UNITARITY BOUNDS IN SUPERSYMMETRIC MODELS

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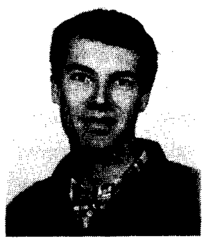

Tree-level, J = 0 partial wave amplitudes for two-body processes are evaluated within a simple supersymmetric extension of the standard model. Perturbative unitarity is violated both in bosonic and in fermionic processes. In the limit of very high energies, this happens for a Higgs mass exceeding a critical value of about 770GeV. In the limit of a very large tree-level Higgs mass, this happens when the center of mass energy is greater than a critical energy around 1TeV. These values are rather smaller compared to the standard model case, where the critical Higgs mass and the critical energy are of the order of 1TeV and 1.7TeV, respectively.

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1. INTRODUCTION

The tree-level Higgs mass M_H or, alternatively, the coupling constant λ of the scalar sector, is a parameter of the standard model which has not yet been determined. Either M_H is small and the Higgs particle will be probably detected in a near future, or M_H is large, opening the possibility of observing strong interaction phenomena [1]. In fact, in the limit of very large energy, tree-level $J = 0$ partial wave amplitudes a_o for scattering processes among scalars and/or longitudinally polarized vector bosons are constants of the kind $c_o(-i)G_F M_H^2/(4\pi\sqrt{2})$. If M_H exceeds a critical value M_H^c, then $|a_o| > 1$, which means that perturbative unitarity is violated. One expects a new strong interacting sector [2]. The same situation can be described in another fashion. Since in the large M_H limit and for sufficiently large energies $\sqrt{s}$, a_o grows linearly with s, at a certain critical energy $\sqrt{s^c}$, unitarity is violated. In the standard model with one Higgs doublet one finds $M_H^c \sim 1\text{TeV}$ [3] and $\sqrt{s^c} \sim 1.7\text{TeV}$. However, in models with a richer Higgs structure, unitarity can be violated even earlier [4]. This last observation stimulated the analysis reported in this talk where the situation in a simple supersymmetric extension of the standard model is investigated [5].

2. A MINIMAL SUPERSYMMETRIC MODEL

The starting point of the present analysis is a renormalizable model with $SU(2)_L \times U(1)_Y$ as gauge group, spontaneously broken to $U(1)_{em}$ at tree-level and possessing an exact $N = 1$, global supersymmetry [6]. At the end, this last assumption must be relaxed in order to make contact with the real world. For such minimal model we have chosen the superpotential :

$$f = \lambda N(H_1^1 H_2^2 - H_1^2 H_2^1) + \mu^2 N + f_s \quad . \tag{1}$$

In (1), $\lambda > 0$, $\mu^2 < 0$ and f_s depends on squarks, sleptons and, linearly, on H_1 and H_2. The Higgs and gauge supermultiplets are those of Table I.

Table 1 : Higgs and gauge supermultiplets

	BOSONS	FERMIONS	$SU(2)_L$	$U(1)_Y$
HIGGS	$H_1 = \binom{H_1^1}{H_1^2}$	$\tilde{H}_1 = \binom{\psi_{01}}{\psi_-}$	$\underline{2}$	-1
	$H_2 = \binom{H_2^1}{H_2^2}$	$\tilde{H}_2 = \binom{\psi_+}{\psi_{02}}$	$\underline{2}$	$+1$
	N	χ	$\underline{1}$	0
GAUGE	V_μ^a	λ^a	$\underline{3}$	0
	V'_μ	λ'	$\underline{1}$	0

The scalar potential has a supersymmetric minimum for : $\langle H_1^1\rangle = \langle H_2^2\rangle = \sqrt{-\mu^2/\lambda} = v$, $\langle N\rangle = \langle H_1^2\rangle = \langle H_2^1\rangle = \langle\text{sleptons}\rangle = \langle\text{squarks}\rangle = 0$. The mass spectrum of the model is supersymmetric and contains two scales : $M_W \sim M_Z$ and M_H. It consists of the following supermultiplets :

SUPERMULTIPLET	$(\text{MASS})^2$	
(λ_Y, A_μ)	0	(2)
$\left(h_o, \Psi_o = \begin{pmatrix}\psi_{o\alpha}\\ \bar{\lambda}_o^{\alpha}\end{pmatrix}, Z_\mu\right)$	$M_Z^2 = (g_1^2 + g_2^2)v^2$	(3)
$\left(h^\pm, \Psi_\pm = \begin{pmatrix}\psi_\pm\\ \overline{\lambda^{\pm\dot\alpha}}\end{pmatrix}, W_\mu^\pm\right)$	$M_W^2 = g_2^2 v^2$	(4)
(h_1, h_2, φ)	$M_H^2 = 2\lambda^2 v^2$	(5)
(h_3, h_4, χ)	$M_H^2 = 2\lambda^2 v^2$	(6)

The gauge bosons Z_μ and $W_\mu^\pm$ have become massive by the usual Higgs mechanism, by absorbing spinless states z and $w^\pm$; g_1 and g_2 are the gauge coupling constants of $U(1)_Y$ and $SU(2)_L$, respectively. The states listed above are appropriate linear combinations of the states of Table 1.

3. PARTIAL WAVE AMPLITUDE ANALYSIS

In the context of the model sketched above, we have assumed $M_H^2 >> M_W^2, M_Z^2$ and we have computed tree-level amplitudes for a chosen set of two-body reactions, obtaining a scattering matrix. For each reaction we have selected the $J = 0$ partial wave amplitude $a_o(s)$ and we have taken the limit $s >> M_H^2$:

$$a_o(s) \xrightarrow{s >> M_H^2} c_o(-i)\frac{G_F M_H^2}{4\pi\sqrt{2}} \tag{7}$$

(c_o is a numerical coefficient). In (7), all contributions depressed by powers of (M_W/M_H) with respect to the leading ones have been omitted. Finally, we have imposed perturbative unitarity by demanding that the maximum eigenvalue of the partial wave scattering matrix obtained in this limit is not larger than 1, in absolute value. In this way we get a critical value M_H^c for M_H.

We have also considered the limit $M_H^2 >> s$. In this case :

$$a_o(s) \xrightarrow{M_H^2 >> s} c_o'(-i)\frac{G_F s}{4\pi\sqrt{2}} \tag{8}$$

and, by imposing perturbative unitarity, a critical value $\sqrt{s^c}$ for the energy is recovered.

In performing the computation, the equivalence theorem [7], which, in the limit $s >> M_W^2$, relates amplitudes with external longitudinally polarized gauge bosons to amplitudes with external Goldsone bosons, has revealed itself very useful.

4. RESULTS AND CONCLUSIONS

In the limit $s >> M_H^2$ we have included all possible two-particle neutral channels which one can form out of the supermultiplets (3), (4), (5) and (6). The scattering matrix separates into a bosonic block t_o^B (only boson-boson channels) and a fermionic block t_o^F (only fermion-fermion channels). As a consequence of supersymmetry, t_o^B and t_o^F have the same eigenvalues with the same multiplicities [8]. We find :

$$-1(2),\ +1(11),\ +2(2),\ \frac{1-\sqrt{17}}{2}(1),\ \frac{1+\sqrt{17}}{2}(1) \quad . \qquad (9)$$

These are the non vanishing, common eigenvalues of t_o^B and t_o^F when the expression $(-i)G_F M_H^2/(4\pi\sqrt{2})$ is factorized out. The multiplicity is indicated in parenthesis. Since the largest eigenvalue is $(1+\sqrt{17})/2$, unitarity is violated for M_H larger than

$$M_H^c = \left(\frac{8\pi\sqrt{2}}{(1+\sqrt{17})G_F}\right)^{1/2} \sim 770\text{GeV}$$

which is rather smaller than the standard model result : $(M_H^c)_{sm} \sim 1\text{TeV}$.

In the limit $M_H^2 \gg s$ we have included only the neutral, light channels related to the supermultiplets (3) and (4). Here again the scattering matrix separates into a bosonic block T_o^B and a fermionic block T_o^F having same eigenvalues and multiplicities [8]. We obtain :

$$-\tfrac{1}{2}(2),\ +\tfrac{1}{2}(1),\ +\tfrac{3}{2}(2) \qquad (10)$$

in units of $iG_F s/(4\pi\sqrt{2})$. The number in parenthesis is the multiplicity. The largest eigenvalue is $\frac{3}{2}$ and unitarity is violated at energies $\sqrt{s}$ larger than

$$\sqrt{s^c} = \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} \sim 1\text{TeV}$$

a smaller value than the standard model one : $(\sqrt{s^c})_{sm} \sim 1.7\text{TeV}$.

It should perhaps be stressed that unitarity is violated in both bosonic and fermionic processes. The latter are relevant when scattering processes among supersymmetric partners of the Higgs are considered. When supersymmetry breaking terms are included, the results of the present analysis are modified by terms of the order of (m/M_H) in the limit $s \gg M_H^2$ (m is a supersymmetry breaking scale). However, at least in simple cases, they are not modified when the limit $M_H^2 \gg s$ is considered [8].

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