Robinson-Bertotti solution from flux compactification

O. Loaiza-Brito

Departamento de Física, DCI, Campus León, Universidad de Guanajuato,C.P. 37150, Guanajuato, México

E-mail: `oloaiza@fisica.ugto.mx`

L. Vazquez-Mercado

Departamento de Física, DCI, Campus León, Universidad de Guanajuato,C.P. 37150, Guanajuato, México

E-mail: `livazquezm@fisica.ugto.mx`

Abstract. We present a 10 dimensional supergravity compactification threaded with a simple flux configuration of Neveu-Schwarz-Neveu-Schwarz (NS-NS) and Ramond-Ramond (RR) 3-form fluxes to obtain the near-horizon geometry of an extremal black hole, namely, the Robinson-Bertotti solution. The constraints and conditions that fluxes must fulfill to attain an $AdS_2 \times S^2$ symmetry in 4d space-time, were found by Einsteins equations, Bianchi identities and by the integrability conditions of 10d chiral spinors.

1. Introduction

The $AdS_2 \times S^2$ geometry of extremal black holes is a solution that has been studied under different approaches [1]. In the context of $\mathcal{N} = 2, d = 4$ ungauged supergravity, this geometry emerges from the compactification of type IIB superstring theory on a Calabi-Yau (CY) threefold. The presence of branes wrapping internal cycles of the CY are considered. The effective theory in 4d space-time is interpreted as an extremal supersymmetric black hole. The $AdS_2 \times S^2$ geometry appears when the near-horizon limit is taken.

We are interested in $AdS_2 \times S^2$ solutions of supergravity theories within a flux compactification scenario. This construction is studied in detail in a previous work by the authors [2]. The aim of it is to study the conditions under which, a compactification in presence of fluxes, yields a 4d space-time of the type $AdS_2 \times S^2$. The solution we present, considers a constant dilaton and hence, a constant warping factor. It consists of type IIB string theory compactification on a $SU(3)$-structure manifold, Ricci-flat and containing torsion terms. The compactification is performed in the presence of RR and NS-NS 3-form fluxes. The flux configuration we use, satisfies the Einstein equations, Bianchi identities and the integrability conditions of the corresponding ten dimensional spinors. Besides, it yields a null contribution to the 4d scalar curvature. Therefore, we provide a new way to obtain the $AdS_2 \times S^2$ geometry as a near-horizon solution. We might mention that a well known solution [3], which includes 5-form RR fluxes is also considered as a solution in our work.

The content of the manuscript is as follows. In section 2, we study the contribution of some configuration of fluxes, within the context of supergravity compactification, which yield a Ricci-

flat space-time formed by the product $AdS_2 \times S^2$. We see how the presence of fluxes modifies (and enriches) the scalar curvatures. So we can have positive, null or negative curvature. While, in the absence of fluxes, the only allowed curvatures are null or negative [4]. The analysis is performed, in spirit of [4], by computing the contribution of the energy-momentum tensor of the n-form fluxes to the scalar curvature. Then, we study the compatibility of such fluxes with Einstein equations and Biachi identities. We find a simple flux configuration, consisting of 3-form RR and NS-NS fluxes, which fulfills these constraints. Section 3 contains the scalar curvatures of spaces AdS_2 and S^2, computed as function of flux numbers from the integrability conditions on the 10d spinors on type IIB supergravity. For this purpose, we choose the simplest flux configuration with the 3-forms from section 2. We look for solutions preserving $\mathcal{N} = 2$ supergravity in 4d. This condition in addition to the assumption of maximally symmetric 2d spaces, imposes the space-time $AdS_2 \times S^2$ to be Ricci-flat. At the end of the manuscript we present our conclusions.

2. Flux supergravity compactification

We begin our study by defining what kind of fluxes are compatible with a compactification of the type IIB string theory on a 6-dimensional manifold into a space formed by the product $AdS_2 \times S^2$. The 6d manifold possesses an $SU(3)$-structure. The symmetry associated to this compactification setup is $SO(1,1) \times SO(2) \times SU(3)$.

Let us start by considering the most generic 10d metric which is invariant under Poincaré transformations in 4 dimensions [5, 6]

$$ds^2 = e^{2A(y)}(\widetilde{g}_{ij}dx^i dx^j + \widetilde{g}_{ab}dx^a dx^b) + h_{mn}dy^m dy^n. \tag{1}$$

We have assigned indices i,j to the coordinates of AdS_2 space, with $i = 0, j = 1$ and indices a,b to the coordinates of S^2 space, with $a = 2, b = 3$. We also have the coordinates of the internal space, whose indices are the letters m,n, with $m, n = 4, ..., 9$. Finally, for a generic 4d coordinate, we use Greek letters $\mu, \nu = 0, ..., 4$.

What follows next is to calculate the curvature of the maximally symmetric 2d spaces. With this purpose, by taking the Einstein trace-reverse equations, the Ricci scalar $R(\widetilde{g}_{ij}) \equiv \widetilde{R}_1$ for AdS_2 satisfies

$$\widetilde{R}_1 + e^{2A}(-T_i^i + \frac{1}{4}T_L^L) = e^{-2A}\nabla^2 e^{2A}, \tag{2}$$

where T_{MN} is the energy-momentum tensor in 10d. Integration over the internal manifold fixes the right-hand side of this equation to vanish. Then, a relation between curvature and the field content contribution from the second term of left-hand side, is established. On the other hand, we have the expression of the energy-momentum tensor for a general n-form [4, 7]

$$\mathcal{T}_1 \equiv -T_i^i + \frac{1}{4}T_L^L = -\mathcal{F}_{iM_1...M_{n-1}}\mathcal{F}^{iM_1...M_{n-1}} + \frac{n-1}{4n}\mathcal{F}^2. \tag{3}$$

A similar result is obtained for $\tilde{R}_2$ and $\mathcal{T}_2$ for S^2. Now, it is necessary to consider specific flux configurations which preserve the $SO(1,1) \times SO(2)$ symmetry in 4d. At this point we can have different configurations of n-forms fulfilling this requirement. We can consider *internal fluxes*, with all their indices filling the internal space, or fluxes with indices in one of the 2d spaces, i.e., fluxes of the general form $\mathcal{F}_n = \omega_2 \wedge f_{n-2}$. The contribution of the later to the curvature can be positive or negative, as we shall see.

2.1. Ricci flat space

We study the contribution to 4d scalar curvature of different fluxes by following [4, 7]. We start with those which have 2 indices in one of the 2d subspaces, then, we continue our analysis with internal fluxes and finally, we take into account fluxes with four indices in the external 4d space. Remember that we are considering fluxes compatible with the symmetry $SO(1,1) \times SO(2)$.

Case I

Fluxes with general form $\mathcal{F}_n = \omega_2 \wedge f_{n-2}$, with 2 indices filling one of the 2d spaces, namely the AdS_2 space. Let ω_2, which is the volume 2-form of the space-time AdS_2, be

$$\omega_2 = \frac{1}{2}\omega_{ij}dx^i \wedge dx^j \tag{4}$$

The first term of the right-hand side of Eq.(3) yields

$$\mathcal{F}_{jL_1...L_{n-1}}\mathcal{F}^{jL_1...L_{n-1}} = \frac{2}{n}F^2 \tag{5}$$

From the above expressions, (4) and (5), the contribution to $\widetilde{R}_1$ for $\mathcal{T}_1$ in terms of a general form flux $\mathcal{F}_n$ is

$$\mathcal{T}_1 = \frac{n-9}{4n}F^2. \tag{6}$$

Since $F^2 \leq 0$, it follows that all n-forms in type IIB theory contribute positively to $\mathcal{T}_1$ and negatively to $\widetilde{R}_1$, except for 9-forms which contribution make the curvature vanishes.

Case II

Fluxes whose general form is $\mathcal{G}_n = \tilde{\omega}_2 \wedge g_{n-2}$, with

$$\tilde{\omega}_2 = \frac{1}{2}\omega_{ab}dx^a \wedge dx^b, \tag{7}$$

where the contribution to $\widetilde{R}_{(2)}$ by $\mathcal{T}_{(2)}$ is given by

$$\mathcal{T}_2 = \frac{n-9}{4n}G_n^2. \tag{8}$$

The contribution to $\mathcal{T}_2$ by the field strengths n-forms in type IIB theory is negative. Hence, the contribution to the S^2 curvature, turns to be positive. For values of $n = 9$, curvature vanishes.

Case III

Finally, we consider internal fluxes and fluxes of the form $\mathcal{G}_n = Vol_4 \wedge h_{n-4}$. The contribution of internal fluxes to $\mathcal{T}$ is,

$$\mathcal{T} = \frac{n-1}{2n}F_n^2 \tag{9}$$

From this expression we see that 1-forms make curvature vanishes while all n-forms of type IIB theory yield a negative curvature. In addition, the contribution to curvature from fluxes $\mathcal{G}$ is given by

$$\mathcal{T} = -\frac{9-n}{2n}\mathcal{G}_n^2. \tag{10}$$

The above cases illustrate the possibility of having positive, negative or null scalar curvatures. Since our goal is to obtain a Ricci-flat 4d space-time, based on

$$\widetilde{R}_4 = \widetilde{R}_1 + \widetilde{R}_2 = -e^{2A}\mathcal{T} = -e^{2A}(\mathcal{T}_1 + \mathcal{T}_2), \tag{11}$$

the natural step is to choose fluxes such that, their contribution to $\mathcal{T}$ leads to a vanishing curvature, i.e. fluxes satisfying $\mathcal{T}_1 + \mathcal{T}_2 = 0$. This constraint is not just satisfied by RR 5-form fluxes $F_5 = \omega_2 \wedge F_3$ [3], but by a wider number of flux configurations. An important remark is the fact that, the fluxes $\mathcal{F}_n$ and $\mathcal{G}_n$, can be chosen such that no tadpole is generated in the internal space X_6, avoiding in this way, the presence of orientifolds. This fact allows to keep two supersymmetries in four dimensions.

2.2. Examples

Some examples will illustrate the obtention of a null curvature by the incorporation of specific fluxes configurations. We will consider the presence of fluxes from type IIB supergravity theory. Let us take 5-form fluxes which general formula is $f_2 \wedge F_3$. The coefficients are F_{ijmnp} for the AdS_2 and F_{abmnp} for S^2. Recall that we are labeling 2d coordinates x on AdS_2 with letters $\{i, j, k, l\}$ and on S^2 with letters $\{a, b, c, d\}$, while internal coordinate are denoted by indices $\{m, n, p, ...\}$. The corresponding curvatures are

$$\widetilde{R}_1 = -\frac{e^{2A(y)}}{5}|F_5|^2, \qquad \widetilde{R}_2 = \frac{e^{2A(y)}}{5}|F_5|^2. \qquad (12)$$

Hence, the 4d scalar curvature vanishes. On the other hand, we can choose a more general flux configuration, let H_n be an n-form with two legs on AdS_2 (and the rest of them on the internal space X_6) and an internal flux $\mathcal{F}_m$. Then, the corresponding 4d scalar curvature vanishes if

$$|H_n^2| = \frac{2n}{9-n}\frac{m-1}{m}\mathcal{F}_m^2. \qquad (13)$$

Now, we shall focus on a simpler case in which NS-NS 3-form fluxes are turned on. Let NS-NS flux H_3 and RR flux F_3 be defined by

$$
\begin{aligned}
H_3 &= \left(Ndx^0 \wedge dx^1 + Mdx^2 \wedge dx^3\right) \wedge d\alpha, \\
F_3 &= \left(Pdx^0 \wedge dx^1 + Qdx^2 \wedge dx^3\right) \wedge d\alpha,
\end{aligned} \qquad (14)
$$

with α being a function of internal coordinates. The next step is to obtain the scalar curvatures, which according to (6) are,

$$
\begin{aligned}
\widetilde{R}_1 &= -2e^{2A(y)}(N^2 + P^2)(\nabla\alpha)^2, \\
\widetilde{R}_2 &= 2e^{2A(y)}(M^2 + Q^2)(\nabla\alpha)^2.
\end{aligned} \qquad (15)
$$

From these expressions we can find that total 4d curvature vanishes if $M^2 + Q^2 = N^2 + P^2$. Under these conditions the 4d space-time with an $AdS_2 \times S^2$ geometry becomes the near-horizon limit of an extremal Reisner-Nordström black-hole.

Some remarks are important. First, because of the $SU(3)$-structure of internal manifold we have assume, it is not possible to expand a flux in terms of internal vector components. For this reason, we take a smeared internal index of the 3-form fluxes as the most general case. Second, we have $H_3 \wedge F_3 = 0$, then our choice is consistent with the absence of RR five-form fluxes. This fact does not force us to have a CY as internal space, because, as we shall further see, some torsion terms arise. In the next section we will study and verify the compatibility of our flux configuration with 10d Einstein equations and Bianchi identities.

2.3. Einstein equations

In this section we will compute the 2d Ricci tensors for each 2d subspace following [7]. First, let us consider the bosonic part of the type IIB superstring action,

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, \sqrt{-G} e^{-2\phi} \left(R - \frac{1}{12 \, Im \, \tau} |G_3|^2 \right), \tag{16}$$

where G_{MN} is the 10d metric and $G_3 = F_3 - \tau H_3$. This action is written in the string frame. It follows now, that the 10d component of the Ricci tensor is given by,

$$R_{MN} = -\frac{1}{Im \, \tau} \left(\frac{G_3^2}{48} G_{MN} - \frac{1}{4} G_{MQR} \bar{G}_N{}^{QR} \right). \tag{17}$$

Recall that we want to preserve the symmetries of the compactification setup, and it is necessary to consider the adequate metric. The most general one we choose is

$$ds^2 = e^{2A(y)} \widetilde{g}_{ij} dx^i dx^j + e^{2B(y)} \widetilde{g}_{ab} dx^a dx^b + e^{-2A(y)} \widetilde{h}_{mn} dy^m dy^n. \tag{18}$$

Then, the corresponding Ricci tensors are

$$R_{ij} = \widetilde{R}_{ij} - e^{4A} \left(\widetilde{\nabla}^2 A + 2\widetilde{\nabla} A \cdot \widetilde{\nabla} B - 2(\widetilde{\nabla} A)^2 \right) \widetilde{g}_{ij}, \tag{19}$$

$$R_{ab} = \widetilde{R}_{ab} - e^{2(A+B)} \left(\widetilde{\nabla}^2 B - 2\widetilde{\nabla} A \cdot \widetilde{\nabla} B + 2(\widetilde{\nabla} B)^2 \right) \widetilde{g}_{ab}, \tag{20}$$

where $\widetilde{\nabla}$ is the covariant derivative with respect to the non-warped metric $\widetilde{h}_{mn}$. From these expressions and the corresponding components of Eq. (17), the Ricci tensors of subspaces AdS_2 and S^2 are, respectively

$$\begin{aligned} \widetilde{R}_{ij} &= e^{4A} \left(\widetilde{\nabla}^2 A + 2\widetilde{\nabla} A \cdot \widetilde{\nabla} B - 2(\widetilde{\nabla} A)^2 \right) \widetilde{g}_{ij} - \frac{1}{Im \, \tau} \left(\frac{G_3^2}{48} G_{ij} - \frac{1}{4} G_{iQR} \bar{G}_j{}^{QR} \right), \\ \widetilde{R}_{ab} &= e^{2(A+B)} \left(\widetilde{\nabla}^2 B - 2\widetilde{\nabla} A \cdot \widetilde{\nabla} B + 2(\widetilde{\nabla} B)^2 \right) \widetilde{g}_{ab} \\ &\quad - \frac{1}{Im \, \tau} \left(\frac{G_3^2}{48} G_{ab} - \frac{1}{4} G_{aQR} \bar{G}_b{}^{QR} \right). \end{aligned} \tag{21}$$

These expressions are written in terms of the nonwarped metric. The next step is to compute the Bianchi identities in order to look for possible constraints that fluxes must fulfill.

2.4. Biachi identities

Bianchi identities are trivially satisfied because we are working with 3-form fluxes having a smeared index on the internal space. Hence, it is necessary to look for more astringent conditions provided by the dual Bianchi identities, $d * F_3 = d * H_3 = 0$. From these we get

$$\begin{aligned} (Q + \tau M) \left(-2\partial_m (2A + B) \tilde{\partial}^m \alpha + \tilde{\partial}^2 \alpha \right) &= 0 \\ (P + \tau N) \left(+2\partial_m (-4A + B) \tilde{\partial}^m \alpha + \tilde{\partial}^2 \alpha \right) &= 0. \end{aligned} \tag{22}$$

It follows that $A = B$ for nonvanishing $Q + \tau M$ and $P + \tau N$. On the other hand, it seems that even though M_2 and $\tilde{M}_2$ are independent, they share the same warping factor in a background threaded with fluxes (14). Then, for $A = B$ both expressions reduce to

$$\tilde{\partial}^2 \alpha = 6e^{-2A}\partial_m A \partial^m \alpha = \frac{3}{2}e^{-6A}(\partial_m e^{4A})(\partial^m \alpha). \tag{23}$$

Comparing (21) with the Bianchi identities, we have, for the AdS_2 space

$$\begin{aligned}
\tilde{\nabla}^2(e^{4A} - \alpha) &= 2\tilde{R}_1 + \frac{1}{2}e^{-6A}(\partial_m e^{4A}\partial^m e^{4A}) - \frac{3}{2}\partial_m e^{4A}\partial^m \alpha \\
&\quad + \frac{1}{4Im\,\tau}\left(-P^2 - \tau\bar{\tau}N^2 + 2(Im\,\tau)PN\right)e^{-2A}\partial_m \alpha \partial^m \alpha.
\end{aligned} \tag{24}$$

The configuration (14) has been taken to compute the flux components. Similarly, by making the comparison of R_{ab} with Bianchi identities, the result is

$$\begin{aligned}
\tilde{\nabla}^2(e^{4A} - \alpha) &= 2\tilde{R}_2 + \frac{1}{2}e^{-6A}(\partial_m e^{4A}\partial^m e^{4A}) - \frac{3}{2}\partial_m e^{4A}\partial^m \alpha \\
&\quad + \frac{1}{4Im\,\tau}\left(Q^2 + \tau\bar{\tau}M^2 + 2(Im\,\tau)QM\right)e^{-2A}\partial_m \alpha \partial^m \alpha,
\end{aligned} \tag{25}$$

for the S^2 space. Since $\widetilde{R}_4 = \widetilde{R}_{(1)} + \widetilde{R}_{(2)}$ must vanish for $AdS_2 \times S^2$, we add (24) and (25). The result is

$$\begin{aligned}
\tilde{\nabla}^2(e^{4A} - \alpha) &= e^{-6A}(\partial_m e^{4A}\partial^m e^{4A}) - 3\partial_m e^{4A}\partial^m \alpha + \frac{1}{4Im\,\tau}\left((Q^2 - P^2) - \tau\bar{\tau}(M^2 - N^2)\right. \\
&\quad \left. + 2(Im\,\tau)(QM + PN)\right)e^{-2A}\partial_m \alpha \partial^m \alpha.
\end{aligned} \tag{26}$$

The left side of (26) vanishes if we integrate over a compact manifold. Consequently, the values for the flux numbers and the warping factor are constrained. There are several possibilities. Among all these, we can choose a nonconstant warping factor but the solution would be more complex and it is beyond the scope of this study. Then, we will concentrate on the simplest solution involving a constant warping factor. We are looking for the minimal conditions under which we can construct the Robinson-Bertotti solution in 4d. There are some cases to consider (all of them satisfying the general condition for $\hat{\mathcal{T}} = 0$, $M^2 + Q^2 = N^2 + P^2$) in which Eq. (26) vanishes:

(i) $(M, N, P, Q) \neq 0$ and $M = -N, Q = P$.
(ii) $P = 0$ and $\frac{M}{Q} = \frac{\tau\bar{\tau} - 1}{2\,Im\,\tau}$
(iii) $Q = 0$ and $\frac{N}{P} = \frac{\tau\bar{\tau} + 1}{2\,Im\,\tau}$
(iv) $N = 0$ and $\frac{Q}{M} = \frac{\tau\bar{\tau} - 1}{2\,Im\,\tau}$
(v) $M = 0$ and $\frac{P}{N} = \frac{\tau\bar{\tau} + 1}{2\,Im\,\tau}$

For the above expressions, $H_3 \wedge F_3$ is satisfied as expected for a configuration without a 5-form RR flux.

We proceed to compute the curvature scalars from the integrability conditions on the 2d components of 10d spinors. We shall concentrate on a configuration of RR and NS-NS 3-form fluxes given by (14) satisfying one of these constraints, it turns out that conditions (2) to (4) are easier to be considered in this procedure.

3. Near-Horizon geometry from 3-form fluxes

Our next task, is to compute the curvature scalars of the 2d spaces from the integrability conditions on the 2d components of 10d spinors within a type IIB supergravity flux compactification scenario. We are working with flux configuration (14). The form of the 10d space-time is $AdS_2 \times S^2 \times X^6$, with an $SU(3)$-structure for X^6 and arbitrary curvatures for AdS_2 and S^2. In principle, we are not assuming any relation among the flux coefficients but we look for solutions preserving $\mathcal{N} = 2$ 4d supergravity. Under this condition, the AdS_2 and S^2 curvatures are restricted to be equal in magnitude. Therefore the 4d space-time will be Ricci-flat.

When supersymmetry is preserved, the gravitino variation is

$$\delta\Psi_M = \nabla_M \epsilon - \frac{1}{4} \slashed{H}_M \sigma^3 \epsilon + \frac{1}{16} e^\phi \slashed{F}_3 \Gamma_M \sigma^1 \epsilon = 0 \tag{27}$$

where σ are Pauli matrices, and $\epsilon = \begin{pmatrix} \epsilon^1 \\ \epsilon^2 \end{pmatrix}$ are the chiral spinors. Since the ten-dimensional chiral spinors $\epsilon^{1,2}$ are correlated by the vanishing of the gravitino variation (27), we shall obtain an independent equation for each one by using the vanishing supersymmetric variation of the fermi fields.

We start by taking the 4d component of $\delta\Psi^1$ in a background threaded with the flux content (14). Using the fact that

$$[\Gamma_i, \Gamma_{jkm}] = \{\Gamma_i, \Gamma_{abm}\} = 0, \tag{28}$$

the M_2-component of gravitino variation is

$$\nabla_i \epsilon - \frac{1}{4} \slashed{H}_i \sigma^3 \epsilon + \frac{1}{16} e^\phi \Gamma_i (\slashed{F}_3^{(1)} - \slashed{F}_3^{(2)}) \sigma^1 \epsilon = 0, \tag{29}$$

where $\slashed{F}_3^{(1)} = F^{01m} \Gamma_{01m}$ and $\slashed{F}_3^{(2)} = F^{23m} \Gamma_{23m}$. The strategy we will follow is commuting Γ_i with $\slashed{F}_3$ in the last term of the above equation by using the dilatino variation for a constant dilaton given by

$$\delta\lambda = -\frac{1}{2} \slashed{H}_3 \sigma^3 \epsilon - \frac{1}{4} e^\phi \slashed{F}_3 \sigma^1 \epsilon = 0, \tag{30}$$

this will allow to decouple the ten-dimensional spinors $\epsilon^{1,2}$. However, this seems difficult to perform unless $\slashed{F}^{(1)}{}_3$ or $\slashed{F}^{(2)}{}_3$ vanishes, but according to section 3, this is an available condition on the fluxes. Then, we choose $P = 0$ corresponding to $F^{01m} = 0$. The i-component of the gravitino variation yields

$$\left(\nabla_i - \frac{1}{4} \slashed{H}_i + \frac{1}{8} \Gamma_i \slashed{H}_3 \right) \epsilon^1 = 0, \tag{31}$$

a similar expression for the a-component is computed.

From now on we are going to concentrate on the equations for ϵ^1, while the expressions for ϵ^2 are similar. It is important to notice that both spinors are decoupled just in the presence of non-trivial fluxes H_3 and F_3.

Then, we can express (31) as

$$\nabla_i^{(T)}\epsilon^1 = (\nabla_i + \kappa_i)\epsilon^1 = 0, \tag{32}$$

with $\kappa_i = -\frac{1}{4}\slashed{H}_i + \frac{1}{8}\Gamma_i\slashed{H}_3$.

The next step is to compute the corresponding components of the connection by using the metric $ds^2 = e^{2A(y)}\widetilde{g}_{\mu\nu}dx^\mu dx^\nu + h_{mn}dy^m dy^n$. The i-component of the covariant derivative of the spinor ϵ^1 is

$$\nabla_i^T\epsilon^1 = \left(\widetilde{\nabla}_i - \frac{1}{2}\gamma_i\widetilde{\gamma}\otimes\widetilde{\sigma}\otimes\slashed{\partial}A + \kappa_i\right)\epsilon^1 = 0, \tag{33}$$

we have denoted the covariant derivative with respect to the metric g_{ij} as $\widetilde{\nabla}$ and $\nabla_i = \widetilde{\nabla}_i - \frac{1}{2}\gamma_i\widetilde{\gamma}\otimes\widetilde{\sigma}\otimes\slashed{\partial}A$. On the other hand, we have an expression for the Riemann tensor in terms of the connection,

$$[\widetilde{\nabla}_i, \widetilde{\nabla}_j]\epsilon^1 = \frac{1}{4}\widetilde{R}_{ij}{}^{kl}\gamma_{kl}\epsilon^1, \tag{34}$$

and together with the variation of gravitino, we will have,

$$[\widetilde{\nabla}_i, \widetilde{\nabla}_j]\epsilon^1 = -(\partial_n A\partial^n A)\gamma_{ij} + \frac{1}{2}[\gamma_i\widetilde{\gamma}\otimes\widetilde{\sigma}\otimes\slashed{\partial}A, \kappa_j] + \frac{1}{2}[\kappa_i, \gamma_j\widetilde{\gamma}\otimes\widetilde{\sigma}\otimes\slashed{\partial}A] + [\kappa_i, \kappa_j]\epsilon^1. \tag{35}$$

If we take the constant warping factor as a solution, the last expression reduces to $[\kappa_i, \kappa_j]\epsilon^1$. Therefore, the new expression for the Riemann tensor is

$$\frac{1}{4}\widetilde{R}_{ij}{}^{kl}\gamma_{kl} - [\kappa_i, \kappa_j] = 0. \tag{36}$$

On this point it is worthy to emphasize some facts. The first one is to notice that in the fluxless case, the Riemann tensor and the contorsion would be zero, leading us to Minkowskian space-time. As a second fact, let us consider the flux case, then, even for a constant warping factor, there are contributions to the curvature because of the contorsion term. Let us compute explicitly the $i = 0, j = 1$-components of $[\kappa_i, \kappa_j]$ which yields

$$
\begin{aligned}
[\kappa_0, \kappa_1] &= \frac{1}{16}[\slashed{H}_0, \slashed{H}_1] + \frac{1}{64}[\Gamma_0\slashed{H}_3, \Gamma_1\slashed{H}_3] - \frac{1}{32}[\slashed{H}_0, \Gamma_1\slashed{H}_3] - \frac{1}{32}[\Gamma_0\slashed{H}_3, \slashed{H}_1] \\
&= -\frac{1}{32}(N^2 + M^2)(\nabla\alpha^2)\gamma_{01}.
\end{aligned} \tag{37}
$$

Finally, the curvature of the AdS_2 subspace is given by

$$R_1 = -\frac{1}{8}(N^2 + M^2)(\nabla\alpha)^2, \tag{38}$$

where expressions (36) and (37) were used and then, there is the maximal symmetry on AdS_2 with symmetry $SO(1,1)$. A similar analysis reveals, that the scalar curvature for the S^2 subspace with symmetry $SO(2)$, is exactly the same that R_1 but with opposite sign, i.e. $R_2 = -R_1$. The total curvature of the 4d space formed by the product $AdS_2 \times S^2$ vanishes. Hence, the result is a unique solution called the near-horizon geometry.

There is a remarkable fact in the analysis carried in sections 2 and 3. In the former, the relation among fluxes was established by hand in order to reproduce the near-horizon geometry. While, in the later, this relation is established by requiring $\mathcal{N} = 2$ supergravity in 4d, which, in consequence lead us to decoupling both spinors, ϵ^1 and ϵ^2. Besides, we have found an alternative way for constructing a 4d space-time with an $AdS_2 \times S^2$ symmetry by turning on 3-form fluxes, including NS-NS, rather than the inclusion of only 5-form fluxes.

Before we continue, there are some remarks which summarize the results so far obtained:

(a) In the fluxless case, i.e. for $M = N = 0$ the result is a Minkowski space-time.

(b) Accordingly with (38), RR fluxes seem not to play a role in the curvature, however they can not vanish because in their absence, the contribution to $\widetilde{R}_4$ from Einstein equations by $\mathcal{T}$ would not be zero.

(c) There exists a metric, $\tilde{g}_{\mu\nu}$, which can be induced from R_1 and R_2. The form of the metric is

$$ds_4^2 = -\frac{x_1^2}{h}dx_0^2 + \frac{h}{x_1^2}dx_1^2 + hdx_2^2 + h\,sin^2 x_2 dx_3^2\,, \tag{39}$$

with $h = 2/|\tilde{R}_1|$. In addition to reproduce the curvature of the space-time in $4d$, this metric must satisfy the $4d$ effective theory. If the presence of moduli scalar fields, as a result of the compactification procedure is ignored, the effective theory contains gravity and fields with two antisymmetric indices induced by the presence of RR and NS-NS fields, which are effectively interpreted as electromagnetic tensor fields with no sources. The NS-NS and RR fluxes have both a smeared leg on the internal manifold. Hence, in the presence of an homogeneous electromagnetic field of the form

$$F_{t\rho} = 2|\widetilde{R}_1| = \frac{1}{4}(N^2 + M^2)(\nabla\alpha)^2, \tag{40}$$

there exists a unique solution for the Einstein-Maxwell equations known as the Robinson-Bertotti solution, which is the near-horizon metric of an extremal black hole. Another noticeable point is that coordinates of the F_3 and H_3 fluxes, are related to the homogeneous electromagnetic field.

(d) The curvatures of the subspaces are proportional to the flux numbers $(N^2 + M^2)$. If the numbers are big, the spaces will be highly curved and its horizon area will be small.

(e) The integrability conditions for the second spinor ϵ^2 yield the same result already discussed. The reason lies in the contorsion term

$$\kappa_i = \frac{1}{4}\slashed{H}_i - \frac{1}{8}\Gamma_i\slashed{H}. \tag{41}$$

Then, the curvatures of both subspaces remain unchanged with respect to those computed by the equations of ϵ^1.

(f) From the cases reviewed at the end of section 2, another possibility is to consider a RR flux configuration in which $Q = 0$. This choice will yield the near-horizon metric as a solution in 4d space-time too.

4. Conclusions

We present a simple way to obtain the Robinson-Bertotti solution by compactifying a type IIB supergravity theory in the presence of only NS-NS, H_3 and RR, F_3 3-form fluxes. We assume a 10d space-time conformed by the product of two maximally symmetric 2d spaces and a 6d

space with $SU(3)$-structure, i.e. $AdS_2 \times S^2 \times X^6$. By requiring a 4d Ricci flat space, we find the conditions that flux configuration must fulfill in order to satisfy this symmetry. To achieve the goal we use Einstein equations and dual Bianchi identities. As a result of this analysis we find that one viable solution involves a constant warping factor and the condition $M^2 + Q^2 = P^2 + N^2$ for the flux numbers.

On the other hand, we use the integrability conditions on the 2d components of the chiral 10d spinors ϵ^1 and ϵ^2, with the purpose of finding the expressions for the 2d subspaces curvatures. An interesting feature extracted from this approach is the fact that, by preserving $\mathcal{N} = 2$ in 4d (which implies the decoupling of spinors in Eq.(27)), the conditions that fluxes must fulfill that renders a Ricci-flat space-time arises naturally, provided by supersymmetry.

Another issue is the possibility to obtain positive and negative curvatures since there are many other different flux configurations that can be considered. It is important to notice that the possibility to achieve different values for the curvature increases by not requesting a maximally symmetric 4d space-time.

As an alternative to the well known case of five forms associated to D3-branes and considered in literature so far, we show a way to construct a 4d space-time with an $AdS_2 \times S^2$ symmetry by turning on 3-form fluxes including NS-NS fluxes.

References
[1] V. Balasubramanian and F. Larsen, Nucl. Phys. B **528**, 229 (1998) [arXiv:hep-th/9802198].
[2] O. Loaiza-Brito and L. Vazquez-Mercado, Phys. Rev. D **84**, 066010 (2011) [arXiv:1012.2338 [hep-th]].
[3] H. Ooguri, C. Vafa and E. P. Verlinde, Lett. Math. Phys. **74**, 311 (2005) [hep-th/0502211].
[4] J. M. Maldacena and C. Nunez, Int. J. Mod. Phys. A **16**, 822 (2001) [hep-th/0007018].
[5] B. de Wit, D. J. Smit and N. D. Hari Dass, Nucl. Phys. B **283**, 165 (1987).
[6] T. Ortin, Cambridge Unversity, Cambridge University Press, 2004
[7] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [hep-th/0105097].