Global Optimization of Bethe–Weizsäcker Coefficients of Semi-Empirical Mass Formula

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Introduction

In 1930, George Gamow formulated the Liquid Drop Model (LDM) [1] and on the basis of that Bethe-Weizsäcker [2] proposed the Semi-Empirical Mass Formula (SEMF) for the calculation of binding energies. Various terms present in this model are derived based on the assumption that nucleus can be considered as a drop of incompressible nuclear fluid with extreme density (≈10^{17} kg/m^3) liquid. This formula was the first framework which successfully described the nuclear binding energies, offering a macroscopic approach that captures essential features of nuclear stability and fission and fusion phenomena occurred in heavy and light nuclei. Although the model is successful in predicted the above said features, its accuracy depends heavily on the choice of model parameters associated with volume, surface, Coulomb, asymmetry, and pairing terms. Optimization of these parameters is crucial to improve the predictive power of the model. In the present work, we perform a systematic optimization of global re-fit of SEMF parameters by minimizing the chi-square (χ^2) deviation between experimental binding energies and the values calculated from the model. The experimental data have been taken from the most recent and comprehensive atomic mass evaluation of Wang *et al.* (2017) [3]. This study aims to refine the parameter set of the SEMF and evaluate its performance against experimental data.

Methodology

The SEMF is also known as Bethe-Weizsacker formula that determines the binding energy of a nucleus based on mass number(A) of nuclei and number of protons(Z). The equation of Semi-empirical formula [2] is written as:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_{as} \frac{(N-Z)^2}{A} + \delta(A)$$

where a_v, a_s, a_c, a_{as} are nuclear shape parameters which can be calculated by fitting of experimental available data of binding energies.

$$\delta(A) = \begin{cases} +a_p A^{-\frac{3}{4}}\ for\ even-even\ nuclei \\ -a_p A^{-\frac{3}{4}}\ for\ odd-odd\ nuclei \\ 0 \quad for\ odd-A\ nuclei \end{cases}$$

Here a_p is a pairing coefficient.

The SEMF is comprise of five terms, i.e., volume term, surface term, coulomb term, asymmetry term and pairing term, which represent different physical effects and contribute to the overall binding energy of the nuclei under discussion. The first three terms are classical in nature; the volume term represents the attractive contribution from strong nuclear force towards binding energy. Second term called a Surface term accounts for the reduced binding energy of nucleons due to the nucleon present on the surface of nucleus. This term is related with saturation properties of nuclear forces and hence according to it a nucleon can interact with only nearest neighbors. Third term is known as coulomb term represents electrostatic repulsion among protons within the nucleus that de-stabilize it and reduces binding energy. Fourth term known as asymmetric term responsible for the energy associates with unequal number of neutrons and protons. Last term i.e. pairing energy term accounts for increase in stability due to even number of protons and neutrons, decreases for odd-odd nuclei and zero for odd-A nuclei. To perform the global fit of these parameters, we write a Python script which minimized the RMS deviation among the calculated and experimental data of binding energies. Standard optimization subroutines from

Scipy library are used to ensure the reliability of the global fit.

Results and Discussions

In the present work, we perform a systematic optimization of global re-fit of SEMF parameters by minimizing the chi-square (χ^2) deviation between experimental binding energies and the values calculated from the model. Figure 1 compares the binding energy per nucleon (BE/A) as a function of mass number A, using masses from the recent evaluation by Wang *et al.* [3], with a five-term Bethe-Weizsäcker (BW) global fit. The curve reproduces the rise in BE/A to a maximum near the Fe-Ni region (A≈60) and the gradual decline toward rare-earth and actinides. Residual oscillations reflect odd-even staggering and unmodeled shell structure, most evident around magic regions (A≈20, 40, 82, 126, 208); scatter is largest for A≲20 where few-body and clustering effects occurred which are beyond the range of SEMF.

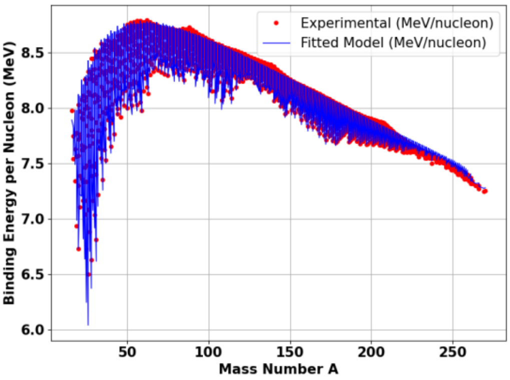

Fig. 1: Global fit of calculated and experimental binding energies.

The globally refitted coefficients differ systematically from commonly quoted values a_v=14.1, a_s=13, a_c=0.595, a_{as}=19, a_p=33.5 [4]. Our global fit yields a_v=15.36 (+8.93%), a_s=16.92 (+30.15%), a_c=0.69 (+15.96%), a_{as}=21.61 (+16.31%), a_p=26.21 (−21.7%). The larger volume and surface terms indicate stronger bulk binding which improves mid-mass and heavy-mass binding energies. Overall, the updated parameters provide an improved binding energies and reduced residual deviations. The kinks present in the curve highlight the need of explicit shell, Wigner, and deformation corrections term for precision mass modeling.

Conclusions

In the present work, we performed a global refit of Bethe–Weizsäcker Coefficients of SEMF and calculated binding energy per nucleon (BE/A) as a function of mass number A, using masses from the recent evaluation by Wang *et al.* [3]. The curve reproduces the rise in BE/A to a maximum near the Fe-Ni region and the gradual decline toward rare-earth and actinides. The globally refitted coefficients differ systematically from commonly quoted values a_v=15.36 (+8.93%), a_s=16.92 (+30.15%), a_c=0.69 (+15.96%), a_{as}=21.61 (+16.31%), a_p=26.21 (−21.7%). We believe that, the updated parameters provide an improved binding energies and reduced residual deviations.

Acknowledgement

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