Einstein's Missing Energy

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Abstract. Einstein worked on General Relativity for a decade before releasing it in 1916. For several of those years he struggled to include gravity's own energy into his equation. He couldn't get it right, so he just dropped it. How do we consistently put gravitational energy back into the equations? In this essay, Einstein's own solution to this problem - the energy pseudo tensor, along with variants are quickly reviewed and found wanting. Quasilocal energy is thus used, and we include this energy into Einstein equations. Perhaps unsurprisingly, general covariance is broken. As an example, a Schwarzschild like solution is developed, but unlike a black hole, this solution shows no horizons and no massive singularity. This also allows for new polarization modes, namely (superluminal) monopolar radiation.

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1. Introduction

Einstein finally, over two hundred years after Newton, found a mechanism for gravity. An unbelievable leap. He spent a decade working on it. A substantial amount of his time was spent trying to find a way to work gravitational energy into the equations. *Noether's Theorems and Energy in General Relativity* by Haro[11] includes a historical timeline, and is well worth a read.

Perhaps the simplest way to describe what's missing in Einstein's equations is to note that energy from pure gravitational phenomena (like gravitational waves, the field outside a black hole, etc) isn't counted automatically in the equations! This fact is largely ignored - so for example by 1973 the 'official word' was not to merely ignore energy conservation, but to actively reject it: Misner Thorne and Wheeler[21]:

> Anybody who looks for a magic formula for "local gravitational energy-momentum" is looking for the right answer to the wrong question. Unhappily, enormous time and effort were devoted in the past to trying to "answer this question" before investigators realized the futility of the enterprise. Toward the end, above all mathematical arguments, one came to appreciate the quiet but rock-like strength of Einstein's equivalence principle. One can always find in any given locality a frame of reference in which all local "gravitational fields" ... disappear.

Misner, Thorne and Wheeler are still be right, no *covariant*, 'magical' expression for gravitational energy has ever been found. Einstein's hope lay in 'pseudo tensors', which are discussed and evaluated for example by Virbhadra[20][19]. It's fairly straight forward using Mathematica to see how unreliable these pseudo tensors are - the same physical Schwarzschild solution in isotropic x, y, z, t vs 'Kerr-Schild' coordinates x, y, z, t give conflicting results[16]. Pseudo tensors don't work.

As a work around, researchers simply plop energy (from for example gravitational waves) into the right side stress energy tensor on an ad hoc basis. But you can't always do it, and there is no reliable formula. That's the point. Einstein (and everyone else) couldn't get that to work.

So, even today, the mantra in physics is (Will Kinney):[22]

> Energy isn't conserved in General Relativity.

Is there a way around this lack of energy conservation? Sure. Drop covariance. (Seems non - trivial!). This is what I'm going to do here, and look at the implications.

2. Quasilocal energy

Golovnev sums up energy covariance in General Relativity[10]:

> As a substitute, and in a fully arbitrary manner, we introduce a fictional Minkowski space, either as a higher dimensional realm full of ghosts or as

a projection to remote invisible heavens, and then we do have a perfectly covariant but pretty esoteric mantra of conservation laws.

There are improvements to be had over the pseudo tensor approach. About the best[11][6][5] formulations for gravitational energy in General Relativity are treatments like the quasilocal energy of Brown and York[4] and Katz[12].

These quasilocal descriptions move the covariance problem from the bulk to the boundary in a manner similar to and connected with the concept of ADM mass.[9] *Covariance is still a problem, but we have swept it aside.*

So, basically, all attempts at covariant energy conservation fail. Some more elegantly than others. Remember though, even if one drops covariance, we find experimentally that 'normal' energy *is* conserved covariantly, i.e: the proper covariant divergence of the stress energy tensor is still zero. $T^{\mu\nu};\nu = 0$. So all our stuff (well at least light and standard model particles), demonstrate covariant energy conservation. I'm going to call this regular matter 'transverse' for reasons that will become apparent later. The speed of light is still the limit for this transverse energy. So now all I have to do is look at the (usually minor) missing energy. What is the missing energy? The mass under gravitational waves, the mass of the gravitational energy around a black hole, the mass of the energy of the universe...

3. A Schwarzschild like solution

As a concrete example, I will build a Schwarzschild like solution that includes quasilocal energy. The expression for quasilocal energy density is, in the Schwarzschild solution in the far field (with isotropic r)[6]:

$$E_{density} = \frac{GM^2}{8\pi r^4}. \tag{1}$$

It's easy to see from equation (1) that this formula is not 'manifestly covariant'.

Plan: Take the quasilocal energy definition from Brown and York[4] or Katz[12], and apply it to a vacuum Schwarzschild - like solution, where this quasilocal energy of the gravitational field is now included in the mass, so that the mass enclosed within a given radius ($m(r)$), is less than M, the distant or ADM total mass.

It's pretty easy to see that the resulting metric will not satisfy the usual Einstein vacuum equations, as the equations will point to a stress energy tensor on the right side. This then is taken to be an indication of the gravitational energy as expressed in the (isotropic used here) coordinate system.

What is the function $m(r)$ - what is the mass inside a radius r? Start answering this question by noting the Lynden-Bell and Katz[6] exact expression for the gravitational energy between r and $r + dr$ in isotropic coordinates:

$$Gm(r)^2/2r^2 \; dr. \tag{2}$$

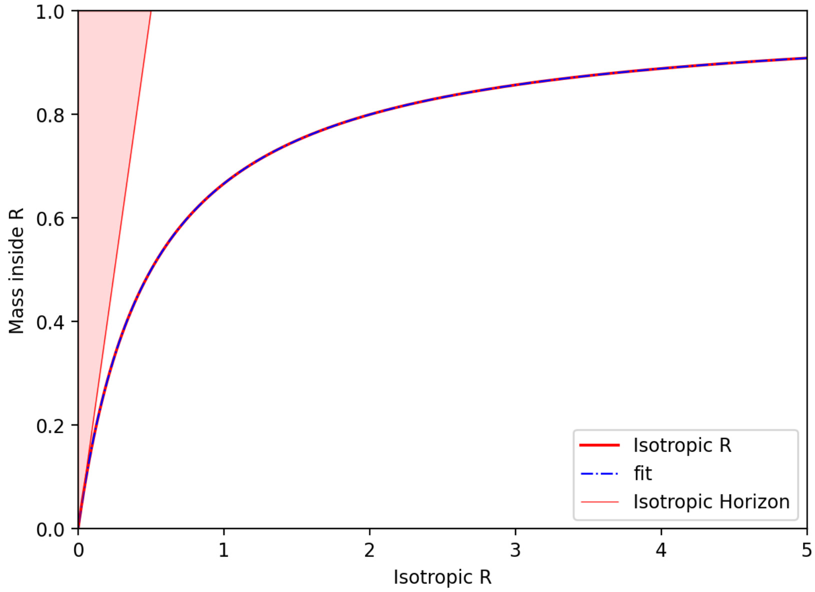

Figure 1. Assuming that building the gravitational energy around a spherical mass requires energy, plotted is the mass inside isotropic r. The fit shows an analytical function describing the mass function which was found. Note that isotropic r behaves badly near $r = 0$, see figure 2 for a better view near the centre.

To obtain the mass function $m(r)$: we start at a distant point out, then walk in, dropping some energy from the 'Birkhoff mass at r' into the gravitational field at every step, thus allowing the newly built gravitational field to have a source of energy.

I have written a Jupyter python script to calculate this mass function. It uses the exact Lynden-Bell-Katz[6] energy density. I assume Birkhoff's theorem at every step, so the gravitational field at (just outside) of r is identical to the Schwarzschild field at that point. See the script online[15].

The mass function that turns out to work is (in isotropic coordinates)

$$m(r) = M - M/(1 + 2r/M), \tag{3}$$

where $m(r)$ is the mass as experienced at r. One can put this into the usual isotropic form of the Schwarzschild metric, turn the crank with Mathematica, and find:

(i) A simple line element (no singularity at r = 0 or elsewhere!)

$$ds^2 = \frac{-r^2}{(M+r)^2}dt^2 + \frac{16(M+r)^4}{(M+2r)^4}(dr^2 + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2), \tag{4}$$

(ii) A contravariant stress energy tensor

$$\begin{pmatrix} \frac{M^2(M+2r)^2}{2r^3(M+r)^3} & \theta & \theta & \theta \\ \theta & \frac{M^3(M+2r)^6}{128r^2(M+r)^9} & \theta & \theta \\ \theta & \theta & \frac{M^3(M+2r)^6}{256r^4(M+r)^9} & \theta \\ \theta & \theta & 0 & \frac{M^3(M+2r)^6 \csc[\theta]^2}{256r^4(M+r)^9} \end{pmatrix} \tag{5}$$

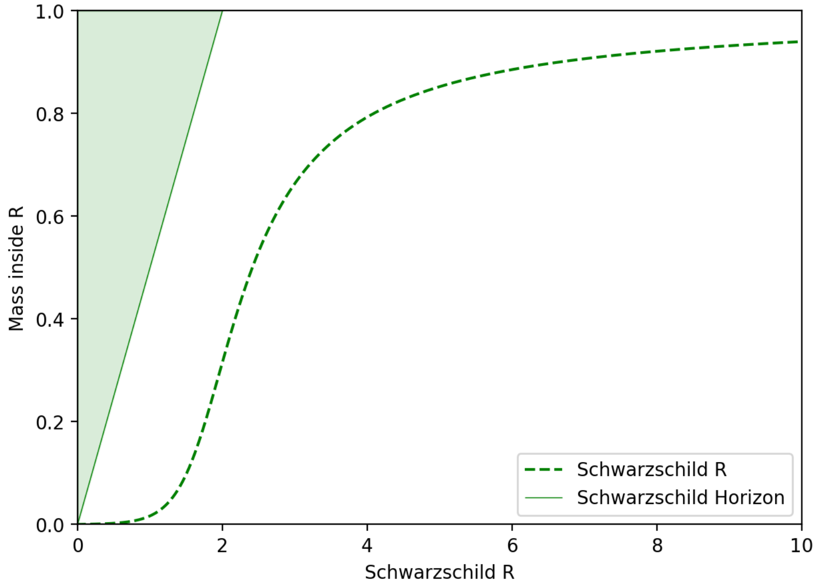

Figure 2. The same data as in figure 1, but with Schwarzschild coordinates. The mass at every radius is below the horizon radius for that mass, so there is no event horizon. Also one can see that the behaviour as $r \to 0$ is not a problem.

(iii) The T^{00} term is of the right form for the field energy density as $r \gg M$. But its not quite right - is this due to pressure contributions?

To summarize: the line element (4) represents a spherical solution of Einstein's equations taking into account the gravitational field energy required to build this 'almost black hole' solution. The solution is diagrammed in figures 1 and 2. There is no room to interpret where the field energy is, no horizon to hide energy behind, no mysterious singularity. The total field energy adds to exactly 1 (see the script). So that's nice.

3.1. Confusion on Gravitational Energy

The extant literature on the subject of 'where the energy is' - even for the Schwarzschild solution - is full of contradictions.

(i) Dyrda, Kinasiewicz, and Kutschera look at equations of state in a neutron star, finding from 20 to 40% of the total energy coming from gravitational field energy. They also point out:[7]

> However, there is some unusual property of the gravitational field in this approach: the field just below the surface of the star has the energy which contributes to the stellar mass, whereas above the stellar surface the field does not contribute to the neutron star mass: no gravitational field energy is localized outside the stellar surface.

Why would gravitational energy drop to zero as matter drops out - how does the field 'know' the energy is from matter and not gravity itself?

(ii) D. Lynden-Bell and Katz on the localization of mass:[6]

> The Penrose mass of a Schwarzschild hole is all within the hole, whereas ours is all outside it...

My conjecture here is that much of the confusion arises because in the original Einstein equations there is no account of the effect of gravitational energy in (even vacuum) solutions, and thus the Schwarzschild black hole has effectively twice the energy needed, one inside and one outside the event horizon. People pick either, or negative of one another.

4. Consequences of Altering the Einstein Equations by Adding a Gravitational Energy Term

The exact form for calculating this energy term is not known, but considering that mass in the universe is almost at rest, a simple sum of equation (1) ($E_{density} = \frac{GM^2}{8\pi r^4}$.) over all nearby masses should be a good starting point. With the $1/r^4$ dependency, it even converges!

It may be that one has to use a Euclidean metric signature of $+, +, +, +$ to work with absolute space and gravitational field energy. Many, if not all solutions the Einstein equations work in any signature. The equations are signature agnostic.

4.1. Monopole waves

(This section does not assume the results of section 3.)

A few moments of thought suggest that if gravitational energy is real, it can be imagined as a sort of 'compressed gas' - like air, and thus monopolar waves (sound waves are monopole) might be possible.

Consider a 'bump' - a small excess of gravitational energy located at some radius r. See figure 3. Again, we are in isotropic coordinates here. It's easy to see that there is not enough (gravitational) energy in the entire diagram to support the measured distant mass $M \equiv M_0 + \Delta M$. We now assume that this missing energy is kinetic energy - that the bump in zone ② is rapidly moving (pure gravitational fields carry energy). Birkhoff's theorem is assumed throughout here. The energy within any r must be that of the measured mass for any observer.

We will need to use the following formula from [6] that tells us the total gravitational field energy outside isotropic r for a mass m:

$$E_{outside\ r} = Gm^2/2r. \tag{6}$$

Back to our diagram. The total energy is $M = M_0 + \Delta M$. The gravitational field energy in zone ① is, using equation (6):

$$M_0 - GM_0^2/2r. \tag{7}$$

I make the assumption that the wave has a small mass so $\Delta M \ll M_0$ and of small radial extent (compared to infinity). One can get the same final expression (even easier)

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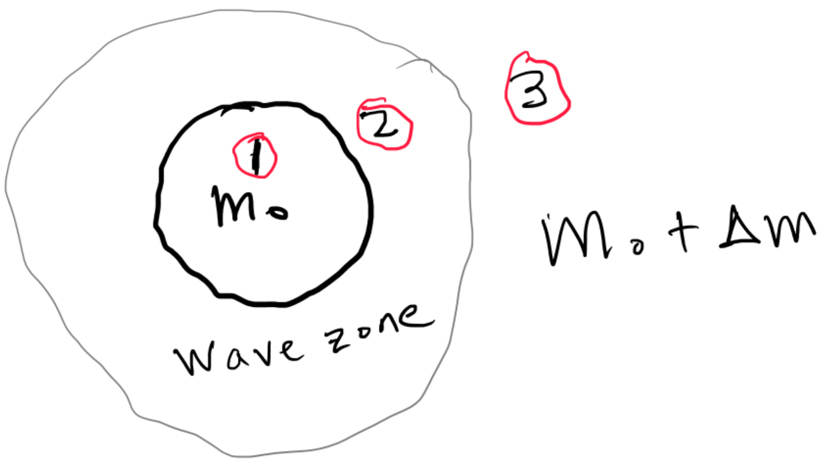

Figure 3. Monopole Wave: There are three zones. In zone ① we are in a Schwarzschild region with mass M_0 In the outer zone (really only at r -¿ infinity) ③ there is a Schwarzschild metric $M_o + \Delta M$. In between is the 'wave zone' ② where the percieved mass drops. Almost all the drop is near the boundary wall at r between ① and ②. We will find that this boundary must be rapidly moving to satisfy global energy conservation.

by assuming the inner zone is Minkowski space - i.e. $M_0 = 0$: this small wave restriction is not vital to the calculation.

The gravitational field energy in the wave zone ② (remember the wave has a small range of r) is almost exactly

$$GM^2/2r. \tag{8}$$

We assume that this wave is moving and carries a kinetic energy - what mass should we use for the KE? We use the excess field energy in the wave zone obtained via subtraction:

$$M_{moving} = GM^2/2r - GM_0^2/2r, \tag{9}$$

$$= G(M_0 + \Delta M)^2/2r - GM_0^2/2r, \tag{10}$$

$$\approx \frac{2GM_0\Delta M}{2r}, \tag{11}$$

where I have dropped the ΔM_0^2 term.

Thus the total energy in the entire spacetime region is, using equations: (7),(8),(11) and Newton's Kinetic Energy formula:

$$M = M_0 - GM_0^2/2r + GM^2/2r + \frac{GM_0\Delta M}{2r}v^2. \tag{12}$$

Rearranging (12) to solve for the speed of the wave front at $v(r)$ still in isotropic coords:

$$v = \sqrt{\frac{2r}{GM} - 2}. \tag{13}$$

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I have taken $M \approx M_0$ at the very end and dropped a ΔM^2 term.

Thus, assuming Birkhoff's theorem, if the bump is moving at v, we have energy conservation. It's obvious that the velocity could be positive or negative, so the pulse could be moving in or out. The calculated velocity here is greater than the speed of light (which is 1 here), except for close to the horizon. The velocity gets to zero just before the $r = GM/2$ isotropic horizon at $r = GM$, perhaps a bounce happens.

For actual physical spacetimes, such as on the surface of the earth, an approximate value of v can be calculated by looking at the nearest masses and picking the slowest speed. (At earth's surface, remembering the black hole size for the earth's mass is about a centimetre, and the earth's radius is 6000km),

$$v_{earth} = \sqrt{\frac{2 * 6000km}{1cm} - 2} \approx 35000c. \tag{14}$$

From the sun on earth:

$$v_{sun} = \sqrt{\frac{2 * 150e6km}{1km} - 2} \approx 17000c. \tag{15}$$

This all assumes a Lorentzian viewpoint of Einstein's ether[8]. Namely that there is a special frame, and these monopolar 'energy balance' waves are defined in the global rest frame (think of the Cosmic Microwave Background - CMB defined frame).

4.2. Quantum Mechanics

From other investigations[1], I surmise these superluminal waves might be the physical underpinning of quantum mechanics. After all, quantum mechanics has been experimentally verified to have non local connections at speeds much greater than c[13]. The theory presented here provides a way for our normal 'transverse' energy world to be subject to Lorentz invariance, while quantum effects such as wave function collapse and entanglement make use of these superluminal monopolar waves. Thus, this, in addition to Einstein's equations might form a starting point to determine how gravity and quantum mechanics work together.

4.3. Tsunami

The monopolar superluminal waves discussed here are similar to a Tsunami wave - Tsunami speed and wavelength are very high on the open ocean, and then become slower and shorter as they encounter shallow water. In fact the equation for speed is about the same $v_t \propto \sqrt{depth}$! In this analogy it's deep space that is like the deep ocean here. Varying speed waves exist in nature, and where they exist, they are 'measuring the depth' so to speak, which in the case of the Schwarzschild solution, means that the wave 'knows' how deep it is in the gravitational well.

4.4. The Stiffness of Space

Does our spacetime support monopolar (longitudinal) waves? The distinction in a physical material between transverse and longitudinal waves is well known. Transverse 'S' waves travel slower inside the earth than 'P' waves. The gravitational ether has a huge - but not infinite stiffness Blair[3]:

> In Newtonian physics spacetime is an infinitely rigid conceptual grid. Gravitational waves cannot exist in this theory. They would have infinite velocity and infinite energy density because in Newtonian gravitation the metrical stiffness of space is infinite. Conversely general relativity introduces a finite coupling coefficient between curvature of spacetime, described by the Einstein curvature tensor, and the stress energy tensor which describes the mass–energy which gives rise to the curvature. This coupling is expressed by the Einstein equation... The coupling coefficient $c^4/(8\pi G)$ is an enormous number, of order 10^{43}.
>
> This expresses the extremely high stiffness of space which is the reason that the Newtonian law of gravitation is an excellent approximation in normal circumstances, and why gravitational waves have a small amplitude, even when their energy density is very high.

4.5. Dark Matter and Dark Energy

Given the previous section, it's perhaps fun to surmise that dark matter may be the mass of the mechanism of an emergent quantum mechanics[2]. The new flexibity of general relativity with the self energy term could provide many avenues to look into for Dark Energy.

5. Discussion

If one takes the step to drop covariance from general relativity, then we arrive at what seems to point to a 'real' ether interpretation of General Relativity, with Lorentzian style special relativity[17][18] for light and standard model particles. Non transverse energy (monopolar waves), are referenced to the universal (aka CMB) rest frame. This monopolar energy is usually almost perfectly absent in normal laboratory experiments. It may be the energy mechanism quantum mechanics uses, since quantum phenomena have been experimentally verified to be non local[13][14].

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