

DESIGN CONSIDERATIONS FOR A HIGH ENERGY
ELECTRON-POSITRON STORAGE RING

H. Wiedemann
Deutsches Elektronen-Synchrotron DESY
Hamburg, Germany

The successful operation of the electron positron storage ring - SPEAR - in the multi-GeV-range encouraged the accelerator physicist to think about the next generation of storage rings for energies of more than 10 GeV. We have learned from the presently working storage rings that it seems not practical to assume an artificial beam enlargement as an essential design aspect. As is well known this artificial beam enlargement was needed to get high luminosities at medium energies where the beam intensities are limited by the beam beam interaction at the collision points. Therefore we have to find other means to increase the luminosity in this energy regime. One possible way is to vary the focusing of the periodic cell structure of a storage ring with energy, thus varying the quantum excitation of betatron oscillations ^{1),2)}. This brings us to the point where we should discuss more general the influence of the beam optics in the periodic cell structure on relevant storage ring parameters. In this note this influence on the energy will be discussed. This can be done in rather general terms since the special design of the long straight section on both sides of the interaction point has no effect on the beam parameters except the amplitude functions at the point where the beams collide. For these amplitude functions we assume values which we know can be realized. Throughout this note we assume only one ring where the two beams counterrotate in the same vacuum pipe and collide head on in the interaction points.

I. Storage Ring Parameters for Numerical Calculations

For numerical calculations in this note we assume the following storage ring parameters which are close to parameters assumed in different laboratories for electron-positron storage ring studies of the next generation:

circumference of the ring	$C = 2304 \text{ m}$
bending radius	$\rho = 206.8 \text{ m}$
average radius of the arcs	$R = 256.7 \text{ m}$
installed rf-power	$P_{\text{rf}} = 5.0 \text{ MW}$
total length of cavities (copper)	$L_c = 240 \text{ m}$
shunt impedance of the cavities at 500 Mc	$R_c = 10 \text{ M}\Omega/\text{m}$

II. Maximum Energy of the Storage Ring

The maximum energy of a storage ring is given when the total installed rf-power is dissipated in the cavities. These cavity losses depend on the cavity length, the shunt impedance per unit length, the rf-frequency and on the necessary rf-voltages as given by the focusing of the periodic cell structure.

The amount of cavity length depends on the amount of money one wants to spend for them. It is not intended to use this length as a parameter here but rather use one length for which the necessary free length in a ring of 2 km circumference can be realized.

- 1) J. Rees, B. Richter, Preliminary design of a 15 GeV Electron-Positron Variable Tune Storage Ring, PEP-Note 69; 1973 PEP-Summer-Study
- 2) H. Wiedemann, e-p Luminosity for different energies in PEP, PEP-Note 58, 1973 PEP-Summer-Study

The shunt impedance per unit length for 500 Mc is conservatively assumed to be $10 \text{ M}\Omega/\text{m}$ which could be increased somewhat using other than iris structures e.g. trift tube cavities. Since the production of the latter are more costly one has to find a cost optimum for the product $R_c \cdot L_c$. The effect of the rf-frequency has been studied in detail ³⁾ with the result of a very flat optimum in the range of 200 to 700 Mc depending somewhat on the energy for which the design is made. In this note we assume 500 Mc. For these rf-parameters the maximum achievable energies as a function of focusing power are calculated. The focusing is varied by varying the cell length keeping the phase advance per cell constant at 90° . This 90° degree phase advance per cell is an essential for the multibunch operation which is discussed later. Characterising the focusing by the transition energy γ_{tr} the maximum achievable energies using copper rf-cavities are shown in fig.1. It becomes clear from fig.1 that the transition energy should be about 20 or larger in order not to loose too much energypotential of the ring. However, the gain between $\gamma_{\text{tr}} = 20$ and $\gamma_{\text{tr}} = 30$ is only 1 GeV if we use copper cavities.

Designing storage rings of the next generation it might be reasonable also to have a look at the next generation technology which in our case means superconducting cavities. In this case the energy isn't limited by cavity losses any more but by the minimum reasonable luminosity which falls off like E^{-10} in this energy regime. If we exchange the 240 m copper cavities by 240 m of superconducting cavities the maximum energy for a luminosity of $L = 10^{31} \text{ cm}^{-2}\text{sec}^{-1}$ is computed and shown in fig.2. Here we find a rather steep increase in energy versus transition energy which calls for the maximum reasonable focusing.

III. Luminosity

It has been shown that increased focusing in the periodic cell structure leads to higher achievable energies. This is a true reflection of the fact that at a fixed energy in the rf-power limited case an increased focusing also results in higher luminosities (fig.3).

At lower energies however, there is the problem of the beam beam incoherent limit. So far it was assumed that each beam is made up by only one bunch or in case of more than two interaction points by half as many bunches as there are interaction points. In this case the bunches meet only at the interaction points and nowhere else in the arcs where the beta-functions and by this the beam beam effect is larger. In fig.3 the circles \odot show the maximum luminosities for one bunch. For energies lower than those indicated by the crosses the beam beam effect is effective. Here the luminosity scales like $L \sim A \cdot \gamma^2$, where A is the beam cross section at the interaction point. If natural beam sizes are used - $A \sim \gamma^2$ - the luminosity scales like $L \sim \gamma^4$. To avoid this fast drop of the luminosity toward lower energies the variation of the focusing was proposed ¹⁾²⁾. In this note another mode of operation is proposed.

For this we divide the whole energy regime into three parts:

- 3) M. Allen, G. Rees, CRISP-Note 72-68, Brookhaven

1. The rf-power limited regime

Here the luminosity is given by: 4)

$$(1) L_1 = \frac{1}{4e^2 f} \frac{P_B^2}{U^2 R \cdot B}$$

where: P_B the rf-power available per beam, U the energy loss per turn, f the revolution frequency; A the beam cross section and B the number of bunches. It is clear that the number of bunches should be as small as possible e.g. $B = 1$. Also the beam cross section should be minimized which requires strong focusing as has been already discussed. Using the scaling laws for FODO-Cell structures 5) the luminosity scales like $L_1 \sim \gamma_{tr}^4$.

2. The rf-power and tune shift limited regime

In this case the beam current per bunch is limited by the incoherent beam beam effect.

$$(2) \Delta v = \frac{r_e}{2ef} \frac{I\beta}{\gamma AB} = \frac{r_e}{2ef} \frac{P_{rf}}{\gamma U} \cdot \frac{\beta}{AB} \leq 0.05$$

(β : amplitude function at the interaction point, B number of bunches).

Together with eq. (1) the luminosity is given by

$$(3) L_2 = \frac{f \Delta v^2}{r_e^2 \beta^2} \frac{A}{B \gamma^2}$$

Since we assume natural beam sizes the number of bunches B has to be varied according to eq.(2) like:

$$(4) B = \frac{r_e}{2ef} \frac{P_{rf}}{\gamma U} \frac{\beta}{A} \sim \gamma^{-7}$$

If we find a way to increase the number of bunches considerably the luminosity scales like $L_2 \sim \gamma^{-3}$ in this regime. Here we neglected the rf-cavity losses. Since it seems very advantageous to increase the number of bunches, a way to do this is discussed in this note. As a result of this discussion we find a maximum number of bunches B_{max} which is much smaller than the harmonic number.

3. The tune shift limited regime

At lower energies where we always have the maximum number of bunches the total beam current is limited only by the incoherent beam beam limit and the luminosity is given by:

$$(5) L_3 = \frac{f \Delta v^2}{r_e^2 \beta^2} \gamma^2 A B_{max}$$

Since there is no easy way to have many bunches and a variable focusing the beam sizes are given by the quantum excitation e.g. $A \sim \gamma^2$ and the luminosity then scales as $L_3 \sim \gamma^4$.

Since this multi bunch mode requires only a phase advance of 90° per cell in the arcs the luminosity can be increased at very low energies by turning off every second quadrupole in the arcs thus increasing the cell length by a factor of two. By this the product $A \cdot B_{max}$ can be increased as will be shown later. In principle this can be continued to increase the luminosity at very low energies but doing so the aperture limit is reached very fast.

IV. Beam Separation

If we use many bunches per beam we have to separate them outside the interaction points. This can be done with an appropriate number of electrostatic separating plates. Many and strong fields however, are needed if we do not match the separation to the focusing structure of the ring. Since there is a small beam beam effect left even if the beams are separated one would like to have the points where the bunches meet at similar places from the beam optics point of view. The easiest way of separating the two beams is given in the case where the phase advance per cell is just 90° . An electrostatic field introduces a regular closed orbit oscillation with a wavelength equivalent to two cell lengths (fig.4). The distance between bunches comes out to be one cell length or a multiple of that length. While for the arcs only one separating plate on either end is needed one may need some more in the straight sections due to the nonregular structure there.

The circumference of the storage ring has to be a multiple of the cell length. Also the distance from any "pass by point" in the arcs to the interaction point has to be a multiple of one half cell length.

In the numerical example a transition energy of $\gamma_{tr} = 31$ is realized by a cell length of $L = 14.4$ m. The maximum number of bunches then is 160.

V. Beam-Beam-Effect

Even with beam separation there is still an electromagnetic interaction of one beam on the particles of the other beam the so called long range forces. It is well known from experience of running storage rings that the beam separation has to be large compared to the standard beam width (a horizontal beam separation is considered). In this note a beam separation is assumed large enough that one beam sees only a force from the other beam which falls off like the inverse of the separation, $F \sim 1/x$. This means the separation x is at least $10\sigma_x$. For that large separations the linear tune shift turns out to be: 6)

$$(6) \delta v = 9.6 \times 10^{-9} \frac{\beta_x I \cdot C}{cp x^2}$$

where β is the betatron amplitude at the "pass by points", I the total beam current in amps, C the ring circumference, cp the beam energy and x the beam separation (C, β, x in meters).

To achieve the shown luminosity in fig.3 for $\gamma_{tr} = 31$, a maximum current of $I = 140$ ma at 15 GeV and $\beta_x = 11.1$ m at a pass by points the linear tune shift is $\delta v \cdot x^2 = 0.023$ cm². For a separation of $x = 1$ cm which is equivalent to a separation of $x = 20.5 \sigma_x$ the total linear tune shift outside the interaction points is only $\delta v = 0.023$. The total beam beam effect at a separation of $x \approx 20 \sigma_x$ is very much reduced to a mere linear tune shift which can easily be corrected by electrostatic quadrupoles. The nonlinear part of the beam beam effect can be neglected for the magnitude of separation discussed here 6).

The total aperture requirement in this case is only $A = x + 14 \sigma_x = 17$ mm. This seems very little however, one needs more at higher energies. At 22 GeV which is the highest energy where the beams have to be

4) M. Sands, Physics of Electron Positron Storage Rings, SLAC-121

5) H. Wiedemann, Scaling of FODO-Cell Parameters, PEP-39, 1973 PEP-Summer-Study

6) SPEAR-Group, Beam Beam Coupling in SPEAR, Proceedings of this conference

separated the standard width is $\sigma_x = 0.71$ mm. A separation of $x = 1$ cm corresponds still to $14 \sigma_x$ and the aperture requirement is only $A = 20$ mm. This small aperture requirements show that the beam separation may be easily increased to twice the assumed value to be safe.

At energies below 15 GeV it is advantageous to double the cell length by turning off every second quadrupole which gives a $\gamma_{tr} = 15.4$. Here the maximum number of bunches is 80 and the maximum current at 12 GeV is $I = 290$ ma. With $\beta_x = 33.2$ m and $x = 2$ cm corresponding to $10 \sigma_x$ the linear tune shift is $\delta\nu = 0.044$. In this case the maximum luminosity is $2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$ for a total installed rf-power of 5 MW (fig.3).

VI. Conclusion

To achieve the highest possible energy for a given installed rf-power the focusing in the periodic cell structure should be as strong as possible. For lower energies where the beam beam effect is effective the total beam current can be pushed up by filling many bunches in one beam, which requires beam separation outside the interaction points. The total linear tune shift due to long range forces is small. Even with beam separation the required apertures seem not to be excessive due to the small beam sizes produced by the strong focusing.

Acknowledgement

The author wishes to thank G.A. Voss for many helpful discussions.

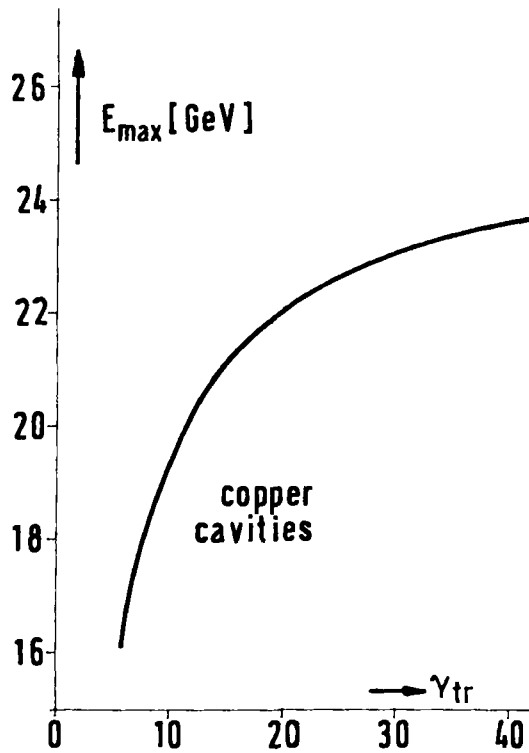


Fig.1: Maximum storage ring energy vs. transition energy.

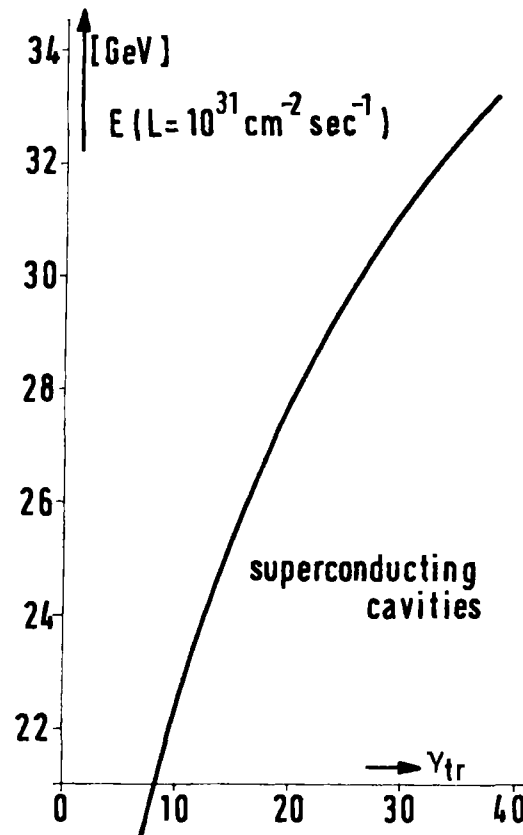


Fig.2: Maximum energy for a luminosity of $L = 10^{31} \text{ cm}^{-2} \text{ sec}^{-1}$ vs. transition energy.

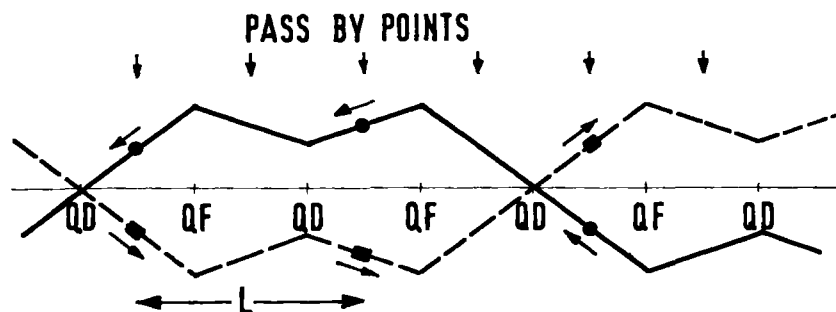


Fig.4: Separation of the beams in the multibunch mode.

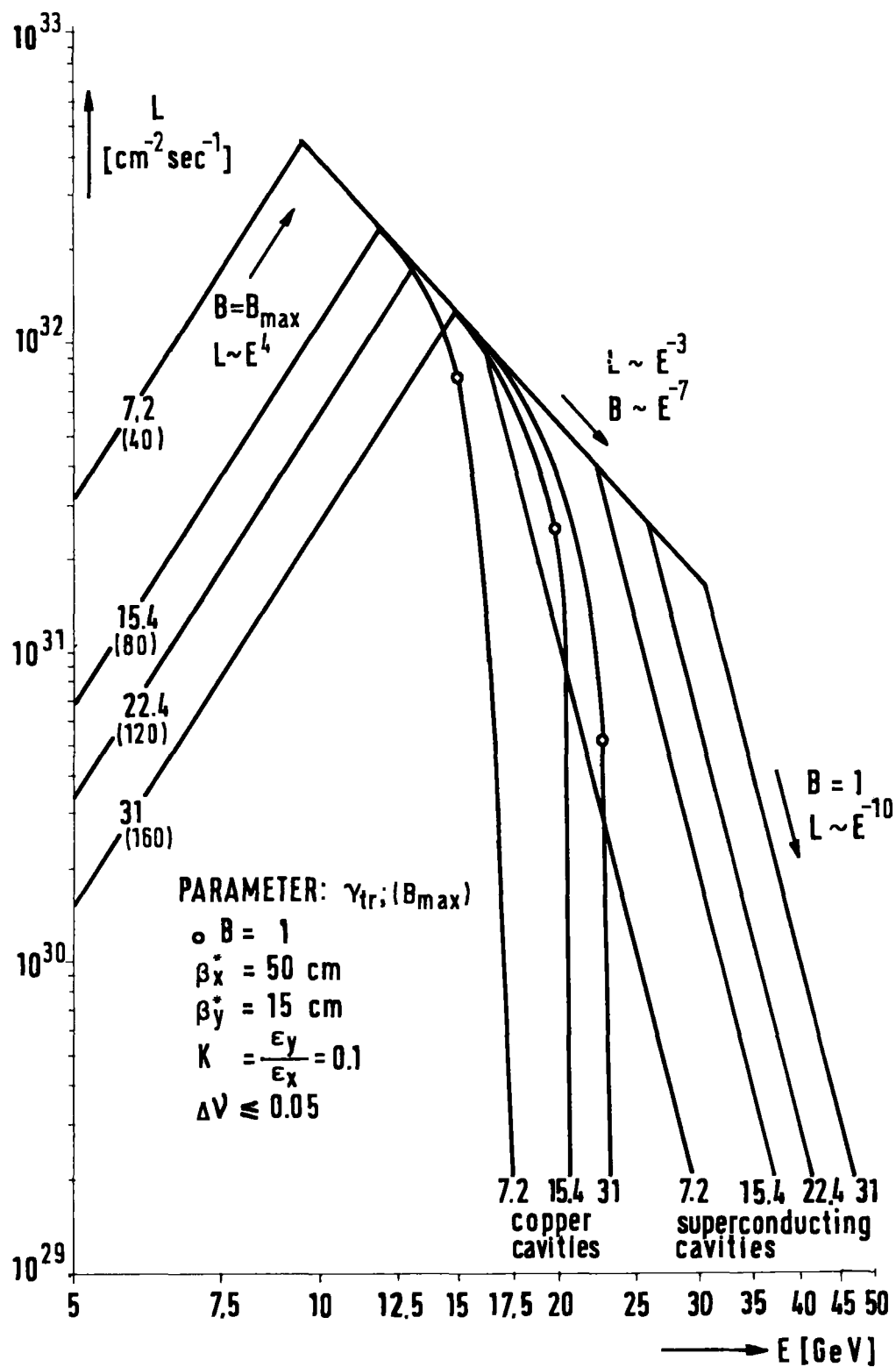


Fig.3: Luminosities for different transition energies and numbers of bunches.