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# NUCLEAR LEVEL DENSITY, QUANTUM CHAOS AND RELATED PHYSICS

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## Abstract.

The knowledge of the level density is necessary for understanding nuclear reactions and many applications of nuclear science. We use the moments method based on the statistical properties of mesoscopic systems (interactions between the constituents and resulting quantum chaos) and the many-body Hamiltonian in a truncated orbital space avoiding the diagonalization of huge matrices. We briefly discuss the method, general results in comparison with other approaches and available data, the underlying physics, collective enhancement phenomenon, and constant temperature description.

## 1. Introduction

It is not necessary to explain the importance of reliable knowledge of the level density for any mesoscopic system of interacting constituents. In nuclei, the level density is a regular part of treating various nuclear reactions, including technological and medical applications and nucleosynthesis in cosmos. In fact, the concept of the level density appeared historically as the first application of statistical ideas in nuclear theory [1, 2, 3].

The nucleus is a self-supporting conglomerate of strongly interacting nucleons. The starting point is the mean field created by averaging the action of individual particles. In the rough approximation, the particles independently occupy various orbitals in this field when the only limitation comes from Fermi-statistics. The excited states (particles and holes) are generated by combining individual promotions of nucleons to higher orbitals. The higher is the excitation the larger the number of possible ways to reach approximately the same total energy, and the level density grows exponentially. This combinatorics was first exploited by Bethe [4] who presented the estimate of the Fermi-gas level density. For a Fermi gas in a spherical box, the level density is a smooth function of excitation energy; its growth can be described by increasing temperature uniquely related to the energy. For a more realistic mean field with the shell structure of single-particle orbitals, the level density is modulated by the density of those orbitals. In a spherical field, the orbitals carry angular momentum quantum numbers and it is important to know the level density for specific classes of many-body states (total spin, isospin, parity). The idea of Bethe was to assume the random coupling of individual angular momenta which results in the



Gaussian distribution of the total spin projection with the width determined by the effective moment of inertia (the so-called spin cut-off parameter).

The next stage of the development is well documented in the review article [5]. The Fermi-gas model now is enriched by the real shell-model structure and pairing effects which significantly change the level density at least at not very high energy. The changes (like “back-shifted Fermi-gas formula” [6]) are phenomenological and contain empirical parameters for each nucleus. Later the pairing correlations were included on a microscopic level through the analog of the BCS or Hartree-Fock-Bogoliubov theory so that the mean field becomes self-consistent and the system superfluid [7, 8]. The level density predicted in this way typically has a smooth background and significant bumps related to the subshell structure.

If we would have a reliable description of interactions inside a nucleus, we would solve the quantum many-body problem and find the theoretical level density. The shell model with effective interactions is just an appropriate instrument. Unfortunately, the orbital states have to be truncated, otherwise the computational problem becomes not tractable. The shell model Monte Carlo approach [9] does not require huge diagonalization and was repeatedly used for finding the level density. It handles however only a limited set of possible interparticle interactions. The modern versions of the shell model (configuration interaction) cover a broad interval of excitation energy and the complete set of possible interaction matrix elements. The real problem is to be able to extract the level density avoiding full diagonalization.

## 2. Quantum chaos and moments method

With excitation energy growing and diminishing level spacings, the stationary wave functions become extremely complicated superpositions of the mean field states (particles and holes). Assuming the interaction being gradually turned on, the wave functions with the fixed global quantum numbers go through the process of multiple avoided crossings. Very fast (even with interaction still weaker than its realistic strength) the states reach the high degree of complexity. The energies inside the given class of states form an “aperiodic crystal” with the weakly fluctuating distances. This is the stage close to the mathematical limit of quantum chaos [10, 11]. There are broad studies of realistic examples which show [13, 14, 15] that indeed the system is locally close to the mathematical limit of the Gaussian Orthogonal Ensemble (GOE).

For our applications we do not need the exact fulfillment of the GOE properties. It is sufficient to satisfy the informal physical description given long ago [10]: in a vicinity of given energy the neighboring states have very similar physical properties (“look the same”) and matrix elements of “simple” physical operators of the same order of magnitude. Then we expect that it is possible to apply the methods of statistical spectroscopy [16, 17].

As possible to check in exactly solvable shell-model versions, the level density in each class is a smooth bell-shaped curve with the maximum in the middle. Because we work in a truncated space, the significant part of this curve characterizes just the applied model rather than realistic physics as at some energy the missing states outside our orbital space should enter. Luckily enough, in practice the initial part of the curve provides a good description of reality up to some excitation energy. In this “realistic region” the complexity of physics allows to find the level density by calculating just the first statistical moments of the distribution.

Starting with the spherical mean field we divide all basis states in partitions  $p$  where the given particle number is distributed in same way over available orbitals. Let  $D_{\alpha p}$  be the dimension of the class of states with exact quantum numbers  $\alpha$  inside the partition  $p$ . For each partition we calculate the lowest moments (traces) of the Hamiltonian. The first moment is just a centroid,

$$M_{\alpha p}^{(1)} = \frac{1}{D_{\alpha p}} \text{Tr}_{\alpha p} H \equiv \overline{H}_{\alpha p}. \quad (1)$$

The second moment is the mean square fluctuation,

$$M_{\alpha p}^{(2)} = \frac{1}{D_{\alpha p}} \text{Tr}_{\alpha p} H^2 - [\overline{H}_{\alpha p}]^2 \equiv \sigma_{\alpha p}^2. \quad (2)$$

The first moment accounts for the mean field energy of the partition and the diagonal matrix elements of the interaction Hamiltonian. The second moment is the mean sum of the off-diagonal matrix elements including the interactions between different partitions. Both moments can be actually read just from the Hamiltonian matrix without any diagonalization procedure.

It was shown long ago [13] that the level density of a partition very fast converges to the Gaussian distribution. Therefore the resulting level density for a given set  $\alpha$  of quantum numbers is given by a sum over partitions of the Gaussians  $G_{\eta}(x - x_0; \sigma)$ , defined by the eqs. (1) and (2),

$$\rho_{\alpha}(E) = \sum_p G_{\eta}(E - (\overline{H}_{\alpha p} - E_{\text{g.s.}}); \sigma_{\alpha p}). \quad (3)$$

Although the procedures are developed for including the higher moments of the Hamiltonian, in practice this is not necessary. The third moment would cut the unphysical tails of the Gaussians. Instead we introduce [18] the parameter  $\eta$  that effectively does the same so that  $G(x - x_0, \sigma)$  is set to zero at  $|x - x_0| < \eta\sigma$ . This is the only parameter that is not determined directly by the Hamiltonian; the shell-model study of the strength functions suggests  $\eta \approx 2.4 - 2.8$ . The ground state energy  $E_{\text{g.s.}}$  should be determined separately either by the calculation (for example, with the Lanczos algorithm) or by application of the exponential convergence method [19]. Figure 1 demonstrates the good agreement of the level density calculated by the two moments procedure with the exact *sd*-shell solution for  $^{28}\text{Si}$ . The fourth moment would improve the result around the center of the curve which is anyway outside the physical region of the *sd*-model.

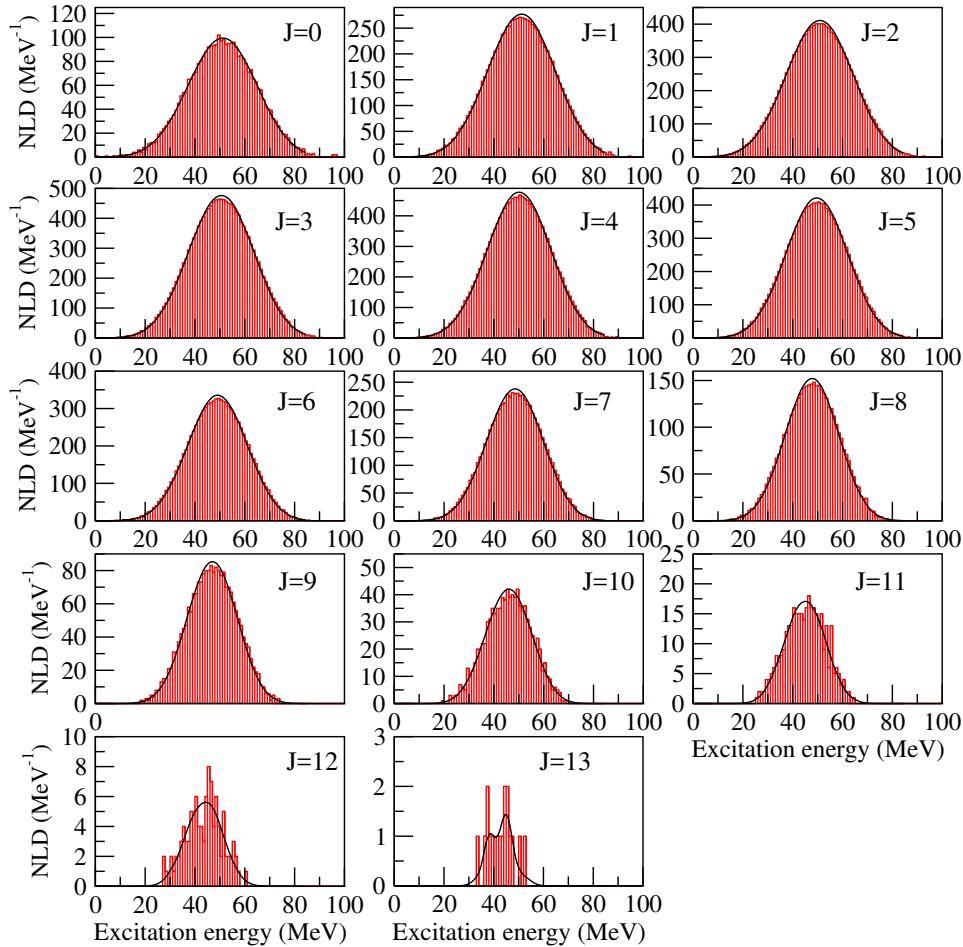
The moments method was recently used for the nuclei of *sd* and *pf* shells, where the reliable two-body interaction Hamiltonians do exist. The discussion of many features of the method and relevant references can be found in [20]; the details of the computational algorithm are given in [21]. For all nuclei studied, the level density in all classes of states is a smooth function with the subshell bumps significantly levelled out. As shown in [20], this is the result of the presence of many incoherent collision-like interactions which are absent in the mean-field, BCS or shell-model Monte Carlo approaches and which in fact form the realistic level density.

### 3. New development

Having at our disposal a realistic model of the many-body level density we can not only count on the better description of nuclear reactions. It becomes possible to turn to some questions of deep meaning. The first desire is to find an analytic approximation that would agree with the exact level density and reduce the theoretical task to predicting the parameters. The analysis of the back-shifted Fermi-gas approach has shown [20] the long expected Rosenzweig effect [5] of the systematic change of the Fermi-gas temperature along the shell filling. Another observation was that the angular momentum coupling described by the random process does not work in the cases similar to  $^{44}\text{Ca}$ . Here the shell model operates with only four protons, and therefore  $T = 2$  states and only the  $T = 1$  component of the two-body forces. This is not sufficient for the randomness of angular momentum coupling.

Currently the best description seems to be given [22] by a constant temperature model: the level density at not very high excitation energy  $E$  is well described as

$$\rho(E) = \frac{1}{T} e^{(E-E_0)/T}. \quad (4)$$

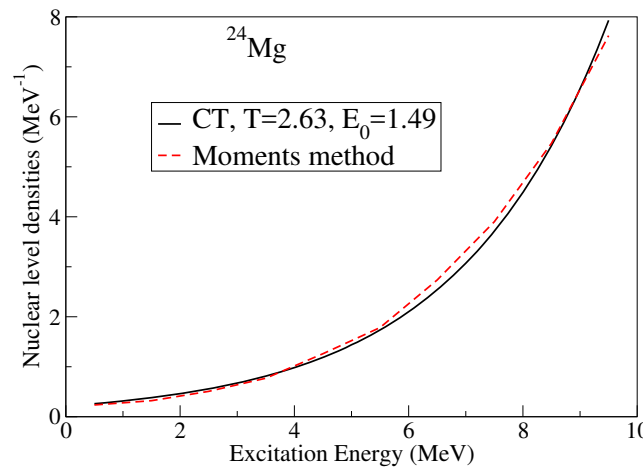


**Figure 1.** The level density of  $^{28}\text{Si}$  in the  $sd$  orbital space calculated with the moments method (smooth dashed curve) and by exact diagonalization (solid line with small wiggles).

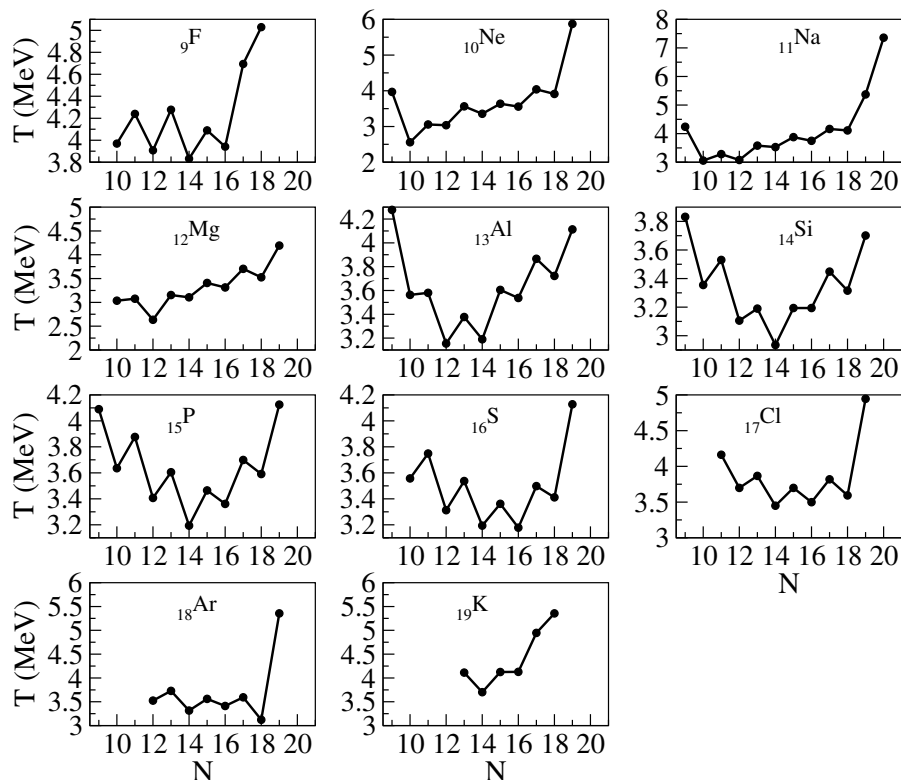
Here  $T$  is a fixed parameter that plays the role of the effective temperature of the ground and low-lying states. Fig. 2 demonstrates the high quality of the fit for the level density by this model for various nuclei. The idea of the ground state temperature is not new. It was long ago suggested by Moretto [23] as a sign of the phase transition from paired to normal Fermi liquid. However, the shell-model studies [24] invariably show a slow fall of the pairing correlators as a function of energy in various classes of states instead of sharp phase transitions. The ground state temperature was also discussed [25] as the fluctuational part of the mean field obtained by averaging over chaotic interactions. This interpretation is in qualitative agreement with the systematics of the parameter  $T$ , Fig. 3. All isotopes reveal the odd-even staggering and clear minima for  $N = Z$  and nearby isotopes. Here pairing and possibly quarteting correlations lead to more “deeply frozen” low-energy configurations. It is also useful to study the changes of the level density varying the interaction. In this way one can see [26] the special role of the interactions mixing single-particle orbitals and inducing the shape deformation along with the related collective enhancement of low-energy level density.

All these properties, as well as dependence on various components of the interaction and on nuclear spin, should be studied in more detail. The consideration has to be generalized to different orbital spaces. For the mixture of shells of different parity one needs to get rid of he

spurious states related to the center-of-mass excitation; there is a well established procedure that excludes such admixtures [27]. In the conclusion we have to mention that the values of the parameter  $T$  for all nuclei in the  $sd$  shell and for all spin classes are tabulated and can be used by experimentalists [28].



**Figure 2.** The level density calculated in the moments method and its description in the constant temperature model, eq. (4).



**Figure 3.** The constant temperature parameter for various nuclei in the  $sd$  shell.

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