The relation between ET Asymmetry, Jet Resolutions and Vertex Resolution

H. Schellman
Northwestern University

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1 Introduction

In this note I will look at the effects of energy and vertex resolutions on observables such as the inclusive jet E_T, jet asymmetries and di-jet masses. I will use a very simple LO model in which there are only two final state particles in order to see the qualitative behavior of the observables as a function of sources of error. Any real measurement would require Monte Carlo studies.

(July 12, 1998 - Freedy Nang pointed out an error in the equation for the angle in the EC. I checked that this was a typo in the note, not in the derivation and it has been corrected, the results are not changed. Thanks to Freedy for his very careful reading of the note and his corrections.)

2 Resolution for a single jet

2.1 Basic Assumptions

If a jet has energy E and polar angle θ then the jet E_T is:

$$E_T = E\sin\theta$$

If the error on E_T is just due to calorimeter energy resolution $r(E, \theta)$, then

$$\frac{\sigma(E_T)}{E_T} \approx \frac{\sigma(E)}{E} = r(E, \theta)$$

However, if the vertex position is mismeasured, the jet angle θ is also mismeasured, leading to an additional error in E_T. This additional error is correlated for all jets in an event and the degree of correlation depends on jet angles.

3 Jet quantities when the vertex is wrong

Assume that the interaction occurs at the origin. For CC showers, the true polar angle is

$$\tan\theta_T = \frac{R_{cal}}{z}$$

where z is the position of the shower in the calorimeter and R_{cal} is the radius at which showers occur. R_{cal} can be considered a constant for most showers.

For EC showers,

$$\tan\theta_T = \frac{r}{Z_{cal}}$$

where r is the position of the shower in the calorimeter and Z_{cal} is the z position in the calorimeter at which showers occur. If the vertex is displaced by an amound Δz due to a reconstruction error, the measured polar angle becomes:

$$\tan\theta_M = \frac{R_{cal}}{z - \Delta z}; CC$$

$$\tan\theta_M = \frac{r_{cal}}{Z_{cal} - \Delta z}; EC$$

Or:

$$\tan\theta_M = \frac{1}{Cot\theta_T - \frac{\Delta z}{R_{cal}}}; CC$$

$$\tan\theta_M = \frac{1}{Cot\theta_T(1-\frac{\Delta z}{Z_{cal}})}; EC$$

We can expand in Δz and find:

$$\sin\theta_M \approx \sin\theta_T \Big(1+\cos\theta_T\sin\theta_T\frac{\Delta z}{R_{cal}}+\frac{1}{2}\sin^2\theta_T(3\cos^2\theta_T-1)(\frac{\Delta z}{R_{cal}})^2+O(\Delta z^3); CC$$

$$\sin\theta_M \approx \sin\theta_T \Big(1+\cos^2\theta_T\frac{\Delta z}{Z_{cal}}+\frac{1}{2}\cos^2\theta_T(3\cos^2\theta_T-1)(\frac{\Delta z}{Z_{cal}})^2+O(\Delta z^3); EC$$

Note that Z_{Cal} has a sign and is negative for negative η. This shift in angle can be a large component of the resolution and, in addition is correlated between objects in an event in a way in which the individual energy errors are not. It should thus be taken into account separately.

Figure 1-4 show the shifts in E_T and η due to z shifts of between 0.05 and 0.25 of the distance to the calorimeter. Gian DiLoreto, using the methods described later in this memo, has measured typical z errors of 13 cm for forward jets, which for calorimeter radius of 91.6 and z position of 178.9 cm corresponds to fractional z shifts of 0.075 to 0.15 at 1 sigma with 5% of events having shifts larger than 0.15-0.30.

4 The Jet Asymmetry

We traditionally use the E_T asymmetry between jets in dijet events to estimate the individual jet resolutions. Although individual jets are not subject to correlations due to vertex shifts, the Asymmetry itself is. Fortunately, different Asymmetry measurements are sensitive to different mixtures of vertex and energy errors.

The Asymmetry is defined as:

$$A = \frac{E_{T1}-E_{T2}}{E_{T1}+E_{T2}}$$

where E_{T1} and E_{T2} are the measured E_T's of the two jets.

In the absence of vertex errors, the rms Asymmetry is just $r/\sqrt{2}$ where r is the fractional energy resolution.

In the more general case, for any given event:

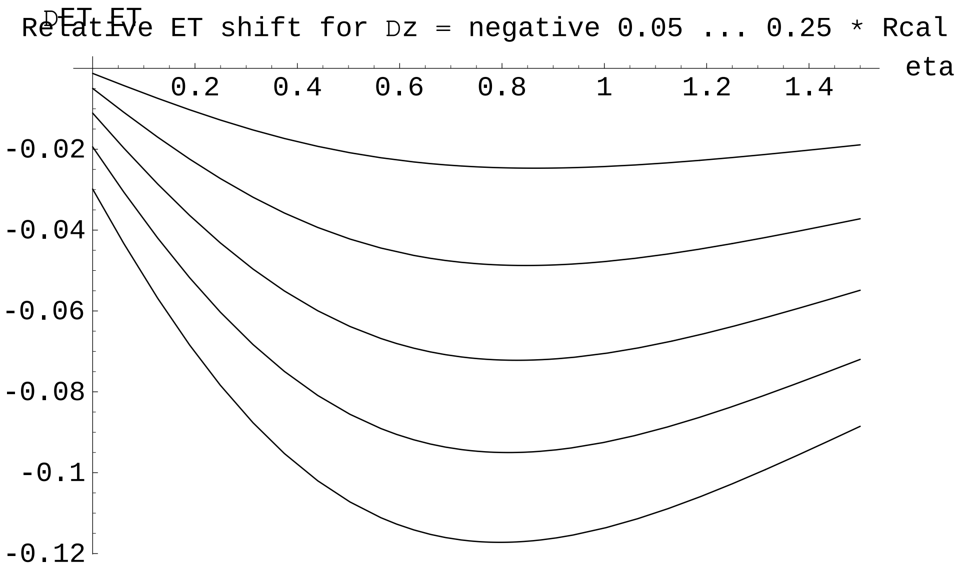

Figure 1: Fractional shift in E_T measured in the Central Calorimeter as a function of η for z shifts of negative 0.05 - 0.25 times R_{Cal}. A typical value of Δz would be around 0.15 R_{Cal} which corresponds to the middle curve. Negative shifts were chosen as η does not change sign for such shifts.

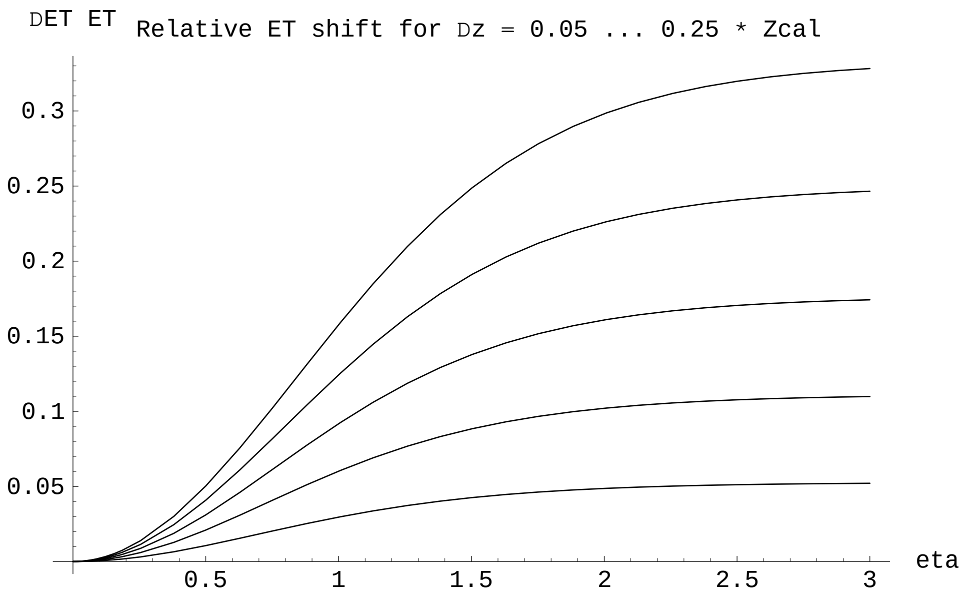

Figure 2: Fractional shift in E_T measured in the End Calorimeter as a function of η for z shifts of positive 0.05 - 0.25 times Z_{Cal}. A typical value would be around 0.10 Z_{Cal} which corresponds to the second curve from the bottom.

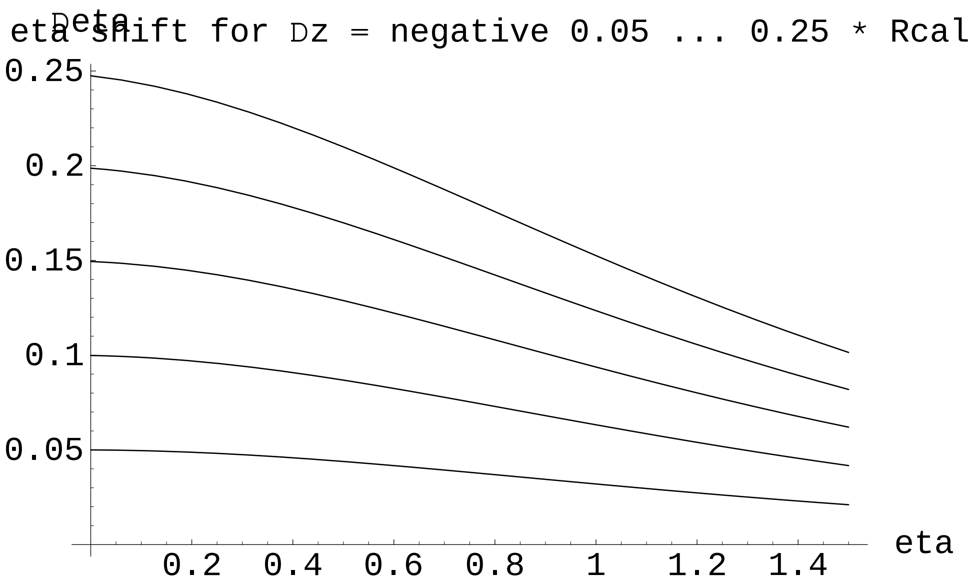

Figure 3: Shift in η measured in the Central Calorimeter as a function of η for z shifts of negative 0.05 - 0.25 times R_{Cal}. A typical value would be around 0.15, the middle curve. Negative shifts were chosen as η does not change sign for such shifts.

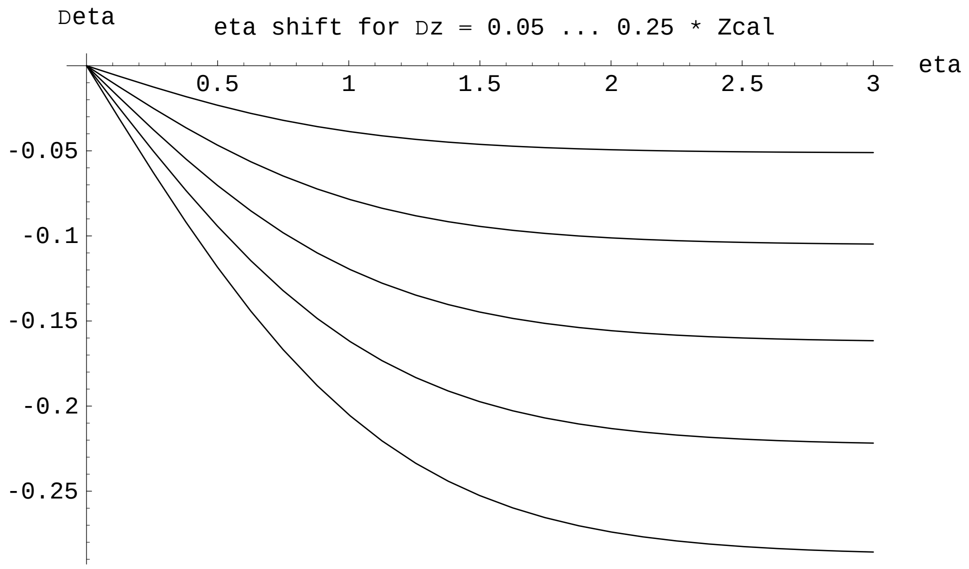

Figure 4: Shift in η measured in the End Calorimeter as a function of η for z shifts of positive 0.05 - 0.25 times Z_{Cal}. A typical value would be around 0.10, the second curve from the top.

$$A = \frac{E(1+r_1)\sin M_1 - E(1+r_2)\sin M_2}{E(1+r_1)\sin M_1 + E(1+r_2)\sin M_2}$$

where r_1 and r_2 are the independent energy smearings of the two jets and M_1 and M_2 are the measured angles. In the special case where the two jets are on the same side of the event and have equal angles, vertex shifts change both angles equally, $M_1 = M_2$, and the asymmetry reduces to:

$$A_S S = \frac{r_1 - r_2}{(1+r_1)+(1+r_2)}$$

and the average Asymmetry squared is:

$$< A_{SS}^2 >= \frac{r^2}{2}$$

In the case where the two angles are equal and opposite, the vertex smearing shifts one angle upwards and one downwards. After much algebra, one finds that:

$$< A_{OS}^2 >= \frac{1}{2}r^2 + \frac{1}{4}(\frac{\sigma_z}{R_{cal}})^2 \sin^2 2\theta$$

for the CC case and

$$< A_{OS}^2 >= \frac{1}{2}r^2 + (\frac{\sigma_z}{Z_{cal}})^2 \cos^4\theta$$

for the EC case. I have dropped terms of order σ_z^4.

Here σ_z is the rms error on the z vertex position. One can thus estimate the vertex contribution to the resolution at a given jet angle θ by taking the difference between the opposite sign and same sign Asymmetries at that angle. This vertex contribution should be independent of jet energy to the extent that the error on the vertex is not energy dependent. One can use this technique to see if vertex errors are important for any given kinematic region.

Since the first version of the note came out, Gian DiLoreto has made this measurement and finds that for jets in the range $1.5 < \|\eta\| < 2.0$, σ_z is around 13 cm.

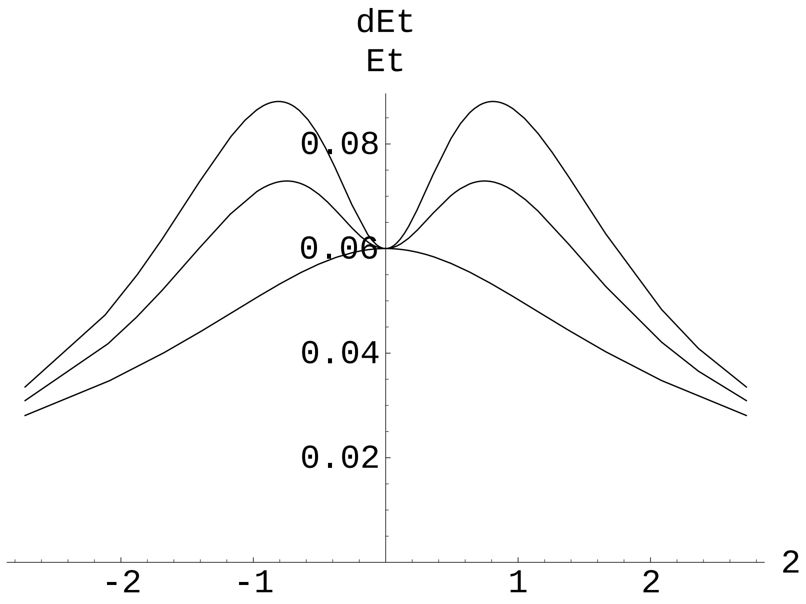

Figure 5: The energy resolutions measured from square roots of the asymmetries vs rapidity η for central jets at $E_T = 100$. The vertex error is assumed to be around 10 cm and the energy resolution error to be 6%. The lower curve is $\sqrt{2A_{SS}^2}$, the top curve is $\sqrt{2A_{OS}^2}$ and the middle curve is $\sqrt{A_{SS}^2 + A_{OS}^2}$. The bottom curve is the resolution if there is no vertex error. As shown in the next section, the middle curve would be the best estimate for the fractional error on the resolution of a single jet.

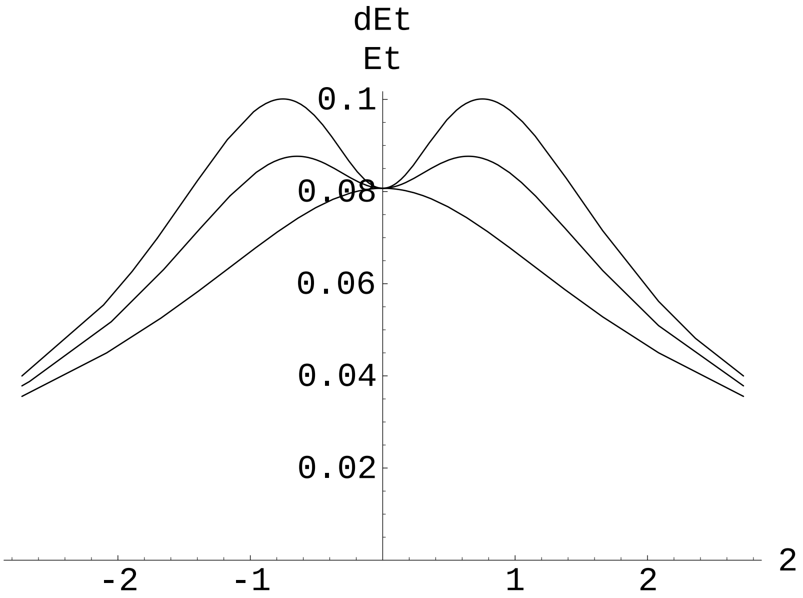

Figure 6: The square roots of the asymmetries vs rapidity η for end-cap jets at $E_T = 50$. The vertex error is assumed to be around 10 cm and the energy resolution error to be 8%. The lower curve is $\sqrt{2A_{SS}^2}$, the resolution if there is no vertex error, the top curve is $\sqrt{2A_{OS}^2}$ and the middle curve is $\sqrt{A_{OS}^2 + A_{SS}^2}$, the best measure of the E_T resolution.

5 The resolution for a single jet in terms of Asymmetry results

The measured E_T of a single jet is:

$$E_{TM} = E_M \sin\theta_M$$

where $E_M = E_T(1+r)$ is the measured energy and θ_M is the measured angle. If one differentiates with respect to the vertex error and resolution error r, one obtains

$$(\frac{\sigma(E_T)}{E_T})^2 \approx r^2 + \frac{1}{4}(\frac{\sigma_z}{R_{cal}})^2 \sin^2 2\theta; CC$$

$$(\frac{\sigma(E_T)}{E_T})^2 \approx r^2 + (\frac{\sigma_z}{R_{cal}})^2 \cos^4\theta; EC$$

One could either use the OS and SS asymmetries to extract the energy and vertex contributions separately or note that the terms on the right are equal to the sum of the squares of the OS and SS asymmetries. A simple asymmetry measurement in which equal numbers of OS and SS events are used at each angle and energy point will yield the full error for a single jet. If the weighting of OS and SS events in the asymmetry sample is far from 1 to 1 the vertex contribution will either be over or underestimated. For small rapidities the vertex contribution is also suppressed by the $\sin 2\theta$ factor.

A reiteration: This argument implies that one should not use a mix of OS and SS events similar to ones data sample to get a resolution measurement for a given sample. For E_T measurements, one should use equally weighted OS and SS samples. One can see this by considering that experimental shifts in the E_T of a jet due to either sampling or vertex position should not depend on where the other jet in the event is on the same or opposite side of the detector. The Dijet Mass in the next section provides a useful and extreme counterexample to the common error of assuming that one should measure the resolutions from the same sample as the data.

6 The Dijet Mass

The Dijet mass is subject to vertex error correlations as well.

The Dijet Mass can be approximated by

$$M_{jj} \approx \sqrt{2E_1E_2(1+\cos(\theta_1+\theta_2))}$$

for event with jets back to back in azimuthal angle.

Here the vertex error contribution will be different than in the Asymmetry. For same side events, a shift in angle due to vertex errors will add constructively for both angles while in opposite side events the shifts will cancel.

For opposite side jets:

$$(\frac{\sigma_M}{M})^2 = \frac{1}{2}r^2$$

For same side jets:

$$(\frac{\sigma_M}{M})^2 = \frac{1}{2}r^2 + \frac{1}{4}(\frac{\sigma_z}{R_{cal}})^2 \sin^2 2\theta; CC$$

$$(\frac{\sigma_M}{M})^2 = \frac{1}{2}r^2 + (\frac{\sigma_z}{Z_{cal}})^2 \cos^4\theta; EC$$

Here, once again, I have left out terms of order σ_z^4.

Interestingly, the fractional resolutions for the mass of same-side jets are the same as the asymmetry errors for opposite side jets and vice versa. If a sample consists of equal numbers of OS and SS events, using the Asymmetry for that sample will yield the correct mass resolutions inclusing vertex effects. However, if the number of OS and SS events are significantly different, the asymmetries should be determined for the two samples separately and applied to the opposite mass sample.

7 Cross Sections

Cross section measurements are more complicated as one now has several independent sources of smearing, some of which are correlated and some of which are not. The best way to estimate biases due to vertex resolution is to take a Monte Carlo sample which closely approximates the kinematics of the data and, for each generated MC event, recalculate the kinematic variables for many different z positions. Comparisons of the MC sample with an unsmeared sample will allow an estimate of the effects of z smearing. The effects should be small, except where cross sections are changing rapidly in E_T or η.

8 Conclusions

An analytical study of the separate effects of energy and vertex resolution on observables indicates that vertex resolution effects could be important. However, measurements of the E_T asymmetries can often be combined, either to extract the different resolution components directly or to form a resolution measurement appropriate for the observable being measured.