VECTOR MESONS AND UNITARY SYMMETRY

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INTRODUCTION

I am going to talk about the vector theory of strong interactions, the universality of the vector meson couplings and then about the various decay modes of the ω meson. The last part of this paper will be concerned with unitary symmetry, especially F and D type couplings and the mysterious mass formula which seems to work rather well.

1. VECTOR THEORY (GAUGE THEORY) OF STRONG INTERACTIONS

The basic philosophy behind the vector theory or gauge theory of strong interactions can be summarized in the following way. It is essentially an attempt to construct a theory of strong interactions in analogy with electromagnetism. We know that, from a certain point of view, quantum electrodynamics is remarkably simple and elegant. The notions of conserved current, universality and what we might call the principle of minimal electromagnetic couplings play important roles. Similarly, in the realm of weak interactions, it has become apparent that the weak interactions are also vectorial, apart from parity non-conservation, and there have been speculations on the divergencelessness of the currents involved in weak processes. Moreover, we know that the notion of universality has been successfully applied to some domains of weak interactions of non-strange particles. Finally, there are conjectures on the possible existence of spin-one particles (W particles) which mediate various weak processes.

If we now turn our attention to the strong interactions, the following questions very naturally arise. Why are the strong interactions also vectorial? Why do we not have a universal theory of strong interactions based on conserved currents? The vector theory of strong interactions is an attempt to answer these questions by constructing a theory of strong interactions which shares the various elegant features of the electromagnetic and weak interactions.

Now let us go back to some speculations made by WIGNER [1] many years ago. He noted that there are essentially two ways to determine the electric charge of a particle.

First, electric charge is regarded as a pure number - a purely additive number - which is conserved in any reaction. For instance, take the reaction

$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e .$$

If we know from some other experiments that the electric charge of the positron is plus one and that the electric charges of the neutrinos are zero, then, by conservation of charge, the electric charge of the μ^+ is determined to be plus one. But the meaning of electric charge is more than that. We can place a beam of charged particles in an electric field and see how much the beam deflects. So electric charge is not only countable but also measurable, and it is in this second sense that we say that the charge of the electron is equal in magnitude to the charge of the proton to a fantastic degree of accuracy, to a few parts in 10^{19}. (This charge equality is one of the most remarkable equalities in modern physics. Quantum electrodynamics says that, if the bare charges are equal, then the corresponding renormalized charges are also equal. Yet nobody can explain the equality of bare charges!).

Wigner argues in the following way. Both the electron and the proton are highly stable. The stability of the electron can be attributed to the conservation law of electric charge since the electron is the least massive particle that bears electric charge. Similarly, the stability of the proton can be attributed to the conservation law of what we might call "baryonic charge" since the proton is the least massive particle with baryon number one. Nobody understands the deep reason for the existence of the conservation laws of electric charges and of baryonic charges, but, says Wigner, let us assume that the two conservation laws have similar causes, and these causes have similar consequences. With this in mind, let us ask what we mean by "baryonic charge"? Take, for instance, the reaction

$$\Lambda \rightarrow p + \pi^- .$$

If the baryonic charge of the proton is one and that of the pion is zero, then we argue that the baryonic charge of the Λ hyperon must also be one. This is how we determine the baryonic charge of a particle. So we are using the notion that baryonic charge is some additive number which is conserved in any reaction. The point to be emphasized is that in the conventional theory there is nothing analogous to Wigner's second way of measuring the charge of a particle, i.e. the notion of coupling constant is completely missing. So although the electric charge and the baryonic charge are similar in the sense that they are both conserved to a fantastically high degree of accuracy (the proton lifetime $> 10^{24}$ yr, the electron lifetime $> 10^{19}$ yr), they are quite dissimilar because in one case the "charge" means both conserved additive number and coupling constant, whereas in the other case the "charge" means just conserved additive number. This asymmetry is quite ugly and disturbing.

The asymmetry between baryonic charge and electric charge can be seen from a somewhat more formal point of view as follows. In the electromagnetic case the charge conservation is an immediate consequence of Maxwell's equations in the sense that the continuity equation

$$(\partial/\partial x_\mu)\, j_\mu = \vec{\nabla}\cdot\vec{j} + (\partial P/\partial t) = 0$$

follows from

$$\vec{\nabla}\cdot\vec{B} - (\partial\vec{E}/\partial t) = \vec{j}, \quad \vec{\nabla}\cdot\vec{E} = P .$$

In the baryonic case, however, baryon conservation stands by itself, so to speak.

Historically, Wigner tried to remove the asymmetry between electric charge and baryonic charge by postulating that the pion is coupled universally to the various baryons. This is the origin of "global symmetry". This analogy, however, is rather superficial, and it cannot be pursued much further. The reason is that the quantity to which the photon field is coupled is a conserved current density, whereas the quantity to which the pion is coupled is a pseudoscalar density which has little to do with baryon conservation.

A much more natural way is to assume that there is a vector meson coupled universally to the baryon current just as the photon is coupled universally to the electric charge current. If the mass of the vector meson were zero, we would get into difficulties because there would be a kind of long-range, anti-gravity effect (analogous to the Coulomb repulsion) between two macroscopic objects, which has been discussed by LEE and YANG [2].

Such an effect, if it exists at all, can be shown to be much weaker than the gravitational interaction; in any case it would have nothing to do with the strong interactions. So we assume that the vector meson coupled to the baryon current is massive.

We may naturally generalize this idea of associating a vector meson to a conserved current to other conserved currents of the strong interactions. For every conserved quantity we postulate the existence of a vector meson coupled linearly to the appropriate conserved current in question. This is the basic idea of the vector theory of strong interactions.

Historically, a number of people have tried to "justify" the vector theory on the basis of what we might call the gauge principle. The requirement that the gauge transformation associated with the conservation law of baryonic charge, etc. be local (space-time dependent) in character demands the existence of a vector field with zero bare mass coupled universally to the baryon current. We can argue endlessly whether or not such an approach makes sense, because the physical mass of the vector meson associated with the vector field must be finite in order that we have a physically interesting theory of strong interactions. But I shall not discuss this very important problem.

From a practical point of view there are a few important points. First, is the idea that for every conserved current there exists a strongly interacting vector meson right? If so, are the vector mesons coupled universally to the appropriate conserved currents in the same sense that the electromagnetic field is coupled universally to the electric charge current? How can we test the universality principle?

I should emphasize at this moment that, given a symmetry of conserved operators, the number and the nature of the vector mesons are determined. If you are just concerned with the exactly conserved currents of the strong interactions, then there are only three - the isospin current, the baryon current and the hypercharge current. Of course, we may take any linear combination of the strangeness current and the baryon current instead of the hypercharge current, but in higher symmetry models, such as the unitary symmetry model or any model in which there is some symmetry between N and Ξ, it is natural to take the hypercharge current, as we shall show later. We can easily verify that the isospin current is isovector and even under G conjugation, whereas the baryon current and the hypercharge current are isoscalar odd under G. So we are led to conjecture that there exist one T=1, even G vector meson and two T=0, odd G vector mesons.

So far we have considered only the exact symmetries of the strong interactions. Perhaps there are hidden symmetries which are approximate. If there are, there may be more currents which are conserved, but only to the extent that this mass difference between the nucleon and the Λ etc. can be ignored. Indeed, in the unitary symmetry model to be discussed later, there is a strangeness changing current with isospin 1/2 which is approximately conserved. So we may conjecture on the existence of a T = 1/2 vector meson coupled to the quasi-conserved strangeness changing current.

Before proceeding, I would like to give credit to the people who are involved in this line of thinking. The first suggestion that there ought to be a vector meson coupled to the isospin current was made by YANG and MILLS [3] as early as in 1954. It was FUJII [4] who first suggested that there should be a strongly interacting vector meson coupled to the baryon current. Subsequently I formulated a theory in which the vector mesons coupled to the baryon current, isospin current and hypercharge current play vital roles in the physics of strong interactions [5]. For the currents generated by gauge transformations of unitary symmetry based on the Sakata triplet, SALAM and WARD [6] have shown that we must have an octet of vector mesons. There is another version of the unitary symmetry model where we again have an octet of vector mesons as shown by GELL-MANN and NE'EMAN [7].

When this kind of theory was proposed, there was no direct experimental evidence for or against the existence of strongly interacting vector mesons. As is well known, there are now two vector mesons whose existence has been firmly established by numerous experiments - the ρ meson with mass $\approx$ 750 MeV with T = 1, G = +1 decaying into three pions. The ρ meson can be identified with the vector meson coupled to the isospin current whereas the ω meson can be one of the candidates to the two T = 0, G = -1 vector mesons proposed by the vector theory of strong interactions. If one subscribes to the philosophy that for every conserved current there should be a vector meson, it would be better to have another T = 0, G = -1 vector meson. In spite of their similarity the two T = 0 vector mesons are quite distinct because the baryon current is very different from the hypercharge current. For instance, the one coupled to the hypercharge current would not be emitted or absorbed by Λ since the Λ has hypercharge = zero, whereas the Λ can emit or absorb the vector meson coupled to the baryon current. This distinction also becomes apparent in the octet version of the unitary symmetry model to be discussed later; the one coupled to the baryon current is an unitary singlet whereas the one coupled to the hypercharge current is a member of a unitary octet. In any case I would like to urge the experimentalists to look for another T = 0, G = -1 vector meson. Perhaps it is relevant to mention that if the conjectured T = 0 meson has mass greater than 2 m_K, then its main decay mode may be $K^+ + K^-$ and $K^0 + \overline{K^0}$. Since the K_1^0 K_1^0 mode and the K_1^0 K_2^0 mode are forbidden by Bose statistics and also by G conjugation invariance, we should see a bump in the K_1^0, K_2^0 Q value distribution but not in the K_1^0 K_1^0 distribution. The conjectured meson may be looked for in the reactions $K + p \to K_1^0 + K_2^0 + \Lambda$, $K^+ + K^- + \Lambda$.

In the unitary symmetry model there is room for a vector meson with $T = \frac{1}{2}$, $S = \pm 1$ which may be identified with the 880 MeV K. There is some preliminary evidence from $p\bar{p}$ annihilation experiments carried out by the CERN - Collège de France group that the spin of K^* is likely to be one.

TABLE I

	Isospin	G	Hypercharge or strangeness	Unitary symmetry classification
Isospin current	1	+	0	Member of unitary octet
Hypercharge current	0	-	0	Member of unitary octet
Baryon current	0	-	0	Unitary singlet
S changing current	$\frac{1}{2}$	no meaning	± 1	Member of unitary octet

The predictions of the vector theory* are summarized in Table I. The existence of $\rho = \omega$ and K^* is gratifying especially if we recall that when the theory was proposed there was no direct evidence for any of these mesons. There are, however, two predictions that have not yet been checked:

(i) The spin of K^* must be one (for which there is some evidence); and
(ii) There must exist another $T = 0$, $J = 1^-$, $G = -1$ vector meson whose major decay modes may well be $K_1^0 + K_2^0$ and $K^+ + K^-$ (but not $K_1 + K_1^0$, $K_2^0 + K_2^0$).

2. UNIVERSALITY

From the quantitative point of view the most important question in the vector theory of strong interactions is the one of the universality of the interactions between the vector mesons and the baryon and meson currents. In the old-fashioned way the interactions of the ρ meson with the nucleon and pion can be written as

$$L_I = f_{\rho\pi\pi}\rho_\mu(\vec{\pi}\times\partial_\mu\vec{\pi}) + f_{\rho NN}\rho_\mu(i\bar{N}\gamma_\mu(\vec{\tau}/2)N) \tag{2.1}$$

Universality means

$$f_{\rho\pi\pi} = f_{\rho NN}.$$

Now we may argue endlessly about whether this kind of equality is supposed to hold at zero momentum transfer, as in the case of the electromagnetism, or on the mass-shell momentum transfer. In this elementary discussion we shall leave aside this question.

In order to test the universality hypothesis we shall calculate from the experimental data $f_{\rho\pi\pi}$ and $f_{\rho NN}$. From the width for the decay $\rho \to 2\pi$, we can obtain $f_{\rho\pi\pi}^2/4\pi$. This procedure is good if the width of the ρ particle is very narrow ($f_{\rho\pi\pi}$ is very small). Actually ρ manifests itself as a resonance in

$J = 1$, $T = 1$, $\pi\pi$ scattering, so we would like to know how to determine $f^2_{\rho\pi\pi}/4\pi$ from the $\pi\pi$ scattering amplitude.

Let us recall how we usually define the coupling constant for the interaction between two pions and a stable particle, which for simplicity we suppose to be scalar. Let us denote this particle by σ and suppose that its mass is smaller than twice m_π. Near the σ pole the T matrix can be written as:

$$T \simeq g^2/s - m^2_\sigma,$$

so

$$\left|\frac{d}{ds}\left(\frac{1}{T}\right)\right|_{s=m^2_\sigma} \approx g^{-2}. \tag{2.2}$$

The rate at which $1/T$ varies with the energy square d near the $(\text{mass})^2$ of the intermediate particle measures the coupling constant. Note that $(1/T)$ vanishes at $s = m^2_\sigma$.

In the unstable σ case (i.e. $m_\sigma > 2m_\pi$), the σ meson manifests itself as a resonance in s-wave $\pi\pi$ scattering.

Since

$$\frac{e^{i\delta}\sin\delta}{k} = \frac{1}{k\cot\delta - ik}$$

and

$$\mathrm{Re}\left(\frac{1}{T}\right) = -\frac{k\cot\delta}{8\pi\sqrt{s}},$$

the real part of $(1/T)$ goes through zero near the resonance just as $1/T$ goes through zero in the stable σ particle case. This suggests the definition

$$\frac{d}{ds}\left[\mathrm{Re}\left(\frac{1}{T}\right)\right] = g^{-2}. \tag{2.3}$$

Now, for a sufficiently narrow resonance, the phase shift is given by

$$e^{i\delta}\sin\delta = -\frac{m_\sigma\Gamma}{(s - m^2_\sigma) + i m_\sigma\Gamma}, \quad s \approx m^2_\sigma,$$

so we obtain

$$\frac{1}{g^2} = \frac{k_{\text{res.}}}{8\pi m^2_\sigma\Gamma}$$

or

$$\Gamma = \left(\frac{g^2}{4\pi}\right)\frac{1}{4}\,\frac{(m^2_\sigma - 4m^2_\pi)^{\frac{1}{2}}}{m^2_\sigma}.$$

It is important to note that exactly the same expression can be obtained by computing the life-time or the decay width by perturbation theory using the effective Lagrangian $g_{\sigma\pi\pi}$. So we see that the well-known formal identity between the pole terms in the sense of dispersion theory and the renormalized Born terms in the sense of perturbation theory can be extended to the case of unstable particles [8].

In a similar manner we can compute the decay width for $\rho \rightarrow 2\pi$. We obtain for the width

$$\Gamma_\rho = \frac{2}{3} \cdot \frac{f^2_{\rho\pi\pi}}{4\pi} \cdot \frac{p^3}{m^2_p} = \frac{1}{12} \frac{f^2_{\rho\pi\pi}}{4\pi} \left(1 - \frac{4 m^2 \pi}{m^2 p}\right)^{\frac{3}{2}} m_p \quad , \tag{2.4}$$

where p is the pion momentum in the rest system of the decaying ρ particle, and Γ_ρ is the full width. Experimentally Γ_ρ is 100-125 MeV; then

$$f^2_{\rho\pi\pi}/4\pi \simeq 2.0 - 2.5. \tag{2.5}$$

How do we get the $f_{\rho NN}$ coupling constant? One possibility should be through nuclear forces, but the related calculations would be very complicated. The potential is best known for p-p scattering but in this case it is not possible to discriminate between the ρ and ω contributions. They appear in the same way. Additional complications come from an anomalous magnetic moment-like term, and, what is more important, we do not have a reliable calculational method for the other contributions (e.g. contributions as a result of the exchange of an uncorrelated pair of pions). The best we can do is to look at π-N scattering. Let us see the contribution from Fig. 1, which gives the product $f_{\rho\pi\pi} f_{\rho NN}$.

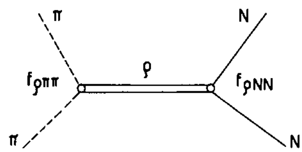

Fig. 1

The effect of the ρ meson on low energy πN scattering has been estimated by many people. Let us first do the most naive thing, i.e., to see the contribution of the above Born graph in the s-wave πN scattering amplitude.

Fortunately the anomalous magnetic moment term does not contribute to s-wave at low energy, and we get something like this:

$$\frac{\tan \delta_3}{k} - \frac{\tan \delta_1}{k} = \frac{3}{4} \frac{f_{\rho\pi\pi} \cdot f_{\rho NN}}{4\pi} \cdot \frac{\omega m N}{W} \cdot \frac{1}{k^2} \log\left(1 + \frac{4k^2}{m^2_\rho}\right), \tag{2.6}$$

where δ_3 is the phase shift of the isospin 3/2 s-wave amplitude and δ_1 the phase shift for isospin 1/2; W is the total energy in the C. M. system. ω is the energy of the pion in the C. M. system. The log term comes from the partial wave projection of the ρ meson propagator, $1/[2k^2(1 - \cos\theta) + m^2_\rho]$.

If we assume that the ρ meson exchange dominates as $k \rightarrow 0$, Eq. (2.6) gives the difference between the two scattering lengths $a_3 - a_1$, from which it follows:

$$f_{\rho\pi\pi} \cdot f_{\rho NN}/4\pi \sim 2.5. \tag{2.7}$$

In a more sophisticated approach HAMILTON, SPEARMAN and WOOLCOCK [9] tried to fit the energy dependence of the phase shift instead of scattering length, and in the notation of Bowckok et al. they obtained:

$$\ell_1 = -\frac{1}{3}\,\frac{f_{\rho\pi\pi}\,f_{\rho NN}}{4\pi} = -0.7 \pm 0.1,$$

or

$$f_{\rho\pi\pi}/4\pi \sim 2.1 \pm 0.3, \tag{2.8}$$

which agrees with (2.7). This shows that $f_{\rho\pi\pi} \simeq f_{\rho NN}$, as required for universality.

It would be nice to test the universality hypothesis in other reactions, for example, in KN and $\overline{K}N$ scattering. To isolate the ρ contribution in these reactions is a very difficult task since we do not know how to calculate other contributions.

We can, however, make an interesting speculation. Whenever the one pion exchange is forbidden by symmetry considerations, then the isospin dependent amplitude for any low energy scattering is dominated by the exchange of a ρ particle coupled universally to the isospin current. This hypothesis can readily be shown to imply the simple rule: the ρ-exchange force is attractive when isospins are antiparallel, repulsive when they are parallel.

It is amusing to notice that this rule works nicely in five cases. Thus in the πN, $T = 1/2$ state we have attraction; in the $T = 3/2$ state, repulsion. In KN, $T = 1$ repulsion is very strong as it is verified in the K^+p scattering experiments.

For KN nucleon scattering, the $T = 0$ state is more attractive than the $T = 1$ state since the Y_0^* resonance of 1405 MeV is most likely an s-wave ($\overline{K}N$) bound state whereas the 1385 MeV Y_1^* resonance is not likely to be related to the s-wave $\overline{K}N$ channel. Also there seems to be an attractive s-wave interaction in $T = 0$ $K\overline{K}$ scattering. Finally, in the $\pi\pi$ case, $T = 0$ is more attractive than $T = 2$.

2.1. ω meson

Let us assume for the sake of argument that the ω meson is coupled to the hypercharge current. And let us do the same kind of thing for the KN, $\overline{K}N$. K and N have hypercharges +1, $\overline{K}$ has hypercharge -1. If in the low energy domain the idea that the ω exchange dominates is correct, then on the average KN is repulsive and $\overline{K}N$ is attractive. This follows because the exchange of a ω meson coupled to the hypercharge current generates Coulomb like interaction at short distances; i.e., for similar hypercharges we have repulsion, for opposite hypercharges, attraction. Then the "potentials" for KN and $\overline{K}N$ can be written in the following form if we take into account only ρ and ω exchange:

$$KN:\; V_\omega + V_\rho\, \vec{\tau}_K \cdot \vec{\tau}_N$$

$$\overline{K}N:\; -V_\omega + V_\rho\, \vec{\tau}_K \cdot \vec{\tau}_N$$

where

$$\vec{\tau}_K \cdot \vec{\tau}_N = \begin{cases} -3 \text{ for } T = 0 \\ 1 \text{ for } T = 1 \end{cases}$$

The signs of V_ω and V_ρ are determined to be positive in the vector theory based on the universality principle (but are arbitrary in any other theory). There is some experimental evidence that the simple description given here corresponds to reality. With the ω meson we may hope to understand nuclear forces at short distances. Again in N-N interaction the exchange of a T = 0 vector meson gives a Coulomb-like repulsion at small distances. This might give rise to the phenomenological hard core in nucleon-nucleon scattering.

To end this section, I would like to show how the universality principle might be formulated on the basis of dispersion theory.

Let us go back to the ρ meson. Suppose that the ρ meson dominates the charge form factors. This means that the nucleon or pion form factors could be approximated by the Fig. 2.

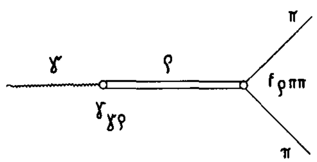

Fig. 2

If we are far from the resonance we can essentially ignore the complications due to the instability of the ρ meson [8]. We then have:

$$F_\pi(q^2) = \gamma_{\gamma\rho} f_{\rho\pi\pi}/(q^2 + m_\rho^2)$$

while for the isovector nucleon charge form factor $F_N^{(v)}$,

$$F_N^{(v)}(q^2) = (\gamma_{\gamma\rho} f_{\rho NN}/2)/(q^2 + M_\rho^2),$$

where $\gamma_{\gamma\rho}$ is the coupling constant of the ρ particle to the photon. Now for $q^2 \to 0$, we have $F_\pi \to \rho$ and $F_N^{(v)} \to \rho/2$, because at zero momentum transfer electric charges are universal. Thus $f_{\rho NN} = f_{\rho\pi\pi}$, which agrees with the concept of universality.

Now there is another final point which is extremely interesting. We have here

$$\gamma_{\gamma\rho} = em_\rho^2/f_{\rho NN} = em_\rho^2/f_{\rho\pi\pi}. \tag{2.9}$$

$\gamma_{\gamma\rho}$ is inversely proportional to the strong interaction constant, a very different result from the perturbation result. In perturbation theory we consider

$$\gamma \to \begin{Bmatrix} \pi + \pi \\ N + N \end{Bmatrix} \to$$

so we would expect $\gamma_{\gamma\rho}$ proportional to the strong interaction constant ($f_{\rho\pi\pi}$, $f_{\rho NN}$). The fact that $\gamma_{\gamma\rho}$ is inversely proportional to the strong coupling constant is analogous to the well-known Goldberger-Treiman relation. There again the pion decay rate is inversely proportional to the strong coupling constant.

3. DECAY MODES OF THE ω MESON

A particle with the quantum numbers $T = 0, J = 1^-$ was predicted by Nambu in 1957 to explain the isoscalar electromagnetic form factor of the nucleon. This means that Fig. 3 must be important.

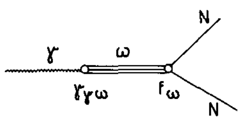

Fig. 3

Then, through the photon which is coupled to any pair of charged particles the ω meson can decay in the following ways:

$$\omega \to \gamma \to \begin{cases} e^+e^- \\ \mu^+\mu^- \\ \pi^+\pi^- \end{cases}$$

If the ω contribution dominates the isoscalar form factor, we can readily show that

$$\gamma_{\gamma\omega} = \frac{e\, m_\omega^2}{2 f_\omega} , \tag{3.1}$$

in complete analogy to what we did for the ρ meson.

The constant f_ω is defined through the interaction

$$L_I = f_\omega \omega_\mu (i\bar{N}\gamma_\mu N + \ldots). \tag{3.2}$$

We can give the following simple rule. Whenever ω dominates, then we insert in the corresponding diagram the factor $\gamma_{\gamma\omega} = e\, m_\omega^2/2 f_\omega$ between γ and ω; ρ occurs, we insert $\gamma_{\gamma\rho} = e\, m_\rho^2/f_\rho$. So the decay rate for $\omega \to e^+ + e^-$, $\mu^+ + \mu^-$ is given by

$$\Gamma\left(\omega \to \begin{matrix} e^+ & e^- \\ \mu^+ & \mu \end{matrix}\right) = \left(\frac{1}{137}\right)^2 \cdot \frac{1}{12} \cdot \frac{m_\omega}{f_\omega^2/4\pi} \cdot \left(1 - \frac{4 m_\rho^2 u}{m_\omega^2}\right)^{\frac{1}{2}}, \tag{3.3}$$

which is inversely proportional to the strong coupling constant f_ω^2. Numerically we get:

$$\Gamma(\omega \to e^+ e^-) \simeq \Gamma(\omega \to \mu^+ + \mu^-) = \frac{3\ \text{keV}}{f_\omega^\rho/4\pi} , \tag{3.4}$$

as shown by NAMBU and SAKURAI [10].

If ω does not dominate, we have to multiply the above expression by $|\alpha_\omega|^2$ where α_ω is the coefficient of the ω contribution to the isoscalar charge form factor.

$$\Gamma_1^{(s)}(q^2) = \frac{m_\omega^2 \alpha_\omega}{q^2 + m_\omega^2} + 1 - \alpha_\omega. \tag{3.5}$$

If the muon had an anomalous interaction, then the $\omega \to \mu^+ + \mu^-$ and $\omega \to e^+ + e^-$ ration would not be equal. If, for example, there exists a meson which interacts somewhat strongly with the muon but not with the electron, then we expect a very different result for the branching ratio

$$[\Gamma(\omega \to \mu + \mu^-)] / [\Gamma(\omega \to e^+ e^-)].$$

This would be a sensitive test of the idea that the muon is a pure Dirac particle. So far there is no experiment on leptonic decays of the ω meson.

Now the situation is somewhat more complicated for the $\omega \to 2\pi$ decay. It is a process where we have violation of T conservation and G invariance. So ω can decay into 2π via electromagnetic interaction:

$$\omega \to \gamma \to 2\pi.$$

In calculating the decay rate, one must take into account the final state interaction between the two pions, and this can be expressed by Fig. 4, where F_π is the pion form factor. The result is

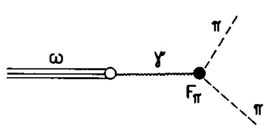

Fig. 4

$$\Gamma(\omega \to 2\pi) \simeq \frac{0.7}{f_\omega^2/4\pi} \text{ keV } \left|F_\pi(q_z^2 = m_\omega^2)\right|^2, \tag{3.6}$$

where the pion form factor F_π is given by

$$F_\pi(q^2) \simeq \frac{m_\rho^2}{(q^2 + m_\rho^2)\, i\Gamma_\rho m_\rho}, \quad \text{for } q^2 \approx -m_\rho^2,$$

if we assume that only the ρ meson contributes to the pion form factor. Since the ρ and ω masses are very near, the enhancement factor $|F_\pi|^2$ in (3.6) can be very large - something like 50.

Let us next consider decays of the ω meson which have been observed. For the $\omega \to \pi^0\gamma$ decay, we assume that the dominant graph is as in Fig. 5. Although we do not know the $\omega\rho\pi$ coupling constant, we can compare this process with the $\omega \to 3\pi$ decay which we suppose to be dominated by the diagram as shown in Fig. 6. In this way, in the branching ratio $[\Gamma(\omega \to \pi^0\gamma)/\Gamma(\omega \to 3\pi)]$ the unknown

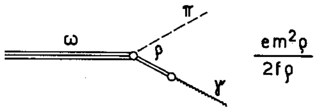

Fig. 5

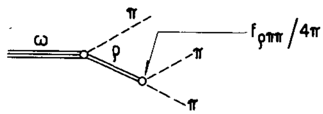

Fig. 6

$\omega\rho\pi$ coupling constant is cancelled out, and only known quantities remain. For $\rho \to \gamma$ we use $\ell m_\rho^2/2f_\rho$ and for $\rho \to 2\pi$ we use $f_{\rho\pi\pi}^2/4\pi \simeq 2.0$, corresponding to $\Gamma_\rho \simeq 100$ MeV.
We then get:

$$\Gamma(\omega \to \pi^0\gamma)/\Gamma(\omega \to 3\pi) \simeq 17\% \text{ [11]}.$$

Experimentally, both the CERN-Paris group ($\vec{p} + p \to K^0 + \overline{K}^0 + \omega$) and the Berkeley group ($\overline{K}+p \to \Lambda + \omega$) give for the above ratio 15 - 20% in excellent agreement with the ρ dominance model. The $\omega\rho\pi$ coupling constant can be calculated if we assume that in the $\pi^0 \to 2\gamma$ decay the dominant graph is as shown in Fig. 7.

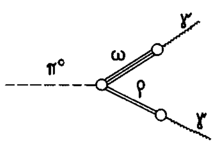

Fig. 7

This was first pointed out by GELL-MANN and ZACHARIASEN [8]. We obtain the absolute value for the decay rate of $\omega \to 3\pi$, which comes out to be approximately 400 keV, if we assume $f_\omega^2/4\pi \simeq 1.5$, as suggested by the unitary symmetry.

Let us consider a more direct method for measuring the ω decay rate. For example, $\Gamma(\omega \to \pi^0\gamma)$ can be obtained from π^0 photo-production. Some preliminary study of $\gamma + p \to p + \pi^0$ has revealed a peculiar angular distribution which cannot be explained with usual phenomenological terms like a reasonable number of powers of $\cos\theta$. There is some evidence that the angular distribution at $E_j \sim 1.1$ BeV is completely dominated by the diagram shown in Fig. 8.

This experiment is still in progress at the California Institute of Technology. Note that the above graph gives the product of f_ω and $f_{\omega\gamma\pi}$.

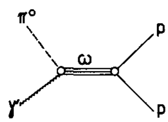

Fig. 8

It was shown experimentally that the process $\pi^{\pm}+p \to p+\omega+\pi^{\pm}$ is very strong so we might try the approach in Fig. 9, and in this way we could obtain the ω-3π vertex. Unfortunately, the experimental results at $p_{\pi}^{(lab)} \sim$ GeV/c do not give any indication of the importance of this one-pion exchange graph. Experimentally, the above processes are dominated by $\omega + N^{*}$. Perhaps at much higher energies the one-pion-exchange mechanism will become important.

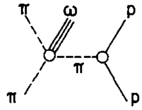

Fig. 9

Finally, there is an experiment proposed by CABIBBO and GATTO in which we study ω production in electron-positron colliding beam annihilation [12]. Then the total cross-section for the process $e^{+}+e^{-} \to$ final state f, where the intermediate state is ω, is given by

$$\sigma\left(e^{+} e^{-} \to f\right) = \frac{3\pi\lambda^{2}\,\Gamma(\omega \to 2e)\,\Gamma(\omega \to f)/4}{(E - m_{\omega})^{2} + \Gamma^{2}/4}, \tag{3.7}$$

where $\Gamma(\omega \to f)$ is the partial width for $\omega \to$ final state in question and $\Gamma(\omega \to 2e)$ is the partial width for the $\omega \to e^{+}+e^{-}$ decay. What one experimentally measures is not the peak but rather the cross-section averaged over some energy interval.

$$\bar{\sigma} = \frac{1}{2\Delta E} \int_{m_{\omega}-\Delta E}^{m_{\omega}+\Delta E} \sigma(E)\,dE. \tag{3.8}$$

Numerically $\bar{\sigma}(e^{+}e^{-} \to 3\pi) \simeq 6.5\ \mu B$ if we assume $\Delta E = 10$ MeV, $\Gamma = 500$ keV and the branching ratio $[\Gamma(\omega \to 2e)]/\Gamma_{tot} \simeq 1\%$. This is much larger than the usual electrodynamic cross-sections.

4. UNITARY SYMMETRY

Let us start with some familiar concepts: charge conservation and isospin conservation. In the charge conservation case, we have a unitary group with one parameter which corresponds to the gauge transformations $e^{i\alpha}$.

For the isospin conservation, we have a three-parameter unitary group which corresponds to rotations in a three-dimensional Euclidean space. It is just an accident that the isospin rotation is usually discussed in analogy with the Euclidean rotation; we can approach isospin rotation in an entirely different way. We first consider two primitive objects, the proton and the neutron:

$$\begin{pmatrix} p \\ n \end{pmatrix}$$

Any isospin rotation can be completely characterized by its effects on the two primitive objects p, n. The usual way of writing the isospin rotation is as

$$\exp\left(i[\tau_e\theta_e/2]\right)\begin{pmatrix} p \\ n \end{pmatrix}, \tag{4.1}$$

where τ_e are the usual Pauli matrices and θ_e specify the rotation. But we can also write

$$\exp\left(i[\tau_n\theta_n/2]\right) = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \tag{4.2}$$

with $|\alpha|^2 + |\beta|^2 = 1$. Note that we again have 3 <u>independent</u> parameters (two complex numbers and one constraint).

The above matrix generates a unitary and unimodular (det = 1) transformation in two-dimensional space. The group of the unitary unimodular 2×2 matrices is denoted by SU_2 (S stands for simple and U_2 for unitary and unimodular).

Then, instead of considering the group of the transformations O_3 (Euclidean rotation in real 3-dimensional space), we may as well consider the equivalent group SU_2. We may note that (4.2) is not the most general unitary two dimensional matrix, but it is the most general unitary unimodular matrix. The most general unitary matrix is obtained by multiplying (4.2) by the one-parameter gauge transformation $e^{i\alpha}$ with real α.

More complicated objects like the pions can be built up from the outer product of $(\bar{p}\,\bar{n})$ and $\binom{p}{n}$:

$$\begin{pmatrix} \bar{p}p & \bar{n}p \\ \bar{p}n & \bar{n}n \end{pmatrix}. \tag{4.3}$$

But this has mixed properties under isospin rotations or equivalently under SU_2: the reason is that the trace part transforms like a singlet. To obtain the triplet we subtract the trace:

$$\vec{\pi} = \begin{pmatrix} \bar{p}p & \bar{n}p \\ \bar{p}n & \bar{n}n \end{pmatrix} - \begin{pmatrix} \frac{\bar{p}p+\bar{n}n}{2} & 0 \\ 0 & \frac{\bar{p}p+\bar{n}n}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\bar{p}p-\bar{n}n}{2} & \bar{n}p \\ \bar{p}n & -\frac{\bar{p}p-\bar{n}n}{2} \end{pmatrix} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & \frac{\pi^0}{\sqrt{2}} \end{pmatrix}. \tag{4.4}$$

Although we can construct all non-strange particles in similar ways, we can never build up strange particles by starting with the primitive objects p and n, which are both non-strange. If we work with SU(2), it is impossible to incorporate the degree of freedom that corresponds to the gauge transformation for strangeness or hypercharge conservation.

Now SU_3 is a slight generalization of SU_2. We can again consider infinitesimal generators $1+i\lambda_i(\delta\theta_i/2)$ instead of $1+i\tau_i(\delta\theta_i/2)$ of SU_2. λ_i here are 3×3 traceless matrices: their representation can be found, for example, in GELL-MANN's paper [7].

For SU_3 we have three primitive objects. For example, in the Sakata model, they are the p, n and Λ. This model was extensively studied especially by Ikeda, Ohnuki and Ogawa. In this approach Σ and Ξ belong to different representations from p, n, Λ. If the $\Lambda\Sigma$ parity were odd, then this would be very promising. There are now good indications that the $\Sigma\Lambda$ parity is even, so it is natural to put Σ and Λ together. Another thing is that in this model the most likely assignment for the cascade spin is 3/2. So far, there is no argument against this assignment, but some preliminary experiments indicate that there is an asymmetry in the Ξ decay with respect to the normal to the production plane; if this persists, the spin 3/2 assignment will be ruled out by the so-called Lee and Yang test. (If we have a higher spin object, there will be more tendency for any decay Λ products to be emitted in the production plane rather than in the direction normal to the production plane.) There are also some predictions on the decay of $\bar{p}p \to 2\pi$, $K\bar{K}$ etc., and some experiments on this seem to contradict the Sakata model as will be discussed in other papers.

We shall discuss the octet model the "eight-fold way" introduced by Gell-Mann and Neéman independently. Here the primitive objects are hidden. For pedagogical purposes, we shall introduce a mathematical lepton multiplet $\ell = \begin{pmatrix} \nu \\ e^- \\ \mu^- \end{pmatrix}$ where νe^- form a doublet and μ^- a singlet with baryon number B = 0. Let us now introduce also a mathematical boson multiplet with baryon number equal to one (B = +1): L = (D^0, D^+, S^+) where D^0, D^+ form a doublet and S^+ is a singlet. Isotopically this multiplet transforms like an antilepton multiplet. In the preceding case of SU_2 where we constructed the pion out of the nucleon-antinucleon doublets, we considered the outer product ($\bar{p}$, $\bar{n}$) $\times$ (p n) out of the nucleon-antinucleon doublets. Let us do the same kind of thing taking the outer product of $\bar{L}$ and l. We then obtain the matrix:

$$\begin{array}{c|ccc|} & D^0 & D^+ & S^+ \\ \nu & D^0\nu & D^+\nu & S^+\nu \\ e^- & D^0e^- & D^+e^- & S^+e^- \\ \mu^- & D^0\mu^- & D^+\mu^- & S^+\mu^- \end{array} .$$

Note that the trace of the above matrix is invariant under unitary transformations. If we subtract from the above matrix the corresponding trace which is a unitary singlet,

$$\begin{vmatrix} \frac{D^0\nu + D^+e^- + S^+\mu^-}{3} & 0 & 0 \\ 0 & \frac{(\text{idem})}{3} & 0 \\ 0 & 0 & \frac{(\text{idem})}{3} \end{vmatrix} \tag{4.5}$$

we obtain

$$\begin{vmatrix} \frac{2D^0\nu - D^+e^- - S^+\mu^-}{3} & D^+\nu & S^+\nu \\ D^0e^- & \frac{-D^0\nu + 2D^+e^- - S^+\mu^-}{3} & S^+e^- \\ D^0\mu^- & D^+\mu^- & \frac{-D^0\nu - D^0e^- + 2S^+\mu^-}{3} \end{vmatrix} . \tag{4.6}$$

Now one can easily identify the various elements of this matrix with stable baryons

$$\left.\begin{matrix} S^+\nu = p \\ S^+e^- = n \end{matrix}\right\} \quad \left.\begin{matrix} D^0\mu^- = \Xi^- \\ D^+\mu^- = \Xi^0 \end{matrix}\right\}$$

since they form an isospin doublet; also $\Sigma^+ = D^+\nu$, $\Sigma^- = D^0e^-$ and by charge independence we identify the neutral member

$$\Sigma^0 = (D^0\nu - D^+e^-)/\sqrt{2}.$$

What is left over must be an isosinglet:

$$\Lambda^0 = (D^0\nu + D^+e^- - 2S^+\mu^-)/\sqrt{6} .$$

Let us note that we obtain the right strangeness for the above particles if we put $S(e, \nu, S^+) = 0$ and $S(D^0, D^+, \mu^-) = -1$, where S is the strangeness. As a result we obtain the baryon octet:

$$B = \begin{vmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{vmatrix} \tag{4.7}$$

We have used l and $\bar{L}$ only as a device to keep track of transformation properties. Once you have obtained the unitary octet, you can forget about them. Any unitary octet must have the same structure.

Now, the observed pseudoscalar mesons also form an octet.

$$m = \begin{vmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{vmatrix}, \qquad (4.8)$$

where η^0 is an isosinglet as the Λ^0, which may be identified as the 560 MeV object. Similarly, the vector mesons might be put in a unitary octet u, and they can be obtained from m by the following substitution:

$$u \Sigma m \begin{pmatrix} \pi \rightarrow \rho \\ K \rightarrow n \\ \eta \rightarrow \omega \end{pmatrix}. \qquad (4.9)$$

In this model K* has spin 1 which may be the 880 MeV $K\pi$ resonance.

5. INTERACTIONS

Now we can write the unitary symmetry Yukawa type interactions. This can readily be done if we recall that the trace of a matrix product is invariant under unitary transformations. Omitting γ matrices, we have the traces:

$$\mathrm{Tr}(\overline{B}mB) \text{ and } \mathrm{Tr}(\overline{B}Bm)$$

where

$$\overline{B} = \begin{vmatrix} \frac{\overline{\Lambda}^0}{\sqrt{6}} + \frac{\overline{\Sigma}^0}{\sqrt{2}} & \overline{\Sigma}^- & \overline{\Xi}^0 \\ \overline{\Sigma}^+ & -\frac{\overline{\Sigma}^0}{\sqrt{2}} + \frac{\overline{\Lambda}^0}{\sqrt{6}} & \overline{\Xi}^0 \\ \overline{p} & \overline{n} & -\frac{2}{\sqrt{6}}\overline{\Lambda}^0 \end{vmatrix}.$$

Let us observe that $\overline{B}$ was chosen in such a way that it is obtained from B by transposing and taking the bar. Otherwise, we do not have conservation of electric charge.

We can consider two types of unitary symmetric interaction:

$$\begin{aligned} &\text{D type: TRACE } (\overline{B}mB + \overline{B}Bm) \text{ and} \\ &\text{F type: TRACE* } (\overline{B}mB - \overline{B}Bm). \end{aligned} \qquad (4.10)$$

These types of coupling are also invariant under the following discrete operations:

D type is invariant under $B \to B^T$, $m \to m^T$ and

F type is invariant under $B \to B^T$, $m \to -m^T$

(where the superscript T stands for "transposed".)
So the most general interaction is a linear combination of the above. To choose an interaction of D or F type means to impose invariance under some discrete operation called R (or hypercharge reflection). The R operation corresponds essentially to interchange N and Ξ and charge conjugate the mesons. The D type couplings have the following properties:

$$g^2_{\pi\Lambda\Sigma} = (4/3)\, g^2_{\pi NN} \;;\; g^2_{\pi\Sigma\Sigma} = 0. \tag{4.11}$$

In the F type couplings the pion is coupled to the pseudo-scalar density that transforms like isospin. Then $g^2_{\pi\Sigma\Sigma} \neq 0$ and $g^2_{\pi\Lambda\Sigma} = 0$. In choosing between the two types of coupling, the D type is probable more reasonable because there is some evidence from hypernuclei that $g^2_{\pi\Lambda\Sigma} = 0$ gets into difficulty. The ΛN forces seem to require some sizable $g^2_{\pi\Lambda\Sigma}$.

In the D type coupling we have $g_{\pi NN} = -g_{\pi\Xi\Xi}$ while in the F type coupling $g_{\pi NN} = g_{\pi\Xi\Xi}$. Then if our interaction is an equal mixture of D and F types, the Ξ would not interact strongly with the pions. This point might be of some interest in the dynamical approach to the recently discovered Ξ resonance.

Whether one has the pure D type, the pure F type or a mixture of both, it is impossible to have the K couplings much weaker than the pion couplings. If one compares pseudoscalar constants, there is some evidence from photoproduction of K mesons that the KΛN and KΣN couplings are weaker than the πN couplings. But it is known that unitary symmetry is broken by large mass ratio, e.g. $m_K, m_\pi = 3.5$. Now, if one uses a pseudovector coupling, this mass ratio is exactly compensated and the pseudovector coupling constants are practically equal for π and K interactions.

Let us now consider the couplings of the vector mesons. Again we have two possible linearly independent couplings:

$$\text{D type : Trace } (\bar{B}vB + \bar{B}Bv)$$

$$\text{F type : Trace } (\bar{B}vB + \bar{B}Bv).$$

The D type couplings bear no resemblance whatsoever to the vector theory (or gauge theory) discussed earlier in which the vector mesons are coupled to the various conserved currents of the strong interactions. On the other hand, with pure F type couplings the vector mesons are coupled to the currents generated by the gauge tranformations of unitary symmetry:

$$1 + i\lambda_i\,(\delta\,\theta_i/2).$$

More precisely, the ρ is coupled to the isospin current and the ω is coupled to the hypercharge current. Moreover, we also have the K* which is coupled to the quasi-conserved strangeness changing current:

$$(-\frac{1}{\sqrt{3}}\overline{N}\Lambda+\overline{N}\vec{\tau}\cdot\vec{\Sigma}+\dots)K^*.$$

As is well known, it might be possible to detect this kind of interaction in associated production experiments provided that reactions such as $\pi^- + p \rightarrow \Lambda^0 + K^0$ are dominated by the exchange of K^*.

In the unitary symmetry scheme with F type couplings of the vector mesons, there is a relation between the coupling constants f_ω and f_ρ:

$$f_\omega^2/4\pi = (3/4)(f_\rho^2/4\pi), \tag{4.12}$$

which does not appear in the usual vector theory without unitary symmetry. From the width of the ρ meson, we have

$$f_\rho^2/4\pi \approx 2.0 \text{ (for } \Gamma_\rho \approx 100\,\text{MeV}),$$

which leads to

$$f_\omega^2/4\pi \approx 1.5.$$

Recently performed nuclear force calculations seem to give a larger value for this coupling constant. This discrepancy might be due to the possible existence of another $T = 0$ vector meson discussed earlier.

It is also interesting to note that the couplings of the vector mesons to the pseudoscalar mesons of the form vmm must be of the F type. For instance, the ρ must be coupled universally to the sum of the π meson isospin current and the K meson isospin current. If the D type couplings were assumed, there would be terms like $\rho^0\eta^0\pi^0$ which would not be invariant under charge conjugation.

There are two couplings of the vector mesons which may "directly" be observed: $\rho \rightarrow 2\pi$ and $K^* \rightarrow K+\pi$. Using (mvm - mmv), we can readily obtain

$$\Gamma(K^*)/\Gamma(\rho) = (3/4)[p^3_{K\pi}/m^2_{K^*}]/[p^3_{\pi\pi}/m^2_\rho].$$

If the K^* mass is assumed to be 880 MeV, then from $\Gamma_\rho = 100$ MeV we obtain $\Gamma(K^*) = 30$ MeV, which is not far from the observed K^* width ($\Gamma_{exp} \sim 47$ MeV).

In the unitary symmetry model there is an open possibility for a vector meson coupled to the baryon current. This will be a unitary singlet vector meson since the baryon current is of the form

$$(\overline{p}p + \overline{n}n + \overline{\Lambda}\Lambda + \dots\dots) = \text{Trace }(\overline{B}B),$$

which is obviously a unitary singlet. Note also that it is impossible to construct a vector current that transforms like a unitary singlet with pseudoscalar mesons (nor with vector mesons). For instance, we cannot construct a vector current bilinear in the η meson. So if there exists a unitary singlet vector meson, it must necessarily be the kind coupled to the baryon current.

Let us now summarize the predictions of the unitary symmetry model based on the Gell-Mann - Neéman octet. First of all, all members of a unitary symmetry multiplet must have the same spin-parity. From this point of view the most crucial test is the spin of Ξ, which in the octet model must be 1/2. Also the K^* spin must be 1 (or else there must be some other $K\pi$ resonance with spin 1).

As for the parities of the baryons and mesons, once we define the parity i. e. $\Gamma\Sigma$ even, and $N\Xi$ even. The K meson must be pseudoscalar with re- to be even by convention, then all baryons must have the same parity, i. e. $\Lambda\Sigma$ even, and $N\Xi$ even. The K meson must be pseudoscalar with respect to both $N\Lambda$ and $N\Sigma$. When the octet model was proposed, the parity was not known to be odd, nor was there any evidence for the $T = 0$ pseudescalar η meson.

The second prediction of the unitary symmetry model is that all members of a unitary symmetry multiplet must have the same mass. Experimentally, we know that this "prediction" is not fulfilled (otherwise unitary symmetry would have been discovered many years ago). However, if unitary symmetry is broken only in lowest order, there are many interesting mass relations that can be checked experimentally.

Let us go back to our "mathematical" model of baryons in which the baryons are composed of ℓ and I particles. If unitary symmetry is broken, the D-S mass difference and the $(e\nu)$-μ mass difference need not be zero. But let us assume that the forces that bind ℓ and I are independent of strangeness and isospin; otherwise, we would be considering higher order violations of unitary symmetry. Assuming for simplicity that the binding energies are zero, we have

$$m_N = m_s + m_e$$

$$m_\Lambda = (2/6)(m_D + m_e) + (4/6)(m_s + m_\mu)$$

$$m_\Sigma = m_D + m_e$$

$$m_\Xi = m_D + m_\mu .$$

From these relations, it follows:

$$(m_N + m_\Xi)/2 = (3m_\Lambda + m_\Sigma)/4, \tag{4.13}$$

as first noted by Gell-Mann. Experimentally, the left-hand side gives 1127 MeV while the right-hand side is equal to 1134 MeV.

A similar relation holds for the pseudoscalar octet, but, as suggested by Feynman, it is better to work with $(\text{mass})^2$.

$$m_K^2 = (3m_\eta^2 + m_\pi^2)/4. \tag{4.14}$$

Experimentally, for the left-hand side we have $(495\ \text{MeV})^2$; for the right-hand side, $(480\ \text{MeV})^2$.

The mass relation is not so good for the vector mesons. The mass formula with the observed ρ and K^* mass predicts the $T = 0$ member of the octet at 920 MeV (rather than at 780 MeV). Perhaps the observed ω

meson is a unitary singlet and a second T = 0 vector meson yet to be discovered is the T = 0 member of the vector meson octet. Or else it may well be that the two T = 0 vector mesons get mixed up in a complicated way; perhaps this kind of mixing is responsible for breakdown of unitary symmetry. But all this is very speculative.

If we take unitary symmetry seriously, baryon isobars must also be classified according to various unitary symmetry multiplets. The representation 8 is obviously inadequate to describe the 3-3 resonance. It is possible to build up multiplets with higher dimensions by decomposing 8 × 8 just as we decomposed 3 × 8 into 1 + 3. It can be shown that

$$8 \times 8 = 1 + 8 + 8 + 10 + \overline{10} + 27.$$

Each representation can further be decomposed into various ordinary multiplets with hypercharge and isospin. This is summarized in Table II.

TABLE II

1	Y = 0,	T = 0
8	Y = 1	T = 1/2
	Y = 0	T = 0, 1
	Y = -1	T = 1/2
10	Y = 1	T = 3/2
	Y = 0	T = 1
	Y = -1	T = 1/2
	Y = -2	T = 0
$\overline{10}$	Y = 2	T = 0
	Y = 1	T = 1/2
	Y = 0	T = 1
	Y = -1	T = 3/2
27	Y = 2	T = 1
	Y = 1	T = 1/2, 3/2
	Y = 0	T = 0, 1, 2
	Y = -1	T = 1/2, 3/2
	Y = -2	T = 1

Various excited baryons can be discussed within the framework of the representations listed above [13]. If we use the following pieces of information taken from experiments:

(i) The Y_1^* (1385) spin is most likely 3/2;

(ii) The Y_0^* (1405) is probably an s-wave $\overline{K}N$ bound state,
$\therefore$ Y_1^* spin $\neq$ Y_0^* spin;

(iii) There is no resonance in K^+p scattering, then it is natural to let the 3-3 resonance and the 1385 MeV Y_1^* belong to the representation 10. Note that we then predict a $T = \frac{1}{2}$, $Y = -1\pi$ Ξ resonance. This may be identified with the recently discovered $\Xi_{\frac{1}{2}}^*$ at 1530 MeV.

Meanwhile Okubo was able to generalize the Gell-Mann mass formula to any unitary symmetry multiplet as follows:

$$m = m_0 \{ 1 + aY + b [T(T+1) - (Y^2/4)]\}. \qquad (4.15)$$

For the representation 8, the formula reduces to (4.13) and (4.14). But for the representation 10, because of the linear relation $T = 1 + Y/2$, the quadratic terms in (4.15) cancel each other. So we are led to the "equal-spacing rule" (emphasized by Gell-Mann at the CERN conference):

$$m = m_0' (1 + a'Y).$$

Experimentally;

$N_{3/2}^*$	1238 MeV
Y_1^*	1385 MeV
$\Xi_{1/2}^*$	1535 MeV

in fantastic agreement with the mass formula. (Moreover, if we assume that the parameters a and b are common for the baryon octet and the 10 isobars, then even the spacing parameter is correctly predicted.) If we take the mass formula seriously, there sould be a $Y = -2$ (strangeness = -3) singlet at $\sim$ 1685 MeV. But the predicted mass of this object (denoted by Z^-) is below the $\bar{K}\Xi$ threshold. Therefore Z^- should be stable against decay via strong interactions.

It may be produced via

$$K^- + p \to Z^- + K^0 + K^+$$
$$\Xi^- + p \to Z^- + p + K^0$$
$$\bar{p} + p \to Z^- + \bar{Z}^- \text{ etc.}$$

It is expected to decay into

$$Z^- \to \pi + \Xi,\ \bar{K} + \Lambda,\ \bar{K} + \Sigma$$

via weak interactions (long lifetime). Should the Z^- be found experimentally, our confidence in unitary symmetry would grow by an order of magnitude.

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