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Review

Clustering as a window on the hierarchical structure of quantum systems

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Abstract Why do quantum particles form a hierarchical structure: quarks, hadrons, nuclei, atoms, and molecules? This is a fundamental question, and its answer is still elusive. Each hierarchical layer is characterized by the constituent particles, which are composite particles except for the quark hierarchy. Such a building block is regarded as a *cluster* and plays a role in forming a hierarchy. In the boundary of the neighboring hierarchies, we may find intermediate hierarchies, called *semi-hierarchies*, where a range of characteristic clusters, such as hadronic molecules, exotic hadrons, neutron halos, α clusters, and Feshbach molecules, appear. Such a cluster structure has some common features throughout the hierarchical layers with different scales. We discuss the role of clusters and their formation in semi-hierarchies.

1 Introduction

Matter in the universe is composed of quantum particles. In the Big Bang, quarks and electrons emerged as the first particles with a finite mass. As the temperature decreased, quarks were confined into nucleons (protons and neutrons); Nucleons congregated, and atomic nuclei were synthesized. In Big Bang nucleosynthesis, ^{4}He nuclei were the primary product, formed from protons and neutrons, while smaller amounts of other light nuclei, such as deuterons, ^{3}He, ^{6,7}Li, were also produced [1]. About 3.8×10^{5} years after the Big Bang, the universe became transparent to photons, when atoms were formed by combining nuclei with electrons. Accordingly, the current universe comprises these quantum particles across a wide range of scales, which form distinctive *hierarchies*: quarks, hadrons, nuclei, atoms, and molecules, as shown schematically in Fig. 1(left).

Why and how do quantum particles in our universe form hierarchies? This fundamental question has remained unsettled since the discoveries of microscopic quantum particles in the twentieth century. Examining the conventional quantum-particle hierarchies in Fig. 1(left), we note that each hierarchy is characterized by particles belonging to it, their constituent particles, and relevant interactions. For instance, atomic nuclei belong to the nucleus hierarchy, and their constituent particles are nucleons, where the relevant interactions are nuclear forces, a form of strong interactions mediated by mesons such as pions. One feature is that a constituent particle itself (nucleon) is a composite particle made of quarks (effectively three constituent quarks) and gluons. Interactions among quarks, which are bare strong interactions mediated by gluons, are substantially stronger than nuclear forces. The quark-gluon degrees of freedom are usually hidden in nuclei.

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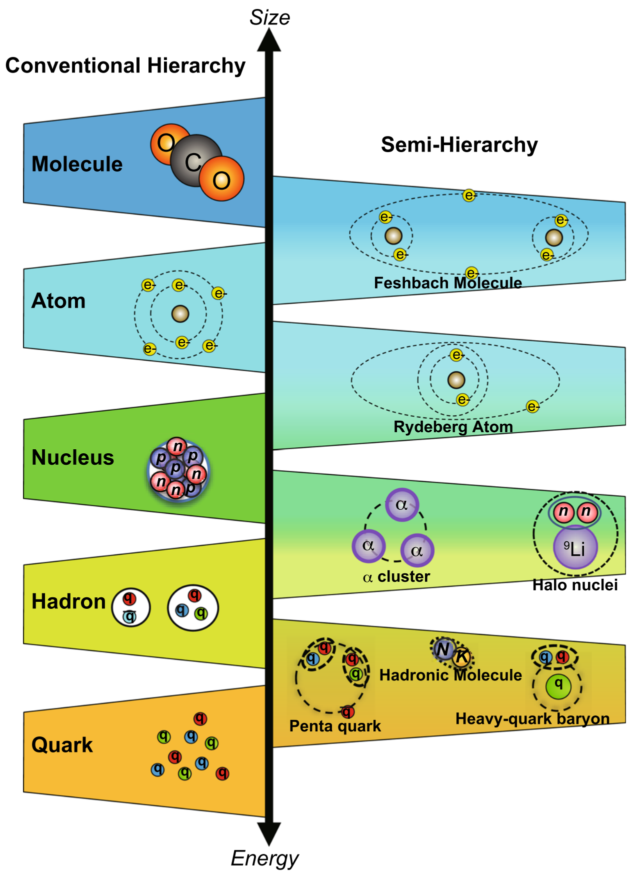

Fig. 1 (Left) Conventional quantum-particle hierarchies are depicted: quarks, hadrons, nuclei, atoms, and molecules are schematically shown from the bottom to the top according to their size scales. The energy scale is, on the other hand, higher from the top to the bottom. (Right) Between conventional hierarchies, we find semi-hierarchy, to which exotic cluster states belong

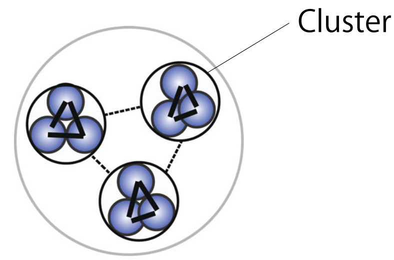

Fig. 2 A concept of cluster is depicted schematically. In this example, a whole particle is formed by the three clusters. Each cluster is a composite particle made of smaller particles. Interactions inside a cluster (solid thick lines) are significantly stronger than inter-cluster interactions (dashed lines)

As such, the nuclear and hadron(nucleon) hierarchies are distinctively separated.

Clusters are composite particles formed of smaller particles in the underlying hierarchy and behave as units of a larger particle. The forces among the smaller particles inside a cluster are significantly stronger than those among clusters inside a larger particle, as schematically shown in Fig. 2. For instance, nucleons inside an atomic nucleus can be regarded as clusters. As mentioned, the interactions inside a nucleon (cluster) mediated by gluons are substantially stronger than the nuclear forces (interactions among nucleons (inter-cluster interactions)) inside a nucleus. In general, the interactions inside a cluster are strong, while those outside are weak. Clusters and relevant interactions provide characteristic features of each hierarchy and relate the two neighboring hierarchies. Hence, understanding clusters is key to comprehending the hierarchical structure of quantum particles.

On the right-hand side of Fig. 1, we illustrate a range of quantum particles which do not belong to the conventional hierarchies. We call such a hierarchy *semi-hierarchy* since particles in such hierarchies are transitional between conventional hierarchies. A particle in a semi-hierarchy is often weakly bound, and its energy (mass) is between those in the neighboring conventional hierarchies, as we discuss in Sect. 3. It often contains novel clusters such as diquark(s), meson(s), dineutron(s), or correlated atoms at the unitary limit.

For instance, baryons called pentaquarks, such as P_c (4380), P_c(4450), and P_c(4312) observed in LHCb, CERN [2, 3], can be located in the semi-hierarchy between the quark- and the hadron hierarchies. Four-quark systems, such as $X(3872)$ [4] and $Y(4260)$ [4], can also be categorized in this semi-hierarchy. Some of these exotic baryons may contain *diquark*, which is not a conventional hadron nor free quarks. We may regard diquark as a cluster. $X(3872)$, on the other hand, is dominated by a hadronic molecule made of D and $\bar{D}^*$ mesons. Reviews on the exotic hadrons and their possible structures are found in [5–7]. Some of the articles in this Topical Article Collections (TAC) on *Clustering as a Window on the Hierarchical Structure of Quantum Systems* discuss exotic hadrons such as diquark and hadronic molecules [8,9].

α clusters belong to the semi-hierarchy between the hadron (nucleon) and nucleus hierarchies [10–12]. As discussed in Sect. 3, α-cluster nuclei appear near the α decay threshold. They can be regarded as being composed of α particles, different from conventional nuclei described as a bound system composed of protons and neutrons. Note that the α particle itself is a composite particle made of four nucleons (two protons and two neutrons). The second 0^+ state of ^{12}C, called the Hoyle state, is a representative example of α clusters as this state is described as a three-α particle state, rather than a twelve-nucleon system. The Hoyle state plays a crucial role in synthesizing carbon-12 from three helium-4 particles (α particles) in the star evolution: $3^4\mathrm{He}\rightarrow {}^{12}\mathrm{C}+\gamma$ [13]. Recently, it was found that ^{10}Be has a molecular structure made of two α's bonded by two neutrons, as demonstrated in Ref. [14]. This is considered a new type of α cluster, and as such, this also belongs to this semi-hierarchy.

The nucleus-hadron semi-hierarchy also contains two-neutron halo nuclei. Such a halo nucleus comprises the core nucleus and the two weakly-bound neutrons: for instance, ^{11}Li has a structure composed of the ^{9}Li core and the two halo neutrons [15–17]. This three-body system has characteristics of the Borromean ring, where any of the constituent two-body systems have no bound state while it is bound as a three-body system. The two neutrons in ^{11}Li have been found spatially correlated, called *dineutron* [18,19]. Hence, we may consider ^{11}Li as a two-body system composed of the ^{9}Li core and the dineutron. In this sense, we may call a dineutron-cluster nucleus, as the dineutron behaves like a constituent particle. In this Topical Article Collections, the dineutron will be discussed [20].

A Feshbach molecule [21] can be a particle in the semi-hierarchy between the atom and molecule hierarchies. As we see later, we find common features of the cluster properties in the semi-hierarchies in different scales. On the other hand, we note that each hierarchy has its own properties. This Topical Article Collection aims to discuss common features and characteristic differences in each semi-hierarchy particle and the relevant interactions.

2 Clustering via charge neutralization

Cluster formation can be associated with the neutralization of charges carried by particles in a system, which can provide one of the mechanisms of clusterization and the formation of quantum-particle hierarchy. For instance, let us consider the hadron and nucleus hierarchies governed by strong interactions. A nucleon is made of quarks and gluons. We find a distinctive difference between the interactions between quarks mediated by gluons and those between nucleons mediated by mesons. Much weaker interactions between nucleons are inherent to the *neutralization* of the color charges among quarks and gluons. Accordingly, an atomic nucleus can be described as a many-body system made of nucleons, and the quark degrees of freedom are mostly hidden and confined inside a nucleon. The interactions among the nucleons can be regarded as inter-cluster interactions, namely, *nuclear force*. As mentioned, inter-cluster interactions are significantly weaker than those among particles inside the cluster. Consequently, the nucleus and hadron (nucleon) hierarchies are totally separated.

We find analogies in the atom-molecule hierarchies governed by electromagnetic (EM) interactions. An atom is formed by the electric forces between the nucleus and the surrounding electrons. The EM interactions between atoms in a molecule are much weaker than those inside the atom. This is because the electric charges are neutralized in one atom, where the positive charge in the nucleus is balanced with the negative charges of the electrons. Inter-atomic interactions in a molecule are inherent to the residual EM ones, such as Van der Waals forces, which we can consider as a slight incompleteness of the charge neutralization.

In a semi-hierarchy, charge neutralization, or in more general, the neutralization of some degrees of freedom, also plays a pivotal role. For an α cluster, the constituent α particle has the spin $S = 0$ and isospin $T = 0$: neutralizations of spin-isospin are realized. Because the spin and isospin operators are the source (charge) of the pion, the pion exchange force, important for nuclear binding, does not exist between α particles. Accordingly, the α particles behave more independent in a nucleus, where the α–α interaction is much weaker than the nuclear forces bonding two protons and two neutrons inside an α particle.

A characteristic feature of the neutralization of charges (degrees of freedom) in semi-hierarchy particles is that the neutralization is often much less complete compared to the conventional hierarchies. For instance, a diquark, in most cases, has spin $S = 0$, while the color charge is non-zero. A dineutron has spin $S = 0$, while the isospin $T = 1$. While a semi-hierarchy particle holds characteristics of the conventional hierarchy, the difference between the interactions inside a cluster and those between the clusters is less distinctive than the conventional one. Accordingly, some features due to a conventional hierarchy remains. For instance, a pentaquark may contain the five-quark component besides the diquark-cluster component.

It is crucial to understand the inter-cluster interactions in terms of particle-particle interactions inside a cluster. This corresponds to understanding the interactions in a specific hierarchy from the more fundamental underlying hierarchy. The inter-cluster interaction can be a residual interaction arising from more fundamental forces. For instance, one of the important subjects in current nuclear physics is understanding nuclear force in terms of quark-gluon degrees of freedom, namely by QCD (Quantum Chromo Dynamics). Such a study bridges over the nucleus and hadron hierarchies. Three-body forces in nuclei and atoms may also provide keys to bridge over hierarchies. For instance, Fujita-Miyazawa type three-nucleon force [22] involves the excitation of a nucleon to the Δ particle, where quark degrees of freedom come into play. This Topical Article Collection discusses the nuclear and atomic three-body forces [23] and experiments on hyperon-nucleon interactions [24]. We also show theoretical approaches for exotic hadrons based on QCD [8].

3 Threshold and semi-hierarchy

The threshold, specifically the energy threshold, is a pivotal concept in understanding the semi-hierarchy. It represents the energy level where the corresponding energy is required for a bound composite particle to decay into its constituents. For instance, the energy level at which a triton decays into a deuteron and a proton is known as the two-particle (deuteron-proton) threshold. Similarly, the three-particle threshold is the one at which a triton decays into a proton and two neutrons.

Figure 3 schematically illustrates that semi-hierarchy particles often appear near the energy threshold in decaying into constituent clusters. The middle of Fig. 3 shows the twelve-nucleon system as an example of the hadron-nucleus semi-hierarchy. The lowest state is the ground state of ^{12}C in the nucleus hierarchy. When it is excited fully, ^{12}C disintegrates into six protons and six neutrons as hadron gas, which belongs to the hadron hierarchy. This illustrates a transition from the nucleus hierarchy to the hadron hierarchy by putting energy. However, when the excitation energy is tuned close to the threshold for decaying into three α particles (7.27 MeV), we find the Hoyle state, the second 0^+ state of ^{12}C ($E_x = 7.65$ MeV), predicted originally by Fred Hoyle [13].

The Hoyle state is nothing but the α-cluster state belonging to the hadron-nucleus semi-hierarchy. Here, the threshold plays a critical role in forming α clusters. Indeed, Ikeda et al. demonstrated that α clusters appear near the relevant α decay threshold [10], as shown in the Ikeda diagram in Fig. 4(left). We also note that low-energy α capture reactions due to such α clusters play a critical role in synthesizing carbon, oxygen, and neon elements in the universe. The threshold represents the boundary between the nucleus and hadron hierarchies; therefore, the semi-hierarchy is relevant to the threshold. In Ref. [10], the relevance of thresholds to α cluster formation was hypothetical. More fundamental understanding of the role of the thresholds in α cluster formation is still a challenge in current nuclear physics.

In the hadron-quark sector, let us take an example of Λ hyperon, which belongs to the hadron hierarchy, as shown in Fig. 3(left). When thoroughly excited (at high temperatures where color charges are deconfined), it becomes a quark soup belonging to the quark hierarchy. However, if the excitation energy is tuned close to the $N\bar{K}$ decay threshold, $\Lambda(1405)$ is formed as a hadronic molecule composed of $\bar{K}$ and a nucleon rather than consisting of three quarks. More recently, several hadronic molecules have been observed. The $X(3872)$ particle is considered to have a hadronic-molecular structure made of $D\bar{D}^*$, and its mass is almost at the threshold decaying into $D + \bar{D}^*$, as shown in Fig.4(right).

For the atom and molecule hierarchies, as shown in Fig. 3(right), we consider the phase transition of ultra-cold atoms. In the uppermost, we have ultra-cold fermionic atoms, while at the bottom, we have a molecule composed of two fermionic atoms. When we tune the interaction between the two fermions such that the s-wave scattering length, a, becomes infinitely large at the unitary limit, a Feshbach molecule can be formed. Such a molecule represents a state

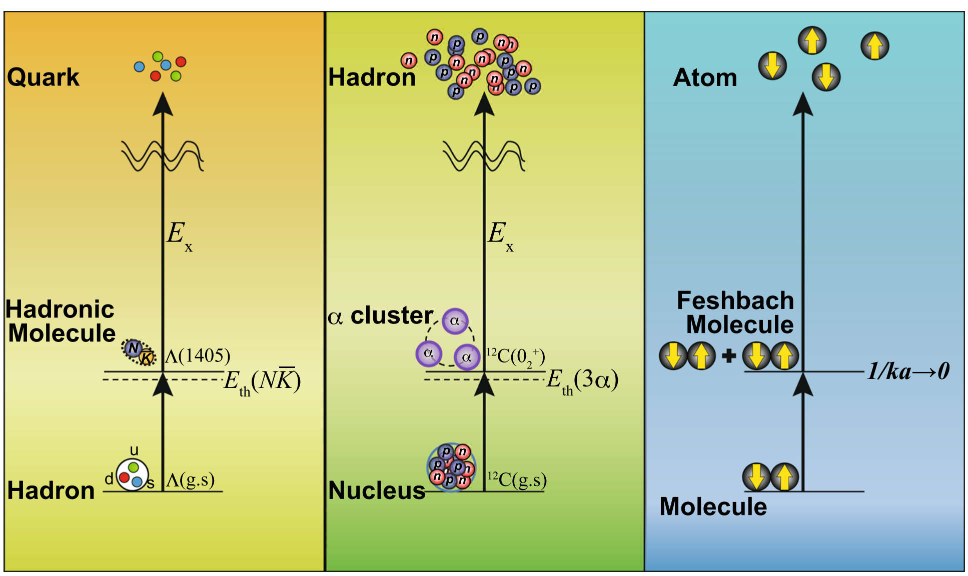

Fig. 3 (Left) $\Lambda(1405)$ is shown schematically as an example of a semi-hierarchy particle between the hadron-quark hierarchies. $\Lambda(1405)$, an excited state of the Λ hyperon, is regarded a $N\overline{K}$ hadronic molecule, whose mass is close to the $\bar{N}K$ decay threshold. (Middle) The Hoyle state, the second 0^+ state of ^{12}C near the three α decay threshold, is shown as an example of a semi-hierarchy particle between the nucleus-hadron hierarchies. The Hoyle state is regarded as a α cluster nucleus composed of three α particles. (Right) Feshbach molecule is schematically shown as an example of a semi-hierarchy particle between the molecule-atom hierarchies. The Feshbach molecule appears near the unitary limit where $1/ka$ is close to zero, where k, a stand for the Fermi momentum and the $s-$wave scattering length of the atoms, respectively

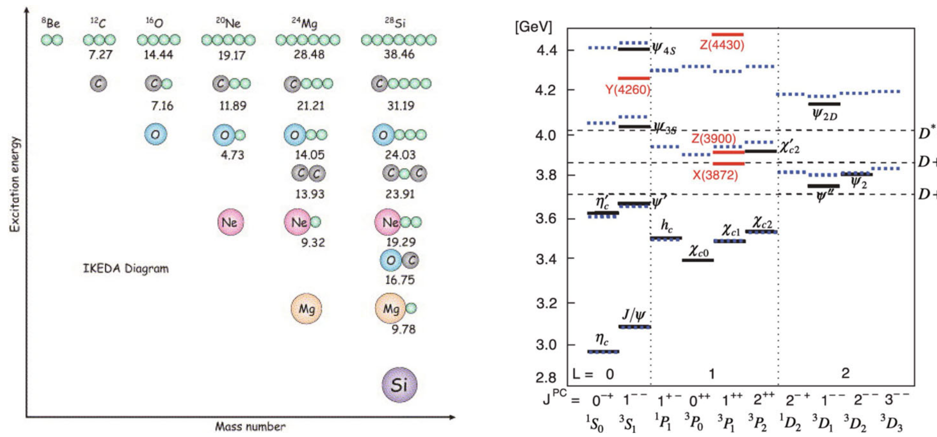

Fig. 4 (Left) Ikeda diagram which shows that α clusters (green spheres) and other molecule-like structures (spheres denoting C, O, Ne, Mg) in nuclei appear at the excitation energy very close to the relevant cluster decay threshold [10]. The figure is adopted from Ref. [11]. (Right) Comparison of the observed states (solid bars) and quark model calculations (dashed bars) for the charmonium sector. Typical X, Y, Z states are shown by red lines. Figure is adopted from Ref. [5]

in the boundary between the atom and molecule hierarchies, and as such it belongs to the semi-hierarchy.

This comparison of semi-hierarchy particles in different scales in Fig. 3 demonstrates that one can understand the semi-hierarchy in a unified picture with the keyword threshold. Theoretical aspects of the role of thresholds are discussed in the article in this Topical Article Collection [9]. We have common features in the boundary (semi-hierarchy region) between the hadron-quark, nucleus-hadron, and atom-molecule hierarchies. This implies that one may study threshold phenomena and semi-hierarchical properties of hadrons and nuclei by ultra-cold atom experiments, where the interactions and conditions are controllable. Such experiments have been performed recently, as shown in the paper in this Topical Article Collection [25]. We also note that some matter properties, such as viscosity, in different scales can be discussed in a more unified way as they are relevant to semi-hierarchy. Viscosity properties in ultra-cold atoms and quark-gluon plasma are discussed in Ref. [26] in this Topical Article Collection.

For a two-body system made of two clusters, one can understand the threshold phenomena and their universality in terms of the low-energy scattering between the two clusters. Indeed, a state just above the decay threshold corresponds to the scattering of one cluster relative to the other at very low energies. In the asymptotic region, where the potential energy is zero, sufficiently small momentum k (low energy $E = (\hbar k)^2/(2\mu)$) corresponds to the large de Broglie wavelength $\lambda\!\!\!^{-} = 1/k$. When $\lambda\!\!\!^{-} \gg r_0$, where r_0 is the range of the interaction, the scattering can be understood solely by the two parameters: the $s-$wave scattering length, a, and the effective range, $r_{\rm eff}$, independent of the detail of the interactions. In the extreme case, where $|a| \gg r_0(\sim r_{\rm eff})$, only a dictates the scattering. The weakly bound system, where the momentum at the asymptotic region is described as $k = i/\lambda\!\!\!^{-}$, is also understood with these two parameters. For an attractive interaction, a positive a corresponds to a bound state, while a negative one corresponds to an unbound state (continuum or a virtual state). Universality can be found, for instance, for weakly bound systems: the wave function is written as

$$\Phi(r) \sim \frac{e^{-r/a}}{r}, \tag{1}$$

with the energy $E = -\hbar^2/(2\mu a^2)$. Examples are found in one-neutron halo nuclei with an s-wave halo neutron, such as ^{11}Be, ^{19}C, deuteron, as well as the weakly-bound atomic systems, such as the ^{4}He dimer. However, some deviations from this simple formula appear due to the correction of $r_{\rm eff}/a$ and higher angular momentum (see also the discussion in Sect. 4). For the unbound two-body system, we find a virtual state at $E = -\hbar^2/(2\mu a^2)$ but with a negative scattering length, which occurs due to a pole on the second sheet in the complex energy plane [27,28].

The concept of scattering length and effective range was initially introduced for the two nucleon systems, pn, pp, and nn systems [29,30]. The nuclear force is short-range, and the wavelength $\lambda\!\!\!^{-}$ is significantly larger than the range of the nuclear interaction. Hadronic molecules, such as $\Lambda(1405)$, have also such a property. In such cases, the scattering phenomena become independent of the specific shape of the potential, leading to the universality of the threshold phenomena regardless of the scales and the type of interactions. The ultra-cold atoms can thus simulate some properties of nuclei and hadrons.

4 Degree of clusterization

Particles in the semi-hierarchy tend to have a mixture of a cluster state and the ones inherent to smaller, more fundamental particles. For instance, α clusters, such as the Hoyle state, have a mixture of an α-cluster state (three α's) and nucleonic states (12-nucleon states). This is because a particle in the semi-hierarchy partially holds a property of the original conventional hierarchy: In this example, the Hoyle state has a property of the nucleus hierarchy. The degree of the α cluster can be quantified as a spectroscopic factor. For instance, Otsuka et al. evaluated the overlap between the Hoyle state and the three-α cluster state using the ab initio type shell model [31], which is reviewed in one of the articles in this Topical Article Collection [32].

Steven Weinberg introduced the concept of compositeness and discussed whether the deuteron is a composite made of a proton and a neutron or elementary (= just one bare particle or six-quark compact state) [33]. This corresponds to quantifying the degree of clusterization in the deuteron. The field renormalization constant Z, called elementarity, was introduced, while the parameter $X(= 1 - Z)$ represents a degree of compositeness, a measure of the proton-neutron structure in the deuteron. Note that the deuteron mass is close to the pn decay threshold: the binding energy is only 2.22 MeV, much smaller than the typical value of 8 MeV for normal stable nuclei.

X is the overlap of the deuteron and the pn scattering state,

$$X = 1 - Z \equiv \int d\boldsymbol{p} |\langle \boldsymbol{p}|B\rangle|^2, \tag{2}$$

where $|B\rangle$ represents the deuteron state, and $\langle \boldsymbol{p}|$ shows the pn scattering state with momentum $\boldsymbol{p}$. Since the pn scattering state is composed of the independent proton and neutron, it represents the cluster component (p-n structure). Naturally, $X = 1$ at the threshold. Hence, one can use X or $1 - Z$ to evaluate the degree of clusterization. In Ref. [33], Weinberg showed for a weakly bound system (near-threshold state),

$$a = \frac{2(1 - Z)}{2 - Z} R + O(m_\pi^{-1}), \tag{3}$$

$$r_{\rm eff} = \frac{(-Z)}{1 - Z} R + O(m_\pi^{-1}) \tag{4}$$

where a is the pn s-wave scattering length, $r_{\rm eff}$ is the effective range, and m_π is the pion mass. R represents the deuteron radius expressed by

$$R = \frac{\hbar c}{\sqrt{2\mu B}} = 4.318\,\text{fm}, \tag{5}$$

where μ and B are the reduced mass of the pn system and the deuteron binding energy $B = 2.2245662(4)$ MeV, respectively. This parameter R also represents the decay length of the asymptotic wave function. Ref. [33] demonstrates that the experimental values of a, r, and R are consistent with $Z \sim 0$ and $X \sim 1$, corresponding to a composite-particle picture of the deuteron made of p and n (two cluster particles).

We can confirm this argument on the deuteron by using the evaluated values of the pn scattering length and the effective range [34]:

$$a = 5.4112(15)\,\text{fm}, \tag{6}$$

$$r_{\rm eff} = 1.7436(19)\,\text{fm}. \tag{7}$$

Hence, $R \sim a$, and a small difference ($\sim$ 1 fm) between these two values is of the order of m_π^{-1}, close to $r_{\rm eff}$.

This demonstrates that the overlap of a particular state with its scattering states (decaying states), as in the parameter X, can represent the degree of cluster formation inside the particle and thus represent how well the hierarchy (in this case, nucleus hierarchy) is formed. The parameter X corresponds to the spectroscopic factor used widely in nuclear physics. Studying compositeness and spectroscopic factors is essential in understanding the cluster phenomena and hierarchical structure of quantum particles. $Z \sim 0$ corresponds to the state near the decay threshold, where $|a| \sim R \gg r_{\rm eff}$. The analysis of compositness is useful in discussing the properties of the semi-hierarchy we are focusing on here. In this Topical Article Collection, Kinugawa and Hyodo discuss the compositeness in describing exotic hadrons, such as hadronic molecules [35]. Recent applications and development of the compositeness concept are reviewed in Refs. [36,37]. Otsuka et al. discuss the spectroscopic amplitude of α clusters [32].

5 Clustering, phase transition, and crossover

The clustering of quantum particles is also discussed in terms of phase transition. For instance, nuclear matter, which is considered a quantum liquid, can be transitioned into a phase where α clusters appear when the density (ρ) becomes lower than a specific density [39,40]. Such a phase transition relevant to clustering corresponds to a transition from one hierarchy to another (semi) hierarchy, providing a universal feature for quantum matter.

The QCD phase diagram, a key to understanding hadronic quantum matter, illustrates symbolic phase transitions associated with clustering [38]. Figure 5 [38] shows the QCD phase diagram in the $\mu - T$ plane, where μ is the chemical potential and T is the temperature. The most notable clustering phenomenon and important phase transition in the QCD diagram is the hadronization from a quark-gluon plasma phase to a hadronic phase. According to the lattice QCD calculation at $\mu \sim 0$, this phase transition is second-order occurring at $T \sim 200$ MeV [41]. There is expected to exist a QCD critical point beyond which the phase transition may become first-order. The experimental evidence for the QCD critical point has yet to be found, and obtaining it is one of the most important goals in the high-energy heavy ion collision experiments at CERN (ALICE) and RHIC. Ref. [38] also explains the liquid–gas phase transition of the nuclear matter, which is the first order near $T = 0$. The phase transition is considered weakened towards higher T to reach a second-order critical point.

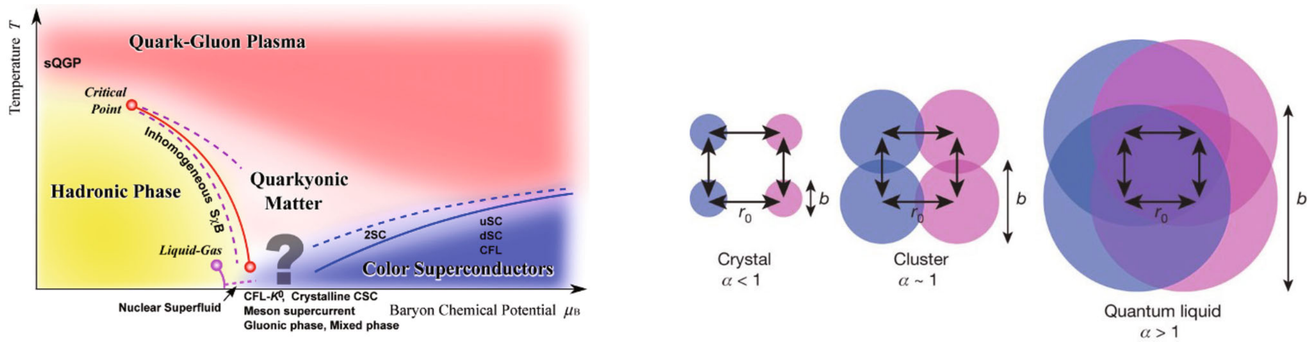

Fig. 5 (Left) QCD diagram in terms of temperature T and chemical potential μ. The figure is adopted from Ref. [38]. (Right) Transition from a crystalline to a quantum liquid phase is schematically shown, which is assessed by the ratio of the dispersion of a nucleon distance b to a typical inter-nucleon distance r_0 (c.a. 1.2 fm). The figure is adopted from Ref. [39]

As mentioned, α clustering is also discussed regarding a phase transition from the quantum liquid [39,40,42]. Ebran et al. [39] used a density functional theory to identify the origin of the clustering: using two different interactions (non-relativistic Skyrme SLy4, relativistic functional DD-ME2), both of which can explain the ground-state properties of the ^{20}Ne such as its matter and proton distribution radii, and binding energy, but only the calculation with the DD-ME2 shows an α cluster structure. They chose the ^{20}Ne ground state as this state is located *not* near the α-decay threshold, which assures that the clustering is not due to the threshold effect. They found that the α clustering occurs not for SLy4 but for DD-ME2 due to its deeper potential, which facilitates the localization of the single-particle nucleon density, a key to clustering. As shown in Fig. 5(right), they found that the degree of the localization α $(= b/r_0$, where b is the dispersion of the nucleon wave function, and r_0 the mean distance between nucleons) for the $DD-ME2$ calculation shows $\alpha \sim 1$ which favors α-clustering, while the quantum liquid should require $\alpha > 1$ and the crystal phase should require $\alpha < 1$. Namely, they consider the clustering transitional between the quantum liquid and crystal, whose criterion may be universal in fermionic many-body systems.

In ultracold Fermi gases, it is known that the quantum system continuously evolves from a normal Fermi gas (NF) to a normal Bose gas (NB) depending on the strength of the attractive interaction, without undergoing a phase transition above the superfluid transition temperature T_c in the BCS-BEC crossover regime [43] as shown in Fig. 6. The intermediate region between NF and NB is called the pseudogap (PG) regime, where pairing fluctuations are significant. Below T_c, the system enters the superfluid phase, and the ground state evolves smoothly from BCS-type superfluidity to Bose-Einstein condensation without a phase transition, known as the BCS-BEC crossover. The crossover from NF to PG may be considered an example of clustering, while the crossover from PG to NB can be interpreted as an example of elementary particle confinement. Therefore, the PG regime corresponds to the intermediate hierarchical region between atoms and molecules, as discussed in Sect. 3, and shares common characteristics with other hierarchical structures. Horikoshi et al. discuss such studies in this Topical Article Collection [25].

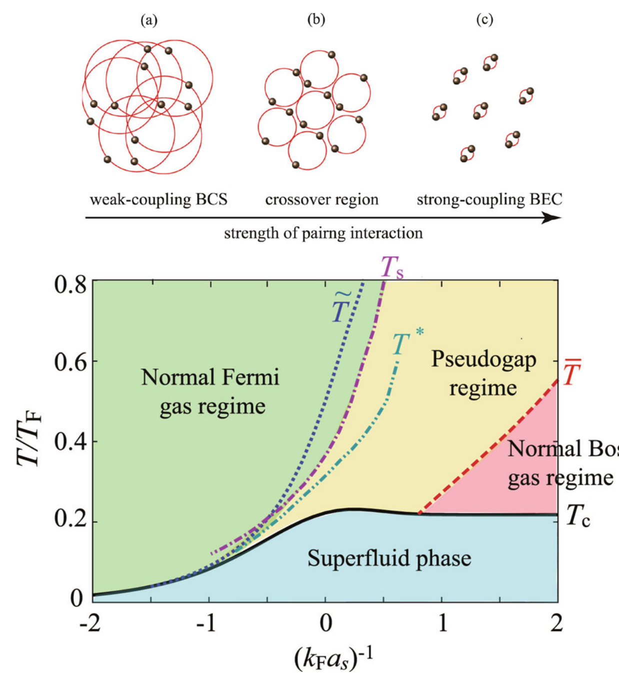

Fig. 6 The phase diagram of the BCS-BEC crossover regime. The solid black curve represents the superfluid transition temperature T_c. The dashed red curve indicates a characteristic temperature $\bar{T}$, marking the crossover region between the normal Bose gas regime and the pseudo-gap regime. The other curves (blue, purple, and green) correspond to different characteristic temperatures that define the crossover region between the normal Fermi gas regime and the pseudo-gap regime. These three curves are determined based on changes in different observable quantities. For more details, see Ref. [43], from which this figure is adapted

The region of $(k_F a_s)^{-1} = 0$, a unitary regime, corresponds to the semi-hierarchy region between atoms and molecules, as discussed in Sect. 3, and has some common features with other hierarchies. For instance, the dineutron correlation has a common feature with the BCS-BEC crossover, discussed theoretically by Matsuo [44]. The thermodynamic properties of neutron stars were investigated through ultra-cold atoms [43,45]. The ultra-cold Fermi gas experiments, where the interactions are tunable, and relevant theories can indeed study the properties of clustering in other hierarchies where the tuning of interactions is impossible. Horikoshi et al. discuss such a study in this Topical Article Collection [25].

6 Summary and perspectives

The question of why the hierarchical structure of microscopic quantum particles - quarks, hadrons, nuclei, atoms, and molecules - exists is fundamental in natural science. However, it has yet to be investigated much in the past partially because each hierarchy appears completely separate, and no clear relationships have been identified. On the other hand, we note that common features that transcend the boundaries of different hierarchies have been recognized in the concept of *clusters*. From 2018 to March 2023, we organized a consortium among researchers in Japan who study quark-gluon plasma physics, hadron physics, nuclear physics, and atomic and molecular physics, entitled "Clustering as a window on the hierarchical structure of quantum systems".

This Topical Article Collection aims to present the recent studies of clusters in the context of the semi-hierarchies based on the research through this consortium. This article is a summary of this Topical Article Collection to show the aim and points of our project. We defined semi-hierarchy as a boundary between the conventional matter hierarchies. In the semi-hierarchies, we find exotic cluster phenomena. This boundary is also the one between the closed and open quantum systems. We also showed that the key elements are thresholds, inter-cluster interactions, neutralization of degrees of freedom (charges), and degree of clusterization (spectroscopic factor, compositness). We showed examples of many-body systems with common features irrespective of scales, such as ultra-cold atoms, weakly-bound/unbound nuclei, and hadronic molecules, where the s-wave scattering length is sufficiently large compared to the range of the interaction and the size of the system.

In our project, we succeeded in doing collaborative research that transcends the matter hierarchies. For instance, the study of three-nucleon forces in nuclei has triggered three- and four-body atomic forces in ultra-cold atom experiments [23,46]. We achieved several mile-stone experiments, including the observation of novel clusters such as multi-neutron systems [47–49] and mesonic nuclei [50,51], research on inter-cluster forces such as precise measurement of hyperon-nucleon interactions [24,52–54], theoretical research based on first-principles calculations and lattice QCD [55–57], and realization of a quantum simulator using ultra-cold atom experiments [25]. Some of these results are discussed in the articles in this Collection.

The physics of clusters and semi-hierarchy will likely be central topics in many-body physics. Owing to the advance of accelerator-based science, ultra-cold atom experiments, and many-body theories using ultra-fast supercomputers will boost such studies. In nuclear and hadron physics, we expect observations of more exotic multi-neutron systems, hypernuclei, hadronic molecules, mesic nuclei, tetraquarks, and pentaquarks. Strong interactions, such as three-nucleon forces and hyperon-nucleon interactions, will be elucidated more accurately. Understanding such interactions using ultra-cold atom experiments that are much more controllable can be useful as simulators for nuclei and hadrons. Theoretical understanding of interactions from more fundamental interactions in the underlying hierarchy will make progress. Accordingly, the mechanism of forming hierarchical structures in quantum particles is expected to be clarified.

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References

1. S. Navas et al., Review of particle physics. Phys. Rev. D **110**(3), 030001 (2024). https://doi.org/10.1103/PhysRevD.110.030001
2. R. Aaij et al., Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \to J/\psi K^- p$ decays. Phys. Rev. Lett. **115**, 072001 (2015). https://doi.org/10.1103/PhysRevLett.115.072001

3. R. Aaij et al., Observation of a narrow pentaquark state, $P_c(4312)^+$, and of the two-peak structure of the $P_c(4450)^+$. Phys. Rev. Lett. **122**, 222001 (2019). https://doi.org/10.1103/PhysRevLett.122.222001
4. S.-K. Choi et al., Observation of a narrow charmoniumlike state in exclusive $B^\pm \to K^\pm\pi^+\pi^- j/\psi$ decays. Phys. Rev. Lett. **91**, 262001 (2003). https://doi.org/10.1103/PhysRevLett.91.262001
5. A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, S. Yasui, Exotic hadrons with heavy flavors: X, Y, Z, and related states. Prog. Theor. Exp Phys. **2016**(6), 062–01 (2016). https://doi.org/10.1093/ptep/ptw045 https://academic.oup.com/ptep/article-pdf/2016/6/062C01/34838356/ptw045.pdf
6. F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, Hadronic molecules. Rev. Mod. Phys. **90**, 015004 (2018). https://doi.org/10.1103/RevModPhys.90.015004
7. Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, S.-L. Zhu, Pentaquark and tetraquark states. Prog. Part. Nucl Phys. **107**, 237–320 (2019). https://doi.org/10.1016/j.ppnp.2019.04.003
8. E. Hiyama, T. Doi, Cluster phenomena using few-body and Lattice QCD theories Eur. Phys. J. A **61**, 270 (2025). https://doi.org/10.1140/epja/s10050-025-01747-7
9. A. Hosaka, Y. Kanada-En'yo, Y. Yamaguchi, Clustering in nuclei and hadrons. Eur. Phys. J. A **61**(5), 103 (2025). https://doi.org/10.1140/epja/s10050-025-01530-8
10. K. Ikeda, N. Takigawa, H. Horiuchi, The systematic structure-change into the molecule-like structures in the self-conjugate 4n nuclei. Prog. Theor. Phys. Suppl. **E68**, 464–475 (1968). https://doi.org/10.1143/PTPS.E68.464 https://academic.oup.com/ptps/article-pdf/doi/10.1143/PTPS.E68.464/5216547/E68-464.pdf
11. W. von Oertzen, M. Freer, Y. Kanada-En'yo, Nuclear clusters and nuclear molecules. Phys. Rep. **432**(2), 43–113 (2006). https://doi.org/10.1016/j.physrep.2006.07.001
12. M. Freer, The clustered nucleus—cluster structures in stable and unstable nuclei. Rep. Prog. Phys. **70**(12), 2149 (2007). https://doi.org/10.1088/0034-4885/70/12/R03
13. F. Hoyle, On nuclear reactions occuring in Very Hot STARS. I. The synthesis of elements from carbon to nickel. Astrophys. J. Suppl. **1**, 121 (1954). https://doi.org/10.1086/190005
14. P.J. Li, D. Beaumel, J. Lee, M. Assié, S. Chen, S. Franchoo, J. Gibelin, F. Hammache, T. Harada, Y. Kanada-En'yo, Y. Kubota, S. Leblond, P.F. Liang, T. Lokotko, M. Lyu, F.M. Marqués, Y. Matsuda, K. Ogata, H. Otsu, E. Rindel, L. Stuhl, D. Suzuki, Y. Togano, T. Tomai, X.X. Xu, K. Yoshida, J. Zenihiro, N.L. Achouri, T. Aumann, H. Baba, G. Cardella, S. Ceruti, A.I. Stefanescu, A. Corsi, A. Frotscher, J. Gao, A. Gillibert, K. Inaba, T. Isobe, T. Kawabata, N. Kitamura, T. Kobayashi, Y. Kondo, A. Kurihara, H.N. Liu, H. Miki, T. Nakamura, A. Obertelli, N.A. Orr, V. Panin, M. Sasano, T. Shimada, Y.L. Sun, J. Tanaka, L. Trache, D. Tudor, T. Uesaka, H. Wang, H. Yamada, Z.H. Yang, M. Yasuda, Validation of the ^{10}Be ground-state molecular structure using ^{10}Be$(p, p\alpha)^6$He triple differential reaction cross-section measurements. Phys. Rev. Lett. **131**, 212501 (2023). https://doi.org/10.1103/PhysRevLett.131.212501
15. I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi, N. Takahashi, Measurements of interaction cross sections and nuclear radii in the light p-shell region. Phys. Rev. Lett. **55**, 2676–2679 (1985). https://doi.org/10.1103/PhysRevLett.55.2676
16. P.G. Hansen, B. Jonson, The neutron halo of extremely neutron-rich nuclei. Europhys. Lett. **4**(4), 409 (1987). https://doi.org/10.1209/0295-5075/4/4/005
17. I. Tanihata, H. Savajols, R. Kanungo, Recent experimental progress in nuclear halo structure studies. Prog. Part. Nucl Phys. **68**, 215–313 (2013). https://doi.org/10.1016/j.ppnp.2012.07.001
18. T. Nakamura, A.M. Vinodkumar, T. Sugimoto, N. Aoi, H. Baba, D. Bazin, N. Fukuda, T. Gomi, H. Hasegawa, N. Imai, M. Ishihara, T. Kobayashi, Y. Kondo, T. Kubo, M. Miura, T. Motobayashi, H. Otsu, A. Saito, H. Sakurai, S. Shimoura, K. Watanabe, Y.X. Watanabe, T. Yakushiji, Y. Yanagisawa, K. Yoneda, Observation of strong low-lying $E1$ strength in the two-neutron halo nucleus ^{11}Li. Phys. Rev. Lett. **96**, 252502 (2006). https://doi.org/10.1103/PhysRevLett.96.252502
19. Y. Kubota, A. Corsi, G. Authelet, H. Baba, C. Caesar, D. Calvet, A. Delbart, M. Dozono, J. Feng, F. Flavigny, J.-M. Gheller, J. Gibelin, A. Giganon, A. Gillibert, K. Hasegawa, T. Isobe, Y. Kanaya, S. Kawakami, D. Kim, Y. Kikuchi, Y. Kiyokawa, M. Kobayashi, N. Kobayashi, T. Kobayashi, Y. Kondo, Z. Korkulu, S. Koyama, V. Lapoux, Y. Maeda, F.M. Marqués, T. Motobayashi, T. Miyazaki, T. Nakamura, N. Nakatsuka, Y. Nishio, A. Obertelli, K. Ogata, A. Ohkura, N.A. Orr, S. Ota, H. Otsu, T. Ozaki, V. Panin, S. Paschalis, E.C. Pollacco, S. Reichert, J.-Y. Roussé, A.T. Saito, S. Sakaguchi, M. Sako, C. Santamaria, M. Sasano, H. Sato, M. Shikata, Y. Shimizu, Y. Shindo, L. Stuhl, T. Sumikama, Y.L. Sun, M. Tabata, Y. Togano, J. Tsubota, Z.H. Yang, J. Yasuda, K. Yoneda, J. Zenihiro, T. Uesaka, Surface localization of the dineutron in ^{11}Li. Phys. Rev. Lett. **125**, 252501 (2020). https://doi.org/10.1103/PhysRevLett.125.252501
20. T. Nakamura, K. Hagino, Y. Kondo, Dineutron clusters (tentative)
21. F. Ferlaino, S. Knoop, R. Grimm, Ultracold Feshbach molecules. arXiv:0809.3920 (2008)
22. J.-I. Fujita, H. Miyazawa, Pion theory of three-body forces. Progress Theor. Phys. **17**(3), 360–365 (1957). https://doi.org/10.1143/PTP.17.360
23. S. Endo, E. Epelbaum, P. Naidon, Y. Nishida, K. Sekiguchi, Y. Takahashi, Three-body forces and Efimov physics in nuclei and atoms. Eur. Phys. J. A **61**(1), 9 (2025). https://doi.org/10.1140/epja/s10050-024-01467-4
24. K. Miwa, K. Nakazawa, H. Tamura, E. Hiyama, T. Takahashi, Nuclear systems with strangeness and baryon–baryon interactions. Eur. Phys. J. A **61**(6), 128 (2025). https://doi.org/10.1140/epja/s10050-025-01571-z
25. M. Horikoshi, Y. Ohashi, D. Kagamihara, Equation of state of quantum cluster matter. Eur. Phys. J. A **61**(6), 141 (2025). https://doi.org/10.1140/epja/s10050-025-01606-5
26. D. Kagamihara, M. Horikoshi, C. Nonaka, Y. Ohashi, K. Shigaki, Shear viscosity and hierarchical structure of matter. Eur. Phys. J. A **61**(7), 180 (2025). https://doi.org/10.1140/epja/s10050-025-01653-y
27. E. Braaten, H.-W. Hammer, Universality in few-body systems with large scattering length. Phys. Rep. **428**(5), 259–390 (2006). https://doi.org/10.1016/j.physrep.2006.03.001
28. H.-W. Hammer, L. Platter, Efimov states in nuclear and particle physics. Ann. Rev. Nucl. Part. Sci. **60**, 207–236 (2010). https://doi.org/10.1146/annurev.nucl.012809.104439
29. E. Fermi, L. Marshall, Interference phenomena of slow neutrons. Phys. Rev. **71**, 666–677 (1947). https://doi.org/10.1103/PhysRev.71.666
30. H.A. Bethe, Theory of the effective range in nuclear scattering. Phys. Rev. **76**, 38–50 (1949). https://doi.org/10.1103/PhysRev.76.38
31. T. Otsuka, T. Abe et al., α-clustering in atomic nuclei from first principles with statistical learning and the Hoyle state character. Nat. Commun. **13**(1), 2234 (2022). https://doi.org/10.1038/s41467-022-29582-0
32. T. Otsuka, N. Itagaki, A. Volya, Clustering in ab initio and other many-body theories (tentative)
33. S. Weinberg, Evidence that the deuteron is not an elementary particle. Phys. Rev. **137**, 672–678 (1965). https://doi.org/10.1103/PhysRev.137.B672
34. R.W. Hackenburg, Neutron-proton effective range parameters and zero-energy shape dependence. Phys. Rev. C **73**, 044002 (2006). https://doi.org/10.1103/PhysRevC.73.044002

35. T. Kinugawa, T. Hyodo, Compositeness of hadrons, nuclei, and atomic systems. Eur. Phys. J. A **61**(7), 154 (2025). https://doi.org/10.1140/epja/s10050-025-01548-y
36. T. Hyodo, Structure and compositeness of hadron resonances. Int. J. Mod. Phys. A **28**(29), 1330045 (2013). https://doi.org/10.1142/S0217751X13300457
37. J.A. Oller, New results from a number operator interpretation of the compositeness of bound and resonant states. Ann. Phys. **396**, 429–458 (2018). https://doi.org/10.1016/j.aop.2018.07.023
38. K. Fukushima, T. Hatsuda, The phase diagram of dense QCD. Rep. Prog. Phys. **74**(1), 014001 (2010). https://doi.org/10.1088/0034-4885/74/1/014001
39. J.-P. Ebran, E. Khan, T. Nikšić, D. Vretenar, How atomic nuclei cluster. Nature **487**(7407), 341–344 (2012). https://doi.org/10.1038/nature11246
40. P. Schuck, T. Sogo, G. Röpke, Quartetting in nuclear matter. Suppl. Prog. Theor. Phys. **196**, 56–71 (2012). https://doi.org/10.1143/PTPS.196.56 https://academic.oup.com/ptps/article-pdf/doi/10.1143/PTPS.196.56/5346946/196-56.pdf
41. Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, The QCD transition temperature: results with physical masses in the continuum limit. Phys. Lett. B **643**, 46–54 (2006). https://doi.org/10.1016/j.physletb.2006.10.021. arXiv:hep-lat/0609068
42. J.-P. Ebran, E. Khan, T. Nikšić, D. Vretenar, Cluster-liquid transition in finite, saturated fermionic systems. Phys. Rev. C **89**, 031303 (2014). https://doi.org/10.1103/PhysRevC.89.031303
43. Y. Ohashi, H. Tajima, P. van Wyk, BCS-BEC crossover in cold atomic and in nuclear systems. Prog. Part. Nucl Phys. **111**, 103739 (2020). https://doi.org/10.1016/j.ppnp.2019.103739
44. M. Matsuo, Spatial structure of neutron Cooper pair in low density uniform matter. Phys. Rev. C **73**, 044309 (2006). https://doi.org/10.1103/PhysRevC.73.044309
45. M. Horikoshi, M. Koashi, H. Tajima, Y. Ohashi, M. Kuwata-Gonokami, Ground-state thermodynamic quantities of homogeneous spin-$1/2$ Fermions from the BCS region to the unitarity limit. Phys. Rev. X **7**, 041004 (2017). https://doi.org/10.1103/PhysRevX.7.041004
46. K. Honda, Y. Takasu, Y. Haruna, Y. Nishida, and Y. Takahashi, Exploring the strongly interacting regime of effective multibody interactions in a trapped ultracold-atom system, Phys. Rev. A **111**, 033303 (2025). https://doi.org/10.1103/PhysRevA.111.033303
47. K.J. Cook, T. Nakamura, Y. Kondo, K. Hagino, K. Ogata, A.T. Saito, N.L. Achouri, T. Aumann, H. Baba, F. Delaunay, Q. Deshayes, P. Doornenbal, N. Fukuda, J. Gibelin, J.W. Hwang, N. Inabe, T. Isobe, D. Kameda, D. Kanno, S. Kim, N. Kobayashi, T. Kobayashi, T. Kubo, S. Leblond, J. Lee, F.M. Marqués, R. Minakata, T. Motobayashi, K. Muto, T. Murakami, D. Murai, T. Nakashima, N. Nakatsuka, A. Navin, S. Nishi, S. Ogoshi, N.A. Orr, H. Otsu, H. Sato, Y. Satou, Y. Shimizu, H. Suzuki, K. Takahashi, H. Takeda, S. Takeuchi, R. Tanaka, Y. Togano, J. Tsubota, A.G. Tuff, M. Vandebrouck, K. Yoneda, Halo structure of the neutron-dripline nucleus ^{19}B. Phys. Rev. Lett. **124**, 212503 (2020). https://doi.org/10.1103/PhysRevLett.124.212503
48. M. Duer et al., Observation of a correlated free four-neutron system. Nature **606**(7915), 678–682 (2022). https://doi.org/10.1038/s41586-022-04827-6
49. Y. Kondo et al., First observation of ^{28}O. Nature **620**(7976), 965–970 (2023). https://doi.org/10.1038/s41586-023-06352-6
50. T. Yamaga, S. Ajimura et al., Observation of a $\overline{K}NN$ bound state in the ^{3}He$(K^-, \Lambda p)n$ reaction. Phys. Rev. C **102**, 044002 (2020). https://doi.org/10.1103/PhysRevC.102.044002
51. S. Ajimura et al., "K^-pp", a $\overline{K}$-meson nuclear bound state, observed in ^{3}He$(K^-, \Lambda p)n$ reactions. Phys. Lett. B **789**, 620–625 (2019). https://doi.org/10.1016/j.physletb.2018.12.058
52. K. Miwa, Others: measurement of the differential cross sections of the $\Sigma^- p$ elastic scattering in momentum range 470 to 850 MeV/c. Phys. Rev. C **104**, 045204 (2021). https://doi.org/10.1103/PhysRevC.104.045204
53. K. Miwa et al., Precise measurement of differential cross sections of the $\Sigma^- p \to \Lambda n$ reaction in momentum range $470 - 650$ mev/c. Phys. Rev. Lett. **128**, 072501 (2022). https://doi.org/10.1103/PhysRevLett.128.072501
54. T. Nanamura, K. Miwa et al., Measurement of differential cross sections for $\Sigma^+ p$ elastic scattering in the momentum range 0.44–0.80 gev/c. Progr. Theor. Exp. Phys. **2022**(9), 093–01 (2022). https://doi.org/10.1093/ptep/ptac101
55. E. Hiyama, K. Sasaki, T. Miyamoto, T. Doi, T. Hatsuda, Y. Yamamoto, T.A. Rijken, Possible lightest Ξ hypernucleus with modern Ξn interactions. Phys. Rev. Lett. **124**, 092501 (2020). https://doi.org/10.1103/PhysRevLett.124.092501
56. K. Sasaki, S. Aoki, T. Doi, S. Gongyo, T. Hatsuda, Y. Ikeda, T. Inoue, T. Iritani, N. Ishii, K. Murano, T. Miyamoto, $\Lambda\Lambda$ and $N\Xi$ interactions from lattice QCD near the physical point. Nucl. Phys. A **998**, 121737 (2020). https://doi.org/10.1016/j.nuclphysa.2020.121737
57. T. Iritani, S. Aoki, T. Doi, F. Etminan, S. Gongyo, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, T. Miyamoto, K. Sasaki, $N\Omega$ dibaryon from lattice QCD near the physical point. Phys. Lett. B **792**, 284–289 (2019). https://doi.org/10.1016/j.physletb.2019.03.050