RELATIVISTIC STABILIZED ELECTRON BEAM

I. PHYSICAL PRINCIPLES AND THEORY

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(presented by A. A. Naumov)

I. Introduction

We work with accelerators in which use is made of magnetic and electric fields of charges inside the accelerator chamber.

In ordinary accelerators, the condition that rot H and div E should be equal to zero in the region of particle movements sets rather strict limits to the shape of fields which can be used for acceleration.

The lifting of this restriction opens up a wide horizon of new possibilities of building accelerators with very strong focusing and much easier ion injection. It thus becomes possible, for example, to accelerate particles in an axial symmetrical magnetic field which grows steeply with the increase in radius. The focusing, both radial and vertical, may in this case prove much stronger than in ordinary accelerators, including those with alternating gradient focusing.

So-called closed "stabilized electron beams" may prove to be useful in creating such fields. This is the name given to an intensive beam of relativistic electrons whose charge is fully or partially compensated by ions and which possesses certain properties which will be discussed below. Due to the magnetic attraction, the force of repulsion of two parallel-moving electrons is reduced γ^2-fold as compared with the repulsion of electrons at rest, $\gamma = (1 - v^2/c^2)^{-1/2}$ being the relativistic factor. As a result, it takes a relatively small amount of ions added to the intensive relativistic electron beam to replace Coulomb repulsion by strong attraction.

Powerful magnetic self-focusing produces a specific electromagnetic radiation which damps transverse oscillations of electrons. This, as well as certain other conditions which have to be fulfilled, causes the beam to shrink to a thin thread with immense electric and magnetic fields on the surface. It seems to be a stable and rather durable formation.

Since electrons cover an enormous distance in the settling (compression) time of the order of seconds and more, a straight-line stabilized beam can form in cosmic space only. Under laboratory conditions, only a closed stabilized is beam obtainable. To that end, it requires to be placed in a magnetic field of definite shape perpendicular to the plane of the apparatus as is the case in a betatron.

As distinct from the betatron, however, the beam surface magnetic field is much greater than the external magnetic field. Such a beam is a limited durable formation of electrons and ions bound chiefly by their own electric and magnetic fields. It is well known that all attempts to create stable classical systems bound by their own electro-magnetic field only have ended in failure. Although a strict theorem about instability of such systems has never been proved, no one to-day doubts that stable existence of such formations is impossible. But a closed stabilized electron beam, is not an absolutely free system as it is in external fields. The latter, which are sufficient to keep the system stable, prove to be many times smaller than the beam field.

The shape of the beam field is such that the space within the beam can be used for the acceleration of ions. When accelerated, the ions are kept on the orbit and focused to it by the beam magnetic and electric field. The fact that the beam field is strong makes us hopeful that very high energies of particles will be obtainable at relatively small accelerator radii and a weak magnetic field keeping the beam at an equilibrum radius. Powerful focusing and the possibility of using for acceleration, part of the ions which compensate the charge of the electrons, afford the prospect of obtaining a very high intensity of accelerated particles limited by the power that can be applied for acceleration.

The present paper deals with the physical principles underlying stabilized beam performance and gives the results of theoretical investigations but without details or mathematical calculations. It also provides data on first experiments in obtaining large ring currents of relativistic electrons.

The following papers are to be published in the near future: a detailed paper on the theory of the stabilized beam, several papers by the author in conjonction with S. T. Beliaev on the theory of relativistic plasma which have a direct bearing on the problem under consideration, and a detailed description of experiments.

The investigations have related to various kinds of accelerators based on a stabilized beam as the system creating a magnetic field and initial ions. We consider it inadvisable, to publish the results of these investigations before obtaining experimental proof that the beam can exist. The point is that though the problems of stability were considered in great detail, only an experiment can provide a final answer to the question of beam stability at parameters necessary for building a high-energy accelerator.

The idea that the existence of a stabilized electron beam was possible was first advanced by the author in 1952. All the main calculations were made in 1953, experimental work has been under way since 1954.

The idea of magnetic self-focusing for weak relativistic beams was first proposed by Bennett in 1934 [1]) and another article by the same author was recently published[2]). However, Bennett, totally ignores the radiation arising from transverse oscillations, wherelas it is its presence which determines the basic properties of a stabilized electron beam.

2. State of equilibrium

If n_1 and n_2 are the densities of electrons and ions in a laboratory system with ions at rest, and n'_1 and n'_2 are densities in a system with electrons at rest, it is easy to see that

$$n'_1 = (1/\gamma)\, n_1;\; n'_2 = \gamma n_2$$

Hence the beam will be in a state of equilibrium only if the following condition is met:

$$1 > \nu_2/\nu_1 > 1/\gamma^2 \tag{1}$$

where we introduced two convenient values ν_1 and ν_2 which are the total number of electrons and ions respectively per unit of beam length, multiplied by the classical electron radius $r_0 = e^2/mc^2$

$$\nu = r_0 \int_0^\infty n(r)\, 2\pi r dr \tag{2}$$

(It should be noted that at $\nu_1 = 1$ the number of electrons per unit of beam length is 3×10^{12} and that the current created by those electrons, which move at a velocity close to that of light, is 17,000 amp).

If this condition is met, in fact, the beam will be negatively charged in a laboratory system and there will therefore be a potential well for ions. In the electron system, the beam is positively charged and hence the electron gas is also in a potential well. If inequality (1) is large enough on both the left and right sides, the depth of both potential wells is of the order of

$$W_2 \sim \nu_1\, mc^2 \qquad W_2' = \gamma \nu_2\, mc^2$$

As will be shown later in this paper, currents have been considered here, where ν is of the order of unity. It follows that the electrons and ions in the beam may have large transverse energies.

The distribution of particle density along the radius and the shape of the porential well are determined by the distribution of particles according to their transverse energies. The distribution of particle density is found by solving a self-confirmed problem of the particle movements in an electric and a magnetic field created by them. Thus if we assume, say, that the electron gas in its coordinate system is in thermodynamic equilibrium with temperature T for an uncharged beam ($\nu_1 = \nu_2$), we obtain a distribution of potential and density normal for such problems:

$$eU'(r) = 2T\, ln \left(1 + \frac{r^2}{r^{*2}}\right)$$

$$n_1(r) = \frac{n_1^0}{\left(1 + \frac{r^2}{r^{*2}}\right)^2} \tag{3}$$

where the beam radius is

$$r^* = \sqrt{\frac{2T}{mc^2 r_0\, \gamma\beta^2\, n_2^0}}$$

and $n_1^0 = n_1(o)$ is the density on the beam axis.

It can be seen from equation (3) that the density drops very quickly as the radius increases. It is therefore advisable to regard the beam as a sharply limited formation. We shall henceforth consider the density to be constant up to radius r* and equal to zero at larger radii.

The electrons are scattered on the ions, thus losing their directed velocity. To maintain this velocity constant, an electric field should be applied which would be parallel to the direction of electron movement.

The most convenient way of considering the problem is in respect of an electron coordinate system. In this system the momentum is not carried away by radiation while the condition that the directed velocity of the electrons should be constant coincides with the condition that the directed momentum should also be constant. The momentum and the energy imparted to the electron gas by the fast ions traversing it during a unit of time in this coordinate system are easily calculated, and equal (per electron):

$$\frac{d\,Px'}{dt} = \frac{1}{\beta c}\,\frac{dE'}{dt'} = \frac{\gamma}{\beta^2}\,\frac{4\pi r_0^2 L n_2}{\widetilde{\gamma}_0}\,mc^2, \tag{4}$$

where Px is the longitudinal momentum

E is the energy of the electrons,

$L = ln\gamma\beta^2 r/r_0$ is the Coulombian logarithm, and $\widetilde{\gamma}$ is the average for electron velocity distribution in the electron coordinate system; its value is

$$\frac{1}{\widetilde{\gamma}} = \frac{1}{\gamma^*}\left(1 + \frac{1}{\gamma^2} - \frac{1}{2\beta^*\gamma^2}\,ln\,\frac{1+\beta}{1-\beta}\right)$$

(the asteriks indicate values belonging to the electron system). At relativistic electron temperatures $1/\widetilde{\gamma} = 1/\gamma^*$ and at non-relativistic ones $\widetilde{\gamma} = 1$.

Thus the required electric field is equal to

$$eE = \frac{\gamma}{\beta^2}\,\frac{4\pi r_1{}^2\,Ln^2}{\widetilde{\gamma}}\,mc^2 \tag{5}$$

The energy accumulated by the electrons in this field is transformed, due to scattering, into the energy of transverse oscillations. This should have led to the widening of the beam. However, a new phenomenon appears under certain conditions—the radiation of electromagnetic waves associated with the transverse oscillations of electrons in the beam field. This radiation damps the transverse oscillations and the beam widening ceases in consequence.

The intensity of this radiation is easily calculated. It equals (per electron)

$$I \simeq 4\,(\gamma\nu_2)^2(1+\gamma\nu_2)^2\,\frac{r_0}{r^{*2}}\,C\;mc^2$$

The radiation is directed along the beam (along a tangent in the case of a closed beam) in a narrow angle $V \sim 1/\gamma$. Its spectrum has the form of a curve with a frequency maximum

$$\omega = 2\gamma(1 + \nu_2\gamma)^3\,c/r^* \tag{6}$$

In the case under consideration, this radiation carries away practically the entire energy accumulated by the electrons from the electric field. Losses due to bremsstrahlung, ionization of residual gas and the so-called betatron radiation consequent to the rotation of the electrons in the closed beam prove much smaller than the losses due to radiation associated with transverse oscillations. It is interesting to note that the power of this radiation may be thousands of kilowatts at infra-red or visible light frequencies. At a given number of particles in a beam and at a given electric field, a state of beam equilibrium sets in at which the momentum gained by the electrons from the field is lost due to collisions with the ions, while the energy gained is carried away by radiation. In this state of the system, the values of the beam parameters γ and r^* are found as functions of the ions number and of the electric field. In this case

$$\gamma = \frac{\varkappa}{\nu_2}$$

$$r^* = \sqrt{\frac{4\varkappa L}{1+\varkappa}}\,\sqrt{\frac{e}{E\ \text{long.}}} \tag{7}$$

where E long. is the accelerating electric field and $\varkappa \simeq 1.7$ is the solution of the equation $\varkappa(1 + \varkappa)^3 = L$, where $L = 35$, which is the Coulombian logarithm.

At first sight, it seems unexpected to find no dependence of electron energy on the electric field or of beam radius on the number of particles. It would seem natural that the larger the electric field the higher the electron energy. If the electric field is increased, indeed the electrons will be accelerated. However, as the electron energy grows, radiation intensifies, and the beam will contract in a denser beam as a result and the force of friction will become greater than that of the electric field. In consequence, the velocity of the electrons will begin to drop until it reaches the original equilibrium value. The radius of the beam will now be smaller in accordance with equation (7). In the case of an uncharged beam $\nu_2 = \nu_1$, equation (7) with numerical coefficient becomes

$$i_{el} = \frac{3}{\gamma}\,10^4\,a$$

$$r^* = \frac{3.6 \times 10^{-3}}{\sqrt{E\,\frac{\text{volts}}{\text{cm.}}}}\ \text{cm.}$$

where i_{el} is the electron current.

Thus a state of equilibrium may set in for example, at the electron energy of 15 Mev and at an electron current of 1,000 amp. It is easily seen from this that the currents in betatrons are far from equilibrium values.

At an accelerating field intensity of 1 volt (which corresponds to an inductive voltage of 600 volts for a closed beam with a radius of about 1 m.), the radius of the beam cross section area works out at about 0.04 mm. At a current of 1,000 amp., a magnetic field of 5×10^4 gauss is formed on the beam surface, whereas the external magnetic field necessary for keeping electrons with an energy of 15 Mev at a radius of 1 m. is 500 gauss.

Radiation damps transverse oscillations over a considerable period, and hence destroys the effect of multiple electron scattering on ions. But if an electron scatters to such a large angle that the amplitude of transverse oscillations becomes greater than the width of the chamber, it will reach the chamber wall before radiation has damped the oscillations. The time of beam existence limited by a single Coulomb scattering of electrons on a larger angle equals :

$$t = \left(1 + 2ln \frac{r_{abs}}{r^*}\right) \gamma \beta^3 \frac{r^{*2}}{2cr_0} \left(1 - \frac{\nu_1 - \nu_2}{\gamma^2 \beta^2 \nu_2}\right),$$

and ranges from fractions of a second for the most compressed beams to thousands of seconds for non-compressed ones. (r_{abs} *is the dimension of the tube which contains* the beam.)

The time during which a required inductive electric field can be maintained equals :

$$t = \gamma\beta \frac{(1 + \varkappa)^2}{4\varkappa h} \frac{r^{*2}}{r_0 c} \frac{\Delta B}{2H},$$

where ΔB is the average change of the magnetic flux induction which creates the inductive electric field and H is the magnetic field necessary to keep the electrons on their orbit. In practice ($\Delta B = 30{,}000$ gauss, from - 15,000 to + 15,000), this time is greater than the time of scattering at a large angle and does not limit the duration of beam existence.

If the beam maintains stability at the selected parameters, the maximum energy of ions obtainable by using the beam magnetic field for keeping the ions on the equilibrium orbit equals :

$$W_{max} = 300\, HR = 1.5 \times 10^4\, i \sqrt{\frac{vR}{2\pi}}\ (eV)$$

where i is the electron current intensity,

v is the intensity of the circuit field necessary to maintain a stationary state;

R is the radius of the turn under electron current (radius of the accelerator);

numerically ($\nu_2 \ll \nu_1$)

$$W_{max} = 2\, I\, (ka) \sqrt{V_{(kv)} \times R_{(m)}}\ (Bev)$$

Assuming that I = 10,000 amp., v = 10 Kv and R = 3 m., we get $W \approx 100$ Bev. Focusing in the beam field can be characterized by the value of $n_{eff.}$ which is that value of $n = - R/H \cdot dH/dR$ in an ordinary accelerator at which the focusing force is equal to the focusing force in the beam. In the case of an uncharged beam,

$$n_{eff} = \frac{R}{r^*}$$

It is easy to see that in this case the focusing forces acting in the vertical and in the radial direction are almost equal. For the example chosen above, $n_{eff.} = 3 \times 10^5$. In a highly charged beam the focusing force for the ions to be accelerated increases $\frac{1 + \beta ion}{\beta ion}$ - fold.

Such a high value of focusing forces ensures that the particles are kept accelerated throughout the entire acceleration cycle in spite of the fact that the cross-sectional area of the accelerating space where the field has the desired shape amounts to fractions of a millimetre.

In conclusion, it should be pointed out that as distinct from ordinary plasma, the stabilized electron beam admits a high-frequency electric field which permits recourse to the usual resonance method of acceleration. This may be accounted for, first, by the fact that the number of particles in the beam is relatively small (strong currents being caused by high velocities of electrons), and, secondly, by a sharp increase at relativistic velocities of the longitudinal mass of electrons ($M_{long} = \gamma^3 m$), the value of which determines the shielding of the longitudinal component of the high-frequency field.

3. Stability problems

One of the most important problems is that of beam stability. Instability of pinched current in plasma in the case of all types of oscillations is well known. Closed ring discharges with strong currents were investigated, among others, and these also possess a number of instabilities. On the other hand, an electron beam in a betatron is absolutely stable.

As $\nu(\nu_1$ and $\nu_2)$ is a dimensionless combination of the number of particles in the beam, the charge, light velocity, and of electron mass whose value in the betatron is much less than unity while in a ring discharge it is much larger than unity, it would be natural to expect that the beam intensity at which $\nu \simeq 1$ (at $\gamma \gg 1$) would be transient from stable to unstable. In fact, the calculations made showed that a stable beam equilibrium is to be expected at relativistic velocities and at $\nu < 1$, whereas an unstable equilibrium is to be expected at non-relativistic velocities and at $\nu > 1$.

It is no accident that we speak of stability so cautiously. The point is that no final theoretical solution to the problem of stability can be found in this case. It can be found only on the basis of thermodynamic laws or on the laws of the integrals of motion, or in simple cases of individual particle movement, as in ordinary accelerators. In systems such as the stabilized beam, which in effect are systems with an infinite number of freedom states, the problem may be one of stability to a definite kind of perturbation the risk of whose occurrence follows from physical considerations. This stability, as a rule, can be practically considered in no higher than the first (linear) approximation. Thus the theoretical results obtained are never sufficiently reliable. They can only give the experimenter a greater or smaller degree of assurance that the stabilized beam he is trying to create can be stable at parameters of practical interest.

Although the problem of stability has been considered in most of the theoretical investigations on the stabilized beam and the regions of stability and instability have been found in some of the hypotheses, it is only by experiment that the problem will finally be solved.

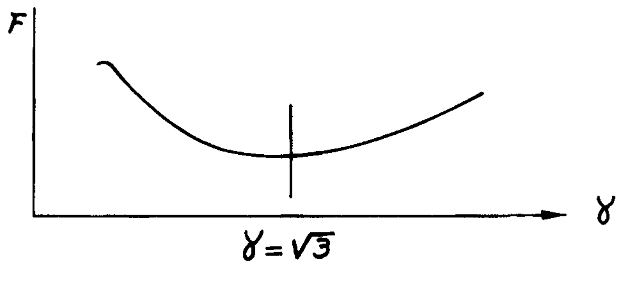
Beam stability to various kinds of perturbations has been considered, three of these are dealt with in the present paper.

(a) Stability to deviation of directed electron velocity from equilibrium velocity

Directed electron velocity in a beam remains constant at the expense of the longitudinal electric field compensating the friction force. Such an equilibrium can be stable only if the friction characteristic is positive, i.e. when the friction force grows as velocity increases. In the opposite case (when the characteristic is negative), a small increase in velocity reduces the friction force below the electric field force, resulting in particle acceleration which in turn produces a further drop of the friction force. When the particles velocity falls below equilibrium, it begins slowing down continuously.

As the cross section area of Coulomb scattering drops when the velocity increases, in the case of non-relativistic as well as relativistic velocities, it seems at first sight that the equilibrium between the electric field and the friction force should be unstable.

In actual fact, however, it follows from equation (4) that in the electron system the force of carrying away the electrons by the ions depends on the velocity as $\gamma/\beta2$ (at small values of v the dependence of $\tilde{\gamma}$ on the velocity may be neglected). The graph of this force may be presented as:



It follows from this that at $\gamma > \sqrt{3}$ the characteristic of friction becomes positive and a stable equilibrium between the friction force and electric field force becomes possible.

From the point of view of an observer in a laboratory system the contradiction of this assertion with the well-known drop of the Coulomb cross-section area as the velocity increases can be accounted for as follows: The electromagnetic radiation which damps the energy of transverse oscillations of electrons carries also away the momentum in the laboratory coordinate system. This radiation friction force grows with the velocity in the relativistic region. The left-hand part of the curve in fig. 1 illustrates the Coulomb friction against the ions which drops when the velocity increases. The right-hand part is the radiation friction which grows as the velocity increases. It should be recalled that when deducing equation (4) we took no account in the electron coordinate system of the presences of radiation (where it does not carry away the momentum). The positive characteristic of friction in a laboratory system results from stationary conditions. Any other influence damping the energy of transverse oscillations in the electron system will inevitably give rise, due to the relativistic invariant conditions, to a friction force with the same dependence on velocity as in the laboratory system.

Moreover, there will be a stable equilibrium even if there is no damping of the transverse oscillations of the electrons which move in the given potential well and scatter on the ions. In this case the process will be non-stationary. In a non-stationary process the condition of the average velocity constant does not coincide with that of the average momentum constant, as the rest mass μ of a beam volume unit increases with time at the expense of random velocity increase. The conditions of equilibrium (i.e. the average velocity constant) may be given as:

$$\mu \frac{dv}{dt} = eE - F_{friction} - v\frac{d\mu}{dt} = O$$

The last term ($v\, d\mu/dt$) must have the same dependence on velocity as radiation friction in a stationary state, which again follows from the relativistic invariant conditions.

(b) Stability to excitation of plasma waves and breaking the beam into separate coagula

It is well-known that a beam of electrons excites plasma oscillations when passed through plasma. The beam is modulated in length and loses much more energy than follows from a theory which takes account of Coulomb pair encounters only.

To begin with, a simpler non-self-confirmed problem was solved which dealt with longitudinal beam oscillations. The beam radius was assumed to be constant for the duration of the oscillations. This simplified problem made it possible to clarify some interesting properties which essentially distinguish the beam from infinite plasma (where, the oscillation problem, incidentally, is usually solved on the basis of the same hypotheses). It was then possible to solve the complete problem of self-confirmed beam oscillations which depend on the longitudinal coordinate z and the cylindrical radius r. After lengthy calculations, the same dispersion equation as in the simplified problem was obtained in certain cases.

The simplified problem consists of solving two kinetic equations, together with Maxwell equations :

$$\frac{\partial f_1}{\partial t} + \nu_1 \frac{\partial f_1}{\partial z} - e E \frac{\partial f_1}{\partial p_1} = 0$$

$$\frac{\partial f_2}{\partial t} + \nu_2 \frac{\partial f_2}{\partial z} + e E \frac{\partial f_2}{\partial p_2} = 0$$

$$\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -4\pi e \, \int f_2 \, dp_2 - \int f_1 \, dp_1)$$

$$\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = \frac{4\pi e}{c} (\int \nu_2 f_2 \, dp_2 - \int \nu_1 f_1 \, dp_1)$$

$$E = - \frac{\partial \varphi}{\partial z} - \frac{1}{c} \frac{\partial A}{\partial t}$$

where f_1 (z, t, p) and f_2 (z, t, p) are functions of the electron and ion distribution in phase space (which we consider, according to our simplification, to be dependent only on $p = p_z$ and on z up to a certain $r = r^*$, and equal to zero for larger r.

A (z, t, r) and φ (z, t, r) are electromagnetic potentials which, apart from the foregoing, also depend on r. Collisions in this problem may be neglected.

Under certain limiting conditions the following dispersion equation is obtained provided thermal velocities are neglected as compared with directed velocity :

$$\frac{4\,\nu_1}{\gamma^3} \frac{1}{(\beta_\varphi - \beta)^2} + \frac{4m\,\nu_2}{M} \frac{1}{\beta^2{}_\varphi} = \frac{\psi^2 [(1-\beta^2{}_\varphi)^{1/2} kr^*] + (1-\beta^2{}_\varphi) k^2 r^{*2}}{1-\beta^2{}_\varphi} \tag{8}$$

where $\beta_\varphi = w/ck$; w and k are frequency and wave vectors $\beta = \nu/c$; ν is the velocity of electrons, and $\psi(x)$ is the solution of the following equation :

$$\psi \frac{I_1(\psi)}{I_0(\psi)} = - x \frac{K_1(x)}{K_0(x)}$$

where $I_1(x)$, $I_0(x)$ and $K_0(x)$, $K_{,}(x)$ are Bressel functions.

The equation is simplified in two extreme cases :

$$Kr^* \gg \frac{1}{\sqrt{1-\beta^2{}_\varphi}}; \qquad Kr^* \ll \frac{1}{\sqrt{1-\beta^2{}_\varphi}}$$

For short waves $\lambda \ll \sqrt{1-\beta^2{}_\varphi}\, r^*$ the equation becomes :

$$\frac{4\nu_1}{\beta^2 \gamma^3 K^2 r^{*2}} \frac{1}{(1-x)^2} + \frac{4m\,\nu_2}{m\beta^2 k^2 r^{*2}} \frac{1}{x^2} = 1$$

At small values of coefficients preceding the terms with unknowns $1/(1-x)^2$ and $1/x^2$, the solutions are effective, and it is not difficult to show that the values obtained from them coincide with Langmuire frequencies in an ion and electron gas in a respective coordinate system.

At a certain value of the coefficients, complex radicals appear, i.e., oscillations begin to rise. The criterion of instability

$$\left(\frac{4\nu_1}{\gamma^3\beta^2}\right)^{1/3} + \left(\frac{4m\nu_2}{M\beta^2}\right)^{1/3} = \left(\frac{r^*}{\lambda}\right)^{2/3}$$

determines the value of wave length $\lambda = \lambda_{crit}$ at which this happens.

It can be thus seen that at this approximation, there will always be long enough waves in respect of whose rise the beam will be unstable. It is, however, necessary that the wave length satisfies the condition $\lambda \ll \sqrt{1-\beta_\varphi{}^2}\, r'' r^*$. The parameters then have to meet certain requirements, namely :

$$\frac{4\nu_1}{\gamma^3\beta^2} \left\{ 1 - \beta^2 \frac{\left(\gamma^3 \frac{m}{M} \frac{\nu_2}{\nu_1}\right)^{2/3}}{\left[1 + \left(\gamma^3 \frac{m}{M} \frac{\nu^2}{\nu^1}\right)^{1/3}\right]^2} \right\}$$

$$\left\{1 + \left(\gamma^3 \frac{m}{M} \frac{\nu_2}{\nu_1}\right)^{1/3}\right\}^3 \gg 1$$

In the most interesting case $\gamma^3\, m/M \cdot \nu_2/\nu_1 \ll 1$, this condition becomes :

$$\frac{4\nu_1}{\gamma^3\beta^2} \gg 1$$

Which means that at currents of

$$i \gg 5 \times 10^3 \gamma^3 \beta^3 \text{ amp.},$$

short waves are excited in the beam. At weaker currents the beam is stable to short-wave excitation. Stability to long waves needs to be considered separately. For this purpose, we shall take the other extreme case

$$\lambda \gg \sqrt{1-\beta_\varphi{}^2}\, r^*$$

In this instance, the equation may be re-written as:

$$\frac{2\nu_1 L}{\gamma^3 \beta^2}\frac{1}{(1-x)^2} + \frac{2m\nu_2 L}{M\beta^2}\frac{1}{x^2} = \frac{1}{1-\beta^2 x^2} \qquad (9)$$

where $L = ln(kr^*\sqrt{1-\beta_\varphi^2})$.

Phase velocity does not depend here on wavelengths (neglecting the small dependence of L), and the equation may also have complex solutions which signify rising oscillations. However, the rise now begins at definite values of the parameters irrespective of the wave length.

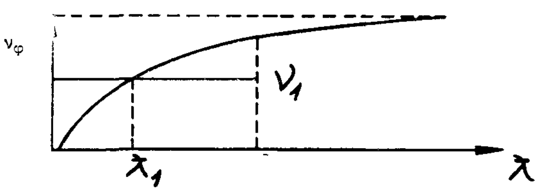
The equation (9) is easily investigated. As a result, we get the following criteria of instability:

$$V > \frac{\gamma^3\beta^2}{2L}, \text{ if } \left(\gamma\frac{m}{M}\right)^{1/3} \ll \frac{2}{\gamma-1}$$

$$V > \frac{1}{8L}\sqrt{\frac{M}{m}}\,\gamma^{3/2}, \text{ if } \gamma \gg \left(\frac{M}{4m}\right)^{1/3}$$

It can be seen from the above that at nonrelativistic velocities instability begins at very low values of current, while at relativistic velocities it begins at currents of over tens of thousands of amperes.

The results obtained can be readily understood if a limited plasma column is considered through which fast particles are flying. In the case of short waves, the plasma may be regarded as infinite. In an infinite plasma, wave frequency does not depend on the wave vector; if some of the charges are deflected from others in a plane, they will in fact oscillate, being a plane capacitor with an external field equal to zero. This means that the perturbation produced will not spread, i.e. the group velocity of the waves is equal to zero. In this case, there is a linear rise in phase velocity which increases as the wave length grows. If, however, the wave length outgrows the cross section area, the field produced by some of the charges deflecting from others is other than zero both between the charges and at the side of each of them. This means that the group velocity is other than zero and, as is shown by calculations, the phase velocity is practically independent of the wave length. The phase velocity may be presented as the following graph:



If the velocity of particles is smaller than the maximum values of phase velocity, there is always a wave length λ, at which $\nu_1 = \nu_\varphi$, but in this case a characteristic resonance braking of particles and wave rise begins as in a travelling-wave tube. If the particle velocity exceeds maximum phase velocity, there is no such wave or rise of oscillations. Assuming the curve to be flattened at a value near $\lambda = r^*$ and substituting for this value the usual expression of the Langmuir frequency $w^2 = 4\pi e^2 n/m$, we obtain the qualitative criterion of instability

$$v^2_\varphi = \omega^2\lambda^2 = \omega^2 r^{*2} = 4\nu_1 c^2 > v^2$$

or

$$\nu_1 > \frac{\beta^2}{4}$$

which coincides with the value previously obtained. For relativistic particles, this gives very high currents which are quite sufficient for the formation of a stabilized beam with large electric and magnetic fields.

(*c*) *Beam stability to transverse bendings*

As is know, an electron beam in a betatron is stable to transverse bending. The reason of this stability is the fact that external transverse focusing forces are much greater than defocusing forces which may arise at bends.

The exact opposite is the case with a current passed through plasma. It is a well-known fact that if the current forms a pinch separated from the chamber walls, it is unstable to bending. This instability is due to the fact that at bends of the current pinch magnetic fields arise of a shape such that their action on the current results in further bending.

The field of the stabilized electron beam is much stronger than the guide fields. Hence the bending magnetic field of the beam may prove larger than external focusing fields, at least for sufficiently short waves.

There is one more mechanism leading to instability of a plasma current thread, this instability being characterized as follows. Let us assume that the beam has bent. The electrons which in the plasma are strongly coupled to the thread will then subject to a centrifugal force. It is not difficult to see that this hydrodynamic force is so directed that it will increase the bending of the current thread. This force can be readily calculated and compared with the electrodynamic one. One finds

$$\frac{F\,hydr}{F\,el.\,dyn} = \frac{\gamma}{\nu_1\, ln\,\frac{8R}{r^*}}$$

For currents in plasma this ratio is well below unity (at least for strong enough currents when defocusing forces become appreciable), which means that in such cases hydrodynamic defocusing forces can always be neglected as compared with the electrodynamic ones.

In a stabilized beam this ratio is of the order of unity or more, i.e. hydrodynamic effects are essential. The stabilized beam also differs, from plasma, however, in the fact that the condition of quasi-neutrality does not apply in this case. At bends of the ion thread the electrons do not move along a curved path at all and, generally speaking, do not necessarily give rise to a hydrodynamic defocusing force. Due to high electron velocity, moreover, the beam acquires a peculiar hydrodynamic resistance to bending, at least in the case of short waves.

Unfortunately, we failed to reach a complete theoretical solution of this problem in respect of internal movements of particles and their distribution in the beam cross-section.

Considering this matter to be of great importance for the problem as a whole, we made an attempt to elucidate certain essential characteristics, confining ourselves to a dummy beam in which each of the gases (electron and ion) presents a uniform system and an absolutely flexible and tensile pinch. When one pinch deflects from the other, forces of attraction arise between them which are proportional to this deflection and equal (per unit of pinch length) to:

$$F \simeq 2\pi e^2 n_1 n_2 r^{*2} x$$

Assuming, in the first approximation, the longitudinal movement to be constant, we write the equation for transverse movement of the electron and the ion beams:

$$\pi r^{*2} n_1 \gamma m \frac{d^2 x_1}{dt} = -2\pi e^2 n_1 n_2 r^{*2}(x_1 - x_2) + F_{1\,\mathrm{extern}} + F_{1\,\mathrm{el.dyn}}$$

$$\pi r^{*2} n_2 M \frac{d^2 x_2}{dt} = -2\pi e^2 n_1 n_2 r^{*2}(x_2 - x_1) + F_{2\,\mathrm{extern}} + F_{2\,\mathrm{el.dyn}}$$

where x_1 and x_2 is deflection of electrons and ions,

F extern are external forces (e.g. focusing forces of an external magnetic field);

$F_{\mathrm{ee\ dyn}}$ are electrodynamic forces of the thread almost equal to:

$$F_{\mathrm{ee\ dyn}} \simeq \pi r^{*2} n_1 \gamma m c^2 k^2 x$$

The hydrodynamic forces are on the left sides of the equations. The equations obtained above can be rewritten as:

$$\frac{d^2 x_1}{dt^2} = -\Omega^2 (x_1 - x_2) + \lambda_1 x_1$$

$$\frac{d^2 x_2}{dt} = \xi \Omega^2 (x_2 - x_1) + \xi \lambda_2 x_2$$

where $\Omega = \frac{\sqrt{2\pi e^2 \eta_2}}{\gamma m}$ = the frequency of electron oscillation in the ion field, $\xi = \gamma m v_1 / m v_2$ = the ratio of the transverse mass of the electron pinch to the transverse mass of the ion pinch, and λ_1 and λ_2 are sums of coefficient of elasticity of the external and electrodynamic forces divided by $\gamma m \pi r^{*2} n_1$.

Let us assume that $v_1 = v$; $v_2 = 0$ $\lambda_1 = \lambda; \lambda_2 = 0$ and try to find a solution $C^{i(kx-\omega t)}$. For dependence of ω on k we obtain the characteristic equation:

$$\omega 4 - 2kv\omega^3 - [\Omega^2(1+\xi) - \lambda - k^2 v^2]\,\omega^2 + 2kv\xi\Omega^2\omega - \xi\Omega^2(k^2 v^2 + \lambda) = 0$$

Investigation of this equation brings us to the conclusion that, a stabilized electron beam, when in a state of equilibrium, is stable to bends at least up to wave-lengths of

$$\lambda \leqslant \frac{\gamma\beta}{\sqrt{2\varkappa}} r^* \qquad \text{where } \varkappa = 1.7$$

with longer waves, there is a rise of oscillations of a hydrodynamic nature.

With high electron velocity therefore electrodynamic forces do not produce pinch bends. Furthermore, weak coupling between electrons and ions (as compared with ordinary plasma) results in the absence of hydrodynamic bending at small wavelengths.

As for bends at large wavelengths, we are hopeful that they can be suppressed by means of Foucault currents within the chamber walls, by the creation of special external focusing systems and by other means.

LIST OF REFERENCES

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