

Recent lattice results with light quarks at zero and nonzero temperature

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Abstract. I review selected recent results of the MILC collaboration (1) determining masses of the light quarks and decay constants of the pion and kaon, (2) analyzing the scalar meson (a_0 and f_0) channel in the presence of two-body thresholds, and (3) determining the equation of state of the quark-gluon plasma with a more realistic quark spectrum than hitherto.

1. Toward high precision lattice calculations

Our ability to solve QCD through numerical simulation has grown dramatically over the past decade, thanks to the development of new algorithms that reduce lattice artifacts substantially and to the continued increase in computer speeds. The Asqtad staggered fermion formulation [1, 2, 3, 4, 5], coupled with advances in our understanding of lattice chiral perturbation theory [6, 7, 8], has been very successful in reproducing a variety of known decay constants and mass splittings to an accuracy of a couple percent [9].

Staggered fermion methods have a comparatively low computational cost, but to achieve a realistic quark spectrum, they require a somewhat controversial approximation: taking the fourth-root of the fermionic determinant. The concern is that the approximation introduces nonlocalities and violations of unitarity, possibly even placing the theory in a different universality class from that of other regularizations of QCD. Fortunately, over the past year there has been considerable progress in understanding the validity of the method. For a review, see [10].

In this talk I present some highlights of recent work by the MILC collaboration using this numerical approach. (1) I will give an update of an effort to match lattice calculations to chiral

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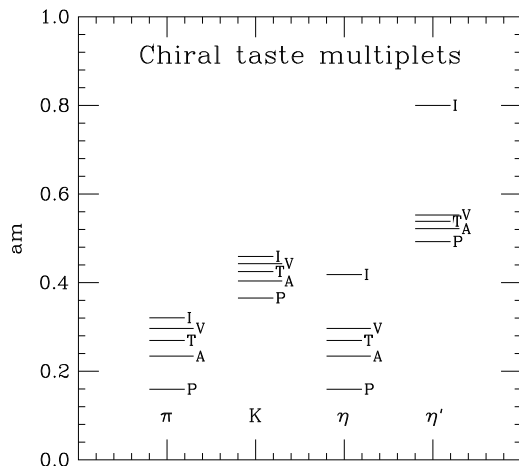


Figure 1. Taste multiplets for π , K , η , and η' for the study ensemble. The letters indicate the taste symmetry assignments, grouped as predicted by Lee and Sharpe [6].

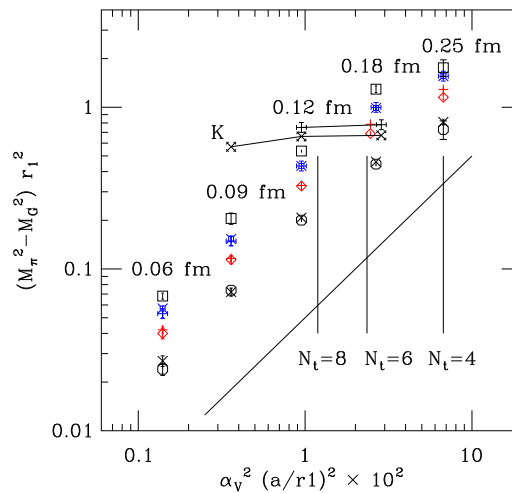


Figure 2. Splittings in the pion taste multiplet as a function of lattice spacing with $m_{ud}/m_s = 0.4$. The vertical scale gives the difference in squared mass between the member of the multiplet and the lowest, Goldstone boson member in units of r_1 . The horizontal scale gives the square of the running color coupling constant times the square of the lattice spacing in units of r_1 . Symbols distinguish nondegenerate members of the multiplet. Staggered chiral perturbation theory predicts a slope of one as indicated by the solid line.

perturbation theory, leading to a determination of the masses of the u , d and s quarks and the pion and kaon decay constants. For more detail see [11, 12] and [13]. (2) I will review a recent study of the difficult scalar meson correlators. In this case, staggered chiral perturbation theory allows us to disentangle the supposed quark-antiquark states from the multitude of two-body channels and provides a good illustration of how artifacts of the fourth-root approximation disappear in the continuum limit. For further detail see [11, 14, 15]. (3) I will present recent results of a determination of the equation of state of the quark-gluon plasma with a more realistic light quark spectrum than hitherto [16, 17, 18].

The determination of quark masses and the analysis of scalar meson correlators is done with the assistance of rooted staggered chiral perturbation theory (rS χ PT) [19, 20]. This formalism accounts for the lattice artifacts of the staggered fermion formalism in the low energy limit. Fitting lattice results to the predicted behavior as a function of the quark masses and lattice spacing makes it possible to do a simultaneous extrapolation to the physical quark mass and continuum limit, and it gives a determination of the parameters of the chiral theory. This procedure is the key to achieving high precision. Armed with the values of the low energy couplings of the chiral theory, we can predict other low energy quantities. Staggered chiral perturbation theory also describes the behavior of various quantities in “partial quenching”, in which the masses of the valence quarks are taken to be different from the sea quark masses, thus

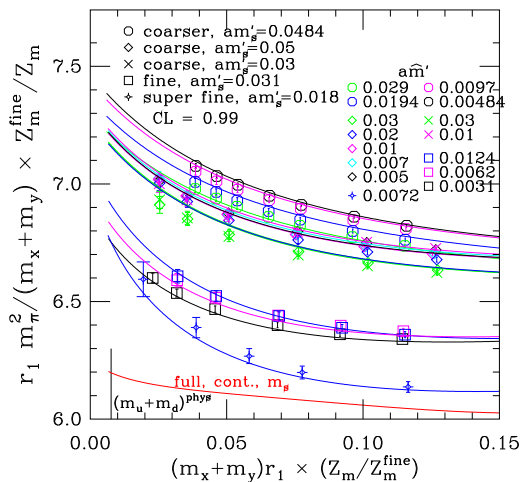


Figure 3. The ratio of the Goldstone pion mass squared to the sum of the light valence quark masses as a function of the sum of the light valence quark masses for the several lattice ensembles in our study. Quark masses are given in lattice units. The red curve labeled “full, cont.” predicts the continuum limit at the physical strange quark mass with valence and sea quark masses set equal. The vertical line marks the physical value of the light quark masses.

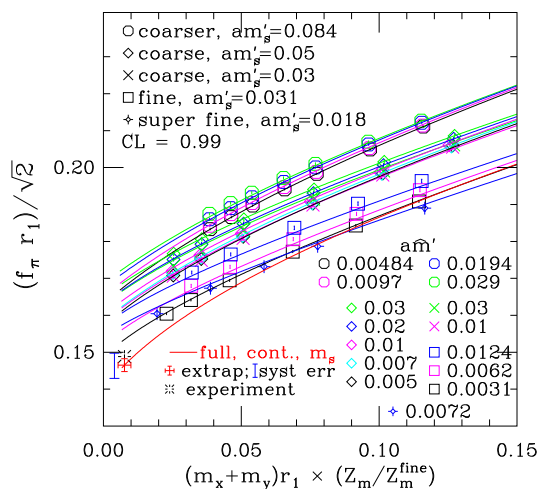


Figure 4. The Goldstone pion decay constant as a function of the sum of the light valence quark masses. Quark masses are given in lattice units. The red curve is as in Fig. 3. The red fancy plus marks the predicted value. The adjacent blue bar indicates the systematic error and the burst plots the experimental value.

enlarging the domain in which lattice results can be matched to chiral perturbation theory.

Staggered chiral perturbation theory provides an understanding of the effects of species doubling which plagues lattice fermions, and staggered fermions in particular. In the staggered scheme each quark flavor appears in four “tastes”, and, as indicated in Fig. 1, each quark-antiquark meson comes in a taste multiplet of sixteen members. They become degenerate in the continuum limit. To be more explicit, in terms of the strong coupling α and lattice spacing a , the improved actions have residual discretization errors of order aa^2 in most quantities, but $\mathcal{O}(\alpha^2 a^2)$ in meson mass splittings. Figure 2 shows splittings in the pion taste multiplet as a function of lattice spacing. (The quantity $r_1 \approx 0.318(7)$ fm is a particularly reliable measure of the lattice scale. It is based on a measurement of the static quark-antiquark potential on the same ensemble.) The solid line gives the slope expected from a splitting proportional to $\alpha^2 a^2$. The trend is consistent with this prediction.

2. Light quark masses

Over the past few years the MILC collaboration has been generating a large archive of publicly available gauge configuration files describing the QCD vacuum in the presence of two flavors of light quarks (u and d) and one strange quark (s). A set of configurations at the same sea quark masses and lattice spacing forms an ensemble. These ensembles are used to measure quantities of importance to physics. The cost of generating them grows very steeply as the lattice spacing a is decreased and as the light quark masses m_{ud} are reduced toward their physical values. Currently, MILC is generating configurations at a resolution of 0.06 fm at $m_{ud}/m_s = 0.2$ and

expects to reach 0.1 at this lattice spacing in the near future.

The pion and kaon masses and their decay constants f_π and f_K are measured as a function of lattice spacing, sea quark masses, and valence quark masses. The lattice ensembles in the study and their approximate lattice spacing are “coarser” (0.15 fm), “coarse” (0.12 fm), “fine” (0.09 fm), and “superfine” (0.06 fm). The strange quark mass is set close to its physical value. The light sea quark mass ratio m_{ud}/m_s ranges from 0.1 to 1. The valence quark mass is varied independently over this range. Figure 3 shows a portion of the results including a fit to the form required by rS χ PT. Altogether 978 data points are included in a fit involving 28 unconstrained and 26 tightly constrained parameters. The fit is excellent.

Similarly Fig. 4 shows the pion decay constant in units of r_1 as a function of the sum of the light valence quark masses.

The light quark masses are determined by requiring that the predicted masses of the π and K mesons match the experimental values at the physical point. The simulations are done with degenerate u and d quarks, so in terms of $\hat{m} \equiv (m_u + m_d)/2$, but with some mild assumptions about electromagnetic mass splittings, it is possible to infer separate masses for the u and d quarks. The most recent determination [13] gives

$$\begin{aligned} m_s^{\overline{MS}} &= 90(0)(5)(4)(0)\text{MeV} \\ \hat{m}^{\overline{MS}} &= 3.3(0)(2)(2)(0)\text{MeV} \\ m_u^{\overline{MS}} &= 2.0(0)(1)(2)(1)\text{MeV} \\ m_d^{\overline{MS}} &= 4.6(0)(2)(2)(1)\text{MeV} \\ f_\pi &= 128.6(0.4)(3.0)\text{MeV} \\ f_K &= 155.3(0.4)(3.1)\text{MeV}. \end{aligned}$$

Quark masses are defined in the modified minimal subtraction scheme at scale 2 GeV.

3. Scalar meson correlators

When up and down quark masses are sufficiently light, the isovector (a_0) and isosinglet (f_0) scalar meson correlators are dominated at large distances by two-body states composed of π , K , and η . Because each meson occurs in a sixteen-member taste multiplet, the multiplet is split at nonzero lattice spacing, resulting in a proliferation of two-body intermediate states, thereby complicating the analysis of the correlators. To make matters worse, many of these states are lattice artifacts with unphysical masses, and some even have ghost (negative) weights. Thus we see nonlocalities in the form of unphysical long-range contributions to the correlators, and we see violations of unitarity in the form of unphysical and negative norm intermediate states. Such artifacts are expected to disappear in the continuum limit.

Fortunately, these complexities are described in detail by rS χ PT [19, 20]. It provides a strict framework in which to analyze these two-body contributions in terms of just a few low energy constants – the same ones determined by fits to the meson masses and decay constants. As we shall see, in rS χ PT the lattice artifacts disappear in the continuum limit.

3.1. Pseudoscalar meson multiplet

The a_0 channel was studied in recent years in staggered fermion QCD by the MILC collaboration [21] and the UKQCD collaboration [22]. Both groups found that the correlator appeared to contain states with energies well below those of possible thresholds involving known mesons. At Lattice 2005 one of us showed that rS χ PT provides a simple explanation [23, 24], namely, lattice artifacts in the staggered fermion scheme introduce unphysically light two-body states in the scalar meson channel.

taste	a_0	f_0
I	2/3	1/4
$I0$	-15/8	-1
V	4/8	4/4
T	6/8	6/4
A	4/8	4/4
P	1/8	1/4

Table 1. Two-body channel weights for $2 + 1$ flavors in the a_0 and f_0 channels. The a_0 weights apply to the $\pi\eta$ contributions for tastes I, P, V, A, T , and the f_0 weights apply to the $\pi\pi$ contributions. Here the taste-singlet η is the physical state. For the a_0 the weight $I0$ denotes the contribution from the taste-singlet pion plus the bare taste-singlet η (unshifted by the anomaly). For the f_0 the weight $I0$ denotes the contribution from two bare taste-singlet etas.

To see this we refer to the meson taste multiplets sketched in Fig. 1. Of particular concern are the η and η' multiplets. They are mixed by the anomaly. But the anomaly is a taste singlet. So only the taste-singlet members are proper candidates for the physical states. The other unphysical taste members are unmixed at tree level and remain degenerate with the pions or nearly so.

Consider the two-meson contributions to the physical taste-singlet a_0 correlator. The lowest energy state should be $\pi\eta$ with a variety of possible taste assignments for the π and η . Taste symmetry requires that π and η tastes be identical. Otherwise all contribute. In particular, the taste-pseudoscalar pion and a taste-pseudoscalar eta have a threshold energy equal to just twice the Goldstone pion mass. Other taste pairs are similarly light. Thus we expect anomalous low-energy contributions to the correlator from these unphysical states.

3.2. Threshold weights

Explicit expressions were presented in [23, 24] for the two-meson contribution to the a_0 correlator, including all tastes. Some of us have extended this analysis to the f_0 correlator [15]. The expressions (not written here) consist of several two-body threshold contributions with weights depending on the number of flavors and the replica factor [7] needed to correct for the unwanted taste multiplicity. If we ignore mixings induced by the taste axial-vector and vector hairpins, the weights for the physical $2 + 1$ flavor case are given in Table 1. In the continuum limit in the a_0 channel, all thresholds but the taste-singlet $\pi\eta$ become degenerate and the axial-vector and vector hairpin mixings vanish. As is evident from the table, the weights add up to zero: the ghost threshold cancels the unphysical thresholds, and only the physical taste-singlet $\pi\eta$ survives. In the f_0 channel, all the $\pi\pi$ thresholds become degenerate leaving a proper weight of three for the three charge assignments for the physical pions.

3.3. Fits and Results

Our initial study used only one “coarse” ensemble with a quark mass ratio $m_{ud}/m_s = 0.1$. Shown in Figs. 5, 6 and 7 are results of fitting the measured correlators at several scalar meson momenta to the expressions

$$\begin{aligned}
 C_{a0}(p, t) &= f_{\text{meson},a0}(p, t) + f_{\text{bubble},a0}(p, t) \\
 C_{f0}(p, t) &= f_{\text{meson},f0}(p, t) + f_{\text{bubble},f0}(p, t)
 \end{aligned}$$

where the two-meson “bubble” contribution is determined completely by the independently measured pseudoscalar meson masses and three chiral low energy constants. The latter are also

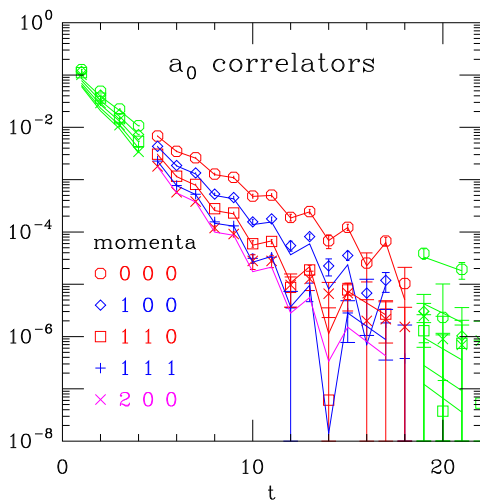


Figure 5. Fit to the a_0 correlator for five momenta. Green points dropped.

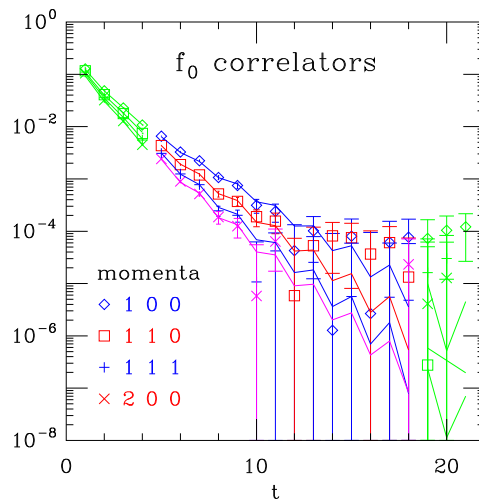


Figure 6. Fit to the f_0 correlator for four momenta.

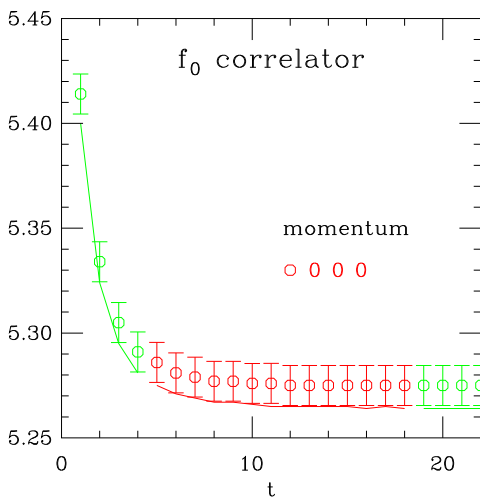


Figure 7. Fit to the f_0 correlator for zero momentum.

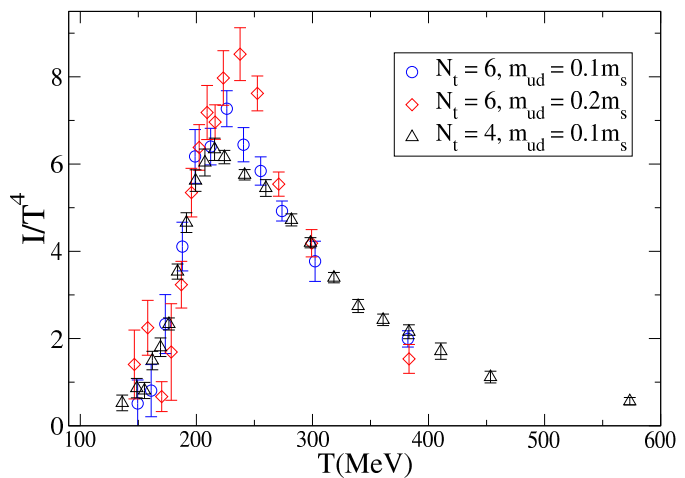


Figure 8. Interaction measure *vs* temperature.

determined in fits to the meson masses and decay constants. The “meson” contribution is a standard set of single particle states, including a possible quark-antiquark a_0 or f_0 .

The three low energy constants are $\mu = m_\pi^2/(2m_{u,d})$ and the axial vector and vector hairpin constants $\delta_A = a^4\delta'_A$ and $\delta_V = a^4\delta'_V$ in the notation of [11]. In this analysis it was convenient to impose a range of values of δ_V by means of a Bayesian prior, based on the analysis of the meson masses and decay constants [11]. The other two values were varied with only loose priors. The resulting values, $r_1\mu = 8.2(1.1)$ and $\delta_A = -0.056(7)$ are to be compared with $r_1m_\pi^2/(2m_{u,d}) = 6.72(2)$ from a direct measurement of the mass spectrum and $\delta_A = -0.040(6)$ from chiral fits to the meson mass spectrum and decay constants. Our results appear to be reasonably consistent with the model and with the previously determined low energy constants.

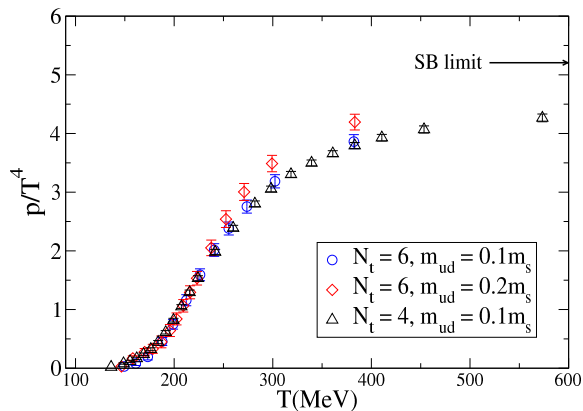


Figure 9. Pressure *vs* temperature.

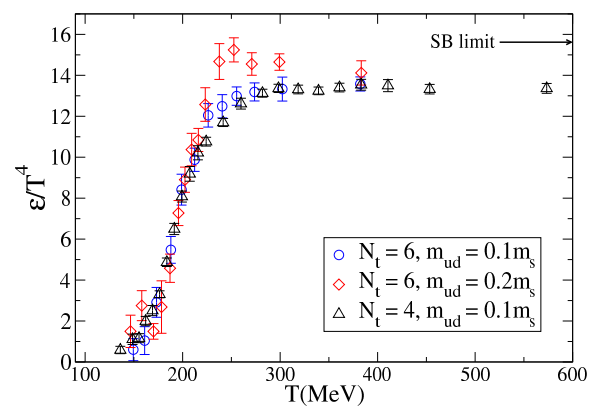


Figure 10. Energy density *vs* temperature.

4. Quark gluon plasma equation of state

The equation of state of the quark gluon plasma is a central ingredient in hydrodynamical models of the plasma dynamics. It is directly calculable in lattice simulations. The popular integral method for determining it [25] begins with the thermodynamic relations

$$\varepsilon - 3p = I = -\frac{T}{V} \frac{d \ln Z}{d \log a} \approx -\frac{\partial p}{\partial \log a}, \quad (1)$$

where ε is the energy density, p is the pressure, I is the interaction measure, T is the temperature, V is the spatial volume, Z is the partition function, and a is the lattice spacing. The interaction measure I is simplest to calculate, since it involves local operators of the type that appear in the lattice action and derivatives of the various lattice parameters with respect to lattice spacing. From the last equality in the chain we see that we can obtain the pressure by integrating the interaction measure, and finally, from the first equality we get the energy density. We choose to integrate along lines of approximately constant $m_{ud}/m_s = 0.1$ and 0.2 .

The energy density and pressure are, of course, always defined relative to their vacuum values. Thus we need the difference between the quantities calculated at the desired temperature and at zero temperature. Since the quantities in the difference are dominated by ultraviolet fluctuations, which grow as a^{-4} as we approach the continuum limit, the calculation becomes increasingly expensive at small a . Thus calculations until now have been limited to somewhat coarse lattices.

In conventional practice the temperature is varied by varying the lattice spacing. Since $T = 1/(N_t a)$, where N_t is the lattice extent in the imaginary time direction, at any fixed N_t we map out a temperature scale. In particular for a given N_t , the lower the temperature, the coarser the lattice spacing. To move closer to the continuum we increase N_t and repeat with a faster computer. There is an extensive body of work at $N_t = 4$ [26, 27], but relatively little at $N_t = 6$ [28, 16, 29].

What is needed is a simulation with reasonably small lattice artifacts and a realistic quark (*i.e.* meson) spectrum. For staggered fermions it would seem important in the interesting region just below the crossover that most of the pion taste multiplet be lighter than the kaon taste multiplet. In Fig. 2 the vertical bars indicate the location of the crossover temperature T_c for $N_t = 4, 6$, and 8 . Also shown is the mass of the lightest kaon at $m_{ud}/m_s = 0.1$ and 0.2 for the Asqtad action. At $N_t = 8$ all members of the pion multiplet are lighter than the kaons at the crossover temperature. At $N_t = 4$, however, the extrapolated lightest kaon mass approximates the second lightest pion at the crossover. Thus $N_t = 6$ is better than 4 , but 8 is better still.

In Figs. 8, 9 and 10 we show our results for the interaction measure, pressure, and energy density for both $N_t = 4$ and 6 . The interaction measure appears to peak more sharply just

above the crossover region at $N_t = 6$, but there are otherwise no dramatic differences between the two N_t values.

This work was supported in part by the US DOE and the US NSF. Computing resources were provided by FNAL, IU, NCSA, NERSC, PSC, ORNL, SDSC, and the Utah CHPC.

References

- [1] Tom Blum *et al.* Improving flavor symmetry in the Kogut-Susskind hadron spectrum. *Phys. Rev.*, D55:1133–1137, 1997.
- [2] Kostas Orginos and Doug Toussaint. Testing improved actions for dynamical Kogut-Susskind quarks. *Phys. Rev.*, D59:014501, 1999.
- [3] Kostas Orginos, Doug Toussaint, and R. L. Sugar. Variants of fattening and flavor symmetry restoration. *Phys. Rev.*, D60:054503, 1999.
- [4] G. Peter Lepage. Flavor-symmetry restoration and Symanzik improvement for staggered quarks. *Phys. Rev.*, D59:074502, 1999.
- [5] J. F. Lagae and D. K. Sinclair. Improving the staggered quark action to reduce flavour symmetry violations. *Nucl. Phys. Proc. Suppl.*, 63:892–894, 1998.
- [6] Weon-Jong Lee and Stephen R. Sharpe. Partial flavor symmetry restoration for chiral staggered fermions. *Phys. Rev.*, D60:114503, 1999.
- [7] C. Aubin and C. Bernard. Staggered chiral perturbation theory. *Nucl. Phys. Proc. Suppl.*, 129:182–184, 2004.
- [8] C. Aubin and C. Bernard. Pseudoscalar decay constants in staggered chiral perturbation theory. *Phys. Rev.*, D68:074011, 2003.
- [9] C. T. H. Davies *et al.* High-precision lattice QCD confronts experiment. *Phys. Rev. Lett.*, 92:022001, 2004.
- [10] Stephen R. Sharpe. Rooted staggered fermions: Good, bad or ugly? *PoS.*, LAT2006:022, 2006.
- [11] C. Aubin *et al.* Light pseudoscalar decay constants, quark masses, and low energy constants from three-flavor lattice QCD. *Phys. Rev.*, D70:114501, 2004.
- [12] C. Bernard *et al.* Update on the physics of light pseudoscalar mesons. *PoS.*, LAT2006:163, 2006.
- [13] C. Bernard *et al.* Low energy constants from the MILC collaboration. 2006. hep-lat/0611024.
- [14] C. W. Bernard, C. DeTar, Ziwen Fu, and S. Prelovsek. Taste breaking effects in scalar meson correlators. *PoS.*, LAT2006:173, 2006.
- [15] C. Bernard, C. DeTar, Ziwen Fu, and S. Prelovsek. Scalar meson spectroscopy with lattice staggered fermions. in preparation, 2007.
- [16] C. Bernard *et al.* The equation of state for QCD with 2+1 flavors of quarks. *PoS.*, LAT2005:156, 2005.
- [17] C. Bernard *et al.* The QCD equation of state with Asqtad staggered fermions. *PoS.*, LAT2006:139, 2006.
- [18] C. Bernard *et al.* QCD equation of state with 2+1 flavors of improved staggered quarks. 2006. hep-lat/0611031.
- [19] C. Aubin and C. Bernard. Pion and kaon masses in staggered chiral perturbation theory. *Phys. Rev.*, D68:034014, 2003.
- [20] C. Bernard. Staggered chiral perturbation theory and the fourth-root trick. *Phys. Rev.*, D73:114503, 2006.
- [21] C. Aubin *et al.* Light hadrons with improved staggered quarks: Approaching the continuum limit. *Phys. Rev.*, D70:094505, 2004.
- [22] Eric B. Gregory, Alan C. Irving, Craig C. McNeile, Steven Miller, and Zbyszek Sroczynski. Scalar glueball and meson spectroscopy in unquenched lattice QCD with improved staggered quarks. *PoS.*, LAT2005:027, 2006.
- [23] S. Prelovsek. Effects of staggered fermions and mixed actions on the scalar correlator. *Phys. Rev.*, D73:014506, 2006.
- [24] Sasa Prelovsek. Effects of partial quenching and staggered fermions on the scalar correlator. *PoS.*, LAT2005:085, 2006.
- [25] J. Engels, J. Fingberg, F. Karsch, D. Miller, and M. Weber. Nonperturbative thermodynamics of SU(N) gauge theories. *Phys. Lett.*, B252:625–630, 1990.
- [26] T. Blum, Leo Karkkainen, D. Toussaint, and Steven A. Gottlieb. The beta function and equation of state for QCD with two flavors of quarks. *Phys. Rev.*, D51:5153–5164, 1995.
- [27] F. Karsch, E. Laermann, and A. Peikert. The pressure in 2, 2+1 and 3 flavour QCD. *Phys. Lett.*, B478:447–455, 2000.
- [28] Claude W. Bernard *et al.* QCD thermodynamics with two flavors at $N(t) = 6$. *Phys. Rev.*, D45:3854–3861, 1992.
- [29] Y. Aoki, Z. Fodor, S. D. Katz, and K. K. Szabo. The equation of state in lattice QCD: With physical quark masses towards the continuum limit. *JHEP*, 01:089, 2006.