VIRTUAL NEUTRINO EFFECTS

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Let us consider the interaction Lagrangian density responsible for the lepton weak interactions in the conventional Fermi current-current form:

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} j_\rho^+(x)\, j^\rho(x) \tag{1}$$

with

$$j^\rho(x) = :\bar{e}(x)\gamma^\rho(1+\gamma_5)\nu_e(x): + :\bar{\mu}(x)\gamma^\rho(1+\gamma_5)\nu_\mu(x):$$

This Lagrangian implies the existence of forces between leptons due to the exchange of neutrinos (see Fig.1a) where ℓ stands for lepton).

Since the Lagrangian in the form (1) is non-renormalizable, it must be modified in an as yet unknown way near $x^2 = 0$ One such modification, as in the intermediate vector boson theory, consists in starting, instead of from (1), from the Lagrangian:

$$\mathcal{L}_W = g(j_\rho^+(x)\, W^\rho(x) + j^\rho(x)\, W_\rho^+(x)) , \tag{1'}$$

which in turn might be made renormalizable [1], and the corresponding neutrino exchange is represented in Fig.1b. These modifications near $x^2 = 0$, and hopefully renormalizations, will only modify the neutrino forces at short distance (high momenta) while their behaviour at large distance (low momenta) will be uniquely determined by the experimentally well-established behaviour of the Lagrangian (1) or (1') at large x^2 .

Their behaviour has been studied [2] and is represented by the behaviour (for $x^2 \neq 0$) of the vacuum expectation value (corresponding to the neutrino loop of Fig.1a):

$$\Pi_{\lambda\rho}(x) = \langle 0|T j_\lambda^{(\nu)}(x)\, j_\rho^{(\nu)}(0)|0\rangle \tag{2}$$

where

$$j_\lambda^{(\nu)} = :\bar{\nu}(x)\gamma_\lambda(1+\gamma_5)\nu(x):$$

At large distance, $\Pi_{\lambda\rho}(x)$ decreases with x as x^{-6} . The corresponding static potential in the three-dimensional space is:

$$V(r) = -\frac{G_F^2}{4\pi^3} r^{-5} , \tag{3}$$

where $r = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}$ and the corresponding r^{-6} - dependent force is repulsive between two leptons yet attractive between lepton and antilepton.

Due to the x^{-6} dependence, the neutrino forces may become strong at short distances. The problem is now up to which (short) distance does the well-established x^{-6} behaviour remain valid.

In the intermediate boson theory one would say up to the Compton wave-length of the W mass. In the current-current theory (1), since the only inbuilt length is $G_F^{\frac{1}{2}}$,it would be reasonable to take this as the lower limit for the validity of the large-distance behaviour (this corresponds to the unitary cut-off in momentum space).

Let us then examine which are the consequences of the following working hypothesis:

Hypothesis: The modification of the neutrino forces due to the modification of the Lagrangian (1) (and hopefully of its renormalization) happens, for leptons, at distances of the order of $G_F^{\frac{1}{2}}$:

$$\begin{array}{ll} \Pi_{\lambda\rho}(x) \text{ is given by (2)} & \text{for } x^2 \gtrsim G_F \\ \Pi_{\lambda\rho}(x) \text{ is not more singular than } x^{-2} & \text{for } x^2 \lesssim G_F . \end{array} \tag{4}$$

This hypothesis does not conflict with the knowledge we have at present of the lepton weak interactions and is particularly harmless for the neutrino forces.

We have in fact that if we admit the generally accepted hypothesis that neutrinos are both massless and chargeless, the function (2) obeys the equation:

$$\partial^\lambda \Pi_{\lambda\rho}(x) = 0$$

and, as a consequence, its Fourier transform can be put in the form

$$\Pi_{\lambda\rho}(k) = (k_\lambda k_\rho - k^2 g_{\lambda\rho})\Pi(k^2) \tag{5}$$

and $\Pi(k^2)$ is represented by only a logarithmically divergent integral instead of quadratically, as one would deduce from power counting. (And this is true even at higher orders in G_F, that is, if we take the neutrino loop in higher orders, inserting into it an arbitrary number of lepton and neutrino loops. (see B.F.II)[2].)

The consequence of the logarithmic divergence of $\Pi(k^2)$ is that the unitary cut-off ($\Lambda \sim G_F^{-\frac{1}{2}} \approx 300$ GeV) or regularization of the neutrino loop is harmless for the higher-order weak lepton processes. It is plausible that (see for example A.T. Filippov's [3] report at the present meeting) the unitary cut-off is harmless for all lepton weak processes. Now precisely the unitary cut-off or regularization corresponds, in the x space, to the assumed hypothesis(4) that the neutrino forces have the behaviour (2) and (3) up to distances $x^2 \sim G_F$ and that the Lagrangian (1) has to be modified only for $x^2 \lesssim G_F$

Obviously, the most interesting consequence of these neutrino forces would be the formation of lepton-antilepton resonances or composite states.

The best instrument we have to examine this possibility is the Bethe-Salpeter equation, which, starting from the Lagrangian density (1), has the general form:

$$(i\hat{\vec{\partial}}_1 - m_1)\chi_P(x_1x_2)(i\hat{\overleftarrow{\partial}}_2 + m_2) = -\frac{G_F^2}{2}\gamma^\rho(1+\gamma_5)$$
$$\langle 0|T\, j_\rho^{(\nu)}(x_1)\ell_1(x_1)\bar{\ell}_2(x_2)j_\lambda^{(\nu)}(x_2)|P\rangle\ \gamma^\lambda(1+\gamma_5) \qquad (6)$$

where the 4×4 function $\chi_P(x_1x_2) = \langle 0|T\ell(x_1)\bar{\ell}(x_2)|P\rangle$ represents the wave function of the possible bound state of total momentum P built up, due to neutrino forces, by the leptons $\ell_1(x_1)$ $\ell_2(x_2)$. The left-hand side member of (6) is usually called the vertex function Γ :

$$\Gamma_P(x_1x_2) = (i\hat{\vec{\partial}}_1 - m)\ \chi_P(x_1x_2)(i\hat{\overleftarrow{\partial}}_1 + m_2)\ , \qquad (7)$$

and, in the ladder approximation, is represented in Fig.2. Because of the projectors $(1+\gamma_5)$ in (6) the vertex (7) can have only the form

$$\Gamma(x_1 x_2) = \gamma^{\rho}(1+\gamma_5)\Gamma_{\rho}(x_1 x_2) \qquad (8)$$

and the Bethe-Salpeter equation in the ladder approximation assumes a particularly simple form both for the vertex Γ and for the bound state wave function χ_P and the sectors for their tensor components are decoupled (see B.F. II).[2] It is well known that the Bethe-Salpeter equation acquires a particularly simple and symmetric form for massless composite states $(P^2 = 0)$.

In our case the equation for the Γ_ρ appearing in (8) becomes, in momentum space and reference system $P_\mu = 0$, (see B.F.II Eq.(39')):[2]

$$\Gamma_\rho(p) = -4G_F^2 \int d^4q \, \frac{\Pi[(p-q)^2]}{(1-q^2)^2} \, (p-q)^2 \, [q^2 g_\lambda^\tau - 2q_\lambda q^\tau] \, \Gamma_\tau(q) \quad , \qquad (9)$$

where p is the relative momentum of the constituents.

In a general reference system the vertex appearing in (9) will be both a function of the total momentum P and of the relative momentum p. It is easy to show that the solution of the equation for $\Gamma_\rho(P,p)$ is invariant against the gauge transformation

$$\Gamma_\rho(P,p) \rightarrow \Gamma_\rho(P,p) + P_\rho \Lambda(P,p) \qquad (10)$$

with $\Lambda(P,p)$ an arbitrary well-behaved function of P and p. This means that one can impose on the vertex $\Gamma_\rho(P,p)$ the condition

$$P^\rho \Gamma_\rho(P,p) = 0 \qquad (11)$$

and, consequently, further decompose it in the form

$$\Gamma_\rho^{(r)}(p) = \varepsilon_\lambda^{(r)}[g_\rho^\lambda \, S(p^2) + (4p^\lambda p_\rho - p^2 g_\rho^\lambda) D(p^2)] \qquad (12)$$

where $\varepsilon^{(r)}$ is a unit polarization vector and the equations for S and D can be reduced to one-dimensional integral equations (B.F.II).[2]

In order to solve Eq.(9) or the corresponding equations for S and D one has now to regularize the logarithmically divergent kernel $\Pi(k)$ representing the neutrino loop. This can be done in many, to some extent equivalent, ways. If one uses the non-polynomial technique in which the regularization depends on a regularizing coupling constant f (of dimensions M^{-2}) one finds that, for different approximations of the kernel and of the integral equations, the condition for Eq.(9) to admit a massless solution is *) (see B.F.I)[2]

$$f \simeq G_F \tag{13}$$

This condition in ordinary space implies that the behaviour of the neutrino forces will deviate from the long-distance behaviour (they will become less singular for $x^2 \to 0$) at distances of the order of

$$x^2 \approx f \approx G_F \tag{14}$$

that is Eq.(13) corresponds to the hypothesis (4).

From (11) and (12) we see further that the lepton-antilepton composite states may only have spin-1. Their coupling to the free leptons can also be computed, properly normalizing the wave function $\chi_P(x_1 x_2)$, and it turns out that in our approximations it is either strong or medium strong (see B.F.I).[1]

We have then that from the Lagrangian (1) the two following possible consequences can be drawn:

a) Hypothesis (4) is valid and as a consequence spin-one lepton composite states exist. Since they are presumably not weakly coupled to the free leptons it should be easy to verify their existence experimentally.

b) Neither the composite states nor resonances exist. But then

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*) The equations for S and D can easily be reduced to the Goldstein or Thirring[4] type of integral equations.

the hypothesis (4) must be discarded and the neutrino forces must deviate from their x^{-6} dependence at a distance much larger than $G_F^{\frac{1}{2}}$. The condition of non-existence of resonances or bound states could establish then a lower distance of validity of the Fermi current-current Lagrangian (1) which should be larger than $G_F^{\frac{1}{2}}$. In the intermediate vector boson theory it would establish an upper bound on the W mass.

Let us keep hypothesis (4) from which (a) follows, and let us draw some further consequences from it.

It is known [5] that a composite state can always be represented by a field, and in this frame the amplitude (8) could be considered to be derived (in the limit of long wavelength: $\lambda \gg G_F^{\frac{1}{2}}$) from the effective Lagrangian:

$$\mathcal{L}^{eff} = e\,\bar{\psi}(x)\gamma^{\rho}(1 + \gamma_5)\psi(x)A_{\rho}(x) \qquad (15)$$

where x stands for the c.m. co-ordinate of the composite system and $A_{\rho}(x)$ is defined, in momentum space, by

$$e\,A_{\rho}(P) = \Gamma_{\rho}(P, p_1^2 = m^2\,,\ p_2^2 = m^2) \qquad (16)$$

Because of (10) and (11) we may impose on A_{ρ} the condition

$$\partial^{\rho}A_{\rho}(x) = 0 \qquad (17)$$

The Lagrangian (15) implies the existence of a spin-1 field quasilocally (in the limit $\lambda \gg G_F^{\frac{1}{2}}$) coupled to the electron and muon fields.

We shall suppose that the bare (with respect to the weak interactions) electron e_0 and muon μ_0 have the same mass. As a consequence, because of weak universality, Γ_{ρ} will obey the same equation, (9), both for electron and muon, and e in (16) will be the same for these two leptons. (15) will be explicitly:

$$\mathcal{L}^{eff} = e[\bar{e}_0(x)\gamma^\rho(1+\gamma_5)e_0(x) + \bar{\mu}_0(x)\gamma^\rho(1+\gamma_5)\mu_0(x)]A_\rho(x) \quad . \qquad (15')$$

Approximate solutions of Eq.(9) (see B.F.I)[2] give for e values between 0.1 and 1 . . If these values are confirmed by further calculations, the composite states represented by A_ρ should be easily detectable.

It is known that the only spin-one boson medium-strongly coupled to the massive leptons is the photon.

One would then be naturally brought to identify the spin-one massless lepton composite state represented by the field A_ρ as the photon. But the difficulty in this interpretation is that from the weak Lagrangian the only permissible vertex of both electron and muon with the composite state is the parity non-conserving (15') while notoriously electromagnetic interactions conserve parity. One must enquire if there might be a mechanism by which the effective Lagrangian (15') deduced from the weak lepton Lagrangian might give rise to the space (and charge) reflection invariant form of the quantum electrodynamical Lagrangian.

The problem here is similar to that encountered in the recent successful attempts [1] to unify electrodynamics and weak interactions; there also, one has to find a mechanism to get rid of the γ_5 term in the neutral weak currents. One can then borrow from those models the mechanism for the restoration of parity (space and charge) conservation in going from weak to electromagnetic interactions.

One such possibility would be to take the Salam-Weinberg model [1] and choose the coupling constants g and g' in the intermediate boson Lagrangian in such a way as to make the diagonal weak Lagrangian parity-conserving for the massive leptons. In this way one would obviously obtain parity-conserving vertex for the free lepton-composite state amplitude. But the mathematical simplicity brought to our model by the projectors $(1 + \gamma_5)$

would be spoiled. Besides, one needs the introduction of more neutral vector bosons.

Another possibility is offered by the recent work of Georgi and Glashow [1] in which no new neutral vector bosons are introduced besides the photon, which would be for us the lepton composite state. New unobserved massive leptons are introduced instead, building up two triplets of electronic states (and further two for the muon). In our case we would have that massless neutrino exchange generates composite states between these leptons; the charged and massive ones (we would bind a charged lepton and a neutral antilepton with different masses) would be the charged W meson and the only neutral massless one the photon. The O(3)-invariant coupling of these vector bosons to the fermions would give both the known weak and the parity-conserving electromagnetic coupling of the composite lepton-antilepton state with the leptons (the charged ones) plus weak and electromagnetic interactions of the unobserved leptons, as in the Glashow-Georgi model. Since the theory is renormalizable, we should obtain a well determined regularization of the kernel $\Pi(k^2)$. But still free parameters like the lepton masses would be left in the theory.

Yet we prefer to think that if hypothesis (4) and its consequence (a) are correct there should be a more economical way to restore parity (P and C) conservation. A possible way could be the following. Take the Konopinsky weak Lagrangian in the two equivalent forms:

$$\mathcal{L}_K^{(1)} = \frac{G_F}{\sqrt{2}} \bar{\mu}^c \gamma^\rho (1 - \gamma_5) \nu \; \bar{e} \gamma_\rho (1 + \gamma_5) \nu + \text{h.c.} \tag{18}$$

$$\mathcal{L}_K^{(2)} = \frac{G_F}{\sqrt{2}} \; \bar{\nu} \gamma^\rho (1 + \gamma_5) \mu \; \bar{\nu} \gamma_\rho (1 - \gamma_5) e^c + \text{h.c.} \tag{18'}$$

Starting from these we could deduce for the vertex amplitude corresponding to (8):

$$\Gamma = \gamma^{\rho}(1 - \gamma_5)\Gamma_{\rho}^{(c)} \tag{8'}$$

both for the electron and for the muon, where the superscript (c) stands for charge conjugate (or, better, lepton conjugate since there is no charge yet).

As (15') was obtained from the amplitude (8) we may think that the amplitude (8') might be derived from the effective Lagrangian

$$\mathcal{L}_K^{\text{eff}} = e[\bar{e}_0^{(c)}(x)\gamma^{\rho}(1 - \gamma_5)e_0^{(c)}(x) + \bar{\mu}_0^{(c)}(x)\gamma^{\rho}(1 - \gamma_5)\mu^{(c)}(x)]A_{\rho}^{(c)}(x) \tag{19}$$

Taking the charge conjugate of this we have

$$C\,\mathcal{L}_K^{\text{eff}}\,C^{-1} = e[\bar{e}_0(x)\gamma^{\rho}(1 - \gamma_5)e_0(x) + \bar{\mu}_0(x)\gamma^{\rho}(1 - \gamma_5)\mu(x)]A_{\rho}(x) \tag{19'}$$

where the hypothesis

$$C\,A_{\rho}^{(c)}C^{-1} = A_{\rho} \tag{20}$$

was adopted.

To obtain the electromagnetic Lagrangian we need only "define" it as the half sum of (15') plus (19') to obtain:

$$\mathcal{L}_0 = e[\bar{e}_0(x)\gamma^{\rho}(e_0(x) + \bar{\mu}_0(x)\gamma^{\rho}\mu_0(x)]A_{\rho}(x) \tag{21}$$

(This "definition" could be justified by the necessity of summing over opposite directions in closed lepton loops (see Fig.2).) The universality of the lepton electric properties follows from the universality of their weak interactions if we admit that the bare electron and muon have equal masses and that the (weak) renormalization will not alter the vector current densities and we obtain

$$\mathcal{L}_{em} = e[e(x)\gamma^{\rho}e(x) + \bar{\mu}(x)\gamma^{\rho}\mu(x)]A_{\rho}(x) \tag{22}$$

Electric charge conservation follows from the gauge invariance (11) and it is different from lepton number conservation which follows from the gauge invariance of the weak Lagrangian (1).

Needless to say, in this model the lepton electrodynamics is finite, the unrenormalized electric charge is zero ($z_3 = 0$) and the standard local electrodynamics should be valid up to distances of the order $G_F^{\frac{1}{2}}$ (if strong interactions are not taken into account).

In this model the electric charge should be computed as a function of G_F after solution of Eq.(9) or the corresponding ones for S and D in (12). It is clear that these solutions will depend on the way the kernel $\Pi(k^2)$ is regularized. In the way we have obtained the electrodynamical Lagrangian (22), this regularization is not defined because the current-current weak Lagrangians (1) and (18) are not renormalizable. If we started instead from a renormalizable Lagrangian like (1') then the renormalization of the weak interactions would fix uniquely (apart from parameters like the W masses which could be fixed by conditions in the frame of weak interactions) the behaviour of the kernel $\Pi(k^2)$ at high values of k^2 and the value of the electric charge should be computable uniquely.

Should the main line of this scheme for the origin of electromagnetic properties of leptons be at least partially true, many more questions remain open and await an answer. One of the first is the explanation of the electromagnetic properties of hadrons. There, because of baryon number conservation, one cannot exchange neutrinos between baryons (or, rather, baryon quarks); one is then unable to think of the photon as a composite of hadron quarks, as it was of lepton quarks (here we use the term lepton quark for bare leptons).

Nevertheless, if one accepts the idea that weak interactions are responsible for binding lepton quarks into the photon, they should also be able to bind hadron quarks into hadrons.*) (In the frame of recent models, one has only to bring the W mass one order of magnitude higher than the

*) One such possibility was examined some time ago by Thirring and coworkers[4].

lower limits accepted at present.) The situation there is much more complicated by the fact that, roughly speaking, the baryon quarks differ much more from baryons than the lepton quarks do from leptons. Besides, one probably has not the $(1 + \gamma_5)$ projector but perhaps $(1 + \alpha\gamma_5)$ and then one will obtain $(\alpha^2 - 1)$ proportional pseudoscalar and tensor vertices besides the (8) vector and axial-vector one. One could nevertheless think that by some mechanism a massive vector composite system of baryon quarks is formed which has the same quantum numbers as the photon. Then, by weak interaction it will have the possibility of transition into the photon (the baryon constituents decay weakly in those of the lepton). This would then be a dynamical basis for the explanation of so-called vector dominance. Naturally, one could always suppose, at low momenta, that the lepton composite state, the photon is directly bound to the hadron and that the vertex obeys (10), from which electric charge conservation and universality follow. The intermediate of the massive vector boson would only be felt in the hadron electric form factor.

But many more problems regarding the renormalizability of this model and its implications for higher-order processes have yet to be investigated.

At this preliminary stage the model has the attractive features of allowing the derivation of electric from weak lepton universality and that for both electric and weak interactions the Fermi coupling constant represents the minimal distance at which the interactions are well represented by the local standard theory, while divergences disappear from electrodynamics and are left only in weak interactions.

But, apart from this and the possible self-consistency of the scheme, only nature, through experimental evidence, can tell us if all this is not only possible but also true.

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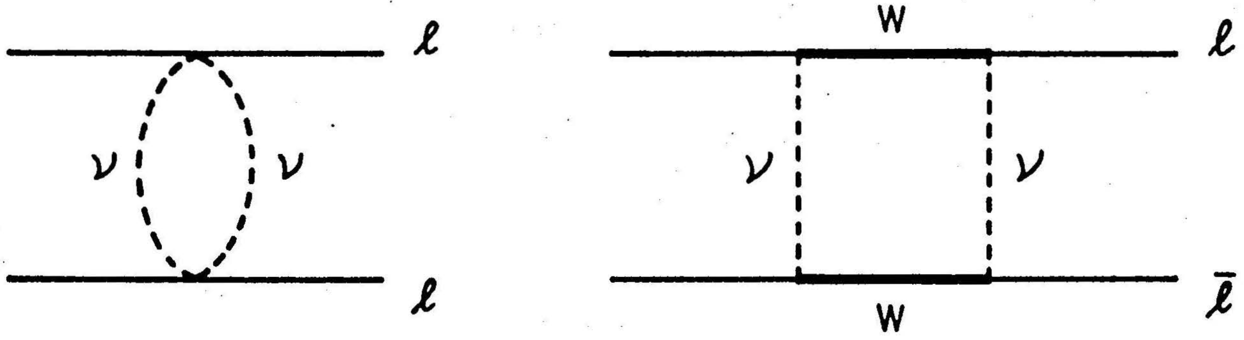

a) b)

Fig. 1

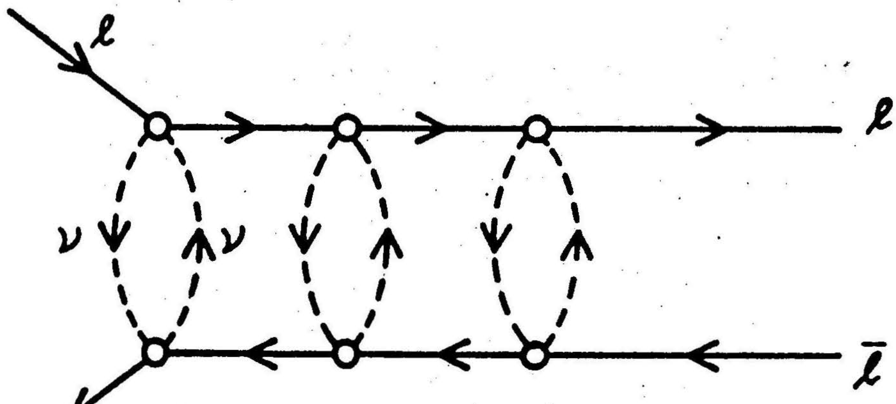

Fig. 2