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Bubble Nucleation from a de Sitter–Planck Background with Quantum Boltzmann Statistics

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Abstract: Every physical theory involving quantum fields requires a model of quantum vacuum. The vacuum associated to quantum gravity must incorporate the prescriptions from both the theory of relativity and quantum physics. In this work, starting from the hypothesis of nucleation of sub-Planckian bubbles from a de Sitter vacuum, we study the necessary conditions to obtain baby universes, black holes and particles. The de Sitter-Planck background is described by an "infinite" Quantum Boltzmann statistics that generates fermions and bosons, and manifests itself as a deformation of the geometry that leads to a generalized uncertainty principle, a unified expression for the generalized Compton wavelength and event horizon size, drawing a connection between quantum black holes and elementary particles, seen as a collective organization of the bubbles of the vacuum described by the generalized Compton wavelength. The quantum thermodynamics of black holes is then outlined and the physical history of each bubble is found to depend on the cosmological constant described in terms of thermodynamic pressure. A treatment of the Casimir effect is provided in the de Sitter-Planck background, and finally wormholes are explored as bubble coalescence processes.

Keywords: de Sitter–Planck background; sub-Planckian bubbles; quantum Boltzmann statistics; generalized uncertainty relations; quantum black holes; cosmological wormholes

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1. Introduction

Bubble theory has an impressive and complex history in theoretical physics that has evolved at the borders of particle physics and quantum cosmology while retaining some features of conceptual and stylistic autonomy linked to the centrality of QFT [1–3]. Recently, proposals for quantum simulation and analogous systems have also arrived alongside the theoretical analyses [4–6]. If we were to try a general definition, we could say that bubble theory concerns the conditions of nucleation of the vacuum and therefore concerns very closely what is the foundation of any theory of everything, at least as regards the top-down mesoscopic aspects. The urgency of a theory of quantum gravity has led today to new hybrid forms of bubble theory with microscopic entities and collective vacuum behaviors [7–9], and the importance of an analysis of bubbles of the vacuum near the Planck scale.

In a series of recent papers [10–12] Carr proposed a version of the Black Hole Uncertainty Principle correspondence—originally introduced by Adler in [13–16]—which imply that there could exist sub-Planckian black holes, namely black holes with mass beneath the Planck scale but a radius of the order of the Compton scale rather than the Schwarzschild radius. Carr's model suggests that a subtle and strict link exists between the microphysics of elementary particles and the macroscopic regime of black holes; in other words, all black holes are, in a sense, quantum and elementary particles and can be seen as sub-Planckian black holes. Moreover, Carr's original idea was developed by Spallucci and Smailagic [17], who introduced a general, physically compelling criterion in order to distinguish between a quantum particle and a quantum black hole in terms of the ratio of the Compton wavelength and the gravitational radius, in the sense that when their ratio is close to one we are in a genuine quantum gravity regime and the geometric, static Schwarzschild horizon does not provide a satisfactory description of a quantum black hole. These two authors introduced a Generalized Uncertainty Principle between the mass of the quantum black hole and its horizon which leads to an "effective" Schwarzschild like-geometry with quantum gravity correction to the Newtonian potential, that nonetheless implies that a quantum black hole can exist only above the Planck mass. These clues are interesting avenues for quantum gravity [18].

In this paper we aim to explore the link between a black hole regime and elementary particles in the context of a generalized uncertainty relation characterizing a de Sitter–Planck background with infinite statistics. In this picture, we will use the term multiverse not in a strictly cosmological sense, as stated in the theory of inflation, but in reference to the generative characteristics of the vacuum on various scales, and we will adopt de Sitter's background for its "virtuous" instability [19–23].

To make the characterization of particles more meaningful by inserting a little more realism, we will hypothesize that the vacuum has a mechanism such as the Boltzmann quantum Statistics to generate fermions and bosons. Finally, we will consider sub-Planckian bubbles as a model for studying a phylogeny of localizations from vacuum, using the third quantization formalism. The production of a nucleation from an "eternal" de Sitter background will be described through a hypersurface feature as "entry into time" [24,25].

The paper is organized as follows. In Section 2 we develop our model of the multiverse in the de Sitter–Planck background with infinite statistics. In Section 3 we analyze how the de Sitter–Planck background with infinite statistics is characterized by a deformation of the geometry at the Planck scale expressed by opportune generalized uncertainty relations. In Section 4 we explore the impact of the generalized uncertainty relations characterizing the de Sitter–Planck background towards a suggestive unifying treatment of the microphysics of elementary particles and the macrophysics of black holes. In Section 5 we analyze the thermodynamics of the black holes in this approach. In Section 6 we explore possible perspectives as regards Casimir energy and cosmological wormholes. Finally, in the conclusions we discuss the implications for particle physics, cosmology and the interpretation of QM.

2. The Multiverse in the de Sitter–Planck Background with Quantum Boltzmann Statistics

In our model of a de Sitter–Planck background, we interpret the appearance of subatomic particles in terms of processes of the creation and annihilation of quanta corresponding, respectively, to the manifestation and de-manifestation of sub quantum bubbles, semi-quantum objects whose dimensions and mass are in the Planck regime and, thus, we assume that there is no distinction between particles and correspondent universes. In this regard, the Wheeler–De Witt equation tells us that objects can be generated by a super-vacuum through a formalism of third quantization. By using the third quantization formalism provided in [26], in particular, we consider that each micro-universe can be described by considering the following version of the Wheeler–De Witt equation:

$$\ddot{\Psi} + \frac{\gamma}{a\sqrt{\alpha}} M_{Pl} \dot{\Psi} + \omega^2(a)\Psi = 0 \quad (1)$$

In this equation, Ψ is the wave function of the bubble (and is a function of the scale factor a and of a massless scalar field φ), $\dot{\Psi} = \frac{\partial \Psi}{\partial a}$. Moreover, the wave function Ψ can be expressed as:

$$\hat{\Psi} = \frac{\gamma M_{Pl} c^2}{\sqrt{2\pi\alpha}\hbar l_p{}^2} \int dM \left(e^{i\frac{\gamma M_{Pl} c^2}{\sqrt{\alpha}\hbar l_p{}^2} M\varphi} \Psi_M(a) \hat{b}_M + e^{-i\frac{\gamma M_{Pl} c^2}{\sqrt{\alpha}\hbar l_p{}^2} M\varphi} \Psi^*_M(a) \hat{b}^\dagger_M \right) \quad (2)$$

In Equation (2):

$$M_{dim} = \frac{\gamma M}{\sqrt{\alpha}} M_{Pl} \tag{3}$$

is the mass of the bubble of the background vacuum, $\gamma = \left(\frac{M\sqrt{G}}{e}\right)^2$ and each amplitude $\Psi_M(a)$ corresponds to a single bubble with a specific value of M. Moreover, $\hat{b}_M$ and $\hat{b}_M^\dagger$ are the annihilation and creation operators which annihilate and create, respectively, bubbles, namely particles of mass M, and satisfy the "quantum Boltzmann statistics" represented by the q deformation of the commutation relations of the oscillators, expressed by the following relation:

$$\hat{b}_k \hat{b}_l^\dagger - q \hat{b}_l^\dagger \hat{b}_k = \delta_{kl} \tag{4}$$

where the cases $q = \pm 1$ correspond to bosons and fermions. As we will see below, the infinite statistics characterizing the bubbles of the vacuum imply that each micro-universe can light up as a boson or fermion depending on the value of the deformation parameter q appearing in the commutation relations of the oscillators (4).

The commutation relations (4) are the essential expression of "Infinite statistics" [27]. A statistic of this type is necessary in order to give a sense of the entropy of a quantum foam and shows that the bubbles are non-local objects [28–33]. The term "infinite statistics" is strictly tied to the non-locality and derives from the fact that a statistic of this type can be thought as a statistic of identical particles with infinite degrees of freedom or a statistic of non-identical particles which are distinguishable by their inner state.

Here the correspondence between the bubbles of the vacuum obeying infinite statistics and opportune cells of this fundamental background—each of them can be in one of two states: "on" if a localization occurs in it, "off" otherwise—can be described as follows. On the basis of the Bekenstein estimation of an upper bound for the information associated with a system endowed with a total energy enclosed in a sphere in three-dimensional space, the production of a bubble can be associated with the creation of an information in a cell given by the following expression:

$$I = \frac{A}{4l^2} \tag{5}$$

where A is the area of the cell and $l \approx 10^{-33}$cm is the Planck length. According to Equation (5), a purely informational interpretation of the Planck scale emerges directly, which suggests interesting perspectives of unification between elementary particle physics (chronon scale) and cosmology (de Sitter background).

In particular, the regime of ordinary Standard Model particles can be obtained by considering the chronon scale $\frac{A^3}{l_p^2} \approx 10^{-13}$cm. As a consequence, by writing $A = c\theta_0$, where c is light speed and θ_0 is the minimum proper time interval between two successive localizations of the same particle, one obtains $(c\theta_0)^3 \approx l_p^2 \cdot 10^{-13}$cm. This means that the minimum proper time interval between two successive localizations of the same particle in the de Sitter–Planck non-local texture characterized by infinite statistics must satisfy the condition $\theta_0 \approx 10^{-25}$ s. The crucial aspect of the approach based on Equation (5) provides a typical particle range and this implies that each cell and each micro-universe can light up as a boson or a fermion depending on the value of the deformation parameter q.

Moreover, as regards the WDW Equation (1), the wave function Ψ_M of the micro-universe can be associated with a mode-depending frequency given by:

$$\omega_M = \frac{1}{\hbar}\sqrt{a^4\Lambda - a^2 + \frac{\gamma^2 {M_{Pl}}^2 c^4 M^2}{\alpha {l_p}^4 a^2}} \tag{6}$$

Λ being the cosmological constant. In particular, in a FRW regime, the behavior of the micro-universes is ruled by the effective Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\hbar^2\omega_M^2}{a^4} = \Lambda - \frac{1}{a^2} + \frac{\gamma^2 {M_{Pl}}^2 c^4 M^2}{\alpha {l_p}^4 a^6} \tag{7}$$

where the last term proportional to a^{-6} is associated with the interaction between the micro-universes. In order to determine the evolution of the micro-universes, we express the mode-depending frequencies (6) as:

$$\omega_M = \frac{\Lambda}{\hbar a}\sqrt{\left(a^2 - a_+^2\right)\left(a^2 - a_-^2\right)\left(a^2 + a_0^2\right)} \tag{8}$$

where $a_0 \leq a_- \leq a_+$,

$$a_+(M) = \frac{1}{\sqrt{3\Lambda}}\sqrt{1 + 2cos\left(\frac{\vartheta_M}{3}\right)} \tag{9}$$

$$a_-(M) = \frac{1}{\sqrt{3\Lambda}}\sqrt{1 - 2cos\left(\frac{\vartheta_M + \pi}{3}\right)} \tag{10}$$

$$a_0(M) = \frac{1}{\sqrt{3\Lambda}}\sqrt{-1 + 2cos\left(\frac{\vartheta_M - \pi}{3}\right)} \tag{11}$$

and

$$\vartheta_M = arccos\left(1 - 2\frac{M^2}{M_{max}^2}\right) = 2arcsin\left(\frac{M}{M_{max}}\right) \in [0, \pi] \tag{12}$$

The maximum value of the mass mode M is:

$$M_{max} = \frac{\pi}{\sqrt{3}}\frac{M_{Pl}^2}{\hbar^2 c \Lambda^2} \tag{13}$$

From Equations (7) and (8) the following re-reading of the evolution of each micro-universe can be provided: one has a re-collapsing baby universe for $a < a_-$ and an asymptotically de Sitter universe for $a > a_+$, while for $a_- < a < a_+$ the micro-universe is in the Euclidean, classically forbidden, region. Although the baby micro-universe and the corresponding asymptotically de Sitter universe are classically disconnected, they can be put in connection through a Euclidean wormhole, in which the scale factor grows from the value a_- to the maximum value a_+. The existence of this wormhole allows the transformation of a baby micro-universe into a new expanding universe, leading to inflation in a new region of the multiverse. By following [34,35], the probability of the tunnelling of a baby micro-universe can be computed as follows:

$$\wp_M(a \to a_+) \approx \begin{cases} exp\left(-\frac{M_{Pl}^2}{\hbar^2\Lambda^2}\right) \; for \; M = 0 \\ exp\left(-\frac{M_{Pl}^2}{\hbar^2\Lambda^2}\left[C_M M\left(\widetilde{k}^2\right) + C_E E\left(\widetilde{k}^2\right) + C_\Pi \Pi\left(\kappa^2 \middle| \widetilde{k}^2\right)\right]\right) \; for \; 0 < \frac{M}{M_{max}} < 1 \end{cases} \tag{14}$$

where $M\left(\widetilde{k}^2\right)$, $E\left(\widetilde{k}^2\right)$, $\Pi\left(\kappa^2 \middle| \widetilde{k}^2\right)$ are the complete elliptic integrals of the first, second and third kind and the linear coefficients C_M, C_E and C_Π are defined as:

$$C_M = 3\pi\Lambda^3\widetilde{k}^2\left[\frac{1}{3} + \frac{1}{\kappa^2} + \widetilde{k}^2\left(\frac{1}{3} - \frac{1}{\kappa^4}\right)\right] \tag{15}$$

$$C_E = -3\pi\Lambda^3\widetilde{k}^2\left[\frac{1}{3} + \widetilde{k}^2\left(\frac{1}{3} - \frac{1}{\kappa^2}\right)\right] \tag{16}$$

$$C_{\Pi} = 3\pi\Lambda^3\widetilde{k}^2\left(1-\frac{\widetilde{k}^2}{\kappa^2}\right)\left(1-\frac{1}{\kappa^2}\right) \tag{17}$$

and $\widetilde{k}^2 = \left(a_+^2 - a_-^2\right)/\left(a_+^2 + a_0^2\right)$ and $\kappa^2 = \left(a_+^2 - a_-^2\right)/a_+^2$. In the light of Equation (13), one can predict that if all the baby micro-universes are nucleated with the same probability, those with larger K are most likely to tunnel through the wormhole and therefore undergo an inflationary era similar to our own patch of the universe.

In the light of the frequencies (6) generating the behavior of the micro-universes, the annihilation and creation operators $\hat{b}_M$ and $\hat{b}_M^{\dagger}$, evaluated in the hypersurface Σ_0 where $a = a_0$ and $\varphi = \varphi_0$, may be expressed as:

$$\hat{b}_M^{\dagger} = \sqrt{\frac{\omega_{0k}}{2\hbar}}\left(\Psi - \frac{i}{\omega_{0k}}\hat{p}_{\Psi}\right);\ \hat{b}_M = \sqrt{\frac{\omega_{0k}}{2\hbar}}\left(\Psi + \frac{i}{\omega_{0k}}\hat{p}_{\Psi}\right) \tag{18}$$

where ω_{0k} is the value derived from (6) on the hypersurface Σ_0 and $\hat{p}_{\Psi}$ is the third quantized momentum conjugated to the wave function operator $\hat{\Psi}$. Equations (6) and (14) imply that, for $M = 0$ one obtains a Lorentzian micro-universe for values $a > \frac{1}{\sqrt{\Lambda}}$ and a Euclid micro-universe for values of the scale factor satisfying $0 < a < \frac{1}{\sqrt{\Lambda}}$. The transition hypersurface $\Sigma_{\frac{1}{\sqrt{\Lambda}}}$ corresponds to the appearance of time.

In this approach, the evolution of the multiverse may be described in terms of the creation and annihilation of bubbles, each of them identified by the mass mode M whose maximum value is (13). The state of the multiverse is linked to the vacuum in terms of the creation and annihilation operators satisfying the Boltzmann quantum statistics (4), in the same way that, in second quantization, a state of the field is connected to the vacuum. One can express this result through the following formalism:

$$|\Psi(a)\rangle = \prod{}_{\mathrm{M}} e^{\omega_M \hat{b}_M^{\dagger} - \omega_M^* \hat{b}_M}|0\rangle_M \tag{19}$$

On the basis of relation (19), one can describe, for example, the creation of pairs of universes or their annihilation, the merging of universes or the stemming of universes from parent universes and other processes quite similar to those known in elementary particle physics. In this regard, an interesting case, strictly related to the geometry of bubbles in a de Sitter background, is the creation of couples of black holes, examined by Bum-Hoon Lee and Wonwoo Lee in [36]. Moreover, it is interesting to make a parallelism with the approach of gravitational instantons, wormholes and baby universes in the context of the Euclidean path integral formalism [37] where the amplitude relating two spatial slices of a given universe, allowing for any number of wormholes to be inserted between initial and final time, is:

$$e^I = \int \frac{d\alpha}{\sqrt{2\pi}} exp\left(-\frac{1}{2}\alpha^2\right) exp\left(\alpha\sqrt{\Delta}\int d^4x\sqrt{g}\mathcal{O}(x)\right) = \langle 0|e^{(a+a^{\dagger})\sqrt{\Delta}\int d^4x\sqrt{g}\mathcal{O}(x)}|0\rangle \tag{20}$$

where $|0\rangle$ is the baby universe vacuum (state with no baby universe), $\alpha = a + a^{\dagger}$, where $a^{\dagger}$ and a are baby universe creation and annihilation operators satisfying the usual commutation relation $\left[a, a^{\dagger}\right] = 1$. By comparing Equation (20) of the Euclidean path integral formalism of [37] with Equation (19) regarding the creation of pairs of universes or their annihilation, the merging of universes or the stemming of universes from parent universes of our approach of de Sitter–Planck background with infinite statistics, we can say that our approach can be seen as a generalization of the Euclidean path integral formalism in the sense that the usual commutation relations regarding baby universe creation and annihilation operators of Euclidean path integral formalism are here substituted by the creation of annihilation operators of bubbles satisfying the more general infinite statistics inside a de Sitter–Planck background, while the quantity $\alpha\sqrt{\Delta}\int d^4x\sqrt{g}\mathcal{O}(x)$ can indeed be associated with the more fundamental frequencies (6) of the de Sitter–Planck background.

In summary, one can say that the vacuum can be seen as a matrix, a sea from which bubbles of various kinds can emerge. Universes would be bubbles with an inflative mechanism, while particles would be "stable" objects. Nonetheless, both micro-universes and elementary particles can be described by the wave function (2), namely at a fundamental level of the de Sitter–Planck background can receive a unifying description. In other words, we can say that, in this picture, the primary physical entity is the wave function (2)—satisfying the WDW Equation (1)—and both micro-universes, the macroscopic regime and elementary particles emerge from it as special structures with their specific features.

In the de Sitter–Planck background with quantum Boltzmann statistics, the vacuum is a generator of a multiverse where each universe is characterized by a cosmological constant which determines its evolutionary–inflationary aspects. Here, a crucial point is that in both micro-universes, a macroscopic regime of black holes and elementary particles can receive a unifying description based on the wave function (2), and this implies that particles and black holes can be defined by an equivalent of the cosmological constant. The action of the cosmological constant is such that, for the black hole regime, gravity has a stronger action than the statistics, while for elementary particles this implies the emergence of the statistics as well as of the spontaneous symmetry breaking.

In particular, as regards the elementary particles of the Standard Model, the introduction of a counterpart of the cosmological constant can be justified as follows. By considering, in the scheme of the electroweak standard model, a scalar potential of the form:

$$V(H) = -\frac{M_{dim}^4}{4\lambda} - M_{dim}^2 H^2 + \lambda\left(\frac{M_{dim}^2}{\lambda}\right)^{1/2} H^3 + \frac{\lambda}{4} H^4 \tag{21}$$

where H is the neutral Higgs boson, λ is the coupling to a ϕ^4 interaction and $M_{dim} = \frac{\gamma M}{\sqrt{\alpha}} M_{Pl}$ is the mass of the bubble, one finds that the contribution of the vacuum energy density plays the role of a cosmological constant for elementary particles:

$$\Lambda = -\frac{2\pi G}{c^4}\frac{M_{dim}^4}{\lambda} \tag{22}$$

Taking account of the link with Fermi constant one finds that:

$$\Lambda \approx -1.3 \times 10^{-33} M_{dim}^2 \tag{23}$$

which turns out to be 10^{52} times than the astronomical value, meaning that spontaneous symmetry breaking requires a large cosmological constant, and thus a large vacuum energy, for elementary particles in the Standard Higgs Model.

A crucial aspect lies just in the perspectives that our approach introduces as regards the interpretation of the elementary particles of the Standard Model. The physical meaning of Equation (2) is that the mass of the bubble $M_{dim} = \frac{\gamma M}{\sqrt{\alpha}} M_{Pl}$ practically determines the skeleton, namely the "bare" state of the elementary particle mass of the observable world. On the basis of Equation (2), the appearance of a particle can be seen as an emerging fact from the mass of the bubbles of the de Sitter–Planck vacuum, and this occurs when the corresponding region of the de Sitter–Planck geometry is characterized by excited cells.

The following picture is outlined:

(1) The particle is initially massless (namely, it corresponds to the vacuum state);
(2) Its localization in an interaction event requires an amount of energy equal to the ratio of $\hbar$ and an opportune duration corresponding to the transition hypersurface $\Sigma_{\frac{1}{\sqrt{\Lambda}}}$ generating the appearance of time; thus, the fluctuations of the quantum vacuum are associated with the appearance of a particle take place;
(3) The particle self-interacts for a duration of $\hbar/M_{dim}c^2$, and therefore on a scale of lengths equal to $\hbar/M_{dim}c$. The total mass of the real particle is therefore the sum of

the "bare" mass associated with the bubbles of the de Sitter–Planck vacuum and the ε/c^2 mass derived from this self-interaction.

In particular, if the localization of a particle, following a collective organization of bubbles of the de Sitter–Planck background obeying infinite statistics, is influenced by gauge fields, the perturbative correction to its mass due to the particle self-interaction can be written through the following relation:

$$\frac{\varepsilon}{c^2} = -\frac{e}{c^2}\int \overline{\phi}\gamma^{\mu}A_{\mu}\phi dV \tag{24}$$

In Equation (23) A_{μ} is the self-field and ϕ is the spinor satisfying the ordinary Dirac equation:

$$i\hbar\gamma^{\mu}\partial_{\mu}\phi = mc\phi \tag{25}$$

By using the language of quantum field theory, the integral appearing in Equation (9) is extended to a volume of diameter $\hbar/mc$ around the vertex corresponding to the appearance of the particle and the minimum interaction distance is linked with the opportune duration corresponding to the transition hypersurface $\Sigma_{\frac{1}{\sqrt{\Lambda}}}$ generating the appearance of time in the associated micro-universe.

3. Bubbles Geometry and the Generalized Uncertainty Relations

A crucial result of the de Sitter–Planck geometry with infinite statistics involving a minimal spatial length at the Planck scale is represented by the breakdown of the Heisenberg uncertainty principle at the Planck scale.

Following Petruzziello and Illuminati [38], if one starts from the following deformed canonical commutation relations:

$$\left[\hat{X},\hat{P}\right] = i\hbar\left(1+\beta l_p^2\frac{\hat{P}^2}{\hbar^2}\right) \tag{26}$$

where β is the deformation parameter, $\hat{P}^2 = \sum_k \hat{P}_k^2$, where,

$$\hat{P}_k^2 = \left(1+\beta l_p^2\frac{\gamma^2\hat{M}^2}{\alpha\hbar^2}{M_{Pl}}^2c^2\right)\hat{p}_k \tag{27}$$

$$X_j = \hat{x}_j + O\left(\beta^2\right) \tag{28}$$

and $\hat{x}_j$ and $\hat{p}_k$ satisfy the ordinary Heisenberg uncertainty relations, at the Planck scale one obtains the following generalized uncertainty relations:

$$\Delta x\Delta p \geq \frac{\hbar}{2}\left(1+\beta l_p^2\frac{\gamma^2M^2}{\alpha\hbar^2}{M_{Pl}}^2c^2\right) \tag{29}$$

In Equation (29) the parameter β is a fluctuating quantity which expresses the fact that here space–time fluctuations fix the minimal length scale only on average, in analogy with what happens in quantum foam scenarios such as loop quantum gravity as well as cellular automaton interpretation of quantum mechanics [39–46].

The fluctuations in β are a characteristic sign of approaching the Planck scale, in epistemological affinity with the treatment made in [47,48], and will depend also on the scale of the appearance of elementary particles (namely the chronon scale invoked, for example, by Chiatti and Licata in their transactional approach of quantum jumps [49–51]).

The generalized uncertainty relations (29) have the merit of introducing new, interesting and unifying perspectives by suggesting a connection between great theories of the XXI century which invoke quantum foams, loops and holographic features at the Planck scale. For example, in loop quantum gravity, one may remark that each loop carries a quantum geometry of the Planck scale, namely, that a smooth geometry cannot be approximated at a physical scale lower than the Planck length, according to the following equation for the metric approximated by the increased loop density:

$$g^{(\mu)}{}_{\mu\nu}\left(\vec{x}\right) = \frac{l_P^2}{\mu^2}\eta_{\mu\nu} \tag{30}$$

where μ is the density of the loops and $\eta_{\mu\nu}$ is the metric of the 3D flat space [52]. Now, this result can be seen as the natural counterpart of the breakdown of the Heisenberg uncertainty principle at the Planck scale, expressed by Equation (26), in our approach of de Sitter–Planck geometry with quantum Boltzmann statistics, in the sense that the density of loops can be associated with the quantity $\beta l_p^2 \frac{\gamma^2 M^2}{\alpha\hbar^2} M_{Pl}{}^2 c^2$ depending on the planckeons as well as the parameter β. In other words, the density of loops can be seen as the consequence of fundamental processes involving the bubbles of the de Sitter–Planck background.

Moreover, the holographic features of loop quantum gravity in spherical symmetry are associated with an uncertainty in the determination of volumes that grow radially, invoked by Gambini and Pullin in [53], which imply that the number ΔN of elementary volumes in a shell with width Δx is:

$$\Delta N = \frac{x\Delta x}{2\gamma\rho l_P^2} \tag{31}$$

where ρ is the coordinate density of the loops, which can be seen themselves as a consequence of a more fundamental breakdown of the uncertainty principle at the Planck scale. In fact, by substituting (28) into Equation (30), one finds:

$$\Delta N = \frac{x\Delta x}{2\gamma\rho l_p^2} \geq \frac{x\hbar}{4\gamma\rho l_p^2 \Delta p}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha\hbar^2} M_{Pl}{}^2 c^2\right) \tag{32}$$

The physical meaning of Equation (32) is that the uncertainty in the number ΔN of elementary volumes in a shell with width Δx depends on a quantity which is linked with the deformation of the geometry of the bubbles at the Planck scale, besides the density of the loops and the uncertainty of the momentum.

Finally, if one considers Ng's model of a fundamental spacetime foam of a holographic nature where the quantum fluctuations manifest themselves in the form of uncertainties in the geometry of spacetime leading to a partition of the spacetime volume into "cells" of size $\left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3}$ [31–33], in our approach the distance l may be assimilated with the quantity Δx appearing in generalized uncertainty relations (2). In other words, we can say that the average minimum uncertainty, and thus the accuracy with which we can measure the geometry of space–time in Ng's model, is determined by the deformation of the geometry of the bubbles of the de Sitter background at the Planck scale. Moreover, since the holographic features of Ng's quantum spacetime foam are associated with infinite statistics, the suggestive perspective is opened that a simple correlation exists between the degree of deformation of the geometry of the bubbles at the Planck scale (expressed by the parameter β) and the infinite statistics.

In our approach based on the generalized uncertainty relation (26), one has the following modified de Broglie relation for bubbles of the de Sitter–Planck background:

$$\vec{p}\,' = \hbar\vec{k} + \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha\hbar^2} M_{Pl}{}^2 c^2\right)\left(\vec{k}\,' - \vec{k}\right) \tag{33}$$

which follows directly from the relation:

$$< \vec{x}\,\vec{x}\,\prime \Big| \vec{p}\,\vec{p}\,\prime >= \frac{1}{\sqrt{\pi\hbar\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha\hbar^2} M_{Pl}{}^2 c^2\right)}} e^{i\hbar\vec{p}\cdot\vec{x}} e^{\frac{i}{\frac{\hbar}{2}\left(1+\beta l_p^2 \frac{\gamma^2 M^2}{\alpha\hbar^2} M_{Pl}{}^2 c^2\right)}\left(\vec{p}\,' - \vec{p}\right)\cdot\left(\vec{x}\,' - \vec{x}\right)} \tag{34}$$

which can be considered as the generalization of the standard expression for the position space representation of a momentum eigenstates:

$$< \vec{x} \mid \vec{p} >= \frac{1}{\sqrt{2\pi\hbar}} e^{i\hbar \vec{p} \cdot \vec{x}} \tag{35}$$

In Equation (35), $\vec{k}$ is the usual de Broglie wave vector, which is associated with the momentum eigenstate of a quantum particle on a classical background space while the term $\vec{k}' - \vec{k}$ can be understood as the possible "kick", given to a point on the plane-wave, due to the transition $\vec{x} \to \vec{x}\prime$ in the de Sitter–Planck background.

Moreover, here one has the following version of the modified energy–frequency de Broglie relation:

$$E = \left[\hbar + \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha \hbar^2} {M_{Pl}}^2 c^2\right)\right]\omega \tag{36}$$

which yields the following expression for the modified quantum dispersion relation for free planckeons of the de Sitter–Planck background:

$$\omega = \frac{\left[\hbar \vec{k} + \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha \hbar^2} {M_{Pl}}^2 c^2\right)\left(\vec{k}' - \vec{k}\right)\right]^2}{2\frac{\gamma M}{\sqrt{\alpha}} M_{Pl}\left[\hbar + \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha \hbar^2} {M_{Pl}}^2 c^2\right)\right]} \tag{37}$$

4. From the Generalized Uncertainty Relations to Sub-Planckian Black Holes

Now, we want to show how the generalized uncertainty relation (29) can be considered the starting point in order to shed new light on the interpretation of the black hole as a "particle".

In this regard, before all, by starting from the generalized uncertainty relation (29), if one makes the substitution $\Delta x \to R$ and $\Delta p \to cM$, dropping the factor 2, one obtains:

$$R \geq \frac{\hbar}{cM}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{\alpha \hbar^2} {M_{Pl}}^2 c^2\right) \tag{38}$$

In Equation (38):

$$R'_C = \frac{\hbar}{cM} + \beta l_p^2 \frac{\gamma^2 M c}{\alpha \hbar} {M_{Pl}}^2 \tag{39}$$

may be considered as a generalized Compton wavelength, while the second term represents a small correction as one approaches the Planck regime. Equation (38) can also be applied for $M \gg M_{Pl}$ and has suggestive implications also for the black hole horizon size, where one obtains:

$$R \geq R'_S = \beta l_p^2 \frac{\gamma^2 M}{\alpha \hbar} {M_{Pl}}^2 c \left(1 + \frac{\alpha \hbar^2}{\beta l_p^2 \gamma^2 M^2 {M_{Pl}}^2 c^2}\right) \tag{40}$$

which represents a small correction to the Schwarzschild radius for $M \gg M_{Pl}$ if the parameters β and γ^2 satisfy the relation:

$$\beta \gamma^2 = \frac{2G\hbar\alpha}{c^3 l_p^2 {M_{Pl}}^2} \tag{41}$$

Equations (39) and (40), then, directly lead to a unified expression for the generalized Compton wavelength and event horizon size:

$$R'_C = R'_S = \sqrt{\left(\frac{\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha \hbar} {M_{Pl}}^2 c\right)^2} \tag{42}$$

The physical meaning of Equation (42) is that there is a zone of interpenetration between the Schwarzschild radius and the Compton wavelength and therefore a possible link between the uncertainty principle on the scale of elementary particles and the regime of black holes in macrophysics. When the Compton wavelength is bigger than the Schwarzschild radius a particle is formed, otherwise one deals with a black hole. In general, there exist quantum black holes, with their characteristic radiation. Moreover, this correspondence between microphysics and black holes involved with the bubbles of the de Sitter–Planck background seems to suggest that there could be sub-Planckian black holes with a size of order of their Compton wavelength and that the origin of these sub-Planckian objects becoming black holes lies just in the collective behavior of the bubbles. One can here make a sort of parallelism with the "loop black hole" solutions, behaving as wormholes, predicted by loop quantum gravity [54–59].

In order to explore in detail the existence of these sub-Planckian black that are simultaneously "black holes" and "elementary particles", we propose a generalization of Equations (38) and (40) in terms of the deformation parameter q appearing in the relation (4) defining the infinite statistics, as follows:

$$R'_C = \frac{q\hbar}{cM}\left(1 + \beta l_p^2 \frac{\gamma^2 M^2}{q\alpha\hbar^2} {M_{Pl}}^2 c^2\right) \tag{43}$$

and

$$R'_S = \beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} {M_{Pl}}^2 c \left(1 + \frac{q\alpha\hbar^2}{\beta l_p^2 \gamma^2 M^2 {M_{Pl}}^2 c^2}\right) \tag{44}$$

Equation (43) is valid in the sub-Planckian regime, while Equation (44) holds for the super-Planckian regime. In this way, one obtains a unified expression of the generalized Compton wavelength and event horizon size of the form:

$$R'_C = R'_S = \sqrt{\left(\frac{q\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} {M_{Pl}}^2 c\right)^2} \tag{45}$$

leading to the approximations

$$R'_C \approx \frac{q\hbar}{cM}\left(1 + \beta^2 l_p^4 \frac{\gamma^4 M^4}{q^2\alpha^2\hbar^4} {M_{Pl}}^4 c^4\right) \tag{46}$$

and

$$R'_S \approx \beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} {M_{Pl}}^2 c \left(1 + \frac{q^2\alpha^2\hbar^4}{\beta^2 l_p^4 \gamma^4 M^4 {M_{Pl}}^4 c^4}\right) \tag{47}$$

for $M \ll M_{Pl}$ and $M \gg M_{Pl}$, respectively.

Equation (45) can be considered the turning key in order to explain in what manner particles (and their corresponding statistics) emerge from the bubbles of the vacuum. By invoking a fruitful consideration of Elementary Cycles Theory [60,61], which states that every isolated elementary constituent of nature (every elementary particle) is characterized by an intrinsic Compton periodicity $T_C = \frac{h}{Mc^2}$, where M is the mass of the particle, thus leading to a unified formulation of relativistic and quantum physics as well as a fully geometrodynamical formulation of gauge interactions, here we can assume that the ordinary subatomic particles of the Standard Model emerge from the collective organization of the bubbles of the vacuum if the generalized Compton wavelength (42) gives rise to an intrinsic periodic phenomenon of generalized Compton periodicity:

$$T_C = \frac{1}{c}\sqrt{\left(\frac{q\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} {M_{Pl}}^2 c\right)^2} \tag{48}$$

As a consequence, one can say that an elementary particle of mass m emerges as a reference elementary clock, or vibrating mode, characterized by the fundamental generalized Compton periodicity (42), namely:

$$m = \frac{h}{T_C c^2} \tag{49}$$

By substituting relation (48) into (49), the mass of an elementary particle of the Standard Model, intended as an emergent entity from the collective organization of the bubbles of the vacuum described by the generalized Compton wavelength (45), is therefore:

$$m = \frac{h}{c\sqrt{\left(\frac{q\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} M_{Pl}{}^2 c\right)^2}} \tag{50}$$

Equation (50) illustrates in what sense the mass of a subatomic particle of the Standard Model emerges from the generalized Compton wavelength giving rise to an intrinsic periodic phenomenon of generalized Compton periodicity. As a consequence of the appearance of a real subatomic particle of mass (50), in correspondence, the wave function Ψ of the micro-universe (2) becomes the wave function of the particle by assuming the following form depending on the mass (50):

$$\hat{\Psi} = \frac{\gamma M_{Pl} c^2}{\sqrt{2\pi\alpha}\hbar l_p{}^2} \int dM \left(e^{i\frac{\gamma M_{Pl} c^2}{\sqrt{\alpha}\hbar l_p{}^2} m\varphi} \Psi_M(a)\hat{b}_M + e^{-i\frac{\gamma M_{Pl} c^2}{\sqrt{\alpha}\hbar l_p{}^2} m\varphi} \Psi_M^*(a)\hat{b}_M^\dagger \right) \tag{51}$$

namely

$$\hat{\Psi} = \frac{\gamma M_{Pl} c^2}{\sqrt{2\pi\alpha}\hbar l_p{}^2} \int dM \left(e^{i\frac{2\pi\gamma M_{Pl} c}{\sqrt{\alpha} l_p{}^2} \frac{1}{\sqrt{\left(\frac{q\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} M_{Pl}{}^2 c\right)^2}}\varphi} \Psi_M(a)\hat{b}_M + e^{-i\frac{2\pi\gamma M_{Pl} c}{\sqrt{\alpha} l_p{}^2} \frac{1}{\sqrt{\left(\frac{q\hbar}{cM}\right)^2 + \left(\beta l_p^2 \frac{\gamma^2 M}{\alpha\hbar} M_{Pl}{}^2 c\right)^2}}\varphi} \Psi_M^*(a)\hat{b}_M^\dagger \right) \tag{52}$$

In Equation (52), the operators $\hat{b}_M$ and $\hat{b}_M^\dagger$ now become the annihilation and creation operators which annihilate and create, respectively, particles of mass M, and satisfy the "quantum Boltzmann statistics" represented by the q deformation of the commutation relations of the oscillators (4), where the case $q = 1$ corresponds to the generation of the mass of bosons, whilst $q = -1$ corresponds to the generation of the mass of fermions. We have thus demonstrated in what sense the infinite statistics characterizing the bubbles of the vacuum imply that each cell, each micro-universe ruled by the WDW Equation (19) can give rise to a subatomic particle of mass (50) and wave function (52) as a result of the collective organization of the bubbles of the vacuum described by the generalized Compton wavelength (45) and this micro-universe can thus switch on as a boson or a fermion, depending on the value of the deformation parameter q that appears in the commutation relations of the oscillators (4).

The relation of the wave function (52)—describing the micro-universe giving rise to a particle as a result of the collective organization of the bubbles–with the dynamics in the de Sitter background space can be so formulated. On the basis of the approach based on Equations (37)–(41), one can define the following form of the Arnowitt-Deser-Misner mass

$$M_{ADM} = M\left(1 + \frac{q\alpha\hbar}{\beta l_p^2 \gamma^2 M M_{Pl}{}^2 c}\right) \tag{53}$$

which implies the generalized uncertainty principle because it satisfies relation

$$\frac{M_{ADM}}{r} = \frac{\Delta p}{c\Delta x} \tag{54}$$

if the momentum term is replaced by the substitution

$$\Delta p \to \Delta p + \frac{\alpha \hbar c}{\beta l_p^2 \gamma^2 \Delta p} \tag{55}$$

The Arnowitt-Deser-Misner mass (53) allows us to build a quantum-modified Schwarzschild metric of the form

$$ds^2 = F(r)c^2 dt^2 - F(r)^{-1} dr^2 - r^2 d\Omega^2 \tag{56}$$

where

$$F(r) = 1 - \frac{2GM}{c^2 {M_{Pl}}^2 r}\left(1 + \frac{q\alpha\hbar}{\beta l_p^2 \gamma^2 M {M_{Pl}}^2 c}\right) \tag{57}$$

Now, in order to analyze the dynamics of the evolution of the bubbles in the de Sitter background space, we take into account that it has been recently demonstrated that a scalar field non-minimally coupled to Einstein's tensor and Ricci scalar in geometries of asymptotically de Sitter space-times leads to regions of instability in the space-time determined by the geometry and field parameters [62]. On the basis of the results obtained in [62], one can propose that the link of the a-dynamical sea of bubbles—which gives rise to particles as a consequence of jumps in the archaic universe—with a perturbative dynamical picture, can be characterized by introducing the perspective that the wave function (52) can be associated to a corresponding probe scalar field Φ (of the type invoked in [62]) which can be treated as a small perturbation that does not produce strong modifications of the fixed geometry. In particular, by following [62], one starts from an action of the form

$$S = \int d^4x \sqrt{-g}\Big(\mathcal{L}_{background}(\mathcal{R}, \Lambda, F^{\mu\nu}) + \mathcal{L}_{perturbative}(\Phi)\Big) \tag{58}$$

where

$$\mathcal{L}_{background}(\mathcal{R}, \Lambda, F^{\mu\nu}) = \frac{\mathcal{R}}{16\pi G} - \frac{6}{L^2} - \frac{F_{\mu\nu}F^{\mu\nu}}{4} \tag{59}$$

$\mathcal{R}$ being the Ricci scalar, L stands for de Sitter radius, related to the cosmological constant by $L^2 = 3/\Lambda$, $F_{\mu\nu}$ are the components of electromagnetic field strength tensor, and

$$\mathcal{L}_{perturbative}(\Phi) = -\frac{1}{2}(g^{\mu\nu} + \eta G^{\mu\nu})\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2 \Phi^2 - V(\Phi) \tag{60}$$

where $g_{\mu\nu}$ and $G_{\mu\nu}$ are the components of the metric tensor and the Einstein tensor, η is the non-minimally derivative coupling parameter. The equation of motion for the scalar field derived from the action (60) is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}(g^{\mu\nu} + \eta G^{\mu\nu})\partial_\nu\Phi\right) - \frac{dV}{d\Phi} \tag{61}$$

Now, if one applies the standard ansatz to separate variables, the scalar field can be expressed in radial-temporal and angular parts as

$$\Phi(t, r, \theta, \phi) = \textstyle\sum_{l, m_\phi} R(r,t) Y_{l,m}(\theta, \phi) \tag{62}$$

By substituting (62) into equation (61) one thus finds the following equation

$$-\frac{\partial^2 R}{\partial t^2} + \alpha\frac{\partial^2 R}{\partial r^2} + \alpha\left(\frac{2}{r} + \frac{df}{dr}\right)\frac{\partial R}{\partial r} - \vartheta(r)R = 0 \tag{63}$$

Here, the crucial point lies in considering the equation of motion (63) inside a quantum Schwarzschild-de Sitter geometry expressed by the metric (56), equipped with the function (57). In this way, as regards the quantities appearing in (63), one has the following results

$$\alpha = [F(r)]^2, f = ln\left(\left(1+\frac{3\eta}{L^2}\right)F(r)\right), \vartheta(r) = F(r)\left(\frac{l(l+1)}{r^2} + \frac{\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2 L^2 + 12\xi}{L^2-3\eta}\right) \tag{64}$$

where ξ is a perturbative factor. As a consequence, the effective potential in the quantum Schwarzschild-de Sitter geometry ruling the evolution of the bubbles can be expressed as

$$V(r) = \left[1 - \frac{2GM}{c^2 {M_{Pl}}^2 r}\left(1+\frac{q\alpha\hbar}{\beta l_p^2\gamma^2 M {M_{Pl}}^2 c}\right)\right]\left[\frac{l(l+1)}{r^2} + \frac{\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2 L^2}{L^2-3\eta} + \frac{2GM}{c^2 {M_{Pl}}^2 r^3} - \frac{2}{L^2}\right] \tag{65}$$

In this equation, it must be remarked that the term $\frac{\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2 L^2}{L^2-3\eta}$ acts as a sort of a new scalar field mass, which shows how the bubbles of the vacuum are affected by the fundamental geometry of the de Sitter background, expressed by the de Sitter radius and the non-minimally derivative coupling parameter. The quantity $\frac{\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2 L^2}{L^2-3\eta}$ can be positive or negative depending on the parameters of the geometry and this generates changes of the effective potential between horizons, thus provoking instabilities for the field evolution. Coherently with the results obtained in [62], in our model of bubbles in a quantum Schwarzschild-de Sitter geometry, one obtains that, while for highly enough η and $l > 0$, the field turns out to be stable, this is not the case however for $l = 0$.

We remember here that the usual evolution of the scalar field after a initial burst in a positive potential is characterized by three distinct possibilities after the ringing phase: (1) if $l > 0$, it decays exponentially, (2) if $l = 0$, it moves towards a constant value that scales the cosmological constant, (3) finally, it oscillates indefinitely as a function of the scalar field mass. In the model suggested in this paper, however, in the light of Equations (61)–(65), when the potential is not entirely positive between horizons, unstable modes can emerge and the geometry overcomes some changes. As regards the non-minimally derivative coupling η of quantum Schwarzschild-de Sitter geometry here considered, the potential is partly or entirely negative (depending on the coupling and geometry parameters) and this implies that an unstable dynamics arises. In particular, by studying the evolution of the field for different L, η and l, inside the constraint $L^2 > 27\left(\frac{\gamma M}{\sqrt{\alpha}}M_{Pl}\right)^2$ corresponding to the causal structure condition for the presence of an encapsulated singularity (by the event horizon) and a cosmological horizon, one finds the following results:

A stable evolution occurs from $\eta = 0$ until $\eta < L^2/3$, showing the expected decay in time, since the corresponding potential is positive;

Instead, for $\eta > L^2/3$, the dynamics is always unstable and, even for asymptotic η, where the potential is partly positive, there is no stable evolution.

On the other hand, the quantum-modified Schwarzschild metric (56)—equipped with the function (57)—leads us to make relevant considerations as regards the behavior of the black hole systems. In fact, the metric (56) is characterized by the horizon size

$$r_H = R'_S = \frac{2GM}{c^2 {M_{Pl}}^2}\left(1+\frac{q\alpha\hbar}{\beta l_p^2\gamma^2 M {M_{Pl}}^2 c}\right) \tag{66}$$

which, in the different regimes of mass, becomes

$$r_H \approx \begin{cases} \frac{2GM}{c^2 M_{Pl}{}^2} \; if \; M \gg M_{Pl} \\ \frac{2G\beta l_p^2\gamma^2 M_{Pl}{}^2 + q\alpha\hbar c}{\beta l_p^2 \gamma^2 M_{Pl}{}^3 c^2} \; if \; M \approx M_{Pl} \\ \frac{2G\beta l_p^2\gamma^2 + q\alpha\hbar c}{\beta l_p^2\gamma^2 M_{Pl}{}^2 c^2} \; if \; M \ll M_{Pl} \end{cases} \tag{67}$$

The first expression coincides with the standard Schwarzschild radius. The intermediate expression gives a minimum of order l_p, so the Planck scale is never actually reached for $q > 0$ and therefore the singularity remains inaccessible. The last expression can be associated with the Compton wavelength.

On the basis of the metric (56), one can also explore the thermodynamics of the black hole solutions in the three limits considered above (super-Planckian, trans-Planckian and sub-Planckian limits). In this regard, by following the treatment of Carr [2,3], if the temperature is determined by the black hole's surface gravity [63], one has:

$$T = \frac{M_{Pl}{}^2}{8\pi M\left(1 + \frac{q\alpha\hbar}{\beta l_p^2 \gamma^2 M M_{Pl}{}^2 c}\right)} \approx \begin{cases} \frac{M_{Pl}}{8\pi M\left[1 - q(M_{Pl} - M)^2\right]} \; if \; M \gg M_{Pl} \\ \frac{M_{Pl}}{8\pi[1 + q/2]} \; if \; M \approx M_{Pl} \\ \frac{M}{4\pi q\left[1 - (M/M_{Pl})^2/q\right]} \; if \; M \ll M_{Pl} \end{cases} \tag{68}$$

On the basis of Equation (68), the large M limit corresponds to the usual Hawking temperature with a small correction. Instead, as the black hole evaporates, the temperature reaches a maximum at around T_{Pl} and then decreases to zero as $M \to 0$.

A possible explanation for the $M \ll M_{Pl}$ behavior lies in invoking the possibility that a decaying black hole makes a temporary transition to a (1 + 1)-D dilaton black hole when the Planck scale is approached, since this naturally encodes a $1/M$ term in its gravitational radius.

If the temperature is given by (68), the black hole entropy can be calculated in the usual fashion as:

$$S = \int_{M_0}^{M} \frac{dM\prime}{T(M\prime)} = 4\pi k\left(\frac{M^2}{M_{Pl}^2} - \frac{M_0{}^2}{M_{Pl}^2} + q ln\frac{M}{M_0}\right) \tag{69}$$

where $M_0 < M_{Pl}$ is some lower bound of integration. In Equation (69), the presence of a logarithmic correction is compatible with the entropy of a (1 + 1)-D Schwarzschild spacetime [64], as well as with the notion that (1 + 1)-D black holes are naturally quantum objects [65], emerging here via the dependence of r_H on M. This is in agreement with a model-independent feature emerging from a variety of approaches to quantum gravity such as string theory [66], loop quantum gravity [67] and ultraviolet gravity self-completeness [68].

5. Thermodynamics of a Quantum Black Hole

Now, on the basis of the black hole temperature (60), the luminosity of the black hole may be expressed as:

$$L = \frac{M_{Pl}^3}{t_{Pl}} M^{-2}\left(1 + \frac{q\alpha\hbar}{\beta l_p^2\gamma^2 M M_{Pl}{}^2 c}\right)^{-2} \tag{70}$$

Although the black hole loses mass on a timescale:

$$\tau \sim \frac{t_{Pl}}{M_{Pl}^3} M^3\left(1 + \frac{q\alpha\hbar}{\beta l_p^2\gamma^2 M M_{Pl}{}^2 c}\right)^2 \tag{71}$$

it never entirely evaporates because the mass loss rate decreases when M falls below M_{Pl}. There are however two values of M which are associated with a timescale τ that is comparable with the age of the universe ($t_0 \sim 10^{17}$ s), namely a super-Planckian value:

$$M_* \sim \left(\frac{t_0}{t_{Pl}}\right)^{1/3} M_{Pl} \sim 10^{15} \text{ g} \tag{72}$$

and a sub-Planckian value

$$M_{**} \sim \beta^2 \left(\frac{t_{Pl}}{t_0}\right) M_{Pl} \sim 10^{-65} \text{ g} \tag{73}$$

The mass M_* is the standard expression for the mass of a primordial black hole that evaporates at the present epoch. The mass M_{**} effectively specifies the lower integration bound in Equation (68), namely $M_{**} = M_0$. The mass cannot actually reach the value M_{**} at the present epoch because of the effect of the cosmic microwave background. This occurs because the black hole temperature is less than the CMB temperature T_{CMB}, suppressing evaporation altogether, below an epoch-dependent mass:

$$M_{CMB} = 10^{-36}(T_{CMB}/3K)\text{g} \tag{74}$$

which represents the value at which the primordial black hole freezes, leading to effectively stable relics which might provide a candidate for the dark matter.

The mass M_* corresponds to a black hole radius:

$$r_H \sim 10^{-13} \text{ cm} \tag{75}$$

and temperature

$$T \sim 10^{12} K \tag{76}$$

while the mass M_{**} corresponds to a radius

$$r_H \sim \left(\frac{t_0}{t_{Pl}}\right) l_p \sim 10^{60} l_p \sim 10^{27} \text{ cm} \tag{77}$$

and temperature

$$T \sim 10^{-28} K \tag{78}$$

Relations (77) and (78) correspond, respectively, to the current cosmological horizon size and to the Hawking temperature for a black hole with the mass of the universe.

Finally, at the end of this section, let us analyze how, by starting from the horizon size (66), one can shed new light on the features of the equivalent of the cosmological constant for black holes. In this regard, by invoking a fruitful consideration made by Kubiznak and collaborators in [69], we can interpret the counterpart of the cosmological constant for black holes as a sort of thermodynamic pressure on the basis of the relation:

$$P = -\frac{\Lambda}{8\pi G_d} = \frac{(d-1)(d-2)}{16\pi r_H^2 G_d} \tag{79}$$

where G_d is the d-dimensional gravitational constant. The thermodynamic pressure (79) can be considered as a fundamental physical entity which determines a dynamical evolution of the black holes. In particular, it makes the black hole grow due to the accretion of the scalar field, and implies that, while the growing black hole tends to pull the cosmological horizon further in, the decaying cosmological constant makes it expand. If one considers a potential interpolating between initial and final values of the cosmological constant, Λ_i and Λ_f, the change in both the black hole and cosmological horizon areas turns out to evolve proportionally to $|\delta\Lambda| = \left|\Lambda_f - \Lambda_i\right|$ times, a factor that depends on properties of the de Sitter–Planck background. By following [70], the change in the area of the black hole horizon can be expressed by the relation:

$$|\delta A_H| = \frac{\mho}{2\pi T_h} \frac{A_H}{A_{tot}} \left(\Lambda_i - \Lambda_f\right) \tag{80}$$

where T_h is the temperature of the black hole horizon, given by Equation (68), $A_{tot} = 4\pi\left(r_c^2 + r_H^2\right)$ is the total horizon area, $\mho = 4\pi\left(r_c^3 + r_H^3\right)/3$ is the thermodynamic volume of the de Sitter black hole region, r_H is the black hole horizon given by (66), Λ_i and Λ_f are the initial and final values of the cosmological constant associated with the black hole system and r_c is the cosmological horizon, which is strictly dependent on the cosmological constant and the black hole horizon through the relations:

$$\Lambda = \frac{3}{\left(r_c^2 + r_H^2 + r_H + r_c\right)} \tag{81}$$

$$\frac{2GM}{c^2} = \frac{r_c r_H (r_c + r_H)}{\left(r_c^2 + r_H^2 + r_H + r_c\right)} \tag{82}$$

M being the mass of the clack hole. In particular, in the limits where the black hole horizon is small or comparable in size to the cosmological horizon, the modification in the black hole horizon area is given, respectively, by:

$$\delta A_H \cong \begin{cases} 2A_H \frac{|\delta\Lambda|}{\sqrt{3}\Lambda_i} \cdot \left(r_H\sqrt{\Lambda_i}\right) \; if \; r_H\sqrt{\Lambda_i} \ll 1 \\ A_H \frac{|\delta\Lambda|}{\Lambda_i} \; if \; r_H\sqrt{\Lambda_i} \sim 1 \end{cases} \tag{83}$$

In the light of relations (83), the fractional growth in the area of the black hole, $\delta A_H/A_H$ turns out to be negligible for small black holes, while it is of order $\frac{|\delta\Lambda|}{\Lambda_i}$ for large ones. Finally, the effect of the black holes on the size of the cosmological horizon is to determine a change in the cosmological horizon area given by the relations:

$$\delta A_c \cong \begin{cases} 12\pi \frac{|\delta\Lambda|}{\Lambda_i^2} \; if \; r_H\sqrt{\Lambda_i} \ll 1 \\ 4\pi \frac{|\delta\Lambda|}{\Lambda_i^2} \; if \; r_H\sqrt{\Lambda_i} \sim 1 \end{cases} \tag{84}$$

According to relations (84), the cosmological horizon growth is negligible for small black holes, while for large black holes the effect of a black hole in the cosmological horizon growth can be reduced by as much as 2/3 [70].

6. Casimir Energy and Cosmological Wormholes

In this section, we want to show how our approach of sub-Planckian black holes in a de Sitter–Planck background can shed new light on a unifying treatment of Casimir effect—which lies in the measurable force between macroscopic objects generated by the vacuum energy in QFT—and cosmological wormholes, which constitute physical connections between two distant regions of the universe according to Einstein's equations of General Relativity. As regards a unifying treatment of the Casimir effect and cosmological wormholes, a pioneering analysis by Sorge [71] investigated the interference of both non-inertial effects and spacetime geometry on the vacuum energy density of a non-massive scalar field present in a small Casimir cavity, which orbits an Ellis–Thorne wormhole [72]. From this perspective, a recent work by Santos, Muniz and Oliveira analyzed the changes in the quantum vacuum energy density of a massless scalar field inside a Casimir apparatus that orbits a wormhole, in the context of a cosmological model with an isotropic form of the Morris–Thorne wormhole, embedded in the FLRW universe. The three authors explored the effects of the global curvature of the universe and its scale factor on the Casimir energy density, finding that the Casimir energy density was higher in a hyperbolic Universe, lower in a spherical one and intermediary in a flat Universe, with the difference between them being higher as more distant the plates are from the wormhole throat [73].

On the other hand, in [74], Sorge found that an observer comoving with a Casimir cavity, freely falling in a Schwarzschild black hole, measures a small reduction in the (absolute) value of the (negative) Casimir energy as the black hole horizon is approached because of the changing spacetime geometry. Working in the Lemaitre coordinates where the Schwarzschild metric takes the form:

$$ds^2 = d\tau^2 - \frac{r_g}{r(\tau,\rho)}d\rho^2 - r^2(\tau,\rho)d\Omega^2 \tag{85}$$

where r_g is the Schwarzschild radius and, near the black hole horizon,

$$r(\tau,\rho_0) = r_g\left(1 - \frac{3\tau}{2r_g}\right)^{2/3} \tag{86}$$

which represents a freely falling particle (in our case the Casimir cavity) whose trajectory intersects the horizon at $\tau = 0$, Sorge found that the overall energy density (as measured by the comoving observer) of the Casimir effect inside a small cavity, freely falling into a Schwarzschild black hole, is made by two contributions, namely a static contribution:

$$\epsilon_{C\ static} = -\frac{\pi^2}{1440L^4} \tag{87}$$

which is related to the vacuum polarization and a dynamical contribution, due to the time dependent background experienced by the quantum field, leading to particle creation inside the Casimir cavity, given by the relation:

$$\epsilon_{C\ dynamical} = \frac{1}{384L^2}\frac{\xi^2}{(1-\xi\tau)^2} \tag{88}$$

where $\xi = \frac{3c}{2r_g}$.

However, inside Sorge's approach, the finiteness of the Casimir plates was not taken into account, since it was assumed $L \ll \sqrt{A} \ll r_g$. Such an assumption is satisfied in any realistic scenario where the gravitational radius of a black hole is undoubtedly many orders of magnitude larger than the cavity's size. Instead, by considering micro-black holes, having a gravitational radius $r_g \sim L$, or even better the sub-Planckian black holes of the model suggested in this paper, Sorge's equations need to be extended and generalized because in these situations the condition $L/r_g \ll 1$ is violated, as in that limit the local frame cannot be considered almost Minkowskian (in these situations the tidal effects, associated with anisotropies in the distribution of the vacuum energy density inside the cavity, will dominate).

In order to develop a generalization of Sorge's approach which is able to take into account the sub-Planckian black holes of the de Sitter–Planck background, let us return, before all, to the generalized uncertainty relations (28), which we express now in the following equivalent form:

$$\Delta x\Delta E \approx \frac{\hbar c}{2}\left(1 + \beta\left(\frac{2\gamma M M_{Pl}\Delta E}{\sqrt{\alpha}p^2}\right)^2\right) \tag{89}$$

and let us see how this relation leads to corrections in the Casimir energy with respect to the standard prediction of quantum field theory (in the simple case of the three-dimensional geometry of two parallel plates separated by a distance $d \ll L$ where L is the side of the plates). By solving Equation (89) with respect to ΔE, one obtains:

$$\Delta E = \frac{\Delta x}{\hbar c \beta} \frac{p^4}{4\frac{\gamma^2 M^2}{\alpha} {M_{Pl}}^2} \left[1 \pm \sqrt{1 - \beta \left(\frac{\hbar c}{\Delta x} \frac{2\gamma M M_{Pl}}{\sqrt{\alpha} p^2} \right)^2} \right] \tag{90}$$

where one must consider only the negative solution in order to obtain a coherent result for vanishing β. After expanding to the first order in β one finds:

$$\Delta E = \frac{\hbar c}{2\Delta x} \left[1 + \frac{\beta}{4} \left(\frac{\hbar c}{\Delta x} \frac{2\gamma M M_{Pl}}{\sqrt{\alpha} p^2} \right)^2 \right] \tag{91}$$

If we now neglect those photons coming from distances greater than the value r_e representing the effective distance beyond which photons have a negligible probability to reach the plate, it is natural to assume the uncertainty position Δx of the single photon to be of the order of r_e and, thus, of the distance d between the plates, according to the relation:

$$r_e \cong 2.6\mathrm{d} \tag{92}$$

which follows from Heisenberg uncertainty principle. Then, by replacing $\Delta x \cong 2.6\mathrm{d}$ into Equation (90), the contribution to the Casimir energy at a given point becomes:

$$|\Delta E(d)| \cong 0.2 \frac{\hbar c}{d} \left[1 + 0.04\beta \left(\frac{\hbar c}{d} \frac{2\gamma M M_{Pl}}{\sqrt{\alpha} p^2} \right)^2 \right] \tag{93}$$

which agrees with the equation

$$\Delta E(d) = -\frac{\pi}{12} \frac{\hbar c}{d} \tag{94}$$

derived from the standard Heisenberg uncertainty relation in the limit $\beta \to 0$. Equation (93) may be thus considered as the generalization of the Casimir energy in the framework of the de Sitter–Planck background based on the generalized uncertainty relation (29).

Now, we propose a generalization of Sorge's model which takes into account the features of the sub-Planckian black holes and their consequent effects on the Casimir energy density. In this regard, in the usual Casimir cavity, freely falling from spatial infinity and adjusting the comoving observer clock so that the proper time $\tau = 0$ when the cavity is at the radial horizon coordinate r_H, we consider the quantum-modified Schwarzschild metric in the Lemaitre coordinates $\{\tau, \rho, \theta, \phi\}$ in which a freely falling test body has a constant value ρ_0 of the radial ρ coordinate, of the following form

$$ds^2 = d\tau^2 - \frac{r_H}{r(\tau)} d\rho^2 - r^2(\tau) d\Omega^2 \tag{95}$$

where

$$r(\tau) = r_H \left(1 - \frac{3\tau}{2r_H} \right)^{2/3} \tag{96}$$

and r_H is the horizon size given by (66). The quantum vacuum fluctuations characterizing the orbiting cavity of the Casimir apparatus may be associated with a massless scalar field $\varphi(x^\mu)$ which obeys Dirichlet boundary conditions at the plates:

$$\varphi(\tau, x, \vec{x}_\perp)|_{x=0} = \varphi(\tau, x, \vec{x}_\perp)|_{x=L} = 0 \tag{97}$$

and, by applying the minimal coupling constraint, satisfies the following Klein–Gordon equation of motion:

$$\left[\eta^{bc}\partial_b\partial_c + \frac{1}{4}\frac{\zeta^2}{(1-\zeta\tau)^2}\right]\varphi = 0 \tag{98}$$

where

$$\zeta = \frac{3c}{2r_H} \tag{99}$$

By using the techniques developed by Sorge in [71,74], we can search for a solution of (98) of the form:

$$\varphi(x^a) \sim e^{i\vec{k}_\perp\cdot\vec{x}_\perp} sin\left(\frac{n\pi}{L}x\right)\chi(\tau) \tag{100}$$

By inserting (89) into (87) we obtain the following equation for the function $\chi(\tau)$ of the proper (local) time:

$$\left[\partial_\tau^2 + \omega_k^2 + \frac{1}{4}\frac{\zeta^2}{(1-\zeta\tau)^2}\right]\chi = 0 \tag{101}$$

By solving Equation (93), χ may be expressed in terms of Hankel functions of the second kind:

$$\chi_k(\tau) = \frac{1}{2}\sqrt{\frac{\pi}{\zeta}(1-\zeta\tau)}H_0^{(1)}\left[\frac{\omega_k}{\zeta}(1-\zeta\tau)\right] \tag{102}$$

which has a Minkowskian behavior at $\tau \to -\infty$ and, when the cavity is at the spatial infinity with respect to the black hole, becomes:

$$\chi_k(\tau) \sim \frac{1}{\sqrt{2\omega_k}}e^{-i\omega\tau},\ for\ \tau \to -\infty \tag{103}$$

By following [74], here the Casimir effect inside a small cavity, freely falling into a Schwarzschild black hole, may be derived from the real part of the action associated with Equation (90), namely:

$$W_H(\nu) = \frac{(-i)^\nu A\pi^{3/2}}{16L^3}\sum\nolimits_k \zeta^{2k}2^k a_k\left(\frac{L}{\pi}\right)^{2(\nu+k)} \times \int_{-\infty}^{T}\frac{d\tau}{(1-\zeta\tau)^{2k}}\Gamma(\nu-3/2+k)\sum\nolimits_{n=1}^{\infty}\frac{1}{n^{(2\nu-3+2k)}} \tag{104}$$

where $a_0 = 1$

$$a_k = \frac{1}{k!8^k}\left[(-1)^2(-3)^2\cdots\left(-(2k-1)^2\right)\right],\ k \geq 1 \tag{105}$$

in the limit $\nu \to 0$, namely:

$$\langle\epsilon_{Cas}\rangle = -\lim_{\nu\to 0}\frac{1}{AL}\frac{\partial}{\partial\tau}ReW(\nu) \tag{106}$$

After a β-power expansion, one obtains:

$$\langle\epsilon_{Cas}\rangle = -\frac{\pi^{3/2}}{16L^4}\sum\nolimits_{k=0}^{\infty}\frac{2^k\zeta^{2k}a_k}{(1-\zeta\tau)^{2k}}\left(\frac{L}{\pi}\right)^{2(\nu+k)}\Gamma(-3/2+k)\varsigma(-3+2k) \tag{107}$$

which yields

$$\langle\epsilon_{Cas}\rangle = -\frac{\pi^2}{1440L^4} + \frac{1}{384L^2}\frac{\zeta^2}{(1-\zeta\tau)^2} + O\left(\zeta^4\right) \tag{108}$$

At the horizon crossing, namely $\tau \to 0^-$, one obtains:

$$\langle\epsilon_{Cas}\rangle_{hor} = -\frac{\pi^2}{1440L^4}\left[1 - \frac{135}{16\pi^2}\left(\frac{L}{r_H}\right)^2\right] \tag{109}$$

Equation (108) tells us how the corrections to the Casimir energy density change with the proper time as the cavity approaches the black hole horizon, both in the adiabatic regime—where the proper time satisfies $\Delta\tau \gg L$—and if $\Delta\tau < L$ holds where the dynamical effects of the de Sitter–Planck background generated by the collective behavior of the bubbles play a dominant role.

The physical meaning of Equation (100) lies in the fact that the small reduction in the (absolute) value of the (negative) Casimir energy measured by a comoving observer near the black hole horizon is linked with the collective behavior of the bubbles of the de Sitter–Planck background, via the dependence of the horizon size expressed by Equation (66). In particular, on the basis of Equation (67), one obtains the following expressions for the Casimir energy at the horizon crossing in the different regimes of mass:

$$\langle\epsilon_{Cas}\rangle_{hor} = \begin{cases} -\frac{\pi^2}{1440L^4}\left[1-\frac{135}{16\pi^2}\left(\frac{Lc^2{M_{Pl}}^2}{2GM}\right)^2\right] \; if\ M \gg M_{Pl} \\ -\frac{\pi^2}{1440L^4}\left[1-\frac{135}{16\pi^2}\left(\frac{L\beta l_p^2\gamma^2{M_{Pl}}^3c^2}{2G\beta l_p^2\gamma^2{M_{Pl}}^2+q\alpha\hbar c}\right)^2\right] \; if\ M \approx M_{Pl} \\ -\frac{\pi^2}{1440L^4}\left[1-\frac{135}{16\pi^2}\left(\frac{L\beta l_p^2\gamma^2{M_{Pl}}^2c^2}{2G\beta l_p^2\gamma^2+q\alpha\hbar c}\right)^2\right] \; if\ M \ll M_{Pl} \end{cases} \quad (110)$$

In summary, one can say that the small reduction observed in the static Casimir energy value is generated by the collective behavior of the bubbles of the de Sitter–Planck background, which induce a peculiar correction in the different regimes of mass. This result turns out to be compatible with the treatment of Sorge [74], where the small reduction in the Casimir energy value was associated with the phenomena of particle creation inside the Casimir cavity. Moreover, at the same time, our model goes further by suggesting a more general scenario that is able to explain the processes in the sense that it provides a deeper explanation of the origin of this energetic correction in terms of a more fundamental background.

To conclude, regarding the interaction of two or more bubbles from the point of view of the ER-EPR conjecture, bubbles are spacetime fluctuations of a spacetime which is described in terms of non-local wormhole connections, viz., the building blocks of the physics of the gravitational field at and below Planck scales. When it is fixed for any given metric g a finite length L down to the Planck length l_p and a characteristic time τ, the generation of one or more bubbles in the neighborhood of Planck's scales can be interpreted as metric fluctuations Δg characterized by a Riemann tensor that, for any given spacetime, from the equivalence principle are defined by the energy and time fluctuations given in Planck units,

$$R_{(4)}(g, L) \;\sim\; \frac{E_P}{\hbar}\left(\frac{\tau_P}{\tau}\right)^2\frac{g^2}{L^2} \quad (111)$$

The curvature tensor depends on the Planck scales the time interval τ and the length L in a spacetime defined by the metric tensor g. These metric fluctuations can in principle be interpreted in terms of space–time relationships connecting different events within a distance L with the effect of generating spacetime from spacetime relationships. Below Planck's scales the events are undistinguishable, physically identical, in principle undetectable and entangled. At lower energies the spacetime connections between two events are interpreted in terms of virtual graviton exchanges as in the ER=EPR scenario [75] with the result of coupling two or more events at larger distances. The connection between two events via Einstein-Rosen bridge (ER) is equivalent to the quantum entanglement described by the Einstein, Podolski and Rosen paradox. As an example, thin-shell wormholes can be used to construct a something that represents a closed and complete universe.

7. Conclusions

In this work we investigated the sub-Planckian aspects of bubble theory, moving a little away from the traditional cosmological meaning of the theme linked to inflation, and returning to some structural characteristics of the WDW equation. As is known [75], the use of this equation requires to choose of specific boundary conditions as in the case of the no-boundary condition imposed by Hartle–Hawking [76,77].

In the case of sub-Planckian bubbles, the WDW is constrained by the de Sitter geometry when close to the Planck scale, as in Formula (1), and thus we find a number of suggestive applications in microphysics. The addition of a generative statistics such as the quantum Boltzmann statistic allowed us to take a few steps forward in this toy model in order to distinguish between particles (fermions or bosons), baby universes and black holes. In particular, in the de Sitter–Planck background with infinite statistics and opportune generalized uncertainty relations (29), which express the deformation of the geometry of the bubbles at the Planck scale, led us to a unified expression for the generalized Compton wavelength and event horizon size, given by (45), which implies the existence of a possible link between the uncertainty principle on the scale of elementary particles and the regime of black holes in macrophysics. We thus demonstrated how in this picture a subatomic particle can be seen as a collective organization of the bubbles of the vacuum described by the generalized Compton wavelength (45) and an intrinsic periodic phenomenon of generalized Compton periodicity (48).

Moreover, by considering the Arnowitt–Deser–Misner mass (53) associated with the generalized uncertainty relations (29), we arrived at a quantum-modified Schwarzschild metric which is characterized by the horizon size (66) whose super-Planckian limit coincides with the standard Schwarzschild radius, the trans-Planckian limit gives a minimum of order l_p and the sub-Planckian limit corresponds to the Compton wavelength. The quantum-modified Schwarzschild metric allowed us to explore the thermodynamics of the black hole solutions in these three limits, while, in the light of the horizon size (66), we found that the change in both the black hole and cosmological horizon areas evolves proportionally to $|\delta\Lambda| = \left|\Lambda_f - \Lambda_i\right|$ times, a factor that depends on the properties of the de Sitter–Planck background, where Λ_i and Λ_f are the initial and final values of the cosmological constant interpreted as a thermodynamic pressure.

Finally, we showed how our approach of sub-Planckian black holes in a de Sitter–Planck background can shed new light on a unifying treatment of the Casimir effect and cosmological wormholes finding that the small reduction observed in the static Casimir energy value is generated by the collective behavior of the bubbles of the de Sitter–Planck background, which induces a peculiar correction in the different regimes of mass.

The idea that space–time is generated by a non-local background now has a wide citizenship in physics, in particular for a de Sitter background [78–82]. In his *Aspects of Symmetry* [83] Sydney Coleman notes that when speaking about a non-perturbative vacuum we cannot say anything because the values of the constants and the parameters that regulate it could be completely different from those on which the physics we know is based. In particular, in a non-local background there is no space–time and therefore we have to ask ourselves some fundamental questions about the localizability of the bubbles.

On the one hand, there is the Planck limit which arises as an extreme limit of observability, on the other hand, using the Bekenstein limit (see Equation (5)), we characterized the maximum amount of information that can be associated with a physical event, roughly speaking on the chronon's scale. A quick consideration about the range of known particle masses suggests that each particle admits a discrete mass spectrum that is bounded from below by a ground state with energy close to Planck's mass, a recurring idea in theoretical physics (see e.g., [84]).

The observation in ref. [17] that the higher angular momentum states adapt to the Regge trajectories seems an indication of the existence in nature of "towers" similar to that hypothesized by Majorana and found in condensed matter and photonics [85]. All this naturally offers the suggestion that bubbles are characterized by a locality similar to those of quasi-particle physics; hence, a semi-locality. In fact, the cells should not be thought of as rigid and separate as in a Rubik's cube, because the bubbles are entangled with the others and therefore described by a type in Equation (1) and a wave function (2) and possibly connected with each other also in a non-local way with metric fluctuations as in Equation (103). Locality in the ordinary sense occurs when the Higgs mechanism and the statistical nature come into play over time, producing a wave function of the type (52). The idea of an original semi-locality in a reticular cell structure leads back to the foundational problems of QFT and to the idea that the locality is a notion derived from the emergence of a physical event in space–time [86].

The third quantization was developed in the late 1980s to study the fluctuations of space–time and proved to be a very powerful and clear framework describing the multiverse and various aspects of its conceptual constellations such as virtual black holes, wormholes and baby universes. This developed from the Wheeler–De Witt Equation (WDW), in which the time variable is not fixed a priori; the possible solutions are therefore incomputable [42]. The choice of the time variable thus implies the selection of an evolutionary mechanism. The quantum jump represents the non-unitary aspect of the interaction of the Standard Model that induces it (e.g., the electromagnetic interaction between electron and photon in quantum jumps of atomic electrons) and consists of a selection of the de Broglie phase in the "multiverse" of possibilities that have previously existed, which is accompanied by the localization of the particle over time. The decoherence is thus generated by the "common" interaction processes and in this context the possibility of geometrically describing the breakdown of unitarity through the exchange of an intermediate micro-universe is considered.

The well-known approach of multiverse and particle-like universes was used here by replacing gravitation with scalar fields, such as that of Higgs, which size the particle micro-universe. The particle, in its "corpuscular" aspect, is seen as an event rather than an object. In this sense, no ontological role of a classical type is assigned to the particle micro-universe, unlike previous similar theoretical elaborations. The investigation into the nature of the "collapse", which remained among epistemological speculations for a long time, thus blends naturally with particle physics to the point of suggesting a relationship between the Higgs mechanism and localization.

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