},*** $F_B=N_B/T$\ and F_r = N_r/T with the photon number density N_r. Next, he claims that to obtain a nonvanishing baryon number one must break the microscopic detailed balance because otherwise the inverse reaction would cancel the baryon number gained. This necessity of simultaneous violation of the baryon number and the *CP*- or T-invariance is one of the points in his suggestion. To illustrate the idea, he adopts a grand unified SU(5) gauge model of the Georgi-Glashow type with six or more flavors of quarks and leptons. The baryon number non-conservation is caused by exchange of the superheavy lepto-quark gauge bosons *W* and also by exchange of colored Higgs scalars H_1 and H_2 (the Yukawa coupling constants / and ***h).*** For CP-violation, he adopts the

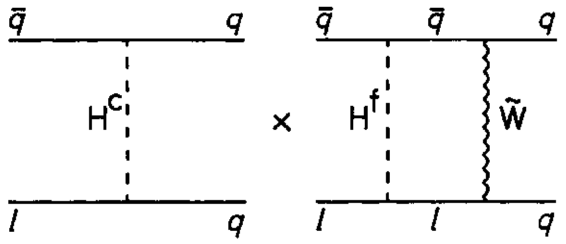

Fig. 1.

Weinberg model[17] in which complex dimensionless parameters, *a* and /3, are introduced in the Higgs propagators by

$$i\langle T(H_1^{\dagger a} H_2^{a})\rangle_{q=0}=\alpha/m_W^2 \text{ for } a=4,\ 5$$
$$=\beta/m^2_{\tilde W} \text{ for } a=1-3.$$

He has found that the dominant contribution to *8* comes from interference of the diagrams in Fig. 1 and that

$$\frac{\delta}{T^2}\simeq\frac{3g^2}{8\pi^2 m^4_{\tilde W}} Im\beta\alpha^*(\mathrm{tr}\, hh^\dagger\, \mathrm{tr} f^2-\mathrm{tr}\, hh^\dagger f^2).$$

Under the reasonable assumption N_B<^N_r, the rate equation is integrated to

$$\frac{N_B(T)}{N_\gamma(T)}\simeq-\left(\frac{8\pi^3 G_N d_F}{5}\right)^{-1/2}\left(\frac{3}{8}\right)^2\frac{\delta}{T^2}$$
$$\times N_\gamma(T_{\mathrm{initial}}).$$

To obtain a rough quantitative idea of this ratio, he has made a drastic extrapolation of this formula up to $T_{initial}=m$^ to get the numerical result

$$\frac{N_B}{N_\gamma}\simeq 2\times10^{-9}\left(\frac{m_t}{10\,\mathrm{GeV}}\right)^2\left(\frac{m_b}{5\,\mathrm{GeV}}\right)^2 A$$
with $A\approx 0(1)$.

An estimate of this ratio[16] from experimental data ranges from 10~^{8} to 10"^{10}, which agrees with this result. Although the numerical result should not be taken too seriously because of crudest approximations involved, I have found that his suggestion is extremely interesting and really working.

Next, I would like to discuss unified models of all elementary-particle forces including gravity. Recently, possible unification with supergravity has been extensively investigated and it has been discussed by Freedman[18] in Session C6. There have been proposed some other ideas on this super-grand unification. The strong gravity proposed by Isham, Salam, and Strathdee and by Zumino[19] is one of them. In the contributed paper, "Emergence of a (possibly) Strong Component in a Poincaré-Quadratic Gauge Theory of Gravity" by HehU Ne'eman, Nitsch, and von der Heyde,[20] a related but quite distinct picture has been proposed for unifying gravity and the confining strong force. It is also a completely new description of gravity which is quite different from Einstein's general relativity. In their gauge approach to the Poincaré group the vierbein *e*" represents the translational potential and the vierbein connection r_{ia}^ = *–Tipa* the rotational potential. The corresponding field strengths are torsion and curvature,

$$F_{ij}{}^{\alpha}\equiv2(\partial_{[i}e_{j]}{}^{\alpha}+\Gamma_{[i\beta}{}^{\alpha}e_{j]}{}^{\beta})$$
$$\text{and } F_{ij\alpha}{}^{\beta}\equiv2(\partial_{[i}\Gamma_{j]\alpha}{}^{\beta}+\Gamma_{[i\gamma}{}^{\beta}\Gamma_{j]\alpha}{}^{\gamma}),$$

respectively. Let (f> and Sf be a matter field and a matter Lagrangian. The total Lagrangian is then given by

$$\mathscr{L}_{\mathrm{tot}}=\mathscr{L}(\psi,\overset{\Gamma}{\nabla}\psi,e)+\mathscr{V}(e,\partial e,\Gamma,\partial\Gamma)$$

it one supposes minimal coupling in accordance with the equivalence principle. In analogy with electrodynamics, they assume the quadratic Lagragian for ^

$$\mathscr{V}=(\det e)\left[\frac{1}{4l^2}(-F^{ij}{}_{\alpha}F_{ij}{}^{\alpha}+2F^{i\gamma}{}_{\gamma}F^{\delta}_{i\delta})+\frac{1}{4\kappa}(-F_{ij\alpha}{}^{\beta}F^{ij\alpha}{}_{\beta})\right],$$

where / is the Planck length and ***K*** is a dimensionless coupling constant. In assuming this Lagrangian, they suppose that the coupled equations derived from it result in a restructuring of the two initial gauge potentials into one long-range Einstein-Newton-like potential with /^{2} dimensional coupling, and a second, Yang-Mills-like, short-range one with dimensionless (strong) coupling. The latter would be asymptotically-free and confining, and would thereby produce a natural "strong gravity" component. They have proved that such a confining contribution does indeed arise in the linearized approximation.

Assume

$$e_i{}^{\alpha}=\delta_i{}^{\alpha}+\frac{1}{2}h_i{}^{\alpha} \text{ with } |h_i{}^{\alpha}|\ll1 \text{ and } |\Gamma_{i\alpha}{}^{\beta}|\ll1,$$

and define

$$\gamma\equiv\gamma_k{}^k \text{ with } \gamma_{ij}\equiv h_{(ij)}-\frac{1}{2}\eta_{ij}h_k{}^k \text{ and } \Gamma_i=\Gamma_{ki}{}^k.$$

Then, the linearized field equations for ***y*** and ***Fi*** turn out to be

$$\Box\Box\gamma = -2\kappa(\Sigma - 2\partial_k\tau^k) - 2l^2\Box\Sigma$$

and

$$\Box\partial_k\Gamma^k = (\kappa/2)(\Sigma - 2\partial_k\tau^k)$$

where *S* and v_k are the trace of momentum and spin current, respectively. For a spinning point particle of mass *m* located at the origin, $r^{fc}=0$ and *Z=—md(r).* Then the first equation yields the solution

$$\gamma(\boldsymbol{r}) = -\frac{l^2}{2\pi}\left(\frac{m}{r}\right) + c_1 - \frac{\kappa}{4\pi}(mr) + c_2r^2$$

for some $r_{min} < r < r_{max}$. The constant multiplying the Newton potential and the one in front of the confinement potential are both fixed. They have further demonstrated that their Lagrangian reproduces the Schwarzschild solution, and therefore proves to be a viable one. There is only one problem: how do leptons avoid strong gravity? They suggest that even leptons might yet display the same features at shorter ranges and larger energies, or alternatively that the color degree of freedom is involved.

In the rest of my talk, I would like to review the recent work by Akama, Chikashige, Matsuki, and myself[21,22] on the unified model of the Nambu-Jona-Lasinio type for all elementary-particle forces including gravity. We start with a nonlinear fermion Lagrangian of the Heisenberg type for a Weinberg-Salam multiplet of massless leptons and quarks:

$$\begin{aligned}\mathscr{L} = {} & \bar{l}_L i\partial\!\!\!/ l_L + \bar{l}_R i\partial\!\!\!/ l_R + \bar{q}_L i\partial\!\!\!/ q_L + \bar{u}_R i\partial\!\!\!/ u_R \\ & + \bar{d}_R i\partial\!\!\!/ d_R + f_1(Y_{l_L}\bar{l}_L\gamma_\mu l_L + Y_{l_R}\bar{l}_R\gamma_\mu l_R \\ & + Y_{q_L}\bar{q}_L\gamma_\mu q_L + Y_{u_R}\bar{u}_R\gamma_\mu u_R + Y_{d_R}\bar{d}_R\gamma_\mu d_R)^2 \\ & + f_2(\bar{l}_L\gamma_\mu\boldsymbol{\tau} l_L + \bar{q}_L\gamma_\mu\boldsymbol{\tau} q_L)^2 \\ & + f_3(\bar{q}_L\gamma_\mu\lambda^a q_L + \bar{u}_R\gamma_\mu\lambda^a u_R + \bar{d}_R\gamma_\mu\lambda^a d_R)^2 \\ & + f_4(b_l\bar{l}_L l_R - b_u\bar{q}_R^G u_L^c + b_d\bar{q}_L d_R) \\ & \times(b_l\bar{l}_R l_L - b_u\bar{u}_L^c q_R^G + b_d\bar{d}_R q_L).\end{aligned}$$

By using the Kikkawa's algorism[23] to analyze this nonlinear Lagrangian and by imposing the massless conditions of Bjorken[24] on vector fields, we construct an effective Lagrangian which combines the unified SU(2) X U(1) gauge theory of Weinberg and Salam for the weak and electromagnetic interactions of leptons and quarks, and the Yang-Mills gauge theory of color SU(3) for the strong interaction of quarks:

$$\begin{aligned}\mathscr{L}'' = {} & \bar{l}_L i\gamma^\mu\left(\partial_\mu + ig'\frac{1}{2}Y_{l_L}B_\mu - ig\frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{A}\right)l_L + \bar{l}_R i\gamma^\mu\left(\partial_\mu + ig'\frac{1}{2}Y_{l_R}B_\mu\right)l_R \\ & + \bar{q}_L i\gamma^\mu\left(\partial_\mu + ig'\frac{1}{2}Y_{q_L}B_\mu - ig\frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{A}_\mu - if\frac{1}{2}\lambda^a G_\mu^a\right)q_L \\ & + \bar{u}_R i\gamma^\mu\left(\partial_\mu + ig'\frac{1}{2}Y_{u_R}B_\mu - if\frac{1}{2}\lambda^a G_\mu^a\right)u_R + \bar{d}_R i\gamma^\mu\left(\partial_\mu + ig'\frac{1}{2}Y_{d_R}B_\mu - if\frac{1}{2}\lambda^a G_\mu^a\right)d_R \\ & - G_l(\bar{l}_L\phi l_R + \bar{l}_R\phi^\dagger l_L) - G_u(\bar{q}_L\phi^G u_R + \bar{u}_R\phi^{G\dagger}q_L) - G_d(\bar{q}_L\phi d_R + \bar{d}_R\phi^\dagger q_L) \\ & - \frac{1}{4}(B_{\mu\nu})^2 - \frac{1}{4}(A_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu}^a)^2 + |D_\mu\phi|^2 - \mu^2|\phi|^2 - \lambda(|\phi|^2)^2.\end{aligned}$$

Let me emphasize the difference between our model and the ordinary gauge models. In our picture of unification, the photon the weak vector bosons $\boldsymbol{W}^{\pm}$ and Z, and the physical Higgs scalar *rj* appear as collective excitations of lepton-antilepton or quark-antiquark pairs while the color-octet gluons $\boldsymbol{G}^a$ appear as those of quark-antiquark pairs. Also, in the ordinary gauge models, the coupling constants *f*, $\boldsymbol{g}$, *g\ K* $\boldsymbol{G}_u$ $\boldsymbol{G}_{v}$, and $\boldsymbol{G}_d$ and the mass parameter $/u^2$ are arbitrary, whereas in our model they are completely fixed by the quantum numbers of leptons and quarks, the cutoff momentum, and the coupling constants in the orginal Lagrangian The most important results of our unified model of the Nambu-Jona-Lasinio type for strong, weak, and electromagnetic interactions are the following: the Weinberg angle 0_w is determined to be

sm²^=(2/3²)/(So²)-=3/8

where / and ***Q*** are the weak isospin and charge of leptons and quarks. The gluon coupling constant is also determined to be 8/3 times the fine-structure constant a. These results coincide with those of Georgi and Glashow[5] in their unified SU(5) gauge model. However, our results are due not to such an assumed higher symmetry as SU(5) but to the Nambu-Jona-Lasinio dynamics in our model with only $SU(3)_{color}$x$[SU(2)xU(1)]_w$ einb$_e$r$_g$.

saiam- The masses of the weak vector bosons are predicted to be

m_w=(7ia/V 2 G_F sin^2 d_wy*=60.9 GeV

and m_z=m $_w$/cos 0 $_w$ = 77.0 GeV.

Entirely new and proper to our model are the following relations between the masses of the physical Higgs scalar and weak vector boson and those of leptons and quarks:

$$m_\eta=2[(\sum m^4)/(\sum m^2)]^{1/2}$$
$$\text{and } m_W=\sqrt{3}[\langle m^2\rangle]^{1/2}$$

where 2 and < > denote the summation and arithmetic average over all leptons and quarks. These relations together with the previous results predict the arithmetic-like-average mass of leptons and quarks to be

$[<m^2>]^{1/2}$-35.2 GeV

and the mass of the physical Higgs scalar to be bounded by

m_v^(2/V~3)m_w=703 GeV.

Another important result is the relation between the fine-structure constant and the sum of the charge squared of leptons and quarks:

$$\alpha=3\pi/(\sum Q^2)\ \ln(\Lambda^2/m^2),$$

where *A* is the universal cutoff momentum and *m* is the geometric-like-average mass of charged leptons and quarks defined by

$$m=\prod m^{Q^2/(\sum Q^2)}.$$

This relation is essentially the old result of Gell-Mann and Low[25] in their renormalization group approach.

In our model of the Nambu-Jona-Lasinio type for gravity, the graviton is also a collective excitation of a fermion-antifermion pair.[26] We start again with a very simple nonlinear fermion Lagrangian

$$\mathscr{L}_0=\bar{\psi}i\frac{1}{2}\overleftrightarrow{\partial}\psi+f_0(T_{\mu\nu})^2$$
$$\text{with } T_{\mu\nu}=\bar{\psi}i\frac{1}{4}(\gamma_\mu\overleftrightarrow{\partial}_\nu+\gamma_\nu\overleftrightarrow{\partial}_\mu)\psi$$

and impose the massless condition on a tensor field, the gravitational field. Then, we derive the effective Lagrangian

$$\mathscr{L}_0''=\bar{\psi}i\frac{1}{2}\overleftrightarrow{\partial}\psi+\frac{1}{4}(\partial_\lambda h_{\mu\nu})^2-g_0T_{\mu\nu}h^{\mu\nu}$$
$$\text{with } g_0=4\pi/(\kappa_0N_0\Lambda^2)^{1/2},$$

where κ_0=2/3 or 1/3 depending on the invariant or Pauli-Villars cutoff procedure. This Lagrangian reproduces the familiar Newtonian gravitational potential if the gravitational constant *G* is related with the total number N_0 of leptons and quarks:

$$G=g_0^2/4\pi=4\pi/\kappa_0N_0\Lambda^2.$$

It is also shown that a more sophisticated model of this type defined on the curved space effectively reproduces the Einstein-Weyl's theory of general relativity.

We further unify the unified model of strong, weak, and electromagnetic interactions and the model of gravity into a unified model of the Nambu-Jona-Lasinio type for all elementary-particle forces including gravity. The most exciting result of this grand unification is a simple relation (the *G~a* relation) between the fine-structure constant and the Newtonian gravitational constant:

$$\alpha=3\pi/(\sum Q^2)\ \ln(4\pi/\kappa_0N_0Gm^2).$$

This relation can be easily derived from combining the above two relations for *a* and *G*. Historically, a relation of this type was conjectured by Landau[27] in 1955. Since this *G-a* relation is very sensitive to the total number of leptons and quarks, we can predict[28] based on it that there exist a dozen leptons (six neutrinos and six charged leptons) and a dozen flavors and three colors of quarks (6x3 up quarks and 6x3 down quarks). The geometric-like-average mass of the charged leptons and quarks is also predicted to be

$$m=(4\pi/\kappa_0N_0G)^{1/2}\exp[-3\pi/2\alpha(\sum Q^2)]$$
$$=23.7\ \text{GeV}.$$

It is now natural to ask why so many leptons and quarks. In concluding this talk, I shall present an answer to this question. It is a "spinor-subquark" model of leptons and quarks in which leptons and quarks are made of three subquarks of spin 1/2,

$$w_i\ (i=1,2),\ h_i\ (i=1,2,\cdots,N),$$
$$\text{and } C_i\ (i=0,1,2,3).$$

The left-handed w_L and the right-handed w_{1R} and w_{2R} are a doublet and singlets of the Weinberg-Salam SU(2), respectively. The /z/s form an Applet of the unknown H symmetry. Also, the C_0 and C/s (z=1, 2, 3) are singlet and triplet under the color SU(3) symmetry. Leptons and quarks are expressed in terms of these subquarks as

$$\nu_j=(w_1h_jC_0),\ l_j=(w_2h_jC_0),\ u_{ji}=(w_1h_jC_i),$$
$$\text{and } d_{ji}=(w_2h_jC_i)$$
$$\text{for } j=1,\ 2,\ \cdots,\ N \text{ and } i=1,\ 2,\ 3.$$

In the unified subquark model of all elementary-particle forces, which is an alternative to the unified lepton-quark model, the gauge bosons 7% *W*, and Z, and the physical Higgs scalar *tj* appear as collective excitations of a *w-w* pair while the color octet gluons G^a appear as those of a *C~C* pair. As a result, we derive the following relations between the masses of *rj, W*, and Z, and those of w's:

$$m_\eta \cong 2m_w,\ m_W \cong \sqrt{3}\,m_w,$$
$$\text{and } m_Z \cong (\sqrt{3}/\cos\theta_W)m_w.$$

These relations predict the masses of the physical Higgs scalar and the *w* subquark to be

m,^70.3GeV and m_w~35.2 GeV.

This result strongly suggests that the masses of the physical Higgs scalar and weak vector bosons may be very close to the threshold of w-subquark pair production, if any (see Fig. 2).

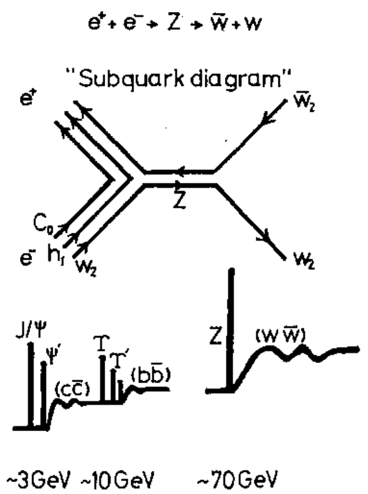

Fig. 2.

I, therefore, strongly urge experimentalists to be still alert for producing possible subquark pairs even after the anticipated exciting discovery of the weak vector bosons in 80's.

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Note added in proof: Recently, J. Arafune has pointed out that the baryon-number non conservation claimed by Yoshimura violates CPT invariance and unitarity.

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C11 Unification: Its Implications for Present and Future High Energy-Experimentation

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§1, Introduction

Since the last International Conference held in Hamburg a year ago, there is one marked difference in the experimental situation regarding the status of the structure of neutral current interactions. At present all experiments, which include neutrino-nucleon-scattering, neutrino-charged lepton scattering as well as parity violation in electron-deuteron-scattering, agree remarkably well with the predictions of the simple gauge-unification based on the symmetry-structure[1] $SU(2)_L xU(1)$.

This raises two important questions:

(1) Do the set of data noted above single out $SU(2)_L xU(1)$ as the only allowed symmetry relevant for low energy electro-weak force, or do they allow for possible alternative symmetries, which would differ from the predictions of $SU(2)_L xU(1)$ even in the low energy regime in areas yet to be explored experimentally?

(2) Given that a gauge unification of the weak and electromagnetic forces is already manifest at present energies through the discovery of neutral current interactions, what new phenomena and correspondingly fundamentally *new physics* may one look forward to discover next at higher energies through high energy accelerators to be completed in the near future and within the decade? Specifically, assuming that the three basic forces—weak, electromagnetic as well as strong—have a common origin, and so also do quarks and leptons,[2,3] one might look forward to discover next *tangible evidence* of such a "grand" unification. This evidence would arise if one could see traces of the new class of interactions (analogous to neutral current interactions) that are needed for putting quarks and leptons into one multiplet. The pertinent question is: can these new interactions and correspondingly "grand" unification manifest at an energy or mass-scale, within experimental reach in the near or conceivable future?

The purpose of my talk is two fold:

(i) First, to note that the present set of