2.5 The Birth and Childhood of the Coupling Impedance and Stability Maps

Vittorio Giorgio Vaccaro
Istituto Nazionale di Fisica Nucleare. Sezione di Napoli
Mail to: vaccaro@na.infn.it

At beginning of sixties important breakthrough innovations were already accomplished in accelerator science, which produced leaps forward in the performances of particle accelerators.

2.5.1 The First Breakthrough Innovation: the Phase Stability

The basic principles of synchrotron design (phase stability) were proposed independently by Vladimir Veksler in the Soviet Union (1944) and Edwin McMillan in the United States (1945). According to this principle, it is possible to accelerate bunches of charged particles of finite dimensions.

Based on the principles stated by Vladimir Veksler and Edwin McMillan a proton synchrotron was built at Brookhaven National Laboratory, named **Cosmotron**. Its construction started in 1948 and it reached its full energy in 1953. It was the first particle accelerator to exceed the GeV wall, accelerating protons to 3.3 GeV. Since when Brookhaven's Cosmotron went into operation in the early 1950's, scientists knew that achieving the higher energies was going to be a difficult problem. Calculations showed that, using existing technology, building a proton accelerator 10 times more powerful than the 3.3GeV Cosmotron would require 100 times as much steel.

2.5.2 The Second Breakthrough Innovation: the Alternating Gradient

While the first strictly used the toroid shape, the strong focusing principle independently discovered by Nicholas Christofilos (1949) and Ernest Courant (1952) allowed the complete separation of the accelerator into the guiding magnets and focusing magnets, shaping the path into a round-cornered polygon. Without strong focusing, a machine as powerful as the Alternating Gradient Synchrotron (AGS) would have needed apertures (the gaps between the magnet poles) between 0.5 m and 1.5 m instead of apertures of less than 0.1 m. The construction of AGS was accomplished in 1960.

2.5.3 The Collider Age. Looking Far

Even before the successful achievements of PS and AGS, the scientific community was aware that another step forward was needed. Indeed, the impact of particles against fixed targets is very inefficient from the point of view of the energy actually available for new experiments: much more efficient could be the **head on collisions** between high-energy particles. With increasing energy, the energy available in the Inertial Frame with fixed targets is incomparably smaller than in the head-on collision (HC), as Wideroe thought some decades before. If we want the same energy in IF, using fixed targets one should build gigantic accelerators. The challenge was to produce intense and high-collimated beams.

2.5.3.1 ***Hamburger Intermezzo***

Touschek was born and attended school in Vienna. Because of racial reasons, he was not allowed to finish high school. However, he could continue is studies in a precarious way. After Anschluss, he moved to Hamburg, where nobody knew of his origins. There he met Rolf Wideroe with whom he started cooperating in building a betatron and discussing on Wideroe's visionary thoughts. However, Touschek was discovered and arrested by the Gestapo in 1945, Wideroe visited him in prison, bringing cigarettes, food and, during these meetings they continued to talk about the betatron. Incidentally, in that context Touschek conceived the idea of radiation damping for electrons. When the Allied army reached Hamburg Wideroe, suspected of collaboration, was arrested. Sometime after, he was found not guilty and released. After the war, Touscheck roamed about Europe. Finally, in 1952 he decided to stay in Rome permanently, receiving the position of researcher at the National laboratories of the Istituto Nazionale di Fisica Nucleare in Frascati, near Rome.

2.5.3.2 ***Collider Contest: Frascati vs. Princeton***

A contest between Princeton and the Frascati Laboratories started: both labs were developing collider programs accelerating electrons and positrons. Princeton chose an eight-shaped structure: two circular rings in which electrons and positron were circulating with the same orientation, meeting at the collision point. Frascati team, which took the field later, was even more audacious: they used a single ring with "counter-rotating" beams of electrons and positrons.

The enterprise began on March 7, 1960, when Bruno Touschek held a seminar at Frascati Laboratories. He was proposing to build an electron-positron storage ring, according to Wideroe's visionary ideas concerning storage rings and colliders. On March 14, a preliminary study demonstrated the feasibility of the proposal. The storage ring was named ADA (Anello Di Accumulazione = Storage Ring). Touschek pointed out the extreme scientific interest of high-energy collisions between particles and antiparticles, and the simplicity of realization of such an accelerator. The machine was conceived as a feasibility experiment to provide a sound basis for the realization of electron-positron colliders of larger center of mass energy and luminosity. The total cost of the project (converted to the present purchasing power) was around 800.000 €.

A first stored beam of few electrons was obtained at the end of May 1961, using the Frascati Electron Synchrotron as an injector. The first electron-positron interactions were observed at the beginning of 1964. The impact of the ADA Collider has been immense in enabling a new chapter of accelerator physics to be established with the machine being the first particle-antiparticle collider and the first electron-positron storage ring. In addition to this grand accomplishment, the machine was also able to prove the idea that one could accelerate and collide a beam of particles and antiparticles in the same machine.

Many laboratories started programs of to accelerate and store particles in order to prove the feasibility to intense beams. Surprisingly enough, a longitudinal instability below transition energy was discovered in 1963 in the *MURA 40 MeV* electron accelerator. At the same time vertical instabilities in the *MURA 50 MeV* were observed [1]. At that time, it was a common place that above transition energy, a beam could be unstable since it was postulated that the prevalent electromagnetic interaction with the vacuum pipe was capacitive. Furthermore, it was not known that there could exist some stabilizing mechanism.

2.5.4 The Analysis of Instabilities. A Step Forward

Two companion papers [2,3] appeared in 1965 on the Review of Scientific Instruments, one concerning longitudinal coherent instability and a second one transverse coherent instability. The approach to the problem was the usual one adopted for modulational instability, which arises from the coupling between the equation of the particle dynamics and the electromagnetic interaction a coasting particle beam with the surrounding pipe of a circular accelerator.

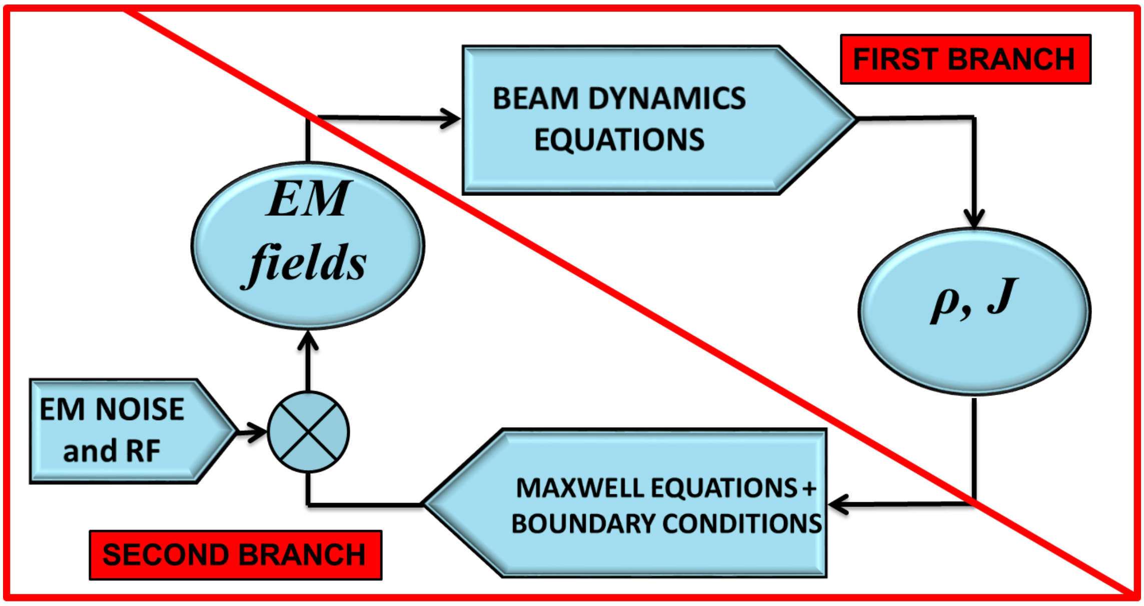

Figure 1: Block diagram of the coupling between e-magnetic equation and dynamics equation.

The novelty was the use of Vlasov equation where it is assumed that the beam particles have an energy distribution function. The problem is solved by means of perturbative techniques that lead to a dispersion relation. The role of Landau damping of the instability coming from the energy spread was emphasized. The pipe is supposed circular, smooth, and lossy and with circular or rectangular cross-section.

In both paper the case in the absence of frequency spread was examined and it was found that the rise time depends on the conductivity of the pipe. However, allowing for a finite spread, the stability criteria obtained from the dispersion relation do not involve the pipe losses. It is worth noting that the stability criteria were derived assuming Gaussian or Lorenzian distribution functions. Four years later, in a CERN internal report [4], a more accurate analysis showed that, taking in account a more realistic distribution function, the stability criteria do involve the pipe losses. This aspect tells the amplitude and the generality of the concepts presented in the two papers. The stream of research born in 1965 and still lasting gave and gives results that have of fundamental importance for particle accelerator.

2.5.5 An Impedance is in the Air. The Banality of Invention

When I was hired by CERN on June 1966, I joined the RF group of the Intersecting Storage Ring (ISR) Department. ISR was under construction and was destined to be the world's first hadron collider. It ran from 1971 to 1984, with a maximum center of mass energy of 62 GeV. At that time at CERN, there was big concern about stability of the beams because of large number and various kind of lumped equipment (300 pairs of clearing electrodes, pick-ups, cavities, etc.), which could be "seen" by the beam. Unfortunately, the stability criteria did not apply to the situation of ISR. I was committed to work on this problem. The task to introduce in the dispersion relation the contribution of a lumped element, e.g. cavity of impedance Z_c (eventually clearing electrodes)

$$I_i = const\langle 2\pi R E_\theta \rangle \int \frac{d\psi_0}{dW} \frac{dW}{[\omega - n\omega_0(W)]}, \quad (1)$$

where I_i is the incipient perturbed current in the beam and ω the frequency of the instability. The procedure is described in Ref. [5]. The impressed voltage at the cavity gaps V_i is calculated assuming that the image current, that loads the cavity is equal to the perturbed beam current I_i. The field distribution in the accelerator is expanded in travelling waves inside the pipe. Then, only the nth harmonic is retained which is riding with the perturbation. Therefore, the mean integral in the above equation may be written as:

$$\langle 2\pi R E_\theta \rangle = -Z_c I_i. \quad (2)$$

The concept of coupling impedance was then extended to pipe with uniform properties. Of course, the above procedure consisted in a brute force approach. Its validity is restricted to wavelengths much larger than the cavity gap and of the pipe radius; however, this limitation does not affect the principle. Fifty years have passed. In the meantime, exact approaches were performed resorting to numerical codes or to analytical-numerical techniques such as the mode matching. Few months after my arrival, Andy Sessler, on leave of absence from LBL, joined the ISR-RF group. I was committed to him and I showed him the manuscript of my results. He reviewed it, making corrections, suggesting integrations and then he stated that the report had to appear with my name only. However, the paper was issued in closed distribution restricted to AR and ISR Scientific Staff. At the same time, he proposed a general treatment of impedance of arbitrary electrical properties [6]. There is another important aspect which should be taken into account: the concept of coupling impedance is a handy concept. This is very well illustrated by Sessler in one of his papers [7]: "It was emphasized - and, it was the main point of [6] - that Z described the impedance of the wall elements and as, thus, amenable to computation - or measurement - by means of all the standard techniques employed in electrical engineering. This engineering technique was applied to a number of problems, such as helical conducting walls [8], and allowed complicated structures to be readily analyzed. Maybe this feature has been one of the factors that determined the success of the coupling impedance concept.

2.5.6 The Universal Stability Charts

The introduction of coupling impedance concept is tightly linked to the analysis of the dispersion relation. The first step was done. The success of the first one influenced the advancement of the second one. Then, when the picture of the longitudinal instability phenomenon was clear, the problem of transverse instability was tackled.

Except in particular cases, Eq. (2) cannot be solved analytically; namely, given the impedance, the distribution function, the harmonic number n and the function $\omega_0(W)$ it is not in general possible to find analytically the frequency ω of the instability, if any. A collaboration with Sandro Ruggiero was set up, which tackled the problem with another point of view: find the coupling impedance for a given value of the complex frequency ω, assuming a linear dependence of ω_0 on W; repeat the procedure for various distribution functions [4]. Therefore, we had to perform analytically the integral for several reasonable distributions:

$$Z(\omega) = -\frac{1}{const \int \frac{d\psi_0}{dW} \frac{dW}{[\omega - n\omega_0(W)]}} \,. \tag{3}$$

This is nothing but a conformal mapping of the complex variable ω into the complex variable Z. The interest is to explore the region where the imaginary part of ω is negative, namely where the oscillation is exponentially increasing. A particular interest was devoted to the mapping of the lines where the frequency is real with a vanishing imaginary part, namely

$$\omega = \omega_r + j0_- \,. \tag{3}$$

This procedure gave quite surprising results:

- The mapping of the lower half-plane covers almost entirely the Z plane.
- The mapping of the upper half plane covers the same region of the Z plane.
- There is a "neutral region", which is not covered by none of the two mapping and is defined as the stable region.
- The stable region is finite if the tails of the distribution function are finite.
- The stable region of a monoenergetic distribution (infinitesimal tails) is the positive imaginary axis.

In Fig. 2 the result of the mapping for a Lorenzian distribution function is reported (similarly to Refs. [2,3]). The impedance is normalized in such a way to get a universal stability diagram. The dashed domain is the stability region, contour of which is a parabola. It is apparent that the stable domain is infinite. The coupling impedance of smooth pipe has small real part due to the pipe resistivity and a large positive imaginary (normalized) part and one could infer that the beam should be stable, which was the same conclusion inferred by the authors. Other distribution functions were considered, such as the Gaussian one (which produces also an infinite stability region), a 2nd, 3rd and 4th order parabola and a truncated cosine. According to the available data, the working point of MURA accelerator was very close to the imaginary axis and had a very large imaginary component. This means that, the detected instability is compatible with the results obtained from the Vlasov equation, provided, that one takes a realistic distribution function. That was an excellent result confirming that correctness of the Vlasov equation approach.

The successful aftermaths stimulated the extension of the research on transverse instabilities. An example is reported in Fig. 3. In this case it is taken into account not only the frequency spread but also the distribution function of the betatron amplitude oscillation. Coupling impedance is fifty this year, but it does not show…

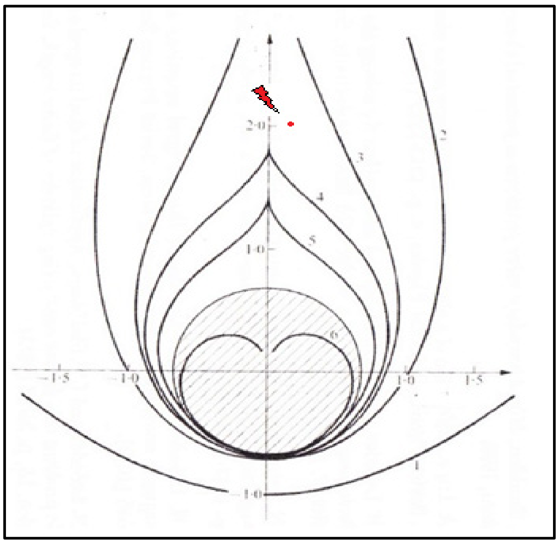

Figure 2: Stability boundaries for various distribution functions.

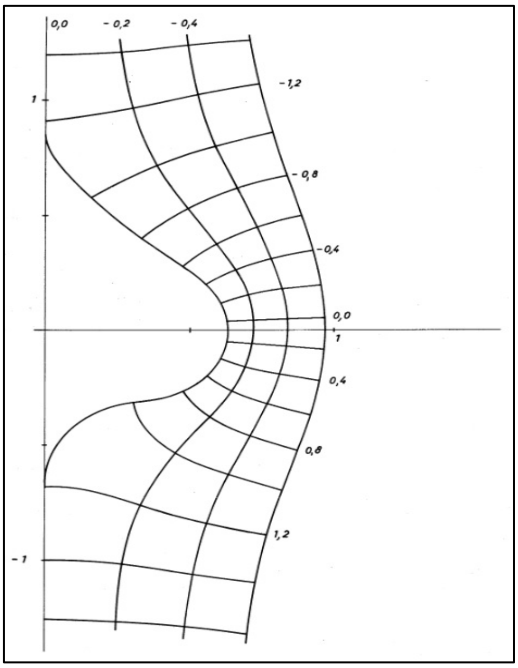

Figure 3: Transverse stability chart with curves at constant rise time.

2.5.7 References

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