Relativistic corrections to large scale structures

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We investigate the relativistic corrections to the standard model of formation of large scale structures. In matter domination and in the Poisson gauge, we use the weak-field approximation which allows to keep compact expressions even for the one-loop bispectrum. Whereas in the Newtonian limit, the choice of gauge is marginally important as all gauge coincides, when relativistic corrections are taken into account, it matters as a change of gauge may induce a change of gravitational potential and introduce fictitious modes in the finial result for the power spectrum. It is precisely what happens in the example presented in this talk as the equivalence principle is not fulfilled in the Poisson gauge and the cancellation of the IR divergence at one-loop does not occur. We will discuss how other choices of gauge may solve this issue.

Keywords: Cosmology, Large Scale Structures, Relativistic corrections, Bispectrum, Non-gaussianities.

1. Introduction

Most of the large scale structures observables can be computed within a Newtonian approximation (see Ref. 1 for a review). On the large scales, at early enough time, the universe is in the linear regime and all gauges coincide with the Newtonian one. On the small scales, local processes for the formation of structures happen and gravity is well described by Newton's theory. However, reporting on Ref. 2, we argue that a proper calculation of the one-loop bispectrum requires to take into account relativistic corrections. Unlike the power spectrum which depends only on one Fourier mode by translation and rotation invariances, the bispectrum couples the scales[a] and therefore could get non-linear relativistic corrections that requires to go beyond the Newtonian gauge. We will present some steps toward calculating the one-loop bispectrum. In section 2, we present a weak field scheme which allows to keep the leading relevant relativistic corrections for cosmological spacetimes. In section 3, we describe the equations of motions for the dark matter perturbations and we solve them using perturbation theory. We present our results for the bispectrum in section 4 and finally present some further subtleties of this calculation and conclude in section 5.

2. Weak field approximation

The task of computing relativistic corrections in full GR is quite challenging even at second order, see *eg.* Refs. 3–5. However, on cosmological scales, gravity is

[a]It is possible to consider triangle configurations that have one large scale and two small scales: the so-called squeezed limit.

weak and to obtain reliable results, one does not need to consider the full GR. Mathematically, the weak field approximation[6–8] relies on the fact that the root mean square of the velocity of dark matter perturbations is small: $v \sim 10^{-3}$ and furthermore the gravitational potential is also small $\phi \sim 10^{-5}$. This allows to tread those two quantities perturbatively in the equation of motion. We note however that to derive the equations of motion of section 3, nothing is assumed regarding the density contrast of the dark matter field: $\delta \sim 1$ can happen in the small scales, at shell-crossing. A more systematic description of the weak field approximation can be found in table 1 of Ref. 2 and in the text around it.

3. Equations of motion

The metric we consider reads:

$$ds^2 = -(1+2\phi)dt^2 + 2\omega_i dx^i dt + a(t)^2 \left[(1-2\psi)\delta_{ij} + h_{ij}\right] dx^i dx^j \,, \qquad (1)$$

where we considered two gravitational potentials ϕ and ψ, frame dragging effects ω_i and gravitational waves h_{ij}. Applying the weak field scheme of section 2 and calculating the first two moments of the Boltzmann equation, supplemented by the Einstein equations, one finds[2]:

$$\dot{\delta}_R + \theta_R = -\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} \alpha(\boldsymbol{k}_1,\boldsymbol{k}_2)(\theta_R(\boldsymbol{k}_1)\delta_N(\boldsymbol{k}_2) + \theta_N(\boldsymbol{k}_1)\delta_R(\boldsymbol{k}_2)) + \mathcal{S}_\delta[\delta_N]\,, \qquad (2)$$

$$\dot{\theta}_R + 2H\theta_R + \frac{3}{2}H^2\delta_R = -2\int_{\boldsymbol{k}_1,\boldsymbol{k}_2} \beta(\boldsymbol{k}_1,\boldsymbol{k}_2)\theta_N(\boldsymbol{k}_1)\theta_R(\boldsymbol{k}_2) + \mathcal{S}_\theta[\delta_N]\,. \qquad (3)$$

We have split the dynamical quantities (δ, $\theta \equiv \partial_i v^i$) between a Newtonian part and a relativistic part, denoted by superscripts N and R. The left hand sides of these equations describe the linear terms, an expression for the relativistic sources is given in appendix C of Ref. 2. The standard kernels α and β describes usual non-linearities terms.

To solve Eqs. (2)-(3), we perform perturbation theory in the usual sense (see *eg.* Ref. 9) and expand the dark matter fields in powers of the linear density contrast δ_l. This allows to define Newtonian kernels F_n together with their relativistic counterparts F_n^R:

$$\delta(\boldsymbol{k},t) = \sum_{n=1}^{\infty} a^n(t) \int_{\boldsymbol{k}_1 \ldots \boldsymbol{k}_n} \Big[F_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) + a^2(t)H^2(t)F_n^R(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\Big]\delta_l(\boldsymbol{k}_1)\ldots\delta_l(\boldsymbol{k}_n)\,. \qquad (4)$$

In appendix D of Ref. 2, expressions for the kernel up to $n = 4$ are presented in two gauges of interest: Poisson and synchronous-comoving.

4. Results

After solving the equations of motion, we are in position to present our main results: a plot of the bispectrum defined as

$$\langle \delta(\boldsymbol{k}_1,t)\delta(\boldsymbol{k}_2,t)\delta(\boldsymbol{k}_3,t)\rangle = (2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(k_1,k_2,k_3,t)\,. \qquad (5)$$

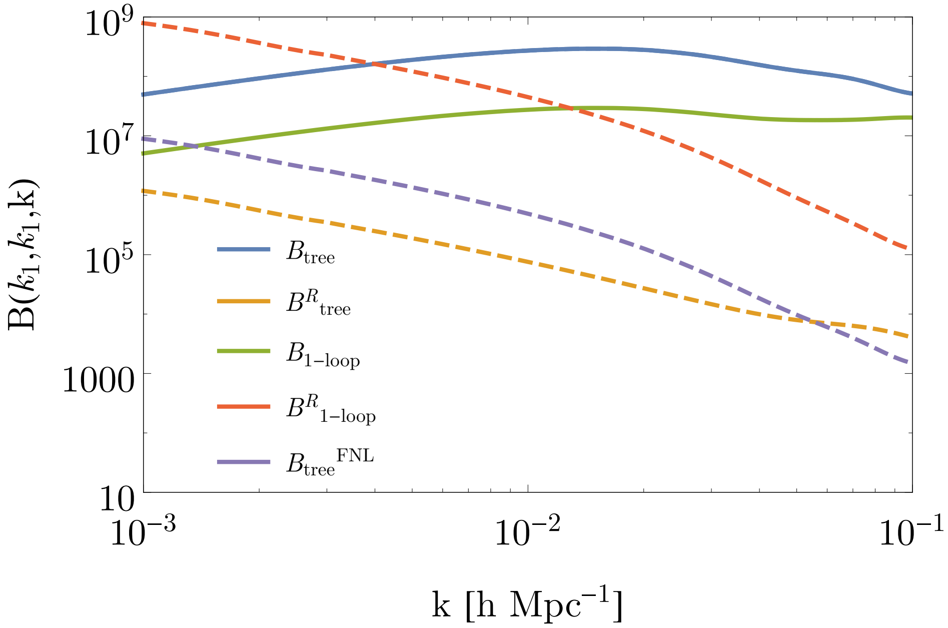

Fig. 1. In the squeezed configuration: $B(k_1, k_1, k)$, one-loop corrections to the linear bispectrum in Poisson gauge, with $k_1 = 0.1$ h Mpc^{-1}. Doted lines denotes a negative contribution. The qualitative reasoning discussed in the introduction holds and indeed relativistic corrections are important at large scales (small k) in squeezed configurations. Observe even that the relativistic one-loop results become larger than the tree-level Newtonian results and the perturbation theory breaks down in the IR. In purple, we ploted a signal that could come from primordial non-gaussianities with $f_{NL} = 1$.

Those results are contrasted with a synchronous gauge in Ref. 2.

5. Conclusions and perpectives

Due to lack of space, several issues were eluded in this proceeding, in particular, in section 5 of Ref. 2, some details are given about the renormalization of the background which is required in order to avoid infinities, the UV and IR behavior of the loop-integrals is also discussed there.

We have reported in this proceeding our new results on the one-loop bispectrum. We argued that taking into account GR is necessary as the bispectrum couples scales. We indeed found that the relativistic corrections becomes of the same order of magnitude than the Newtonian (linear) results.

These encouraging results invite to perform several steps in order to observe the bispectrum:

- Take into account, not only dark matter but galaxies which is done by introducing a set of bias parameters[10,11].

- Take into account the fact that what is observed in our telescopes is a redshift and a direction in the sky and not a position and a time. To do so, one goes to redshift space and takes into account the propagation of photons in our cuspy universe: those effects are sometimes dubbed *redshift space distortions* [12,13].

Other interesting roads includes to more precisely model th short scales, either with effective field theory [14], resummation [15] or renormalization group [16] techniques. The inclusion of a cosmological constant [17] Λ is also in order opening also the door to further generalize to modified gravity scenario for cosmology.

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