# VOID MERGING TREE IN HIERARCHICAL CLUSTERING

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In this study, we formulate an analytical model to construct a merger tree of large and small void populations in terms of the two-barrier EPS formalism suggested by Sheth and van de Weygaert. To do this, we apply Lacey and Cole's tree algorithm based on the extended Press Schechter formalism for the merging history of dark halos to void structures. We extend their formalism into an analytical framework to describe complex void merging history which is strongly correlated with the environment in terms of large and small void populations.

## 1 Introduction

Observational studies show that a large fraction of Universe is dominated by empty regions called voids which evolve from under-densities in the primordial Universe. N-body simulations and theoretical models propose that complex hierarchical evolution of voids can be modeled by the extended Press Schechter [1] (EPS) formalism [3] and the evolution of voids is highly dependent on their internal substructure. In the hierarchical evolution procedure, voids exhibit two different behaviors related with their surroundings and environments, they can merge (void in void problem in the EPS formalism) or collapse (void in cloud problem in the EPS formalism). To construct the void merging tree formalism, we modify Lacey and Cole [2]'s tree algorithm for haloes and apply it to a two-barrier EPS formalism for spherical voids in the Einstein de Sitter (EdS) Universe ($\Omega = 1$).

## 2 Void Merging Tree

Sheth and van de Weygaert suggested that the collapse and merging of voids can be described by a two-barrier EPS formalism [3]. In our study, we use two barriers represented by the merging threshold of spherical voids with a fixed threshold value $\delta_v = -2.81$ and the void collapse of an under-density embedded within a contracting over-density with a fixed threshold value for the collapse barrier $\delta_c = 1.686$ in the EdS Universe. The analytical evaluation of the two-barrier random walk problem takes into account a prediction of the distribution function $f_v(M)$ for voids on a mass scale $M$ corresponding to a fractional under density function given as

$$f_v(M)dM = \frac{1}{\sqrt{2\pi}} \frac{v(M)}{\sigma^2} \exp\left[-\frac{v^2(M)}{2}\right] \exp\left[-\frac{|\delta_v|}{\delta_c}\frac{D^2}{4v^2(M)} - \frac{2D^4}{v^4(M)}\right] dM \qquad (1)$$

where $v(M) = |\delta_v|/\sigma(M)$ is the scaled mass function, $\sigma(M)$ is the mass variance function, $\delta_v$ is the spherical density function and $D$ is the void and cloud parameter $D \equiv 1 - \delta_c/(\delta_c + |\delta_v|)$ which parameterizes the impact of the halo evolution on the evolving population of voids including two barriers for over-dense $\delta_c$ and under-dense $\delta_v$ regions [3]. Mass fraction function in Eq. 1 represents the four dynamical stages of the EPS formalism with respect to the two barriers called "cloud in cloud", "cloud in void", "void in cloud" and "void in void".

### 2.1 Merging Tree of Large Voids

The model of the merging tree of isolated spherical large voids not embedded in over-dense regions is directly obtained by the "void in void" process. This means that we can assume the density function of the over dense regions as zero $\delta_c \to \infty$ which causes the void in cloud parameter $D$ to tend to unity ($D \to 1$) then mass fraction function in Eq. 1 is reduced to the following form including one barrier $\delta_v$ for large voids

$$f_v(S, \delta_v) dS = \frac{1}{\sqrt{2\pi}} \frac{|\delta_v|}{S^{3/2}} \exp\left[-\frac{|\delta_v^2|}{2S}\right] dS \tag{2}$$

where $S$ is the mass scale function which is defined in terms of the mass variance function $\sigma(M)$ as $S(M) = \sigma^2(M)$. Hence the merging rate of the large voids is interpreted as the probability of a void with mass $M_1$ which will merge with another void with mass $\Delta M = M_1 - M_2$ at the corresponding time interval $d|\delta_v|$ is calculated as

$$\frac{d^2 p}{d\ln \Delta M d\ln t} = \sqrt{\frac{2}{\pi}} \frac{\Delta M}{M_2} \frac{|\delta_v(t)|}{\sigma_2} \left|\frac{d\ln \sigma_2}{d\ln M_2}\right| \left|\frac{d\ln |\delta_v(t)|}{d\ln t}\right| \frac{1}{(1-{\sigma_2}^2/{\sigma_1}^2)^{3/2}} \exp\left[-\frac{|\delta_v(t)|^2}{2}\left(\frac{1}{{\sigma_2}^2} - \frac{1}{{\sigma_1}^2}\right)\right] \tag{3}$$

where $\sigma_1 = \sigma(M_1)$ and $\sigma_2 = \sigma(M_2)$ are mass variance functions and $\delta_v(t)$ is linear density function of the under-dense regions in the EdS Universe. We find that the merger rate of the large voids in Eq. 3 has the same appearance as the merger rates of the dark matter halos and the merger rates of the large voids are independent from the redshift value.

### 2.2 Merging Tree of Small Voids

The model of the merging tree of small voids embedded into over dense regions includes four barriers which are reduced to the two barriers in the EdS Universe while the model of large voids includes one barrier. The merging tree model of small voids is constructed by using mass fraction function Eq. 1 but unlike the model of large voids, the model of small voids takes into account the void in cloud parameter $D$. Hence the merging rate of small voids is given as

$$\begin{aligned}\frac{d^2 p}{d\ln \Delta M d\ln t} &= \sqrt{\frac{2}{\pi}} \frac{\Delta M}{M_2} \frac{|\delta_v(t)|}{\sigma_2} \left|\frac{d\ln \sigma_2}{d\ln M_2}\right| \left|\frac{d\ln |\delta_v(t)|}{d\ln t}\right| \frac{1}{(1-{\sigma_2}^2/{\sigma_1}^2)^{3/2}} \exp\left[-\frac{|\delta_v(t)|^2}{2}\left(\frac{1}{{\sigma_2}^2} - \frac{1}{{\sigma_1}^2}\right)\right] \\ &\exp\left[-\frac{1}{(\delta_c + |\delta_v|)}\left(\frac{1}{4}\frac{|\delta_v|}{\delta_c}\left({\sigma_2}^2 - {\sigma_1}^2\right) + \frac{2}{(\delta_c + |\delta_v|)}\left({\sigma_2}^4 - {\sigma_1}^4\right)\right)\right]\end{aligned} \tag{4}$$

where $\delta_c(t)$ linear density function of spherical over-dense regions in the EdS Universe. We conclude that merger rate of small void populations is strongly dependent on the redshift value unlike the merger rate of large voids.

## References


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