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THE CHERENKOV WAKEFIELD ACCELERATOR*

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The possibility of employing a dielectric-filled cavity as a high gradient wakefield accelerator is investigated. It is demonstrated that accelerating gradients on the order of 140 MV/m can be achieved by using modest drive-beam currents of 7.5 kA, and that large transformer ratios can be attained by driving the system with a beam that has a linearly ramped current distribution. This device can be operated in the 20 GHz frequency range, thus making a very small structure possible. Self-consistent particle-in-cell simulations are presented in support of the theory.

During the past several years there has been increasing interest in using wakefield acceleration techniques to achieve TeV energies with the next generation of linear colliders.[1,2] A wakefield accelerator uses the radiation excited by an intense electron beam propagating in a slow wave structure to accelerate a low-current load beam to a much higher energy. Fundamentally, any slow wave structure can be employed in this context. The plasma wakefield accelerator employs a plasma as the accelerating cavity,[4–9] a modified disc-loaded cavity serves as the slow wave structure for another wakefield accelerator, the Voss–Weiland radial transformer.[1,3] One can think of many other structures that could be used for wakefield acceleration. Because of its inherent simplicity we have focused our studies on a dielectric waveguide.[10,11]

The Cherenkov wakefield accelerator is a very simple device. The geometry, shown in Fig. 1, consists of a gapless cavity filled with a dielectric material. A hole is drilled in the center of the dielectric to allow passage of the drive and load beams. As the drive beam propagates through the cavity it generates Cherenkov radiation, and all those modes having a phase velocity of $v_\phi = \beta c/\varepsilon^{1/2}$ are excited. The relative amplitudes of these modes depend upon the pulse length and the current profile of the drive beam; these characteristics can be adjusted so that the fundamental mode dominates. The expanding wave fronts superpose to create, an axial accelerating electric field that has a phase velocity βc. This high phase velocity allows for the loading and acceleration of a properly phased, trailing electron bunch. The law of conservation of energy requires that the load beam

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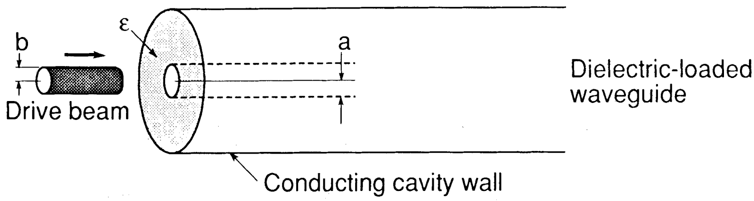

FIGURE 1 Geometry of the Cherenkov wakefield accelerator.

carry much less charge than the drive beam in order to achieve a very high energy; the load beam's effect on the wakefield is therefore negligible.

The wavelength of the fundamental mode is given approximately by $2\pi a(\varepsilon - 1)^{1/2}/p_{0,1}$, where a defines the radius of the cavity, ε is the dielectric constant, and $p_{0,1} \cong 2.4$ represents the first zero of the Bessel function, $J_0(x)$. By varying a or ε one can tune the frequency of the generated radiation. The amplitude of the accelerating field, $E_a \sim I_p/a$, where I_p defines the peak drive beam current. In the interest of building a relatively compact accelerator one would like to make E_a as large as possible; this requires driving the system with multikiloampere currents.

I. THEORY OF THE CHERENKOV WAKEFIELD ACCELERATOR

It is straightforward to arrive at a general expression for the axial wakefield by simply considering the well-known properties of Cherenkov radiation. The radial eigenfunctions are given by $J_0(k_\perp^\nu r)$, where r is radial distance and J_0 defines a Bessel function of the first kind. The perpendicular wavevector is determined from the requirement that $E_z\,(r = a)$ vanish. This boundary condition yields $k_\perp^\nu = p_{0,\nu}/a$, where $p_{0,\nu}$ defines the νth zero of $J_0(x)$. Given $k_\perp^\nu$, one can readily find $k_\parallel^\nu$ by employing the Cherenkov angle relationship, $k_\parallel^\nu/k^\nu = \varepsilon^{-1/2}$. This gives $k_\parallel^\nu = p_{0,\nu}/a(\varepsilon - 1)^{-1/2}$.

The parallel phase velocity is given by $\omega^\nu/k_\parallel^\nu = \beta c$. The frequency of the generated *rf* is thus $f_{0,\nu} = 2\pi c p_{0,\nu}/a(\varepsilon - 1)^{-1/2}$. The frequency of the fundamental eigenmode can be approximated by $f_{0,1} \cong 11.5\text{ GHz}\,[a(\varepsilon - 1)^{-1/2}]^{-1}$. For example, for $a = 0.5$ cm and $\varepsilon = 2.3$, $f_{0,1} = 20$ GHz, corresponding to a wavelength, $\lambda_{0,1}$ equals 1.5 cm.

Given the perpendicular and parallel wavevectors and the frequency, the axial electric field can be expressed in the general form

$$E_z(r, z, t) = \sum A_\nu J_0(p_{0,\nu} r/a) \cos\left[p_{0,\nu}(\varepsilon - 1)^{-1/2}(\beta c t - z)/a\right]$$

where the modal amplitudes, A_ν, must be determined. To evaluate these coefficients we first consider a simple disc current source of the form

$$\mathbf{j}(r, z, t) = \mathbf{z} c\sigma(r)\delta(ct - z) = \mathbf{z} c\sigma_0\Theta(b - r)\delta(ct - z) \tag{1}$$

where Θ defines the Heaviside step function, b is the beam radius and c is the speed of light. The approximation that the beam moves at c ($\beta = 1$) results in the elimination of terms $O(\gamma^{-2})$. (For sufficiently relativistic beams these terms are insignificant.) Given this source, and Fourier-transforming Faraday's and Ampere's laws in z and t, one obtains the following equations for the fields:

$$\frac{1}{r}\frac{\partial}{\partial r} r B_\theta = \frac{4\pi}{c}\sigma_0 \delta(\omega - kc) - i\frac{\varepsilon\omega}{c} E_z \tag{2}$$

$$B_\theta = \frac{\varepsilon\omega}{kc} E_r \tag{3}$$

$$ikE_r - \frac{\partial E_z}{\partial r} = i\frac{\omega}{c} B_\theta \tag{4}$$

In these equations k refers to the parallel wave vector. The delta function appearing on the RHS of Eq. (2) indicates that only those modes having a parallel phase velocity equal to the beam velocity (c) will be excited. For these modes Eq. (3) indicates that in the vacuum region $B_\theta = E_r$. Propagating the beams in vacuum as opposed to a medium such as a plasma means that the beams will experience a negligible radial wakefield. This is an important characteristic of this device; in particular, it allows the current-profile of the drive beam to be maintained. This is crucial for establishing and maintaining a large transformer ratio. Manipulating Eqs. (2)–(4) results in the following inhomogeneous wave equation for the accelerating wakefield:

$$\left\{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\varepsilon\omega^2}{c^2} - k^2\right\} E_z(r, \omega, k) = i\frac{4\pi c}{\varepsilon\omega}\sigma(r)\left(k^2 - \frac{\varepsilon\omega^2}{c^2}\right)\delta(\omega - kc) \tag{5}$$

Once this equation is solved, the other field components can be obtained via Eqs. (3) and (4). To solve Eq. (5) we ignore the hole in the dielectric (a valid approximation provided that is much smaller than λ_1) and expand $E_z(r, \omega, k)$ as a Fourier–Bessel series:

$$E_z(r, \omega, k) = \sum_{v=1}^{\infty} A_v(\omega, k) J_0(p_{0,v} r/a), \tag{6}$$

where $p_{0,v}$ is a zero of $J_0(x)$. This field automatically satisfies the boundary condition that E_z vanish at the conducting cavity wall. Substituting Eq. (6) into Eq. (5) and using the orthogonality condition for Bessel functions on a finite interval yields the coefficients,

$$A_v(\omega, k) = i8\pi\sigma_0 c \frac{b\, J_1(p_{0,v} b/a)}{a\, p_{0,v} J_1^2(p_{0,v})}\frac{1}{\varepsilon\omega}\left\{k^2 - \frac{\varepsilon\omega^2}{c^2}\right\}\frac{\delta(\omega - kc)}{\dfrac{\varepsilon\omega^2}{c^2} - k^2 - \dfrac{p_{0,v}^2}{a^2}} \tag{7}$$

Inverse Fourier-transforming $A_v(\omega, k)$ then gives the accelerating field:

$$E_z(r, z, t) = \sum_{v=1}^{\infty} A_v J_0(p_{0,v} r/a) \cos(k^v(ct - z)), \tag{8}$$

where the coefficients are given by

$$\mathbf{A}_\nu = \frac{8\pi\sigma_0}{\varepsilon}\frac{b}{a}\frac{J_1(p_{0,\nu}b/a)}{p_{0\nu}J_1^2(p_{0,\nu})}. \tag{9}$$

For sufficiently large ν (such that $p_{0,\nu}b/a > 1$), A_ν decreases as $p_{0,\nu}^{-1/2}$. In order to generate a coherent wake and not expend beam energy by exciting deleterious modes, a faster decrease in the modal amplitudes is necessary. This can be accomplished by driving the system with a beam of finite length.

There are several longitudinal current profiles for which the wakefield can be simply analyzed, including a step function, a parabola, and a linear ramp. Of these profiles only the asymmetric distribution results in a transformer ratio larger than two.[6,7,8] The transformer ratio determines the maximum energy a load beam electron can achieve. A large transformer ratio implies that the drive beam experiences a decelerating field that is much smaller than the accelerating field seen by the load beam. This results in a slow energy loss for the drive beam, thus allowing the load beam to travel a long distance and gain maximum energy. We consider a linearly ramped current source of the form

$$\mathbf{j}(r, y) = \mathbf{z}\frac{I_p}{\pi b^2}\frac{y}{L}\Theta(b-r)\Theta(L-y), \tag{10}$$

where L is the pulse length and $y = ct - z$. Using the convolution theorem in conjunction with Eq. (8) results in an axial field behind the beam ($y > L$) that is given by

$$E_z(r, z, t) = \frac{8I_p(\varepsilon-1)^{1/2}}{c\varepsilon}\frac{1}{a}\sum_{\nu=1}^{\infty}\frac{a}{b}\frac{B_\nu J_1(p_{0,\nu}b/a)}{p_{0,\nu}^2 J_1^2(p_{0,\nu})}J_0(p_{0,\nu}r/a)\cos[k_\nu(ct-z)-\phi_\nu] \tag{11}$$

where

$$B_\nu = \left(1 - \frac{2\sin k_\nu L}{k_\nu L} + \frac{2-2\cos k_\nu L}{k_\nu^2 L^2}\right)^{1/2}, \qquad k_\nu = \frac{p_{0,\nu}}{a(\varepsilon-1)^{1/2}} \tag{12}$$

and $\phi_\nu(k_\nu L)$ is a phase factor. Defining dimensions in centimeters and current in kiloamperes, E_z is expressed in megavolts per meter by the following:

$$E_z(r, z, t) = 25\frac{I_p(\varepsilon-1)^{1/2}}{a\varepsilon}\sum_{\nu=1}^{\infty}\frac{a}{b}\frac{B_\nu J_1(p_{0,\nu}b/a)}{p_{0,\nu}^2 J_1^2(p_{0,\nu})}J_0(p_{0,\nu}r/a)\cos[k_\nu(ct-z)-\phi_\nu] \tag{13}$$

For large $k_\nu L$, $B_\nu \sim 1$, and the coefficient under the summation falls off as $p_{0,\nu}^{-3/2}$. Thus the higher-order modes are effectively suppressed, and the wakefield is dominated by the fundamental. As an example of the size of accelerating field that can be generated in a Cherenkov accelerator, consider the following set of parameters: $a = 0.5$ cm, $b/a = 0.1$, $\varepsilon = 2.3$, and $I_p = 7.5$ kA. This results in an accelerating field $E_{\nu=1} = 140$ MV/m. The fundamental wavelength is 1.5 cm, corresponding to a frequency of 20 GHz. It can be shown that the transformer ratio R for a linearly ramped current distribution is approximately $0.5k_1L$. Therefore, $R \cong 12.5$ for a pulse length, $L = 4\lambda_1$.

It is important to note that when driving the system with a long pulse beam a Cherenkov instability may arise because of a transverse perturbed current. Our analysis has not treated this possibility, and it may be particularly severe for very high current beams. Indications of such an instability have been observed in our simulations.

II. SIMULATIONS OF THE CHERENKOV WAKEFIELD ACCELERATOR

In support of the theory we have simulated the Cherenkov wakefield accelerator using the fully self-consistent particle-in-cell model ISIS.[12] These simulations were performed in r–z geometry with parameters corresponding to the above example. The drive beam was ramped over 200 picoseconds, or four fundamental wavelengths. In addition to current ramping, the profile had a linear fall time of 12.5 picoseconds. A load beam was injected to serve as a probe of the wakefield. Both beams were injected into the cavity with energies of 20 MeV. To suppress Cherenkov instability from developing a 17 kG magnetic field was superimposed on the system.

Figures 2a and 2b show the axial wakefield generated by the drive beam, and its Fourier spectrum. The peak field measured directly behind the drive beam is 132 MV/m, and has wavelength of 1.39 cm. Both the amplitude and the

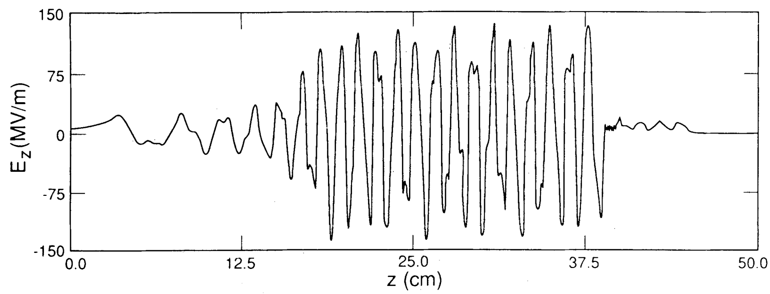

FIGURE 2a The axial wakefield (E_z vs. z).

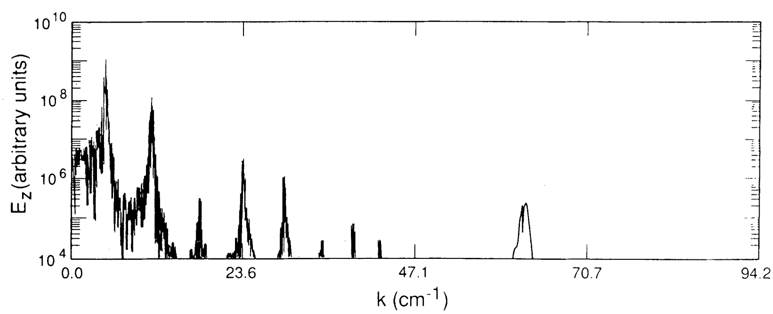

FIGURE 2b Fourier k-spectrum of the axial wakefield.

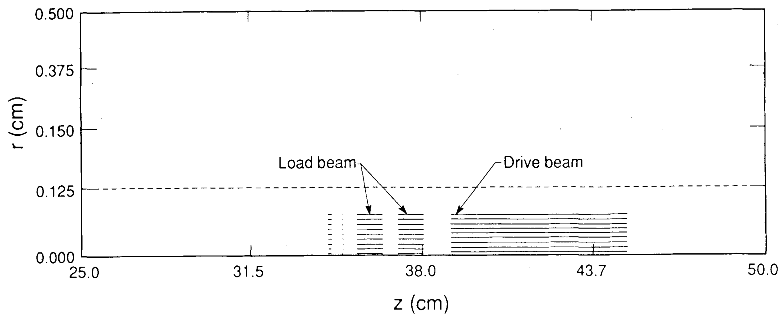

FIGURE 3a Configuration space (r vs. z) for the drive-beam and load-beam electrons.

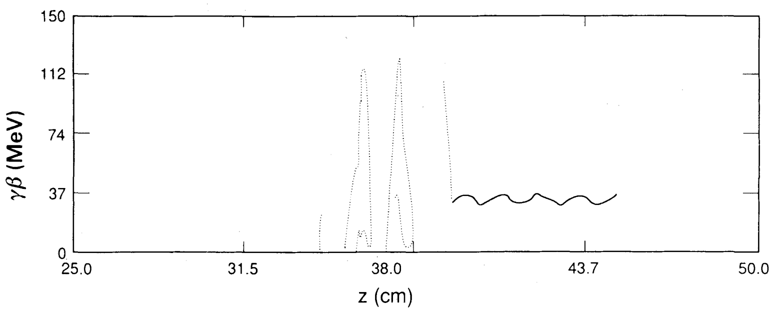

FIGURE 3b Axial phase ($\gamma\beta$ vs. z) for the electrons.

wavelength are within a few percent of the theoretical predictions. Figure 2a indicates that the decelerating field experienced by the drive beam is much smaller than the accelerating field seen by the load beam, implying that a large transformer ratio is achieved.

Figures 3a and 3b show the r–z configuration space and the axial phase space for the two beams. The load experiences strong bunching because parts of the beam are in a decelerating phase of the wakefield. This strong deceleration can be seen in the phase space plot (Fig. 3b). The decelerated electrons rapidly fall back into an accelerating phase where they become trapped and reaccelerated. The peak energy achieved by the load electrons is 67.9 MeV, representing a maximum energy gain of 47.9 MeV in only 37 cm. The tail of the drive beam has also been accelerated; this explains the slight decrease in the amplitude of the wakefield behind the beam. The transformer ratio can be defined as the maximum energy gain of the load beam divided by the maximum energy loss of the drive beam. According to Fig. 3b, the drive beam electrons have lost a maximum of 5.1 MeV. Therefore the achieved transformer ratio R is approximately 9.4.

III. Summary

We have demonstrated analytically and with particle-in-cell simulations that a dielectric-filled, gapless accelerating cavity can serve as an effective wakefield accelerator. An attractive feature of the Cherenkov wakefield accelerator is its simplicity. This allows relatively easy experimental verification of the underlying principles. Such a proof-of-principle experiment has been successfully performed at the Advanced Accelerator Test Facility at Argonne National Laboratory.[13]

There are potential problems in implementing this concept. One is dielectric charging, which can occur if electrons from the drive beam become embedded in the dielectric. By using a sufficiently large guide magnetic field it may be possible to provide enough lateral stability for the beam that this will not occur. If a guide field proves insufficient, it may be necessary to use a larger radius rod, allowing for a larger hole to provide more beam clearance. In the above example, increasing the rod radius to 1 cm would drop the fundamental frequency to 10 GHz and halve the accelerating gradient. To achieve the same accelerating gradient would require doubling the peak-beam current. A second potential problem is breakdown at the vacuum-dielectric interface. Under very high field stresses, dielectrics tend to flash over, and this could produce a high enough plasma density within the center hole of the accelerator to short the electric field. However, it may be that breakdown is suppressed at the high frequencies at which this accelerator will operate. At present there is not experimental data pertaining to dielectric flashover at gigahertz frequencies.

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