Chapter 6

Summary and outlook

In this thesis, we calculated nucleon polarizabilities in Chiral Perturbation Theory with explicit $\Delta(1232)$ degrees of freedom. We calculated and analysed static, energy-dependent and Q^2-dependent polarizabilities up to $\mathcal{O}(q^4 + \epsilon^3)$ order in the small scale expansion. Moreover we calculated the static polarizabilities in a Δ-less approach up to $\mathcal{O}(q^4)$ and compared the convergence of both.

After the introduction to Chiral Perturbation Theory in chapter 2, we used the electric and magnetic radii as well as the polarizabilities α_{E1} and β_{M1} to calculate numerical values for the LECs $\{d_6, d_7, e_{54}, e_{74}\} = \{-1.38(0.05)m^2, -0.51(0.01)m^2, -0.15(0.12)m^3, 1.23(0.31)m^3\}$ as well as $\{e_{91}, e_{92}, 2e_{90} + e_{94} + e_{117}, 2e_{89} + e_{93} + e_{118}\} = \{-0.46(0.23)m^3, -0.22(0.8)m^3, -1.89(0.5)m^3, 1.80(1.2)m^3\}$ in sections 3.2 and 5.2.

We gave numerical results for all spin-independent polarizabilities up to octupole level and spin-dependent polarizabilities up to quadrupole level in section 5.3. For the spin-independent proton quadrupole polarizabilities we calculated $\{\alpha^{(p)}_{\mathrm{E2}}, \beta^{(p)}_{\mathrm{M2}}\} = \{29.5(2.3), -27.1(2.3)\} \times 10^{-4}\mathrm{fm}^5$ and for the dispersive corrections to those dipole polarizabilities we got $\{\alpha^{(p)}_{\mathrm{E1}\nu}, \beta^{(p)}_{\mathrm{M1}\nu}\} = \{-3.9(0.6), 10.2(0.9)\} \times 10^{-4}\mathrm{fm}^5$.[1] The results are in good agreement with fixed-t dispersive relations. We also compared our calculations with similar result obtained in ChPT with δ-counting [130]. Moreover, we gave first predictions for the octupole polarizabilities of the proton $\{\alpha^{(p)}_{\mathrm{E3}}, \beta^{(p)}_{\mathrm{M3}}\} = \{247.2(17.3), -199.8(16.3)\} \times 10^{-4}\mathrm{fm}^7$ as well as higher dispersive polarizabilities $\{\alpha^{(p)}_{\mathrm{E2}\nu}, \beta^{(p)}_{\mathrm{M2}\nu}\} = \{-40.4(3.0), 30.4(3.0)\} \times 10^{-4}\mathrm{fm}^7$ and $\{\alpha^{(p)}_{\mathrm{E1}\nu^2}, \beta^{(p)}_{\mathrm{M1}\nu^2}\} = \{8.4(0.6), -3.7(0.5)\} \times 10^{-4}\mathrm{fm}^7$.

[1] The errors for all polarizabilities in this summary stem the uncertainties in the variables and a Bayesian error estimate and are calculated as the root of the squares.

We also calculated dipole, quadrupole and dispersive spin-dependent polarizabilities

$$\begin{aligned}
\{\gamma^{(p)}_{\mathrm{E1E1}}, \gamma^{(p)}_{\mathrm{M1M1}}\} &= \{-4.65(0.24), 4.80(0.48)\} \times 10^{-4}\mathrm{fm}^4\,,\\
\{\gamma^{(p)}_{\mathrm{E1M2}}, \gamma^{(p)}_{\mathrm{M1E2}}\} &= \{-0.08(0.13), 2.47(0.26)\} \times 10^{-4}\mathrm{fm}^4\,,\\
\{\gamma^{(p)}_{\mathrm{E1E1}\nu}, \gamma^{(p)}_{\mathrm{M1M1}\nu}\} &= \{-3.72(0.18), 2.35(0.22)\} \times 10^{-4}\mathrm{fm}^6\,,\\
\{\gamma^{(p)}_{\mathrm{E1M2}\nu}, \gamma^{(p)}_{\mathrm{M1E2}\nu}\} &= \{-0.95(0.08), 1.83(0.13)\} \times 10^{-4}\mathrm{fm}^6\,,\\
\{\gamma^{(p)}_{\mathrm{E2E2}}, \gamma^{(p)}_{\mathrm{M2M2}}\} &= \{-8.74(0.41), -9.42(1.17)\} \times 10^{-6}\mathrm{fm}^6\,,\\
\{\gamma^{(p)}_{\mathrm{E2M3}}, \gamma^{(p)}_{\mathrm{M2E3}}\} &= \{6.84(0.34), -0.59(0.55)\} \times 10^{-6}\mathrm{fm}^6\,.
\end{aligned}$$

For the neutron spin-independent polarizabilities, we calculated

$$\begin{aligned}
\{\alpha^{(n)}_{\mathrm{E2}}, \beta^{(n)}_{\mathrm{M2}}\} &= \{28.7(2.3), -27.0(2.3)\} \times 10^{-4}\mathrm{fm}^5\,,\\
\{\alpha^{(n)}_{\mathrm{E1}\nu}, \beta^{(n)}_{\mathrm{M1}\nu}\} &= \{-2.6(0.6), 10.2(0.9)\} \times 10^{-4}\mathrm{fm}^5\,,\\
\{\alpha^{(n)}_{\mathrm{E3}}, \beta^{(n)}_{\mathrm{M3}}\} &= \{248.8(17.4), -199.7(16.3)\} \times 10^{-4}\mathrm{fm}^7\,,\\
\{\alpha^{(n)}_{\mathrm{E2}\nu}, \beta^{(n)}_{\mathrm{M2}\nu}\} &= \{-43.2(3.1), 30.6(3.0)\} \times 10^{-4}\mathrm{fm}^7\,,\\
\{\alpha^{(n)}_{\mathrm{E1}\nu^2}, \beta^{(n)}_{\mathrm{M1}\nu^2}\} &= \{10.1(0.6), -4.0(0.9)\} \times 10^{-4}\mathrm{fm}^7\,.
\end{aligned}$$

Moreover, for the neutron spin-dependent polarizabilities, we got

$$\begin{aligned}
\{\gamma^{(n)}_{\mathrm{E1E1}}, \gamma^{(n)}_{\mathrm{M1M1}}\} &= \{-6.17(0.30), 5.22(0.55)\} \times 10^{-4}\mathrm{fm}^4\,,\\
\{\gamma^{(n)}_{\mathrm{E1M2}}, \gamma^{(n)}_{\mathrm{M1E2}}\} &= \{-0.75(0.17), 3.59(0.30)\} \times 10^{-4}\mathrm{fm}^4\,,\\
\{\gamma^{(n)}_{\mathrm{E1E1}\nu}, \gamma^{(n)}_{\mathrm{M1M1}\nu}\} &= \{-5.17(0.24), 2.67(0.24)\} \times 10^{-4}\mathrm{fm}^6\,,\\
\{\gamma^{(n)}_{\mathrm{E1M2}\nu}, \gamma^{(n)}_{\mathrm{M1E2}\nu}\} &= \{-1.00(0.08), 2.19(0.14)\} \times 10^{-4}\mathrm{fm}^6\,,\\
\{\gamma^{(n)}_{\mathrm{E2E2}}, \gamma^{(n)}_{\mathrm{M2M2}}\} &= \{-1.99(0.15), -10.33(1.28)\} \times 10^{-6}\mathrm{fm}^6\,,\\
\{\gamma^{(n)}_{\mathrm{E2M3}}, \gamma^{(n)}_{\mathrm{M2E3}}\} &= \{6.67(0.33), -1.55(0.52)\} \times 10^{-6}\mathrm{fm}^6\,.
\end{aligned}$$

We already could accomplish good agreement with other theoretical models such as fixed-t dispersion relations for the dipole and quadrupole polarizabilities. For octupole polarizabilities other values for comparison are still missing and are hoped for in the future. We also checked the convergence of the dipole polarizabilities in the heavy-baryon expansion.

We also calculated the energy-dependent polarizabilities $\alpha_{\mathrm{E1}}(\omega)$, $\beta_{\mathrm{M1}}(\omega)$, $\alpha_{\mathrm{E2}}(\omega)$, $\beta_{\mathrm{M2}}(\omega)$, $\gamma_{\mathrm{E1E1}}(\omega)$, $\gamma_{\mathrm{M1M1}}(\omega)$, $\gamma_{\mathrm{E1M2}}(\omega)$ and $\gamma_{\mathrm{M1E2}}(\omega)$ in section 5.4.

In addition to the already calculated values at $q^4+\epsilon^3$ level, the full ϵ^4-calculation is needed to complete the one-loop calculation of the nucleon polarizabilities and therefore, it is left to be seen, how the contributions of $\mathcal{O}(q^4)$ split after introducing the Δ at this level as well. Especially, for the Q^2-dependent virtualities additional data (such as from LQCD) is needed to compare our predictions with.

Although we derived analytic expressions for twelve structure functions, we mainly concentrated on the six structure functions that are known from Real Compton Scattering. Further investigation of the other six functions might lead to a better understanding of the underlying structure of the nucleon.