

Bianchi type-II, VIII and IX perfect fluid cosmological model with domain walls in $f(R, T)$ gravity

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Abstract: This paper deals with the Bianchi type - II, VIII & IX metrics in presence of domain walls in the frame work of the modified $f(R, T)$ theory of gravitation, where R is Ricci scalar and T is the trace of energy momentum tensor. Also, some important features of the models, thus obtained, have been discussed.

Keywords: Bianchi type - II, VIII & IX metrics, $f(R, T)$ theory of gravitation and Domain walls.

1. Introduction: The late time accelerated expansion of the universe is one of the most challenging problems in modern cosmology. It is assumed that the expansion is due to some unusual type of matter, referred to as dark energy; however, in the literature, it is clear that it can arise from a modification of gravity and is not due to dark energy. This has encouraged eminent



researchers to explore modified theories of gravitation. Among the various modification theories, the $f(R)$ theory of gravity is one important theory developed during the last decade. It has been suggested that the addition of a nonlinear function of the Ricci scalar R to the Einstein–Hilbert action is the cause of acceleration for a wide variety of $f(R)$ functions W. Hu and I. Sawicki [1]. $f(R)$ Theory also describes the transition phase of the universe from deceleration to acceleration M. F. Shamir [2]. The $f(R)$ theories, including the $1/R$ function, are incompatible with the solar system test of general relativity. However, many critics claim that the solar system test is not ruled out of any form of gravity Chiba et al [3]. Despite the debate on modified gravity, Harko et al. Harko et al [4] have proposed a $f(R, T)$ modified theory, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor. They have discussed some particular models corresponding to a specific choice of the function $f(R, T)$. Subsequently, Rao and Sireesha [5], Sahoo et al. [6], Reddy et al. [7], Mishra and Sahoo [8], Rao et al. [9], santhi et al [10,11], Aditya et al. [12] have discussed some cosmological models in $f(R, T)$ theory of gravity and also A. Y. Shaikh, & K. S. Wankhade [13] have investigated Domain Walls Cosmological Model in $f(R, T)$ Theory of Gravity

Bianchi type space- times play a vital role in understanding and description of the early stages of evolution of the Universe. In particular, the study of Bianchi types II, VIII & IX Universes are important because familiar solutions like FRW Universe with positive curvature, the de Sitter Universe, the Taub- Nut solutions etc. correspond Bianchi type II, VIII & IX space-times. Rao et al. [14,15,16] have obtained Bianchi types II, VIII & IX string cosmological models, perfect fluid cosmological models in Saez- Ballester theory of gravitation and string cosmological models in general relativity as well as self creation theory of gravitation respectively. Rao and Sireesha [17] have discussed Bianchi type II, VIII & IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation, Rao et al [18] have investigate Bianchi Type-II, VIII and IX Interacting Holographic Generalized Chaplygin Gas Cosmological Models in a Scalar Tensor Theory and recently Rao et al [19] explored Bulk viscous string cosmological models in Saez – Ballester theory of gravitation.

Motivated by the above investigations and discussions, in this paper we study the spatially homogeneous Bianchi type - II, VIII & IX metrics in $f(R, T)$ theory of gravitation in presence of domain walls.

2. Field equations of $f(R, T)$ with domain walls:

The field equations of $f(R, T)$ gravity are derived from the Hilbert-Einstein type variation principle. The action for the $f(R, T)$ gravity with domain walls,

$$s = \frac{1}{16\pi} \int \sqrt{-g} f(R, T) d^4x + \int \sqrt{-g} L_m d^4x \quad (2.1)$$

where $f(R, T)$ is an arbitrary function of Ricci scalar (R), (T) is the trace of stress-energy tensor of the matter (T_{ij}) and (L_ϕ) is the matter Lagrangian density. The stress energy tensor of matter T_{ij} is defined as

$$T_{ij} = \frac{-2\delta(\sqrt{-g} L_\phi)}{\sqrt{-g} \delta(g^{ij})} \quad (2.2)$$

Here the dependence of matter Lagrangian is merely on the metric tensor (g_{ij}) is considered rather than on its derivatives and we obtain

$$T_{ij} = g_{ij} L_\phi - \frac{\delta(L_\phi)}{\delta(g^{ij})} \quad (2.3)$$

Now varying the action with respect to metric tensor, $f(R, T)$ gravity field equations are obtained as

$$f(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + f_R(R, T)(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_i) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij} \quad (2.4)$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g_{ij} \partial g^{\alpha\beta}}$$

Here $f_R = \frac{\delta f(R, T)}{\delta R}$, $f_T = \frac{\delta f(R, T)}{\delta T}$ (2.5)

$\Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g_{ij}}$ and ∇_i is the covariant derivative.

The problem of the perfect fluids delineated by associate degree energy density, pressure p and four velocities u^i is difficult since there's no distinctive definition of the matter Lagrangian. However, here it is assumed that the strain energy tensor of the matter is given by

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p \omega_i \omega_j \quad (2.6)$$

and the matter Lagrangian may be taken as $L_m = -p$ and that we have

$$u^i \nabla_j u_j = 0 \quad u^i u_j = 1 \quad (2.7)$$

Then with the employment of equation (2.5), for the variation of stress-energy of perfect fluid the expression is

$$\theta_{ij} = -2T_{ij} - p g_{ij} \quad (2.8)$$

Generally, the field equations also depend, through the tensor, on the physical nature of the Θ_{ij} matter field. Hence in the case of $f(R, T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for gravity. However, Harko et al. [4] gave three classes of these models:

3. Metric and Energy Momentum Tensor:

$$f(R, T) = \left\{ \begin{array}{l} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{array} \right\} \quad (2.9)$$

Here the first case is considered, i.e., where is an arbitrary function of the trace of stress-energy tensor.

Using these relation $f(R, T)$ gravity field equations (2.4) reduced to

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2(T_{ij} + \Theta_{ij})f'(T) + f(T)g_{ij} \quad (2.10)$$

where a prime denotes differentiation with respect to the argument.

Generally, the field equations additionally rely through the tensor θ_{ij} , on the physics nature of the matter field. Hence within the case of $f(R, T)$ gravity reckoning on the character of the matter supply, we tend to acquire many theoretical models reminiscent of every alternative of $f(R, T)$.

Victimization equation (2.8), in equation (2.10) become

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \quad (2.11)$$

We consider a spatially homogeneous Bianchi type-II, VIII & IX metrics of the form

$$ds^2 = -dt^2 + R^2[d\theta^2 + f^2(\theta)d\varphi^2] + S^2[d\varphi + h(\theta)d\psi]^2 \quad (3.1)$$

where (θ, φ, ψ) are the Eulerian angles, R and S are functions of t only.

It represents

Bianchi type-II if $f(\theta) = 1$ and $h(\theta) = \theta$

Bianchi type-VIII if $f(\theta) = \cosh \theta$ and $h(\theta) = \sinh \theta$

Bianchi type-IX if $f(\theta) = \sin \theta$ and $h(\theta) = \cos \theta$

The energy momentum tensors of the Domain walls is taken as

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p\omega_i \omega_j \quad (3.2)$$

where p & ρ , are pressure and energy densities.

In a co moving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = \rho, T_4^4 = -p \quad (3.3)$$

where the quantities p and ρ are functions of 't' only.

4. Solutions of the Field equations:

The field equations of (2.11) for the metric (3.1) using (3.2) and (3.3) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} = -(8\pi + 5\lambda)\rho - \lambda p \quad (4.1)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{3S^2}{4R^4} = -(8\pi + 5\lambda)\rho - \lambda p \quad (4.2)$$

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{S^2}{4R^4} + \frac{\omega}{2}\phi^r\dot{\phi}^2 = (8\pi + \lambda)p - 3\lambda\rho \quad (4.3)$$

Here the overhead dot denotes differentiation with respect to 't'.

The field equations (4.1) to (4.3) are only three independent equations with four unknowns R, S, ρ & p , which are functions of 't'. Since these equations are non-linear in nature, in order to get a deterministic solution we take the following possible physical condition:

The shear scalar σ is proportional to scalar expansion θ , so that we can take a linear relationship between the metric potentials R and S , i.e.,

$$R = S^n \quad (4.4)$$

where n is an arbitrary constant.

From equations (4.1), (4.2) and (4.4), we get

$$(n-1)\frac{\ddot{S}}{S} + 2n(n-1)\frac{\dot{S}^2}{S^2} + \frac{\delta}{S^{2n}} - \frac{1}{S^{4n-2}} = 0 \quad (4.5)$$

Bianchi type-II ($\delta = 0$) cosmological model:

If $\delta = 0$, equation (4.5) can be written as

$$(n-1)\frac{\ddot{S}}{S} + 2n(n-1)\frac{\dot{S}^2}{S^2} - \frac{1}{S^{4n-2}} = 0 \quad (4.6)$$

From equations (4.4) and (4.6), we get

$$R = (\gamma \cos 2t)^{\frac{1}{4}}$$

$$S = (\gamma \cos 2t)^{\frac{1}{2}} \quad (4.7)$$

where γ is an integration constant.

The density ρ is

$$\rho = \frac{-24\pi \sec^2 2t - 2\pi \tan^3 2t - 20\pi + \frac{\lambda}{2} \tan^2 2t \left(5 - \frac{1}{2} \tan 2t\right) - 3\lambda \sec^2 2t - 3\lambda + \frac{(8\pi + 3\lambda)\delta}{\sqrt{\gamma \cos 2t}}}{(16\pi + 10\lambda)(8\pi + \lambda) + 6\lambda^2} \quad (4.8)$$

The pressure p is

$$p = \frac{4\pi(5 \tan^2 2t - 1) + \lambda \left[\frac{\tan^2 2t}{2} \left(25 + \frac{3}{2} \tan 2t\right) + 3 \sec^2 2t + 5 \right] + \frac{(16\pi + 7\lambda)\delta}{\sqrt{\gamma \cos 2t}}}{6\lambda^2 + (16\pi + 10\lambda)(8\pi + \lambda)} \quad (4.9)$$

The metric (3.1), in this case can be written as

$$ds^2 = dt^2 - (\gamma \cos 2t)^{\frac{1}{2}} (d\theta^2 + d\varphi^2) - (\gamma \cos 2t) (d\varphi + \theta d\psi)^2 \quad (4.10)$$

Thus the metric (4.10) together with (4.7) - (4.9) constitutes a Bianchi type-II $f(R, T)$ gravity model in presence of domain walls.

Bianchi type-VIII ($\delta = -1$) cosmological model:

If $\delta = -1$, equation (4.5) can be written as

$$(n-1) \frac{\ddot{S}}{S} + 2n(n-1) \frac{\dot{S}^2}{S^2} - \frac{I}{S^{2n}} - \frac{I}{S^{4n-2}} = 0 \quad (4.11)$$

From equation (4.11), we get

$$R = \frac{\gamma}{w} \cos \frac{wt}{2} \quad (4.12)$$

From equations (4.4) and (4.12), we get

$$S = \left(\frac{\gamma}{w} \cos \frac{wt}{2} \right)^2 \quad (4.13)$$

where $\gamma^2 = -\frac{4}{3}$, $w^2 = 1$.

The density ρ is

$$\rho = \frac{\left(\tan^2 \frac{\omega t}{2} - 1\right) \left(10\omega^2 + \frac{5\lambda\omega^2}{4}\right) - (8\pi + 3\lambda) \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \omega t - \frac{3}{2}\lambda \tan^2 \frac{\omega t}{2} - 4\pi - \lambda}{(16\pi + 10\lambda)(8\pi + \lambda) + 6\lambda^2} \quad (4.14)$$

The pressure p is

$$p = \frac{\left[-\frac{3}{4}\tan^2 \frac{\omega t}{2} - \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \frac{\omega t}{2} - \frac{1}{4}\right] (16\pi + 10\lambda) - 3\lambda \left[\frac{5}{4}\omega^2 \left(\tan^2 \frac{\omega t}{2} - 1\right) - \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \frac{\omega t}{2} - \frac{1}{2}\right]}{(16\pi + 10\lambda)(8\pi + \lambda) + 6\lambda^2} \quad (4.15)$$

The metric (3.1), in this case can be written as

$$ds^2 = dt^2 - (\gamma \cos 2t)^{\frac{1}{2}} (d\theta^2 + d\varphi^2) - (\gamma \cos 2t) (d\varphi + \theta d\psi)^2 \quad (4.16)$$

Thus the metric (4.10) together with (4.7) - (4.9) constitutes a Bianchi type-IIV $f(R,T)$ gravity model in presence of domain walls.

Bianchi type-IX ($\delta = 1$) cosmological model:

If $\delta = 1$, equation (4.5) can be written as

$$(n-1) \frac{\ddot{S}}{S} + 2n(n-1) \frac{\dot{S}^2}{S^2} + \frac{I}{S^{2n}} - \frac{I}{S^{4n-2}} = 0 \quad (4.17)$$

Equation (4.17), with suitable substitution and for $n = 2$, we get

$$R = \frac{\gamma}{w} \cos \frac{wt}{2} \quad (4.18)$$

From equations (4.4) and (4.18), we get

$$S = \left(\frac{\gamma}{w} \cos \frac{wt}{2}\right)^2 \quad (4.19)$$

The density ρ is

$$\rho = \frac{\left(\tan^2 \frac{\omega t}{2} - 1\right) \left(10\omega^2 + \frac{5\lambda\omega^2}{4}\right) + (8\pi + 3\lambda) \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \omega t - \frac{3}{2}\lambda \tan^2 \frac{\omega t}{2} - 4\pi - \lambda}{(16\pi + 10\lambda)(8\pi + \lambda) + 6\lambda^2} \quad (4.20)$$

The pressure p is

$$p = \frac{\left[\frac{-3}{4}\tan^2 \frac{\omega t}{2} + \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \frac{\omega t}{2} - \frac{1}{4}\right] (16\pi + 10\lambda) - 3\lambda \left[\frac{5}{4}\omega^2 \left(\tan^2 \frac{\omega t}{2} - 1\right) + \left(\frac{\omega}{\gamma}\right)^2 \sec^2 \frac{\omega t}{2} - \frac{1}{2}\right]}{(16\pi + 10\lambda)(8\pi + \lambda) + 6\lambda^2} \quad (4.21)$$

The metric (3.1), in this case can be written as

$$ds^2 = dt^2 - (\gamma \cos 2t)^{\frac{1}{2}} (d\theta^2 + d\varphi^2) - (\gamma \cos 2t) (d\varphi + \theta d\psi)^2 \quad (4.22)$$

Thus the metric (4.10) together with (4.7) - (4.9) constitutes a Bianchi type-IX

$f(R, T)$ gravity model in presence of domain walls.

5. Physical and geometrical properties:

Bianchi type-II cosmological model ($\delta = 0$):

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = \gamma \cos 2t \quad (5.1)$$

The average scale factor for the model is

$$a(t) = V^{\frac{1}{4}} = (\gamma \cos 2t)^{\frac{1}{4}} \quad (5.2)$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = -2 \tan 2t \quad (5.3)$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{14}{9} \tan^2 2t \quad (5.4)$$

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i} u^i + \frac{1}{3} \theta^2) = 3 \operatorname{cosec}^2 2t - 1 \quad (5.5)$$

The deceleration parameter appears with negative sign implies accelerating expansion of the Universe, which is consistent with the present day observations.

The Hubble's parameter H is given by

$$H = -\frac{2}{3} \tan 2t \quad (5.6)$$

The mean anisotropy parameter A_m is given by

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{1}{8} \text{ where } \Delta H_i = H_i - H \quad (i = 1, 2, 3) \quad (5.7)$$

The jerk parameter

$$j = (3 \operatorname{cosec}^2 2t - 1)(6 \operatorname{cosec}^2 2t - 1) - 18 \operatorname{cosec}^2 2t \cot^2 2t \quad (5.8)$$

Red shift

$$1 + z = \frac{a_0}{a} = \left(\frac{\cos 2t_0}{\cos 2t} \right)^{\frac{1}{4}} \quad (5.9)$$

The luminosity distance

$$\begin{aligned} d_L &= r_1 (1 + z) a_0 \\ &= \int_T^{T_0} (\gamma \cos 2t)^{-\frac{1}{4}} a_0 \left(\frac{\cos 2t_0}{\cos 2t} \right)^{-\frac{1}{4}} \end{aligned} \quad (5.10)$$

where r_1 is the radial coordinate distance of the object at light emission and is given by

$$r_1 = \int_T^{T_0} \frac{1}{a} dT = \int_T^{T_0} (\gamma \cos 2t)^{-\frac{1}{4}}$$

The distance modulus

$$\begin{aligned} D(z) &= 5 \log d_L + 25 \\ &= 5 \log \left(\int_T^{T_0} (\gamma \cos 2t)^{-\frac{1}{4}} a_0 \left(\frac{\cos 2t_0}{\cos 2t} \right)^{-\frac{1}{4}} dt \right) + 25 \end{aligned} \quad (5.11)$$

The vorticity tensor $w_{ij} = u_{i,j} - u_{j,i}$ is a measure of the rotation of the local rest-frame relative to the compass of inertia, is identically zero. Hence the fluid filling the Universe is non-rotational.

Bianchi type-VIII ($\delta = -1$) & IX ($\delta = 1$) cosmological model:

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = \left(\frac{\gamma}{w} \cos \frac{wt}{2} \right)^4 \quad (5.12)$$

The average scale factor for the model is

$$a(t) = V^{\frac{1}{4}} = \frac{\gamma}{w} \cos \frac{wt}{2} \quad (5.13)$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = -2w \tan \frac{wt}{2} \quad (5.14)$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{14}{9} w^2 \tan^2 \frac{wt}{2} \quad (5.15)$$

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i} u^i + \frac{1}{3} \theta^2) = \frac{3}{4} \operatorname{cosec}^2 \frac{wt}{2} - 1 \quad (5.16)$$

The deceleration parameter appears with negative sign implies accelerating expansion of the Universe, which is consistent with the present day observations.

Jerk parameter

$$j = \left(\frac{3}{4} \cos ec^2 \frac{wt}{2} - 1 \right) \left(\frac{3}{2} \cos ec^2 \frac{wt}{2} - 1 \right) - \frac{9}{8} \cos ec^2 \frac{wt}{2} \cot^2 \frac{wt}{2} \quad (5.17)$$

The Hubble's parameter H is given by

$$H = -\frac{2}{3} w \tan \frac{wt}{2} \quad (5.18)$$

The mean anisotropy parameter A_m is given by

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{1}{8} \text{ where } \Delta H_i = H_i - H \quad (i = 1, 2, 3) \quad (5.19)$$

Red shift is given by

$$1 + z = \frac{a_0}{a} = \left(\frac{\cos \frac{wt_0}{2}}{\cos \frac{wt}{2}} \right) \quad (5.20)$$

Luminosity distance is given by

$$\begin{aligned} d_L &= r_1 (1 + z) a_0 \\ &= \int_T^{T_0} \left(\frac{\gamma}{w} \cos \frac{wt}{2} \right)^{-1} \left(\frac{\cos \frac{wt_0}{2}}{\cos \frac{wt}{2}} \right)^{-1} \end{aligned} \quad (5.21)$$

where r_1 is the radial coordinate distance of the object at light emission and is given by

$$r_1 = \int_T^{T_0} \frac{1}{a} dT = \int_T^{T_0} \left(\frac{\gamma}{w} \cos \frac{wt}{2} \right)^{-1} \quad (5.22)$$

The distance modulus is given by

$$\begin{aligned} D(z) &= 5 \log d_L + 25 \\ &= 5 \log \left(\int_T^{T_0} \left(\frac{\gamma}{w} \cos \frac{wt}{2} \right)^{-1} \left(\frac{\cos \frac{wt_0}{2}}{\cos \frac{wt}{2}} \right)^{-1} \right) + 25 \end{aligned} \quad (5.23)$$

The vorticity tensor $w_{ij} = u_{i,j} - u_{j,i}$ is a measure of the rotation of the local rest-frame relative to the compass of inertia, is identically zero. Hence the fluid filling the Universe is non-rotational.

6. Discussion and Conclusions:

In this paper we have presented non static plane symmetric cosmological models filled with domain walls in the frame work of $f(R, T)$ gravity proposed by Harko et al.

For all the three models we can observe that the spatial volume increases with the increase of time ' t ' and also the models have no initial singularity at $t = 0$. We can see that the expansion scalar θ , shear scalar σ and the Hubble parameter H decrease with the increase of time ' t '. From (5.7) and (5.19), we can observe that $A_m \neq 0$ and this indicates that the Universe is anisotropic. For Bianchi type-II, VIII and IX models, we can observe that the density and the pressure of holographic dark energy are decreases with the increase of time. From equations (5.5) and (5.16), we can see that the deceleration parameter appears with negative sign implies accelerating expansion of the Universe, which is consistent with the present day observations. We have obtained expressions for astrophysical parameters like, red-shift, distance modulus $D(z)$ and luminosity distance d_L versus red-shift. We have also reconstructed the potentials and the dynamics of the scalar field for these anisotropic accelerating models of the Universe.

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