SLAC-R-750

Cosmological Aspects of String Compactifications

L. P. McAllister

Stanford Linear Accelerator Center
Stanford University
Stanford, CA 94309

SLAC-Report-750

Prepared for the Department of Energy
under contract number DE-AC02-76SF00515

Printed in the United States of America. Available from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Road, Springfield, VA 22161.

COSMOLOGICAL ASPECTS
OF
STRING COMPACTIFICATIONS

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF PHYSICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Liam Patrick McAllister
March 2005

Acknowledgements

It is a great pleasure to acknowledge the many people who have contributed to this dissertation, and, more broadly, to my education in theoretical physics.

First of all, I am deeply indebted to my advisor, Shamit Kachru, for his truly outstanding guidance. He has been exceptionally generous with his time and his knowledge, and his foresight in suggesting research directions has been invaluable. Shamit's tireless work and high scientific standards continue to be an inspiration for me. I feel extremely fortunate to have worked with him.

I am likewise most grateful to Eva Silverstein, who has at many points acted as a co-advisor. By her unique example she has convinced me that choosing topics that inspire laughter and joy is the best guide to superior research. Her insistence on working out details on the blackboard has substantially improved my own style of work.

Furthermore, I am happy to thank Stephen Shenker for his help and guidance. Before I had a formal adviser, Steve was kind enough to integrate me into the theory group, and I felt fortunate to work under his watchful eye.

I am also grateful to Savas Dimopoulos, Renata Kallosh, Andrei Linde, Michael Peskin, Leonard Susskind, Scott Thomas, and Sandip Trivedi for sharing their insights and accumulated wisdom with me.

Most of my education at Stanford has been due to the efforts of my colleagues, who have shaped and refined my understanding of physics with their countless answers and questions. Among these friends, I owe the most to Michal Fabinger and Xiao Liu, for innumerable insights. I am likewise grateful to Lukasz Fidkowski, Ben Freivogel, Jon Hsu, Ben Lillie, Sergey Prokushkin, Darius Sadri and Alex Saltman. I had fewer years to learn from Chad Davis, Alexander Giryavets, Peter Graham, and Marc Schreiber, but I have been improved nonetheless. I would also like to thank those more senior colleagues who took the time to explain many things to me: Allan Adams, Stephon Alexander, Marcus Berg, Keshav Dasgupta, Michael

Haack, Amir Kashani-Poor, Alex Maloney, John McGreevy, Aaron Pierce, Michael Schulz, and Jay Wacker.

The results presented in this dissertation are the work of many hands. Great thanks are due to my collaborators: Sergei Gukov, Shamit Kachru, Renata Kallosh, Lev Kofman, Andrei Linde, Juan Maldacena, Alex Maloney, Indrajit Mitra, Eva Silverstein, and Sandip Trivedi.

The formatting of this work was substantially simplified by a TeX package prepared by Michael Schulz, whose efforts I appreciate.

I carried out this research with essential support from an NSF Graduate Research Fellowship, and with partial support from the DOE under contract DE-AC03-76SF00515. I am sincerely grateful to the U.S. government for its willingness to invest in theoretical physics.

My time in the theory group at Stanford and at SLAC would not have been the same without the generous and efficient assistance of Sharon Jensen, Mariel Haag, and Karin Slinger. Karin, in particular, went to extraordinary lengths to take care of all her children, from students to faculty, and I am in her debt.

I am grateful to all my friends for their companionship, and most specially to Henry Fu, Kaiwen Kam, and Albert Pan.

None of this could have been without the constant love and support of my parents, Timothy and Noreen, and my sister, Tara, who have sustained me through this, as through all other labors.

Most of all, I am deeply grateful to my wife, Josephine, for her love.

Stanford University
March 2005

Liam McAllister

Contents

1. Introduction

1.1 The Status of Cosmology

1.1.1 Dark Energy

Spectacular advances in observational cosmology have revolutionized our view of the universe. We now understand that ordinary matter in all its forms makes up a negligible fraction of the mass of the cosmos: the visible stars and galaxies are an insignificant foam drifting on a vast ocean of dark energy. This mysterious substance pervades the vacuum and forces the expansion of the universe to accelerate. No one knows what it is.

The first sign of dark energy came from measurements of type Ia supernovae, which are explosions of a white dwarf following accretion of matter from a companion star. We have a rudimentary understanding of the intrinsic luminosity of these events, so the discovery that very distant supernovae were unexpectedly dim [1,2] presented a problem. After eliminating alternative explanations, the authors of [1] proposed that the most distant supernovae, and the galaxies that contain them, must be accelerating away from us. This amounts to a modification of the famous Hubble Law, which states that the recession velocity of a galaxy is proportional to its distance from us. The supernova observations showed that the expansion rate of the cosmos must have been different, and indeed smaller, in the past.

This accelerating expansion was a profound shock to most theorists. A universe full of any sort of known matter and radiation cannot accelerate, any more than a stone tossed into the air can accelerate upward in flight. A cosmos exploding outward from the violence of the Big Bang may expand indefinitely, but the universal attraction of gravity will inevitably slow the expansion. The only way to accomodate the observed acceleration was to invoke the infamous cosmological constant, the 'energy of the vacuum'.

The cosmological constant problem has mythic status among the deep problems of fundamental physics. Einstein first invoked a constant term in his field equations to create eternal, stationary cosmological solutions, which he found philosophically appealing. The added term was necessary to keep the universe from contracting and collapsing under its own gravitation. Later, faced with Hubble's observation that our own universe is expanding, Einstein recanted and called the addition of a cosmological constant the biggest blunder of his life.

The crisis, however, is not simply that the Einstein equations contain an unknown constant. The problem is that quantum field theory makes a prediction about the vacuum energy, and it is nearly impossible to square this prediction with cosmology. This is very important, because we understand quantum field theory extremely well in the range of energies accessible to particle accelerators.

Precise predictions are difficult or impossible, but essentially any scheme for computing the vacuum energy in quantum field theory will give an energy density $\rho_{QFT}^{1/4} \gtrsim 10^3$ GeV. Typical schemes suggest that in fact $\rho_{QFT}^{1/4} \sim 10^{19}$ GeV. However, although the vacuum energy measured in our universe does dwarf the energy density of matter, it still amounts to only $\rho_\Lambda^{1/4} \sim 10^{-3}$ eV. The theoretical prediction exceeds the measured value by more than 120 orders of magnitude!

The cosmological constant problem is thus a deep conflict between the macroscopic and the microscopic, and between the two great structures of twentieth-century physics: general relativity and quantum field theory. We believe we understand each theory separately, but our inability to understand the vacuum energy proves that we do not understand how to combine them. After decades of effort, the cosmological constant problem still overshadows every scenario in which gravity couples to vacuum energy to source accelerating expansion. Even worse, the new cosmological paradigm requires a gradual transition between stages of acceleration at utterly different scales, from the cataclysmic stretching of inflation to the gentle tug of dark energy today, so the problem is more complex and more acute than ever.

String theory is a theory of quantum gravity: among many other virtues, it provides a complete and consistent description of the quantization of the gravitational field. Should we not expect string theory to resolve the cosmological constant problem and predict, or at least accommodate, the observed value?

Unfortunately, string theory has *not* provided any means of predicting the observed vacuum energy, at least not in any conventional sense of prediction. However, advances in moduli stabilization with fluxes [3,4,5] have provided a method of accommodating the smallness of the vacuum energy within string theory, i.e. of constructing string vacua that contain a minuscule amount of dark energy. This is dramatic progress from the point of view of string theory, but it has not yet shed any light on the observed universe.

In this work we will not provide a solution to any aspect of the cosmological constant problem. The issue, however, is powerful and pervasive, and it underlies all our discussions of moduli stabilization, in Chapter 5, and of string cosmology, in Chapters 6 and 7. Our only concrete step toward accommodating the cosmological constant in string theory is our construction, in Chapter 5, of the first stable solutions of the weakly-coupled heterotic string with non-vanishing vacuum energy. The positive sign and small value of this energy remain out of our reach.

1.1.2 Inflation

A second great advance in our knowledge of the large-scale universe comes from studies of the cosmic microwave background (CMB) radiation. This is the afterglow of the Big Bang, but it has been redshifted by the subsequent expansion down to a mere 2.7 K. The CMB is the first 'light' available to us: most of its microwave photons last scattered off matter when the universe first became transparent, when it was roughly 300,000 years old, so there is no direct way to see farther back using the electromagnetic spectrum. The CMB photons are pervasive and surprisingly numerous: at 411 per cubic centimeter, they far outnumber baryons. The discovery of the CMB by Penzias and Wilson was dramatic evidence for the Big Bang theory, and the detailed properties of the CMB are now providing essential clues about the very early universe.

The most surprising thing about the CMB is its uniformity: the fractional temperature difference between various points on the sky is no more than one part in 10^5. Even before this result was known precisely, the near-uniformity presented a striking problem for the Big Bang model. Each patch of sky of size roughly one degree is a region that was in causal contact at the moment that the universe became transparent: physical signals would have been able to cross this region in the available time, so causal processes could establish thermal equilibrium. Thus, it would not be at all surprising to find that the temperature does not vary within a patch of size one degree or smaller. Amazingly, it is the entire sky that has a very nearly uniform temperature! The challenge was to explain why the temperature of the sky is uniform, and, in a related vein, why the distribution of matter is so nearly homogeneous and isotropic. Without a causal mechanism to smooth the cosmos, how could all these regions have contrived to look so similar? Moreover, what could explain the immense entropy, age, and size of the universe?

The theory of inflation [6] provides a superb answer to all of these questions. An initial epoch of tremendously violent, accelerated expansion could stretch causal signals across a gigantic space, smoothing inhomogeneities and establishing causal relations between widely-separated points. When the positive energy driving this inflation decayed to more ordinary quanta, the resulting temperature was very nearly uniform, with a predictable spectrum of minute temperature anisotropies.

Since its creation, the inflationary scenario has had outstanding explanatory power. However, the source of much recent excitement is the prospect of testing the predictions of inflation, particularly the spectrum of temperature anisotropies, through precision measurements of the CMB [7]. Certain models have already been ruled out, and there is a limited possibility of confirming the whole scenario by finding traces of gravitational waves in the CMB. This is an irreplaceable opportunity for contact between inflation and reality: no terrestrial experiment is likely to probe the energy scales relevant for inflation, so cosmological data is the only means of testing the theory.

As a theoretical structure, inflation is appealing but incomplete. For example, many well-studied models require field expectation values larger than the Planck mass; although this is arguably acceptable even without an ultraviolet completion for the theory, it would be most reassuring to check this assertion in a full theory of quantum gravity. Furthermore, inflationary potentials need to be exceptionally flat, but this is hard to achieve in most settings: various corrections, particularly terms suppressed by the Planck mass, tend to curve the potential. Thus, a complete computation of an inflating potential in a theory of quantum gravity, such as string theory, would be invaluable.

1.2 Prospects for String Cosmology

String theory and cosmology have much to gain from each other. In the preceding section I have reviewed two of the deep questions of theoretical cosmology, the nature of the dark energy and the fundamental physics underlying inflation. As a mathematically consistent theory of quantum gravity, string theory ought perhaps to have something to offer towards solutions of these puzzles, but no clear answer has yet emerged.

However, in recent years there has been very significant progress toward theoretically satisfying, and experimentally testable, string cosmology. For many years the most significant barrier to meaningful contact between string theory and cosmology has been the moduli problem. Correspondingly, most of the progress in recent years is due to advances in moduli stabilization, culminating in the first stable string vacua with positive cosmological constant [5]. We will therefore turn our attention now to the problems posed by compactification moduli, and then, in §1.2.2, to the solution to these problems: moduli stabilization with fluxes and nonperturbative effects.

1.2.1 The Moduli Problem in String Cosmology

A modulus is a massless scalar field, often one which parameterizes the couplings of a field theory or the deformations of a geometry. Geometric moduli are endemic in string compactifications: the preferred compactification manifolds, most notably Calabi-Yau spaces, have complicated topology, and admit correspondingly numerous deformations of the complex structure and of the Kähler parameters. Before accounting for the superpotentials arising from fluxes and from nonperturbative effects, each of these scalars appears with vanishing potential in the four-dimensional theory. Their couplings are of gravitational strength.

Moduli present several problems for cosmological models. First of all, gravitational experiments place strong constraints on the existence of light scalars with gravitational interactions. In addition, some moduli, especially the string dilaton and the compactification volume, affect the couplings of Standard Model fields. Variations in the Newton constant [8] or in the electromagnetic fine-structure constant [9] are strongly bounded, so once again we find that typical moduli are incompatible with the results of experiment. Taken together, these constraints suggest that moduli must somehow be removed from any workable model.

Finally, among the great successes of the Big Bang model, and of early universe cosmology, are the predictions of Big Bang nucleosynthesis. In the first few minutes of expansion, protons and neutrons combined to form light nuclei, primarily helium, but with predictable relative abundances of deuterium and lithium. This provides a powerful tool for excluding new particles that could disrupt this relatively delicate process of synthesis. Moduli are problematic because they almost inevitably store energy during inflation, by being displaced from their zero-temperature minima. Unless the moduli acquire rather large masses, this energy causes one of two problems. If the moduli mass m_χ is smaller than around 100 MeV, the moduli will not have decayed by the present day, and the energy they stored during inflation will overclose the universe. If $100\text{MeV} \lesssim m_\chi \lesssim 30\text{TeV}$ then the moduli will have decayed, but in the process will have released enough entropy to dilute the products of nucleosynthesis [10]. Only if all the moduli have masses above 30 TeV can we maintain the success of nucleosynthesis. Thus, we conclude that in the absence of a mechanism for generating such masses, models based on string compactifications are incompatible with cosmological observations.

Of great importance for us will be one particularly dangerous modulus, the overall compactification volume. In the presence of a positive energy density, this field develops an instability: it becomes energetically favorable for the internal space to expand. This lifts the volume modulus, but the result is not a stable model: instead we find a runaway decompactification.

This observation is significant for cosmological models because both inflation and the present-day acceleration require positive energy density. These two features, arguably the most important of the new cosmological paradigm, simply cannot be achieved in string compactifications without moduli stabilization.

1.2.2 Techniques of Moduli Stabilization

Moduli stabilization is a procedure that generates a potential for the moduli. This potential is usually required to have a minimum in a reasonable range of field values. For example, the introduction of a spacetime-filling positive-energy source creates a potential for the volume modulus whose minimum is at infinite volume. This actually destabilizes the volume modulus, decompactifying the internal space, and certainly does not qualify as volume stabilization!

In order to understand what effects are needed to lift the moduli, it will be helpful to have a partial classification of the moduli of a string compactification on a Calabi-Yau threefold. The geometric moduli are the complex structure moduli, which parameterize the space of choices of complex structure on the manifold, and the Kähler moduli, which govern the sizes of even-dimensional cycles. In heterotic string compactifications and type I or type II compactifications with D-branes, there will also in general be bundle moduli, which control the deformations of the corresponding vector bundles. Finally, there can be open string moduli associated with the locations of D-branes.

In the best-understood setting of type IIB orientifolds, the most numerous and important moduli are the complex structure and Kähler moduli. Intuitively, complex structure moduli control the shapes of cycles, so what is needed is a physical ingredient that associates an energy cost to changes of shape. Three-form fluxes, the field strengths of the two-form Neveu-Schwarz and Ramond-Ramond potentials B_{ij} and C_{ij}, provide just such an effect [4]. This flux is integrally quantized, so inclusion of a real three-form flux amounts to a choice of one integer for each three-cycle in the internal space. There are actually two real three-form fluxes in the type IIB theory, $H_3 = dB_2$ and $F_3 = dC_2$, and it is useful to form the complex combination $G_3 = F_3 - \tau H_3$, where τ is the axio-dilaton. Turning on generic G-flux usually fixes all the complex structure moduli and the dilaton. (In certain special cases such as tori this may not be true.)

Kähler moduli, in contrast, are not lifted by the inclusion of flux. In fact, they cannot appear in the perturbative superpotential: the combination of holomorphy of the superpotential and the shift symmetry [11] of the axion paired with each volume modulus implies that the superpotential can acquire volume-dependence only nonperturbatively. We must therefore seek nonperturbative effects that are sensitive to the volume moduli. Two effects are suitable for this purpose [5]: gaugino condensation on a stack of D7-branes wrapping a divisor, and Euclidean D3-branes wrapping a divisor [12]. The former requires a suitably small matter content in the D7-brane gauge theory, and the latter is possible (in the absence of flux) only if the divisor satisfies a certain topological condition [12].

We conclude that to achieve complete stabilization of the geometric moduli [5], one must introduce generic three-form fluxes and verify that every independent divisor admits either gaugino condensation or Euclidean D3-branes.

1.2.3 Inflationary Models in String Theory

Given a reliable technique for constructing stable string vacua with positive energy, it is quite easy to imagine concrete inflationary models. Slow-roll inflation does not require much more than the relaxation of an overdamped scalar field in a potential with a large positive energy. One therefore needs to search for a relatively flat potential that interpolates between a high-energy configuration and one of approximately zero energy. Ideally, quantum corrections to this potential would be computable and under control. The scale and slope of this potential rather directly determine the magnitude and the spectral index of the density perturbations that we see as CMB temperature anisotropies. One might also hope to explain the very small amplitude of these perturbations, i.e. the near-isotropy of the CMB.

As we will see in Chapter 6, the separation of a D3-brane and an anti-D3-brane in a warped deformed conifold [13] can provide an interaction potential with appropriate properties. This is merely a concrete example, a toy model for string inflation; it seems quite clear that much more generic models remain to be discovered.

One striking consequence of brane inflation models is the possibility of forming networks of stable cosmic superstrings [14,15]. The literature on cosmic strings had largely discounted the possibility of a connection to string theory, in part because the tensions of ordinary F-strings and D-strings in superstring theory are much too large to be compatible with observations. However, progress in brane inflation, particularly in warped models, led to the realization that the strings of string theory could potentially be cosmic in scale.

Cosmic strings stretch for light-years and are so massive that they bend starlight, creating distinctive lensing signals. Moreover, they emit a powerful flux of gravitational radiation, with occasional high-intensity bursts. These metric fluctuations affect the travel time of pulsar signals, so pulsar timing experiments can be used to put bounds on the cosmic string tension. Lensing surveys and direct observation of gravitational waves are other promising routes to discovering or ruling out cosmic strings. In the event of a discovery, it is just possible that we could distinguish a network of F-strings and D-strings from the more conventional cosmic string scenarios that are unrelated to string theory. This thrilling prospect of direct contact between string theory and experiment is another important opportunity for string cosmology.

1.3 Organization of this Thesis

The organization of this work is as follows. In Chapters 2,3, and 4 we discuss the dynamics of unfixed moduli in quantum field theory and in string theory, with emphasis on applications to cosmological model-building. Moduli-stabilizing effects are not included, because our goal is to understand the evolution of moduli whose potentials remain approximately flat. In Chapter 5 we develop a technique for stabilizing moduli in the heterotic string, and we construct the first stabilized, weakly-coupled heterotic vacua. In Chapter 6 we utilize advances in the stabilization of type IIB compactifications to build the first models of inflation in stabilized string vacua. Finally, in Chapter 7 we examine the effects of volume-stabilization on the inflaton mass in closely-related scenarios of string inflation.

Chapter 2 presents a bouncing cosmology seen by an observer on a moving D3-brane: this is a universe whose scale factor decreases to a minimum value and then smoothly re-expands [16]. Singularity theorems usually forbid solutions of this form [17], so we explain the rather surprising way in which our system circumvents these theorems. The relevant modulus in this model is the position of the D3-brane along the radial direction of a warped deformed conifold [13]. This particular modulus will be lifted in the presence of nonperturbative stabilization of the compactification volume, a result that will have great importance in Chapters 6 and 7.

In Chapter 3 we consider quantum corrections to the dynamics of a system of coupled moduli in quantum field theory. Certain particles are light at special points in moduli space, which often exhibit enhanced symmetry. (For example, the light fields could be gauge bosons whose mass is large away from the special points, and in this case the enhanced symmetry would be gauged.) We discover that quantum production of these light particles traps moving moduli at these points of enhanced symmetry [18]. This effect, which we call moduli trapping, has a variety of implications for cosmology. Moduli trapping may ameliorate the cosmological moduli problem by situating moduli at extrema of their effective potential during inflation. It can lead to a short period of accelerating expansion, which we call trapped inflation. The most interesting result is that moving moduli are most powerfully attracted to the points with the highest degree of symmetry. Given suitable initial conditions, this could help to explain why our universe exhibits a relatively large degree of (spontaneously broken) symmetry. Some of the surprising symmetries of our world might have an explanation in the dynamics of moduli.

In Chapter 4 we study an interesting special case [19] of the moduli trapping scenario. When the moduli in motion correspond to the relative positions of D-branes, the quantum effect relevant for moduli trapping is pair-production of stretched open strings. For slow-moving D-branes this is captured by the analysis of Chapter 2. However, relativistic D-branes exhibit a tremendously strong trapping effect. We show that the brane trajectory receives strong corrections from copious production of highly-excited open strings, whose typical oscillator level is proportional to the square of the rapidity. This purely stringy effect makes relativistic brane collisions exceptionally inelastic. We trace this surprising effect to velocity-dependent corrections to the open string mass, which render open strings between relativistic D-branes surprisingly light. Our analysis has applications to cosmological scenarios in which branes approach each other at very high speeds: pair production of open strings could play an unexpectedly strong role in the brane dynamics.

Next, in Chapter 5, we present a technique for stabilizing the moduli of perturbative heterotic string compactifications on Calabi-Yau threefolds [20]. We show that fractional flux from Wilson lines, in combination with a hidden-sector gaugino condensate [21], generates a potential for the complex structure moduli, Kähler moduli, and dilaton. This potential has a supersymmetric AdS minimum at moderately weak coupling and large volume. In this way we construct the first stabilized heterotic string models. Our solutions have a nonvanishing, although negative, cosmological constant, so our methods are a step toward controllable de Sitter vacua of the heterotic string. Our technique circumvents a well-known problem [21] arising from flux quantization by introducing a Chern-Simons invariant that does not have an integer quantization condition. The necessary Chern-Simons invariant can arise naturally from the GUT-breaking Wilson lines that are already present in most phenomenologically appealing models.

In Chapter 6 we use earlier, fundamental advances [5] in stabilization of type IIB compactifications to build the first concrete model of inflation in a stabilized string compactification [22]. Our construction involves a D3-brane moving down a warped deformed conifold [13] geometry in a Calabi-Yau orientifold stabilized by fluxes [4] and by nonperturbative effects [5]. Condensation of a brane-antibrane tachyon ends inflation, so our model is a string embedding of hybrid inflation. One particularly appealing feature is the possibility of light, stable cosmic strings.

In Chapter 7 we reconsider the effect of nonperturbative volume stabilization on inflation [23], and observe that certain geometric shift symmetries constructed to protect the inflaton mass are broken by threshold corrections [24,25]. We conclude that in typical configurations, some degree of fine-tuning is still required. This presents a mild but relevant challenge for inflationary model-building in such scenarios.

Note on Collaborative Research

Modern theoretical physics is a science built on collaborations. Most progress in string theory, in particular, results from the work of small groups, not of individuals in isolation. The unwritten rule governing this system is that each co-author is expected to contribute in some way to every major aspect of a paper.

I was intimately involved in all the research reported in this dissertation. Furthermore, in each project I was continually involved in the writing and rewriting of our results. My contributions and those of my collaborators have been woven together to create complete works, and there is no meaningful way to partition the finished product.

2. Bouncing Brane Cosmologies

ABSTRACT OF ORIGINAL PAPER

We study the cosmology induced on a brane probing a warped throat region in a Calabi-Yau compactification of type IIB string theory. For the case of a BPS D3-brane probing the Klebanov-Strassler warped deformed conifold, the cosmology described by a suitable brane observer is a bouncing, spatially flat Friedmann-Robertson-Walker universe with time-varying Newton's constant, which passes smoothly from a contracting to an expanding phase. In the Klebanov-Tseytlin approximation to the Klebanov-Strassler solution the cosmology would end with a big crunch singularity. In this sense, the warped deformed conifold provides a string theory resolution of a spacelike singularity in the brane cosmology. The four-dimensional effective action appropriate for a brane observer is a simple scalar-tensor theory of gravity. In this description of the physics, a bounce is possible because the relevant energy-momentum tensor can classically violate the null energy condition.

2.1 Introduction

There has recently been considerable interest in the properties of string theory cosmology. A generic feature of general relativistic cosmologies is the presence of singularities, which is guaranteed under a wide range of circumstances by the singularity theorems [17]. Since string theory has had great success in providing physically sensible descriptions of certain timelike singularities in compactification geometries, one can hope that it will similarly provide insight into the spacelike or null singularities which arise in various cosmologies. Proposals in this direction have appeared in e.g. [26,27,28,29,30,31,32,33,34,35,36,37].

In a slightly different direction, the possibility of localizing models of particle physics on three-branes in a higher-dimensional bulk geometry has motivated a great deal of work on brane-world cosmology (see [38,39,40,41,42] and references therein for various examples). Of particular interest to us will be the "mirage" cosmology [38] which is experienced by a D3-brane observer as he falls through a bulk string theory background. In this chapter, we present a simple and concrete example where such an observer would describe a cosmology which evades the singularity theorems: his universe is a flat FRW model which smoothly interpolates between a collapsing phase and an expanding phase.

The background through which the D3-brane moves is a Klebanov-Strassler (KS) throat region [13] of a IIB Calabi-Yau compactification. Compactifications including such throats, described in [4], yield models with 4d gravity and a warp factor which can vary by many orders of magnitude as one moves in the internal space (as in the proposal of Randall and Sundrum (RS) [43]). The backgrounds discussed in [4] would also admit, in many cases, some number of wandering D3-branes. Such a brane can fall down the KS throat and bounce smoothly back out, as the supergravity background has small curvature everywhere. The induced cosmology on this probe, as described by an observer who holds particle masses *fixed*, is a spatially flat Friedmann-Robertson-Walker universe which begins in a contracting phase, passes smoothly through a minimum scale factor, and then re-expands.[1] A D3-brane probe in this background satisfies a "no-force" condition which makes it possible to control the velocity of the contraction; in addition, the background can be chosen so that the universe is large in Planck units at the bounce. For this reason, the calculations which lead the brane observer to see a bounce are controlled and do not suffer from large stringy or quantum gravity corrections. It is important to note that in this scenario, the effective 4d Newton's constant G_N varies with the scale factor of the universe; this results from the varying overlap of the graviton wavefunction with the D3-brane.

The KS solution is actually a stringy resolution of the singular Klebanov-Tseytlin (KT) supergravity solution [45], which ends with a naked singularity in

[1] A different approach to using the KS model to generate an interesting string theory cosmology recently appeared in [44].

the infrared. A brane falling into a Klebanov-Tseytlin throat would therefore undergo a singular big crunch. In this sense, the cosmology we study involves a stringy resolution of a spacelike singularity, from the point of view of an observer on the brane.

Although one can describe the cosmological history of these universes using the behavior of the induced metric along the brane trajectory, it is also interesting to consider the 4d effective field theory that a brane resident could use to explain his cosmology. We construct a simple toy model of these cosmologies using a 4d scalar-tensor theory of gravity. The scalar can be identified with the open string scalar field Φ_r (corresponding to radial motion down the warped throat) in the Born-Infeld action for the D3-brane. It is well known that such scalar-tensor theories can classically violate the null energy condition, making a bounce possible. Related facts about scalar field theories coupled to gravity have been exploited previously by Bekenstein and several subsequent authors [46,47,48,49].

The organization of this chapter is as follows. In §2.2 we use the construction of [4] to study the cosmology on a brane sliding down the KS throat. In §2.3 we provide a discussion of the effective scalar-tensor theory of gravity a brane theorist would probably use to explain his observations. We close with some thoughts on further directions in §2.4.

Several previous authors have investigated the possibility of bounce cosmologies in scalar-tensor theories and in brane-world models. For FRW models with spherical spatial sections ($k = +1$), examples in various contexts have appeared in [46,47,48]. As we were completing this work, other discussions of bounces in brane-world models appeared in [50,51]. To the best of our knowledge, this chapter provides the first controlled example in string theory of a bouncing, spatially flat FRW cosmology with 4d gravity.

2.2 Brane Cosmology in a Warped Calabi-Yau Compactification

2.2.1 The Compactifications

In [52,4,53], warped string compactifications were explored as a means of realizing the scenario of Randall and Sundrum [43] in a string theory context. It was shown that compactifications of IIB string theory on Calabi-Yau orientifolds provide the necessary ingredients. In such models, one derives a tadpole condition of the form

$$\frac{1}{4}N_{O3} \;=\; N_{D3} + \frac{1}{2(2\pi)^4(\alpha')^2}\int_X H_3 \wedge F_3 \;. \tag{2.2.1}$$

Here X is the Calabi-Yau manifold, N_{O3} and N_{D3} count the number of orientifold planes coming from fixed points of the orientifold action and the number of transverse D3-branes, and H_3, F_3 are the NSNS and RR three-form field strengths of the IIB theory.[2] In general, the left-hand side of (2.2.1) is nonzero and can be a reasonably large number, giving rise to the possibility of compactifications with large numbers of transverse D3-branes or internal flux quanta. Since both of these lead to nontrivial warping of the metric as a function of the internal coordinates, (2.2.1) tells us that these Calabi-Yau orientifolds provide a robust setting for finding warped string compactifications [52,4,53].

We can make this somewhat vague statement much more precise in the example of the warped deformed conifold. The conifold geometry is defined in $\mathbb{C}^4$ by

$$z_1{}^2 + z_2{}^2 + z_3{}^2 + z_4{}^2 = 0. \tag{2.2.2}$$

It is topologically a cone over $S^2 \times S^3$; we will refer to the direction transverse to the base as the "radial direction" (with small r being close to the tip and large r being far out along the cone). The *deformed* conifold geometry

$$z_1{}^2 + z_2{}^2 + z_3{}^2 + z_4{}^2 = \epsilon^2 \tag{2.2.3}$$

has two nontrivial 3-cycles, the A-cycle S^3 which collapses as $\epsilon \longrightarrow 0$, and the dual B-cycle. Klebanov and Strassler found that the infrared region of the geometry which is holographically dual to a cascading $SU(N+M) \times SU(N)$ $\mathcal{N}=1$ supersymmetric gauge theory is precisely a warped version of the deformed conifold geometry, with nontrivial 3-form fluxes

$$\frac{1}{(2\pi)^2\alpha'}\int_A F = M, \quad \frac{1}{(2\pi)^2\alpha'}\int_B H = -k \tag{2.2.4}$$

and $N = kM$. In particular, the space (2.2.3) is non-singular and the smooth geometry dual to the IR of the gauge theory reflects the confinement of the Yang-Mills

[2] In an F-theory description, the left-hand side of (2.2.1) is replaced by $\frac{\chi(X_4)}{24}$, where X_4 is the relevant elliptic Calabi-Yau fourfold.

theory (with the small parameter ϵ mapping to the exponentially small dynamical scale of the gauge theory). In a cruder approximation to the physics, Klebanov and Tseytlin had earlier found a dual gravity description with a naked singularity [45]; this heuristically corresponds to the unresolved singularity in (2.2.2).

In [4], the warped, deformed conifold with flux (2.2.3), (2.2.4) was embedded in string/F-theory compactifications to 4d. The small r region is as in [13], while at some large r (in the UV of the dual cascading field theory), the solution is glued into a Calabi-Yau manifold. The fluxes give rise to a potential which fixes (many of) the Calabi-Yau moduli (and in particular the ϵ in (2.2.3)), while the fluxes plus in some cases wandering D3-branes saturate the tadpole condition (2.2.1). If one considers one of the cases with $N_{D3} > 0$, then it is natural to imagine a cosmology arising on a wandering D3-brane as it falls down towards the tip of the conifold (2.2.3).

2.2.2 The Klebanov-Strassler Geometry

The KS metric is given by (we use the conventions of [54])

$$ds^2 = h^{-1/2}(\tau)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(\tau)ds_6^2 \tag{2.2.5}$$

where ds_6^2 is the metric of the deformed conifold,

$$\begin{aligned} ds_6^2 = \frac{1}{2}\epsilon^{4/3}K(\tau)\Big(\frac{1}{3K^3(\tau)}[d\tau^2 + (g^5)^2] + \cosh^2(\frac{\tau}{2})[(g^3)^2 + (g^4)^2] \\ + \sinh^2(\frac{\tau}{2})[(g^1)^2 + (g^2)^2]\Big). \end{aligned} \tag{2.2.6}$$

Here

$$\begin{aligned} g^1 &= \frac{e^1 - e^3}{\sqrt{2}}, \quad g^2 = \frac{e^2 - e^4}{\sqrt{2}} \\ g^3 &= \frac{e^1 + e^3}{\sqrt{2}}, \quad g^4 = \frac{e^2 + e^4}{\sqrt{2}} \\ g^5 &= e^5 \end{aligned} \tag{2.2.7}$$

where

$$\begin{aligned} e^1 &= -sin(\theta_1)d\phi_1, \quad e^2 = d\theta_1 \\ e^3 &= cos(\psi)sin(\theta_2)d\phi_2 - sin(\psi)d\theta_2 \\ e^4 &= sin(\psi)sin(\theta_2)d\phi_2 + cos(\psi)d\theta_2 \\ e^5 &= d\psi + cos(\theta_1)d\phi_1 + cos(\theta_2)d\phi_2\ . \end{aligned} \tag{2.2.8}$$

ψ is an angular coordinate which ranges from 0 to 4π, while (θ_1, ϕ_1) and (θ_2, ϕ_2) are the conventional coordinates on two S^2s. The function $K(\tau)$ in (2.2.5) is given by

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh(\tau)} \ . \tag{2.2.9}$$

Clearly in (2.2.5) τ plays the role of the "radial" variable in the conifold geometry, with large τ corresponding to large r.

Finally, the function $h(\tau)$ in (2.2.5) is rather complicated; it is given by the expression

$$h(\tau) \ = \ (g_s M\alpha')^2 2^{2/3} \epsilon^{-8/3} I(\tau) \tag{2.2.10}$$

where

$$I(\tau) \ = \ \int_\tau^\infty dx \frac{x\coth(x) - 1}{\sinh^2(x)} \Big(\sinh(2x) - 2x\Big)^{1/3} . \tag{2.2.11}$$

It will be useful to note that this reaches a maximum at $\tau = 0$ and decreases monotonically as $\tau \to \infty$. There are also nontrivial backgrounds of the NSNS 2-form and RR 2-form potential; their detailed form will not enter here, but they are crucial in understanding why the D3-brane propagates with no force in the background (2.2.5).

Since the form of $h(\tau)$ will be important in what follows, we take a moment here to give some limits of the behavior of formulae (2.2.10),(2.2.11)[54]. For very small τ, one finds $I(\tau) \sim a_0 + O(\tau^2)$, with a_0 a constant of order 1. In this limit the complicated metric (2.2.5) simplifies greatly (c.f. equation(67) of [54]):

$$ds^2 \to \frac{\epsilon^{4/3}}{2^{1/3} a_0^{1/2} g_s M\alpha'} dx_n dx_n + a_0^{1/2} 6^{-1/3} (g_s M\alpha') \Big(\frac{1}{2} d\tau^2 + \frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4)^2 + \frac{1}{4}\tau^2[(g^1)^2 + (g^2)^2]\Big) \ . \tag{2.2.12}$$

This is $R^{3,1}$ times (the small τ limit of) the deformed conifold. In particular, the S^3 has fixed radius proportional to $\sqrt{g_s M}$, and so the curvature can be made arbitrarily small for large $g_s M$. In the opposite limit of large τ, the metric simplies to Klebanov-Tseytlin form. Introducing the coordinate r via

$$r^2 = \frac{3}{2^{5/3}} \epsilon^{4/3} e^{\frac{2\tau}{3}} \tag{2.2.13}$$

and using the asymptotic behavior $I(\tau) \sim 3 \times 2^{-1/3}(\tau - \frac{1}{4})e^{-\frac{4\tau}{3}}$, one finds

$$ds^2 \rightarrow \frac{r^2}{L^2\sqrt{ln(r/r_s)}}dx_n dx_n + \frac{L^2\sqrt{ln(r/r_s)}}{r^2}dr^2 + L^2\sqrt{ln(r/r_s)}ds^2_{T^{1,1}} \quad (2.2.14)$$

where $ds^2_{T^{1,1}}$ is the metric on the Einstein manifold $T^{1,1}$ and $L^2 = \frac{9g_s M\alpha'}{2\sqrt{2}}$. This means that up to logarithmic corrections, the large τ behavior gives rise to an AdS_5 metric for the x^μ and τ directions. This is the expected behavior from the field theory dual, since large τ corresponds to the UV, where the theory is approximately the Klebanov-Witten $\mathcal{N} = 1$ SCFT [55].

2.2.3 Trajectory of a Falling Brane

We will start the D3-brane at some fixed $\tau = \tau^*$ and send it flying towards $\tau = 0$ with a small initial proper velocity v in the radial τ direction. Before describing the trajectory we will briefly explain our notation. τ always indicates the radial coordinate in the KS geometry (2.2.5) and is dimensionless in our conventions. We will reserve t for proper time (for the infalling brane) and $\dot{}$ for $\frac{d}{dt}$, while ξ represents the coordinate time, in terms of which the metric is

$$ds^2 = h(\tau)^{-\frac{1}{2}}(-d\xi^2 + \sum_i dx_i^2) + g_{\tau\tau}d\tau^2 + angles \quad (2.2.15)$$

and thus

$$(\frac{dt}{d\xi})^2 = h(\tau)^{-\frac{1}{2}}\left(1 - h(\tau)^{\frac{1}{2}}g_{\tau\tau}(\frac{d\tau}{d\xi})^2\right) . \quad (2.2.16)$$

To leading order in the velocity we have $(\frac{dt}{d\xi})^2 \approx h(\tau)^{-\frac{1}{2}}$.

Proper distance is given by $d = \int d\tau' g_{\tau\tau}^{1/2}$, and proper velocity by $v \equiv \dot{d} = \dot{\tau} g_{\tau\tau}^{1/2}$. The initial values of the position, proper distance, coordinate velocity, and proper velocity are denoted by $\tau_*, d_*, \dot{\tau}_0$ and v_0, respectively.

The D3-brane trajectory is determined by the Born-Infeld action

$$S_{BI} = \frac{-1}{g_s^2 l_s^4}\int d^3\sigma d\xi \left(h(\tau)^{-1}\sqrt{1 - h(\tau)^{\frac{1}{2}}g_{\tau\tau}(\frac{d\tau}{d\xi})^2} - h(\tau)^{-1}\right) \quad (2.2.17)$$

where we have neglected contributions from the U(1) gauge field on the brane. At leading order in a low-velocity expansion, rewritten in terms of derivatives with respect to proper time,

$$S_{BI} = \frac{1}{2g_s^2 l_s^4}\int d^3\sigma d\xi \; h(\tau)^{-1}g_{\tau\tau}\dot{\tau}^2 \quad (2.2.18)$$

where the cancellation of the potential $h(\tau)^{-1}$ is the realization of the no-force condition. Conservation of energy then yields

$$\dot{\tau}(t)^2 = \dot{\tau}_0^2 \frac{h(t)}{h(\tau_*)} \frac{g_{\tau\tau}(\tau_*)}{g_{\tau\tau}(t)} \tag{2.2.19}$$

From the profile of $\frac{h}{g_{\tau\tau}}$ it follows that the brane accelerates gradually toward the tip of the conifold. For large τ we may use the KT radial coordinate r (2.2.13), in terms of which (2.2.19) is $\frac{d^2 r}{d\xi^2} = 0$, which is another expression of the balancing of gravitational forces and forces due to flux.

2.2.4 The Induced Cosmology

An observer on the brane naturally sees an induced metric

$$ds^2_{\text{brane}} \;=\; -dt^2 + h^{-1/2}(\tau)(dx_1^2 + dx_2^2 + dx_3^2)\ . \tag{2.2.20}$$

But given that the brane trajectory is a function $\tau(t)$, (2.2.20) gives rise to a standard FRW cosmology

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2) \tag{2.2.21}$$

with $a(t)$ given by

$$a(t) = h^{-1/4}(\tau(t)). \tag{2.2.22}$$

Notice that the graviton wavefunction has a τ-dependent overlap with a brane located at various points in the metric (2.2.5). This is simply the effect exploited in [43]. The dimensionless strength of gravity therefore scales according to

$$G_N(t) m^2_{open} \sim h(\tau(t))^{-\frac{1}{2}} \sim a(t)^2 \tag{2.2.23}$$

where m_{open} is the mass of the first oscillating open string mode. A physicist residing on the brane may choose to fix *one* of the dimensionful quantities G_N, m_{open} in order to set his units of length. Grinstein et al. [56] have shown that a brane observer who uses proper distance to measure lengths on the brane will necessarily find fixed masses and variable G_N. One can argue for the same system of units by stipulating that elementary particle masses should be used to define the units, and should be considered fixed with time. In this model we will use the mass of the first excited open string mode to fix such a frame; in a more realistic model, one would want other (perhaps "standard model") degrees of freedom to be the relevant massive modes.

A brane observer following an inward-falling trajectory in the background (2.2.5) would therefore make the following statements.

1. Elementary particle masses, e.g. m_{open}, are considered fixed with time.

2. In these units, the proper distance between galaxies on the brane scales with $a(t)$ as in standard FRW cosmology. In consequence, for the infalling brane (moving towards $\tau = 0$) one observes blueshifting of photons.

3. The gravitational coupling on the brane is time-dependent,

$$G_{\rm N}(t) \sim a(t)^2 \ . \qquad (2.2.24)$$

Therefore, as the universe collapses, the strength of gravity decreases.

In fact, (2.2.22) together with (2.2.24) imply that in 4d *Planck* units, the size of the universe remains *fixed*. From this "closed string" perspective, the cosmology is particularly trivial; the brane radial position is described by a scalar field Φ_τ in the 4d action which is undergoing some slow time variation (and, for small brane velocity, carries little enough energy that backreaction is not an issue). However, in this frame particle masses vary with time. We find it more natural, as in [56], for a brane observer to view physics in the frame specified by 1-3 above; we will henceforth adopt the viewpoint of such a hypothetical brane cosmologist. In §2.3.1 we describe the field redefinition which takes one from the "brane cosmologist" frame to the "closed string" frame in a toy model.

The Bounce

As the brane falls from τ^* towards zero, the scale factor decreases monotonically. It hits $\tau = 0$ in finite proper time. However, as is clear from the metric (2.2.5), there is no real boundary of the space at this point; $\tau = 0$ is analogous to the origin in polar coordinates. The brane smoothly continues back to positive τ, and the scale factor re-expands. Although it is hard to provide an analytical expression for $a(t)$ given the complexity of the expressions (2.2.10) and (2.2.11), we can numerically solve for a; a plot appears in Figure 1.

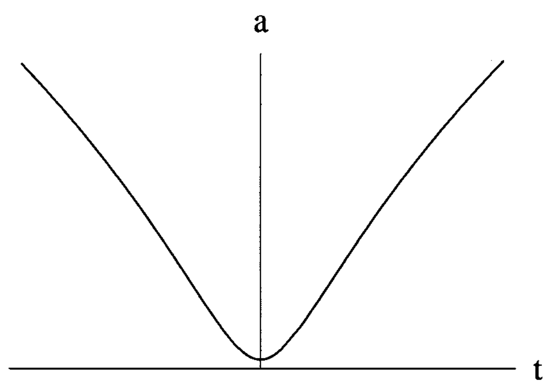

Fig. 1: The scale factor $a(t)$ as a function of proper time for a brane near the tip of the Klebanov-Strassler geometry. This particular bounce begins from radial position $\tau = 4$.

In the approximate supergravity dual to the cascading gauge theory studied in [45], there is instead a naked singularity in the region of small τ, which is deformed away by the fluxes (2.2.4). In the KT approximation to the physics, then, the cosmology on the brane would actually have a spacelike singularity at some finite proper time. The evolution in this background agrees with Figure 1 until one gets close to the tip of the conifold; then, in the "unphysical" region of the KT solution, the brane rapidly re-expands, and a singularity of the curvature scalar of the induced metric arises at a finite proper time. A plot of $a(t)$ for this case appears in Figure 2.

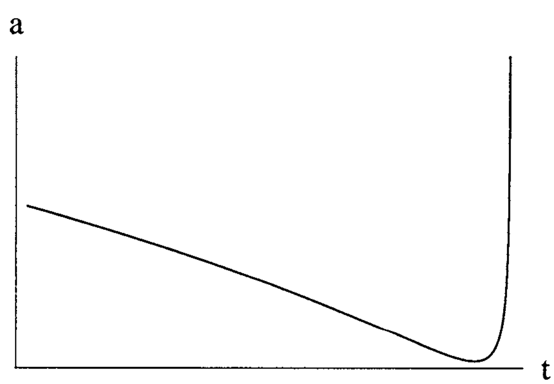

Fig. 2: The scale factor $a(t)$ as a function of proper time for a brane near the singularity of the Klebanov-Tseytlin geometry. The explosive growth of $a(t)$ on the right coincides with a curvature singularity in the induced metric.

Hence, we see that string theory in the smooth KS background gives rise to a bouncing brane cosmology, while the KT approximation would have given rise to a cosmology with a spacelike crunch. There has been great success in understanding the resolution of timelike singularities in string theory, so it is heartening to see that in some special cases one can translate those results to learn about spacelike singularities as well.

Limiting behaviors

In the two asymptotic regimes of $\tau \sim 0$ and very large τ, the formulae simplify [54] and the behavior of $a(t)$ can be given explicitly. For small τ, the geometry is just the product (2.2.12). Hence, in this limit, the brane is effectively falling in an *unwarped* 5d space, and the cosmology is very simple:

$$a(t) = \text{constant} + \mathcal{O}(t^2) \ . \tag{2.2.25}$$

In the large τ regime, the metric (2.2.14) differs from AdS_5 by logarithmic corrections, and so the brane trajectory deviates very gradually from that of a D3-brane in AdS. For simplicity we present here the induced cosmology on a D3-brane in AdS; the logarithmic corrections require no new ideas but lead to more complicated formulae. From (2.2.18), using the D3-brane form of the AdS_5 metric

$$ds^2 = r^2(-d\xi^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{dr^2}{r^2} \tag{2.2.26}$$

we find, in terms of proper time,

$$a^2(t) = a^2(0)(1 + 2\frac{\dot{r_0}}{r_0}t) \tag{2.2.27}$$

for a brane with initial position and velocity r_0, $\dot{r_0}$ at $t = 0$. It follows that

$$(\frac{\dot{a}}{a})^2 = \frac{C}{a^4} \tag{2.2.28}$$

where $C = a^4(0)(\frac{\dot{r_0}}{r_0})^2$. Because the right hand side of (2.2.28) scales like the energy density of radiation, this has been termed "dark radiation" [57,58]. In the language of [38] it might also be called "mirage matter with equation of state $\rho = 3p$."

The Friedmann equation (2.2.28) has been thoroughly investigated in the context of Randall-Sundrum models. In particular, just such a law was found to arise on a visible brane which is separated from a Planck brane by an interval whose length varies with time (see [59] and references therein). This is entirely consistent with our scenario, as the Calabi-Yau provides an effective Planck brane and the bulk motion of the probe changes the length of the interval between the branes.

As the brane proceeds to larger τ, eventually it will reach the region where the KS throat has been glued onto a Calabi-Yau space. Beyond that point it is no longer possible for us to say anything universal about the behavior of the brane cosmology.

2.2.5 Issues of Backreaction

There are several issues involving backreaction that merit consideration. To argue that the bounce we have seen in §2.2.4 accurately describes the behavior of the brane as it propagates in from τ_* and back out again, we must ensure that the state with nonzero $\dot{\tau}$ on the brane does not contain enough energy to significantly distort the closed string background geometry. In fact we must check both that a motionless brane in the throat creates a negligible backreaction, and that the kinetic energy on the brane does not undergo gravitational collapse (yielding a clumpy brane) on the relevant timescales. It is also important to understand the extent of the backreaction from semiclassical particle production. Finally, the presence of nonzero energy density on the brane leads to a potential for the Calabi-Yau volume modulus (as in §6 of [60]). We will imagine that this modulus has been fixed and will neglect this effect.

The first concern can be dismissed quickly. In the limit of small g_s the backreaction on the closed string background is small. The second concern needs to be discussed in somewhat more detail. The falling brane necessarily has energy density localized on its worldvolume. After a sufficiently long time this initially uniform energy can become inhomogeneous because of the Jeans instability. In this subsection we demonstrate that, for a suitable choice of the parameters of the KS geometry, this instability is negligible during the bounce portion of the history of the brane universe.

Jeans Instability

For a uniform fluid of density ρ, the Jeans instability appears at length scales greater than $L_{Jeans} \equiv \frac{v_s}{\sqrt{\rho G_N}}$, where v_s is the velocity of sound. Perturbations with this wavelength could destabilize the brane given a time $t_{instability} \geq L_{Jeans}$. In terms of the volume V_6 of the Calabi-Yau,

$$G_N = g_s^2 l_s^8 V_6{}^{-1} h(\tau_{UV})^{\frac{1}{2}} h(\tau)^{-\frac{1}{2}} \tag{2.2.29}$$

where we choose τ_{UV} such that r_{UV} (as given in (2.2.13)) is of order one (so the throat extends slightly into the KT regime before gluing into the Calabi-Yau). For the compactifications of interest $V_6 \geq l_s^6$,[3] so that for $\tau \leq \tau_{UV}$

$$G_N \leq {g_s}^2 {l_s}^2 \ . \tag{2.2.30}$$

From (2.2.18), (2.2.19), we see that the energy density on the brane is constant,

$$\rho = \frac{1}{2g_s^2 l_s^4} h(\tau_*)^{-1} g_{\tau\tau}(\tau_*) \dot{\tau_0}^2 \tag{2.2.31}$$

so

$$t_{instability} \geq \frac{v_s}{\dot{\tau_0}} h(\tau_*)^{\frac{1}{2}} g_{\tau\tau}(\tau_*)^{-\frac{1}{2}} l_s \ . \tag{2.2.32}$$

Because the brane accelerates toward the tip of the conifold, to fall from d_* to the tip and rebound requires a time

$$t_{bounce} \leq \frac{2d_*}{v_0} \ . \tag{2.2.33}$$

This leads to (we now drop numerical factors of order one)

$$\frac{t_{bounce}}{t_{instability}} < \frac{d_*}{l_s} h(\tau_*)^{-\frac{1}{2}} \ . \tag{2.2.34}$$

Using the asymptotic form of $I(\tau)$, $K(\tau)$ we find

$$\frac{t_{bounce}}{t_{instability}} < \frac{1}{\sqrt{g_s M}} {\tau_*}^{\frac{3}{4}} {l_s}^{-2} (\epsilon^2 e^{\tau_*})^{\frac{2}{3}} \ . \tag{2.2.35}$$

[3] In fact, as discussed in [4], warped compactifications really reproduce the RS scenario when the volume is not very large in string units (since the flux and brane backreaction which produce the warping become larger effects at small Calabi-Yau volume). We are assuming we are at the threshold volume where the warping becomes a significant effect, which should justify the estimate (2.2.29).

Because we have glued the KS throat into the Calabi-Yau geometry at a location where $r = r_{UV}$ of (2.2.13) is of order one, we see that $\epsilon^2 e^{\tau_*} = \mathcal{O}(1)$. This leads to

$$\frac{t_{bounce}}{t_{instability}} < \frac{1}{\sqrt{g_s M}} \tau_*^{3/4} \ . \tag{2.2.36}$$

Finally, since the hierarchy between the UV and IR ends of the throat is exponential in τ_*, it is natural to take τ_* to be a number of order 5-10 (in the language of RS scenarios, τ_* controls the length of the interval in AdS radii, up to factors of π). Therefore, in the supergravity regime where $g_s M \gg 1$, (2.2.36) demonstrates that we can neglect the Jeans instability on the brane in discussing the dynamics during the bounce.

Particle Creation

Because the bounce cosmology is strongly time-dependent, it is also important to consider the spectrum of particles created semiclassically by the bounce. We will argue that the energy density due to such particle production is small enough that its backreaction is negligible.

The bounce geometry (2.2.21) is conformally trivial, so massless, conformally coupled scalar fields will not be produced by the cosmological evolution. Massive fields break the conformal invariance. The relevant massive scalar fields on the brane are excited string states with mass $m \geq \frac{1}{l_s}$. Quite generally we expect that modes with frequencies $\omega \gg \frac{\dot{a}}{a} \equiv H$ will not be significantly populated by the bounce, i.e. the probability that a comoving detector will register such a particle long after the bounce is exponentially small in $\frac{\omega}{H}$. The cases of interest involve slow-moving branes, so the maximum value of H is far below the string scale. Thus we expect the energy density due to particle creation should be quite small.

Concrete calculations of the production of massive scalar and fermion fields in a bouncing $k = 0$ FRW cosmology were carried out in [61] (though the system in consideration there did not satisfy Einstein's equations). The scale factor in [61] has the same limiting behaviors as our own, and the results there are consistent with our expectations. It would be interesting to carry out the relevant particle creation calculation directly in string theory. A particle creation calculation in closed string theory was described in worldsheet (2d conformal field theory) language in [62].

2.3 Four-dimensional Lagrangian Description

2.3.1 Effective Lagrangian

In the limit of low matter density on the probe brane, the cosmology is determined entirely by the bulk geometry. The D3-brane trajectory is determined by the Born-Infeld action, and the induced metric along this trajectory provides a time-dependent mirage cosmology. The mirage cosmology proposal of [38] includes another step: one can write down the Friedmann equations for the cosmology and identify the right hand side with mirage density and mirage pressure.

This is not yet an ideal formulation from the perspective of a brane resident. One would like a four-dimensional Lagrangian description of the mirage matter, of the cosmological evolution, and of the variation of G_N. In particular, since a bounce in a flat Friedmann-Robertson-Walker universe necessitates violation of the null energy condition, it would be interesting to understand this violation in terms of a 4d Lagrangian and energy-momentum tensor. In this section we will propose a toy scalar-tensor Lagrangian which admits cosmologies reproducing the basic features of our "bouncing brane" solutions; similar Lagrangians have arisen in the study of RS cosmology [63].

The massless fields in our 4d theory include a 4d graviton and the massless open strings on the D3-brane: a U(1) gauge field A_μ, a scalar Φ_r corresponding to radial motion in the compactified throat, and scalars Φ_i, $i = 1, \cdots, 5$ parametrizing motion in the angular coordinates. All other scalar fields are massive. (In fact without a no-force condition there can be a potential and a mass for Φ_r. For simplicity we will work only with the BPS case, but the trajectory of anti-branes in the KS throat would also yield an interesting time-dependent solution.[4]) We will choose to fix the Φ_i, and the requirement of negligible energy density in open string modes on the brane means that A_μ is not relevant for cosmological purposes. This leaves Φ_r and $g_{\mu\nu}$ as the only massless fields entering the 4d Lagrangian.

Our goal in this section is to show explicitly how an observer who sees particle masses which depend on Φ_r could change his units of length and see an FRW cosmology with varying G_N. (In §2.2.4 we provided several arguments motivating

[4] In particular, anti-branes near the tip of the conifold can annihilate by merging with flux [60]. This could potentially lead to a cosmology which begins or ends with a tunneling or annihilation process.

this choice of frame.) Because the full Lagrangian for a brane observer in the KS background, including all massive fields, is quite complicated, it will be most practical to work with a simpler Lagrangian which has the correct schematic features. In particular, all particle masses depend on Φ_r in the same way, so it will suffice to consider a single massive field χ (which could be, for example, an excited open string mode).

A "mass-varying" Lagrangian with the appropriate features is

$$L = \int d^3x\sqrt{-g}\,\Big(\frac{R}{16\pi G_N} - \frac{R}{12}\Phi_r^2 - \frac{1}{2}g^{\mu\nu}\nabla_\mu\Phi_r\nabla_\nu\Phi_r \\ -\frac{1}{2}g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - \frac{1}{2}m^2(\Phi_r)\chi^2 - V(\chi)\Big) \tag{2.3.1}$$

where χ is a matter field on the brane whose mass depends on Φ_r as

$$m^2(\Phi_r) \equiv \Omega^2(\Phi_r)\mu^2 \tag{2.3.2}$$

for fixed μ. The form of the potential for χ and the coupling of χ to the curvature scalar will be unimportant for this analysis, and we will henceforth omit these terms. Note that Φ_r is conformally coupled.

As discussed in §2.2.4, an observer confined to the brane most naturally holds fixed the masses of fields on the brane. This can be accomplished by performing the change of variables

$$\tilde{g}_{\mu\nu} = \Omega^2(\Phi_r)g_{\mu\nu} \tag{2.3.3}$$

$$\tilde{\Phi}_r = \Omega^{-1}(\Phi_r)\Phi_r \tag{2.3.4}$$

$$\tilde{\chi} = \Omega^{-1}(\Phi_r)\chi \tag{2.3.5}$$

The resulting "mass-fixed" Lagrangian is

$$L = \int d^3x\sqrt{-\tilde{g}}\,\Big(\frac{\tilde{R}}{16\pi G_N\Omega^2(\Phi_r)} + \frac{3}{8\pi G_N\Omega(\Phi_r)^4}\tilde{g}^{\mu\nu}\nabla_\mu\Omega\nabla_\nu\Omega - \frac{\tilde{R}}{12}\tilde{\Phi}_r^2 \\ - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\tilde{\Phi}_r\nabla_\nu\tilde{\Phi}_r - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\tilde{\chi}\nabla_\nu\tilde{\chi} - \frac{1}{2}\mu^2\tilde{\chi}^2\Big)\,. \tag{2.3.6}$$

We have discarded terms which look like $(\nabla\Omega)^2\tilde{\chi}^2$ because $\dot{\Omega} \ll \mu$ (at least in our example, where χ represents a massive string mode). Terms which look like $(\nabla\Omega)^2\tilde{\Phi}_r^2$ cancel due to the conformal coupling of Φ_r.

The effective gravitational coupling is given by

$$G_N{}^{eff} = G_N \Omega^2(\Phi_r) \; . \tag{2.3.7}$$

According to the discussion in §2.2.4, we expect that $\Omega^2(\Phi_r) = h(\tau(\Phi_r))^{-\frac{1}{2}}$, so indeed the strength of gravity scales as required by (2.2.24). (We will not need the explicit relation between τ and Φ_r.)

We are interested in the limit where the backreaction due to $\tilde{\Phi}_r, \tilde{\chi}$ is small, so in particular $\tilde{\Phi}_r, \tilde{\chi} \ll m_{Planck}^{eff}$. This means that for the purpose of solving the Einstein equations in the mass-fixed frame we may neglect terms which are suppressed by a factor of G_N. Defining

$$\gamma = \sqrt{\frac{3}{4\pi G_N}} \Omega^{-1}(\Phi_r) \tag{2.3.8}$$

we may write the effective Lagrangian

$$L = \int d^3x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{12}\gamma^2 + \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\gamma\nabla_\nu\gamma + \mathcal{O}(\frac{\Phi_r}{m_{Planck}}) \right) . \tag{2.3.9}$$

Observe that the kinetic energy term is now negative semidefinite (we are using signature $-+++$), so it is easy to violate the null energy condition which is relevant (via the singularity theorems) in constraining the behavior of the metric $\tilde{g}_{\mu\nu}$.[5]

The equation of motion which follows from this Lagrangian is

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu\gamma - \frac{\tilde{R}}{6}\gamma = \mathcal{O}(\frac{\Phi_r}{m_{Planck}}) \tag{2.3.10}$$

Now let us see that this system reproduces our expectations from §2.2.4. Given an FRW cosmology specified by $a(t)$, if we set $\gamma(t) = ca^{-1}(t)$ for some constant c then (2.3.10) is satisfied identically. From (2.3.3), (2.3.4), (2.3.5) it is clear that we should identify

$$a(t) \propto \Omega(\Phi_r(t)). \tag{2.3.11}$$

Then the Einstein equations for (2.3.9) are satisfied if the varying-mass metric $g_{\mu\nu} = \eta_{\mu\nu}$ and the mass-fixed metric $\tilde{g}_{\mu\nu} = a^2(t)\eta_{\mu\nu}$. So as discussed in §2.2.4, we have two complementary perspectives: the brane observer uses the mass-fixed action (2.3.6) and sees an FRW cosmology with varying G_N, while the "closed string" observer sees gravity of fixed strength in Minkowski space.

[5] Notice that because of the non-minimally coupled scalar, it is also possible to violate the null energy condition which governs the behavior of $g_{\mu\nu}$.

2.3.2 Relation to Warped Backgrounds

We can be slightly more explicit about how the toy model of §2.3.1 would be related to a given warped background. Given any function $a(t)$, we can construct a warped background $h(r)$ such that a no-force brane probe of that geometry experiences an induced cosmology specified by $a(t)$. We simply define $\xi = \int \frac{dt}{a(t)}$, $r = v\xi$ (v constant), and $h(r) = a(r)^{-4}$.

A few comments are in order:

1. Very few backgrounds $h(r)$ will correspond to solutions of IIB supergravity. One which does, and indeed corresponds to a D3-brane in the warped deformed conifold, is given by taking $\tau(t)$ to solve (2.2.19) and setting $a(t) = h^{-\frac{1}{4}}(\tau(t))$ with h given by (2.2.10).
2. The no-force condition is only a convenience. We could instead take $r(\xi)$ to be any function of ξ. This would correspond to a brane which accelerates due to external forces. Again, very few systems of this sort arise from known branes of string theory moving in valid supergravity backgrounds.

2.4 Discussion

As demonstrated in general terms in §2.3, and in a special example in string theory in §2.2, in the presence of scalar fields it is easy to evade the singularity theorems (from the perspective of a reasonable class of observers), even with a $k = 0$ FRW universe. It therefore seems likely that many examples of such constructions, arising both as cosmologies on D-branes and perhaps even as closed string cosmologies, should be possible. The cosmology we presented is just a slice of evolution between some initial time when we join the brane moving down the throat, and a final time when it is heading into the Calabi-Yau region. The later evolution of our model is then non-universal; it depends on the details of the Calabi-Yau model (or in the language of [43], the detailed structure of the Planck brane). It would be very interesting to write down models with 4d gravity whose dynamics can be controlled for an eternity; some controlled, eternal closed string cosmologies were recently described in [62].

The cosmology discussed here is far from realistic. As a first improvement, one would like to study probe branes with a spectrum of massive fields below the scale $\frac{1}{l_s}$ (which could be called "standard model" fields). It may be possible to construct such examples by using parallel D3-branes which are slightly separated in the radial direction, wrapped Dp-branes with $p > 3$, or anti-branes in appropriate regimes. It is also important to control the time-variation of G_N during/after nucleosynthesis, since this is highly constrained by experiment (see for instance [8]). To improve the situation, one can envision a program of "cosmological engineering." That is, one could try to design IIB solutions with background fields specifically chosen to give rise to interesting mirage cosmologies (various authors have already proposed mirage models of closed universes [64], inflation with graceful exit [65], asymptotically de Sitter spaces [50], etc., though most of these models do not include 4d gravity). Each desired feature of the cosmology would result in a new condition on the closed string fields. Then one would simply impose these conditions along with the field equations of IIB supergravity.

3. Moduli Trapping at Enhanced Symmetry Points

ABSTRACT OF ORIGINAL PAPER

We study quantum effects on moduli dynamics arising from the production of particles which are light at special points in moduli space. The resulting forces trap the moduli at these points, which often exhibit enhanced symmetry. Moduli trapping occurs in time-dependent quantum field theory, as well as in systems of moving D-branes, where it leads the branes to combine into stacks. Trapping also occurs in an expanding universe, though the range over which the moduli can roll is limited by Hubble friction. We observe that a scalar field trapped on a steep potential can induce a stage of acceleration of the universe, which we call trapped inflation. Moduli trapping ameliorates the cosmological moduli problem and may affect vacuum selection. In particular, rolling moduli are most powerfully attracted to the points with the largest number of light particles, which are often the points of greatest symmetry. Given suitable assumptions about the dynamics of the very early universe, this effect might help to explain why among the plethora of possible vacuum states of string theory, we appear to live in one with a large number of light particles and (spontaneously broken) symmetries. In other words, some of the surprising properties of our world might arise not through pure chance or miraculous cancellations, but through a natural selection mechanism during dynamical evolution.

This chapter is reprinted from Lev Kofman, Andrei Linde, Xiao Liu, Alexander Maloney, Liam McAllister, and Eva Silverstein, "Beauty is Attractive: Moduli Trapping at Enhanced Symmetry Points," JHEP **0405** (2004) 030, by permission of the publisher.

3.1 Introduction

3.1.1 Moduli Trapping Near Enhanced Symmetry Points

Supersymmetric string and field theories typically contain a number of light scalar fields, or moduli, which describe low-energy deformations of the system. If the kinetic energy of these fields is large compared to their potential energy then the classical dynamics of the moduli is described by geodesic motion on moduli space.

At certain special points (or subspaces) of moduli space, new degrees of freedom become light and can affect the dynamics of moduli in a significant way [66,67,68,69,70]. These extra species often contribute to an enhanced symmetry at the special point. We will refer to any points where new species become light as ESPs, which stands for extra species points, and also, when applicable, for enhanced symmetry points.

A canonical example is a system of two parallel D-branes. When the branes coincide, the two individual $U(1)$ gauge symmetries are enhanced to a $U(2)$ symmetry, as the strings that stretch between the branes become massless [71]. Similar points with new light species arise in many contexts; examples include the Seiberg-Witten massless monopole and dyon points in $\mathcal{N} = 2$ supersymmetric field theories [72], the conifold point (2.2.3) and ADE singularities in Calabi-Yau compactification [73], the self-dual radius of string compactifications on a torus, small instantons in heterotic string theory [74], and many other configurations with less symmetry.

Classically, there is no sense in which these ESPs are dynamically preferred over other metastable vacuum states of the system. We will argue that this changes once quantum effects are included. In particular, quantum particle production of the light fields alters the dynamics in such a way as to drive the moduli towards the ESPs and trap them there.

The basic mechanism of this trapping effect is quite simple. Consider a modulus ϕ moving through moduli space near an ESP associated to a new light field χ. For example, ϕ could be the separation between a pair of parallel D-branes, and χ a string stretching between the two branes – in this case the ESP $\phi = 0$ is the point where the branes coincide and χ becomes massless. As ϕ rolls through moduli space, the mass of χ changes; χ gets lighter as ϕ moves closer to the ESP and heavier as ϕ moves farther away. This changing mass leads to quantum production of χ particles; as ϕ moves past the ESP some of its kinetic energy will be dumped into χ particles.

As ϕ rolls away from the ESP, more and more of its energy will be drained into the χ sector as the χ mass increases, until eventually ϕ stops rolling. At this point the moduli space approximation for ϕ has broken down, and all of the original kinetic energy contained in the coherent motion of ϕ has been transferred into χ particles, and ultimately into all of the fields interacting with χ (including decoherent quanta $\delta\phi$). As we will see in detail, the χ excitations generate a classical potential for ϕ which drives the modulus back toward the ESP and traps it there.[6]

In the example of the pair of moving D-branes, the consequences of this are simple: two parallel branes that are sent towards each other will collide and remain bound together. The original kinetic energy of the moving branes will be transferred into open string excitations on the branes and eventually into closed string radiation in the bulk.

In §3.3.2 we will describe the general trapping mechanism and study its range of applicability using a few simple estimates. In §3.3 we will write down the equations of motion governing trapping in more detail, and describe the numerical and analytic solutions of these equations in a variety of cases.

It is important to recognize that this trapping effect is in no way special to string theory. Flat space quantum field theory with a moduli space for ϕ and an ESP is an ideal setting for the trapping effect, and it is in this setting that we will perform the analysis of §3.2 and §3.3. In §3.4 we will generalize this to incorporate the effects of cosmological expansion, and in §3.5 we will discuss the possibility of significant effects from string theory. Having established the moduli trapping effect in a variety of contexts, we will then study its applications to problems in cosmology.

The most immediate application is to the problem of vacuum selection. As we will see in §3.6, the trapping effect can provide a dynamical vacuum selection principle, reducing the problem to that of selecting one point within the class of ESPs. This represents significant progress, since the vast majority of metastable vacua are not ESPs. Trapping at ESPs may also help solve the cosmological moduli

[6] There are also corrections to the effective action for ϕ from loops of χ particles, including both kinetic corrections and a Coleman-Weinberg effective potential. Both effects will be subdominant in the weakly-coupled, supersymmetric, kinetic-energy dominated regimes we will consider.

problem, as we will see in §3.7. In particular, trapping strengthens the proposal of [75] by providing a dynamical mechanism which explains why moduli sit at points of enhanced symmetry.

Finally, as we will explain in §3.8, the trapping of a scalar field with a potential can lead to a period of accelerated expansion, in a manner reminiscent of thermal inflation [76]. This effect, which we will call trapped inflation, can occur in a steeper potential than normally admits such behavior.

From a more general perspective, moduli trapping gives us insight into the celebrated question of why the world is so symmetric. The initial puzzle is that although highly symmetric theories are aesthetically appealing and theoretically tractable, they are also very special and hence, in an appropriate sense, rare. One expects that in a typical string theory vacuum, most symmetries will be strongly broken and most particles will have masses of order the string or Planck mass, just as in a typical vacuum one expects a large cosmological constant. Vacua with enhanced symmetry or light particles should comprise a minuscule subset of the space of all vacua.

Nevertheless, we observe traces of many symmetries in the properties of elementary particles, as spontaneously broken global and gauge invariances. Moreover, all known particles are hierarchically light compared to the Planck mass. Given the expectation that a typical vacuum contains very few approximate symmetries and very few light particles, it is puzzling that we see such symmetries and such particles in our world.

For questions of this nature, moduli trapping may have considerable explanatory power. Specifically, the force pulling moduli toward a point of enhanced symmetry is proportional to the number of particles which become massless at this point, which is often associated with a high degree of symmetry. This means that the most attractive ESPs are typically the ones with the largest symmetry, and rolling moduli are most likely to be trapped at highly symmetric points, where many particles become massless or nearly massless. Moreover, the process of trapping can proceed sequentially: a modulus moving in a multi-dimensional moduli space can experience a sequence of trapping events, each of which increases the symmetry. These effects suggest that the symmetry and beauty we see in our world may have, at least in part, a simple dynamical explanation: beauty is attractive. We will discuss this possibility in §3.6.

3.1.2 Relation to Other Works

Similar effects have been described in the literature. There has been much work on multi-scalar quantum field theory in the context of inflation, especially concerning preheating in interacting scalar field theories. Some of our results will be based on the theory of particle production and preheating developed in the series of papers [66,67,68], which explores many of the basic phenomena in scalar theories of the sort we will consider. Likewise, Chung et al. [69] have explored the effects of particle production on the inflaton trajectory and on the spectrum of density perturbations. Although we will derive what we need here in a self-contained way, many of the technical results in this chapter overlap with those works, as well as with standard results on particle production in time-dependent systems as summarized in e.g. [77]. Although we will not study the case in which χ goes tachyonic for some range of ϕ, our results may nevertheless have application to models of hybrid inflation [78,79], including models based on rapidly-oscillating interacting scalars [80,81,82].

In strong 't Hooft coupling regions of moduli spaces which are accessible through the AdS/CFT correspondence, virtual effects from the large numbers of light species dramatically slow down the motion of ϕ as it approaches an ESP, with the result that the modulus gets trapped there [70]. This also provides a mechanism for slow roll inflation without very flat potentials. In the present work, which applies at weak 't Hooft coupling, it is quantum production of *on-shell* light particles which leads to trapping on moduli space.

Other works in the context of string theory have explored the localization of moduli at ESPs. The authors of [83,84] studied the evolution of a supersymmetric version of the $\phi - \chi$ system arising near a flop transition using an effective supergravity action. They showed that, given nonvanishing initial vevs for both ϕ and χ, the fields will settle at the ESP even if one formally turns off particle production effects. Our proposal, by contrast, is to take into account on-shell quantum effects which dynamically generate a nonzero $\langle\chi^2\rangle$. In works such as [85] attention was focused on the boundaries of moduli space, while here we focus on ESPs in the interior of moduli space. In [86], production of light strings was studied in the context of D0-brane quantum mechanics; as we explain in §3.2.3, this has some similarities, but important differences, with our case of space-filling branes. Scattering of Dp-branes was also studied in [87].

Dine has suggested that enhanced symmetry points may provide a solution to the moduli problem, as moduli which begin at an enhanced symmetry minimum of the quantum effective potential can consistently remain there both during and immediately after inflation [75]. One would still like to explain why the moduli began at such a point. As we discuss in §3.7, our trapping mechanism provides a natural explanation for this initial configuration.

Horne and Moore [88] have argued that the classical motion on certain moduli spaces is ergodic, provided that the potential energy is negligible. This means that all configurations are sampled given a sufficiently long time, and in particular a given modulus will eventually approach an ESP. We will argue that quantum corrections to the classical trajectory are significant, and indeed lead to trapping, whenever the classical trajectory comes close to an ESP. Combining these two observations, we expect that in the full, quantum-corrected system the moduli are stuck near an ESP at late times. This means that the quantum-corrected evolution is not fully ergodic: the dynamics of [88] (see also [89]) implies that the modulus will eventually approach an ESP, at which point quantum effects will trap it there, preventing the system from sampling any further regions of moduli space.

3.2 Moduli Trapping: Basic Mechanism

We will now describe the mechanism of moduli trapping in more detail. Our discussion in this section will be based on simple estimates of particle production and the consequent backreaction, generalizing the results of [66,67,68] to the case of a complex field. A more complete analysis, along with numerical results, will be presented in §3.3.

We will consider the specific model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\bar{\phi} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{g^2}{2}|\phi|^2\chi^2 \qquad (3.2.1)$$

where a complex modulus $\phi = \phi_1 + i\phi_2$ interacts with a real scalar field χ. We are restricting ourselves to the case of a flat moduli space which has a single ESP at $\phi = 0$, where χ becomes massless, and a particularly simple form for the χ interaction. This simple case illustrates the basic physics and can be generalized as necessary, for example to include supersymmetry.

We will consider the case where ϕ approaches the origin with some impact parameter μ, following a classical trajectory of the form

$$\phi(t) = i\mu + vt. \tag{3.2.2}$$

Classically, if χ vanishes then (3.2.2) is an exact solution to the equations of motion, and the presence of the ESP will not affect the motion of ϕ.

Quantum effects will alter this picture considerably, because the trajectory (3.2.2) will lead to the production of χ particles, as we discuss in §3.2.1. The backreaction of these particles on the motion of ϕ will then lead to trapping, as we will see in §3.2.2. In §3.2.3 we will illustrate this effect with the example of colliding D-branes.

3.2.1 Quantum Production of χ Particles

Let us first study the creation of χ particles without considering how they may backreact to alter the motion of ϕ. In this approximation we may substitute (3.2.2) into the action (3.2.1) to get a free quantum field theory for χ with a time-varying mass

$$m_\chi^2(t) = g^2|\phi(t)|^2. \tag{3.2.3}$$

This time dependence leads to particle production.

Consider a mode of the χ field with spatial momentum k, whose frequency

$$\omega(t) = \sqrt{k^2 + g^2|\phi(t)|^2} \tag{3.2.4}$$

varies in time. This mode becomes excited when the non-adiabaticity parameter $\dot{\omega}/\omega^2$ becomes at least of order one. This parameter vanishes as $t \to \pm\infty$, indicating that particle creation takes place only while ϕ is near the ESP. It is straightforward to see that, for the trajectory (3.2.2), $\dot{\omega}/\omega^2$ can be large only in the small interval $|\phi| \lesssim \Delta\phi$ near the ESP, where

$$\Delta\phi = \sqrt{\frac{v}{g}}, \tag{3.2.5}$$

and only for momenta

$$\frac{k^2 + g^2\mu^2}{gv} \lesssim 1. \tag{3.2.6}$$

When the quantity on the left hand side is small, particle creation effects are very strong. They are strongest if the modulus passes sufficiently close to the ESP, i.e. if

$$\mu \lesssim \sqrt{v/g}. \tag{3.2.7}$$

In this case χ modes whose momenta k fall in the range (3.2.6) will be excited.[7] Qualitatively, we expect that the occupation numbers n_k of such modes will vary from zero (no real particles) for modes with vanishing non-adiabaticity to of order unity for modes with very large non-adiabaticity. The full computation of n_k given in Appendix 3.A yields

$$n_k = \exp\left(-\pi\frac{k^2+g^2\mu^2}{gv}\right), \tag{3.2.9}$$

which agrees with this qualitative expectation. Note that even when (3.2.7) is not satisfied, there is generically a nonvanishing, though exponentially suppressed, number density of created particles; even in this case we will find a nontrivial trapping effect.

Before discussing the backreaction due to the production of χ particles, it is crucial to control other effects from the χ field. In particular, there is another important quantum effect which arises in motion toward the origin: loops of light χ particles give corrections to the effective action. These include both kinetic corrections and the Coleman-Weinberg potential energy. The latter we will subtract by hand, as we will explain in §3.3.1. This gives a good approximation to the dynamics in any situation where kinetic energy dominates.

The kinetic corrections are organized in an expansion in v^2/ϕ^4 [70]. The parameters controlling both remaining effects – the nonadiabaticity controlling particle production and the kinetic factor v^2/ϕ^4 controlling light virtual χ particles –

---

[7] This may be checked as follows. We have argued that unsuppressed particle production occurs only when the modulus is sufficiently close to the ESP, $|\phi| \lesssim \sqrt{v/g}$. The modulus remains within this window for a time

$$\Delta t \sim \frac{\sqrt{v/g}}{v} \sim (gv)^{-1/2}. \tag{3.2.8}$$

The uncertainty principle implies in this case that the created particles will have typical energy $E \sim (\Delta t)^{-1}$ and thus momenta $k \sim (gv - g^2\mu^2)^{1/2}$. This agrees with the estimate (3.2.6).

diverge as we approach the origin. However, at weak coupling, the nonadiabaticity parameter is parametrically enhanced relative to the kinetic corrections, i.e. $v^2/g^2\phi^4 \gg v^2/\phi^4$, so we can sensibly focus on the effects of particle production. More specifically, we can ensure that the kinetic corrections are insignificant by including a sufficiently large impact parameter μ.

We will also analyze the case of small μ, including $\mu = 0$. This relies on the plausible assumption that the effects of the kinetic corrections remain subdominant as we approach very close to the origin, and that in particular in our weak coupling case they do not by themselves stop ϕ from progressing through the origin. It would be interesting to develop theoretical tools to analyze this issue more directly and check this hypothesis.

3.2.2 Backreaction on the Motion of ϕ

One might expect *a priori* that any description of the motion of ϕ which fully incorporates backreaction from particle production would be immensely complicated. Fortunately, this turns out not to be the case, and a simple description is possible. The key simplification is that creation of χ particles happens primarily in a small vicinity of the ESP $\phi = 0$, so one can treat this as an instant event of particle production. These particles induce a very simple linear, confining potential acting on ϕ, $V \sim |\phi|$. The motion of ϕ in this potential between successive events of particle production can be described rather simply.

Let us now explore this in more detail. We have seen that as ϕ moves in moduli space, some of its energy will be transferred into excitations of χ. This leads to a quantum vacuum expectation value $\langle\chi^2\rangle \neq 0$. As ϕ rolls away from the ESP, the mass of the created χ particles increases, further increasing the energy contained in the χ sector. At this point the backreaction of the χ field on the dynamics of ϕ becomes important, and the moduli space approximation breaks down.

We will concentrate on the backreaction of the created particles on the motion of the field ϕ far away from the small region of non-adiabaticity, i.e. for $\phi \gg \Delta\phi \sim \sqrt{v/g}$. At this stage the typical momenta are such that the χ particles are nonrelativistic, $k \lesssim \sqrt{gv} \ll g|\phi|$. Therefore the total energy density of the gas of χ particles is easily seen to be

$$\rho_\chi(\phi) = \int \frac{d^3k}{(2\pi)^3} n_k \sqrt{k^2 + g^2|\phi(t)|^2} \approx g|\phi(t)| n_\chi, \qquad (3.2.10)$$

where n_χ is the number density of χ particles,

$$n_\chi = \int \frac{d^3k}{(2\pi)^3} n_k = \frac{(gv)^{3/2}}{(2\pi)^3} e^{-\pi g\mu^2/v} \tag{3.2.11}$$

As ϕ continues to move away from the ESP $\phi = 0$, the number density of χ particles remains constant, as particles are produced only in the vicinity of $\phi = 0$. However, the energy density of the χ particles grows as $g|\phi(t)|n_\chi$. This leads to an attractive force of magnitude gn_χ, which always points towards the ESP $\phi = 0$.

This force of attraction slows down the motion of ϕ, and eventually turns ϕ back toward the ESP. This reversal occurs in the vicinity of the point ϕ_* at which the initial kinetic energy density $\frac{1}{2}\dot{\phi}^2 \equiv \frac{1}{2}v^2$ matches the energy density ρ_χ contained in χ particles. We find

$$\phi_* = \frac{4\pi^3}{g^{5/2}} v^{1/2} e^{\pi g\mu^2/v}. \tag{3.2.12}$$

Observe that for $g \ll 1$ the trapping length on the first pass is always much greater than the impact parameter μ, which means that the motion of the moduli after the first impact is effectively one-dimensional.

After changing direction at ϕ_*, ϕ falls back toward the origin. On this second pass by the ESP, more χ particles are produced, leading to a stronger attractive force. This process repeats itself, leading ultimately to a trapped orbit of ϕ about the ESP, in a trajectory determined by the effective potential and consistent with angular momentum conservation on moduli space.

We conclude that, in this simplified setup, a scalar field which rolls past an ESP will oscillate about the ESP with an initial amplitude given by (3.2.12).

In fact, in many cases the amplitude of these oscillations will rapidly decrease due to the effect of parametric resonance, similar to the effects studied in the theory of preheating [66], and the field ϕ will fall swiftly towards the ESP. This important result will be described in more detail in §3.3.3.

So far we have not incorporated the effects of scattering and decay of the χ particles. These could weaken the trapping potential (3.2.10) by reducing the number of χ particles. Specifically, the energy density ρ_χ contained in a fixed number of χ particles (3.2.10) grows at late times, since the χ mass increases as ϕ rolls away from the ESP. However, if the number density of χ particles decreases due to annihilation or decay into lighter modes, this mass amplification effect is lost. It is therefore important to determine the rate of decay and annihilation of the χ particles.

In Appendix 3.B we address these issues and demonstrate that the trapping effect is robust for certain parameter ranges, provided that the light states are relatively stable. This stability can easily be arranged in supersymmetric models, and in fact occurs automatically in certain D-brane systems.

Rescattering effects, in contrast, may actually strengthen the trapping effect. Once χ particles have been created, they will scatter off of the homogeneous ϕ condensate, causing it to gradually decay into inhomogeneous, decoherent ϕ excitations [66,90,91]. However, we will not consider this potentially beneficial effect here.

3.2.3 The Example of Moving D-branes

Before proceeding, it may be illustrative to discuss these results in terms of a simple, mechanical example – a moving pair of D-branes. The moduli space of a system of two D-branes is the space of brane positions. In terms of the brane worldvolume fields the separation between the two branes can be regarded as a Higgs field ϕ. The off-diagonal components of the $U(2)$ gauge field are the W bosons. At the ESP of this system, $\phi = 0$, the W bosons are massless. Away from $\phi = 0$ the W bosons acquire a mass by the Higgs mechanism, breaking the symmetry group from $U(2)$ down to $U(1) \times U(1)$. If we identify χ with the W field[8] and $g^2 \sim g_{YM}^2 \sim g_s$ with the string coupling, then we find that the brane worldvolume theory contains a term like (3.2.1). We therefore expect this system to exhibit moduli trapping.

The trapping effect is a quantum correction to the motion of D-branes. As the D-branes approach each other, the open strings stretched between them become excited. When the D-branes pass by each other and begin moving apart the stretched open strings become massive and pull the D-branes back together. We depict this in Figure 3.

This effect can be a significant correction to the dynamics of any system with a number of mobile, mutually BPS D-branes. Consider, for example, N D3-branes which fill spacetime and are transverse to a compact six-manifold M. Let us take these branes to begin with small, random, classical velocities in M. The classical

[8] For simplicity we ignore the superpartner of the χ boson.

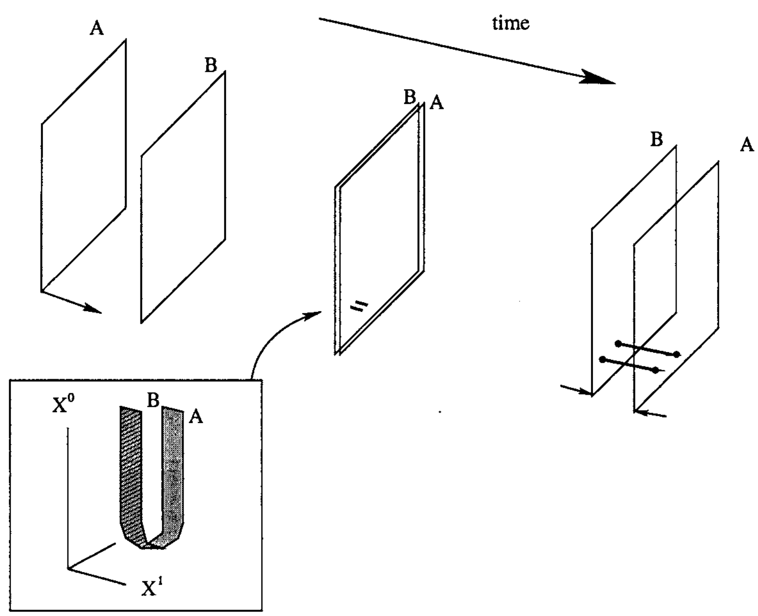

Fig. 3: This figure illustrates the creation of open strings as two D-branes pass near each other. The left corner shows the target space picture of the creation of the open strings.

dynamics of this system is similar to that of a nonrelativistic, noninteracting, classical gas. When we include quantum production of light strings, the branes begin to trap each other, pairwise or in small groups, then gradually agglomerate until only a few massive clumps of many branes remain.

One interesting consequence is that such a system will tend to exhibit enhanced gauge symmetry, with gauge group $U(N)$ if the final state consists of a single clump. (Hubble friction may bring the branes to rest before the aggregation is complete, in which case the gauge group will be a product of smaller factors; we will address related issues in §3.4.1.) Another important effect of massive clumps is their gravitational backreaction: a large cluster of D-branes will produce a warped throat region in M, which may be of phenomenological interest [92].

There are additional corrections to the classical moduli space approximation of the D-brane motion which come from velocity-dependent forces. These correspond in the D-brane worldvolume field theory to higher-derivative corrections generated by virtual effects. When this field theory is at weak 't Hooft coupling, open string production is the dominant effect as one approaches an ESP. However, sufficiently large clusters of branes will be described by gauge theories at strong 't Hooft coupling, where the dynamics of additional probe branes is governed instead by the analysis of [70].[9]

A similar interaction was studied in the context of the scattering of D0-branes in [86]. There is a crucial difference between that system and the case of interest here, in which the branes are extended along $3+1$ dimensions. In the D0-brane problem, there is a nontrivial probability for the D0-branes to pass by each other without getting trapped: because the D0-brane is pointlike, there is some probability for no open strings between them to be created or for those created to annihilate rapidly. This is the leading contribution to the S-matrix. In our case, there is always a nonzero number *density* of particles created. As we argue in Appendix 3.B, for certain ranges of parameters these particles do not annihilate rapidly enough to prevent trapping.

3.3 Moduli Trapping: Detailed Analysis

In the previous section we gave an intuitive explanation of the trapping effect, which we will now describe in more detail. In §3.3.1 we will present the equations of motion which govern the trajectory of the modulus ϕ, including the backreaction due to production of light particles. These equations are difficult to solve exactly, so in §3.3.2 we will integrate the system numerically. In §3.3.3 we focus on the special case $\mu = 0$, where the modulus rolls directly through the ESP. In this case analytic techniques are available, and as we will see the trapping effect is considerably stronger than in the $\mu \neq 0$ case.

3.3.1 Formal Description of Particle Production Near an ESP

The full equations of motion are found by coupling the classical motion of ϕ to the time-dependent χ quantum field theory defined by (3.2.1).[10]

In general, the presence of an ESP will alter the moduli dynamics in two ways. First, any χ excitations produced by the mechanism described above will backreact on the classical evolution of ϕ. In particular, as we saw in (3.2.10), a non-zero

[9] A further correction to our dynamics could arise if, as we will discuss in §3.5, the branes keep moving until the system is beyond the range of effective field theory.

[10] We remain in flat space quantum field theory, reserving gravitational effects for §3.4.

expectation value $\langle\chi^2\rangle \neq 0$ arising from particle production effectively acts like a linear potential for ϕ and drives the moduli towards the origin. This is the effect we wish to describe. Second, virtual χ particles generate quadratic and higher-derivative contributions to the effective action as well as an effective potential for a spacetime-homogeneous ϕ.

As we discussed in §3.2.1, we can neglect the kinetic corrections in our weakly-coupled situation. The interaction in (3.2.1) also induces important radiative corrections to the effective potential. Specifically, it leads to a Coleman-Weinberg effective potential and three UV-divergent terms:

$$V_{eff}(\phi) = \Lambda_{eff} + g^2 m_{eff}^2 \phi^2 + g^4 \lambda_{eff} \phi^4. \tag{3.3.1}$$

These UV divergences could be subtracted by hand using appropriate counterterms. In a supersymmetric system these divergences are absent.

In order to isolate the effects of particle production at the order we are working, we will subtract by hand the entire Coleman-Weinberg effective potential for ϕ that is generated by one loop of χ particles. This mimics the effect of including extended supersymmetry, which is a toy case of interest in string theory and supergravity. For the more realistic $\mathcal{N} = 1$ supersymmetry in four dimensions, radiative corrections do generically generate a nontrivial potential energy. Nevertheless, particle production effects can still dominate the virtual corrections to the potential after spontaneous supersymmetry breaking. The reason is that bosons and fermions contribute with opposite signs in loops, but on-shell bosons and fermions, such as those produced by the changing mass of χ, contribute with the same sign to backreaction on ϕ.

To describe the production of χ particles, we first expand the quantum field χ in terms of Fock space operators as

$$\chi = \sum_k a_k \chi_k + a_k^\dagger \chi_k^* \tag{3.3.2}$$

where the χ_k are a complete set of positive-frequency solutions to the Klein-Gordon equation with mass

$$m_\chi^2(t) = g^2 |\phi(t)|^2. \tag{3.3.3}$$

Expanding in plane waves

$$\chi_k = u_k(t) e^{ik\cdot x} \tag{3.3.4}$$

the equation of motion is

$$\left(\partial_t^2 + k^2 + g^2|\phi(t)|^2\right)u_k = 0. \tag{3.3.5}$$

The modes (3.3.4) are normalized with respect to the Klein-Gordon inner product, which fixes

$$u_k^* \dot{u}_k - \dot{u}_k^* u_k = -i. \tag{3.3.6}$$

The wave equation (3.3.5) has two linearly-independent solutions for each k, so in general there will be many inequivalent choices of positive-frequency modes χ_k. Each such choice of mode decomposition defines a set of Fock space operators via (3.3.2), which in turn define a vacuum state of the theory. The wave equation depends explicitly on time, so there is no canonical choice of Poincaré invariant vacuum. Instead, there is a large family of inequivalent vacua for χ.

We can choose a set of positive frequency modes u_k^{in} that take a particularly simple form in the far past,

$$u_k^{in} \rightarrow \frac{1}{\sqrt{2\sqrt{k^2+g^2|\phi|^2}}} e^{-i\int^t \sqrt{k^2+g^2|\phi(t')|^2}dt'} \qquad \text{as } t \rightarrow -\infty. \tag{3.3.7}$$

This choice of mode decomposition defines a vacuum state $|in\rangle$. In the far past the phases of the solutions (3.3.7) are monotone decreasing with t, indicating that the state $|in\rangle$ has no particles in the far past. This state, known as the adiabatic vacuum, evolves into a highly excited state as the modulus ϕ rolls past the ESP.

We can now write down the classical equation of motion for ϕ including the effects of χ production. Including a subtraction δ_M, to be determined shortly, it is

$$\left(\partial^2 + g^2(\langle\chi^2\rangle - \delta_M)\right)\phi = 0. \tag{3.3.8}$$

The expectation value $\langle\chi^2\rangle$ depends on time and is calculated in the adiabatic vacuum $|in\rangle$. At time t

$$\langle in|\chi^2(t)|in\rangle = \int \frac{d^3k}{(2\pi)^3}|u_k^{in}(t)|^2. \tag{3.3.9}$$

where the u_k^{in} are determined by the boundary condition (3.3.7) in the far past.

In order to subtract the Coleman-Weinberg potential, we must remove the contribution to $\langle\chi^2\rangle$ coming from one loop of χ particles, replacing the χ mass-squared with $g^2|\phi(t)|^2$. That is, the subtraction δ_M can be written as

$$\delta_M \equiv \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{k^2+g^2|\phi|^2}}. \tag{3.3.10}$$

With this form it is straightforward to see that when the impact parameter is very large, $(\langle\chi^2\rangle - \delta_M)$ is negligible and ϕ follows its original trajectory (3.2.2).

To summarize, the effects of quantum production of χ particles on the classical motion of the modulus ϕ are governed by:

$$\begin{aligned} \left(\partial^2 + g^2(\langle\chi^2\rangle - \delta_M)\right)\phi &= 0 \\ \left(\partial_t^2 + k^2 + g^2|\phi(t)|^2\right)u_k^{in} &= 0 \\ \langle\chi^2(t)\rangle &= \int \frac{d^3k}{(2\pi)^3}|u_k^{in}(t)|^2. \end{aligned} \tag{3.3.11}$$

The above equations of motion can be reformulated in terms of the energy transferred between the two systems. In particular, it is straightforward to show that the coupled equations (3.3.11) are equivalent to the statement

$$\frac{d}{dt}H_\phi = -\frac{d}{dt}\langle in|H_\chi|in\rangle. \tag{3.3.12}$$

The left-hand side of (3.3.12) involves the classical energy of the rolling $\phi(t)$ fields, whereas the right hand side is an expectation value of the time-dependent χ Hamiltonian calculated in quantum field theory. This is the more precise form of energy conservation which applies to our rough estimate in §3.2.2.

Furthermore, the angular momentum on moduli space is conserved, since the action (3.2.1) is invariant under phase rotations $\phi \to \phi e^{i\theta}$. In the present case (3.2.1), the χ particles do not carry angular momentum, so the orbit of ϕ around the ESP will have fixed angular momentum. The result is an angular momentum barrier which keeps the modulus at a finite distance from the ESP.

More complicated scenarios allow for the exchange of angular momentum between ϕ and χ. This includes the case of colliding D-branes, where the strings stretching between the two D-branes can carry angular momentum. Moreover, as we will see in §3.4, the situation changes once gravitational effects are included, as angular momentum is redshifted away by cosmological expansion. This leads to scenarios where the moduli are trapped exactly at the ESP, rather than orbiting around it at some finite distance.

3.3.2 Moduli Trapping: Numerical Results

The coupled set of integral and differential equations (3.3.11) governing the trapping trajectory is hard to solve in general. Some analytic results can be obtained through an expansion in the non-adiabaticity parameter $\dot{\omega}/\omega^2$, combined with a systematic iteration procedure. However, as time goes on, the mass amplification of the χ particles makes higher-order terms as well as non-perturbative terms in the adiabatic expansion crucial for the motion of the moduli. This makes it very hard to proceed analytically to obtain the detailed evolution of the system.

We have therefore numerically integrated the coupled equations (3.3.11) in Mathematica, using a discrete sum to approximate the momentum integral k, and implementing the subtraction of the Coleman-Weinberg potential described above.

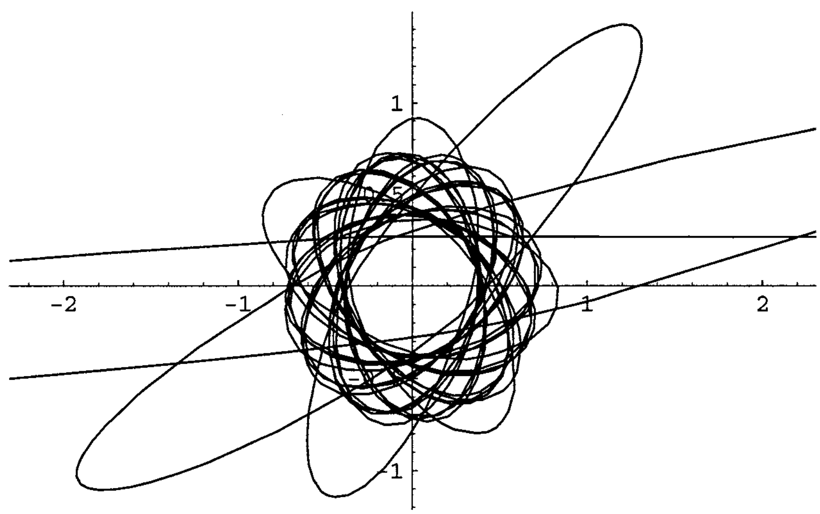

Fig. 4: This figure shows the evolution, in the complex ϕ plane, of a system with parameters $g^2 = 20, \mu = 0.3, v = 1$. The field rolls in from the right and gets trapped into the precessing orbit exhibited in the plot. The orbit is initially an elongated ellipse, but gradually becomes more circular. In an expanding universe, the field would lose its angular momentum, so that the radius of the circle would eventually shrink to zero.

In Figure 4 we plot a trajectory for the case $\mu > 0$, where ϕ becomes trapped in a spiral orbit around the ESP. The radius of the orbit varies with the parameters, but the qualitative features shown are typical.

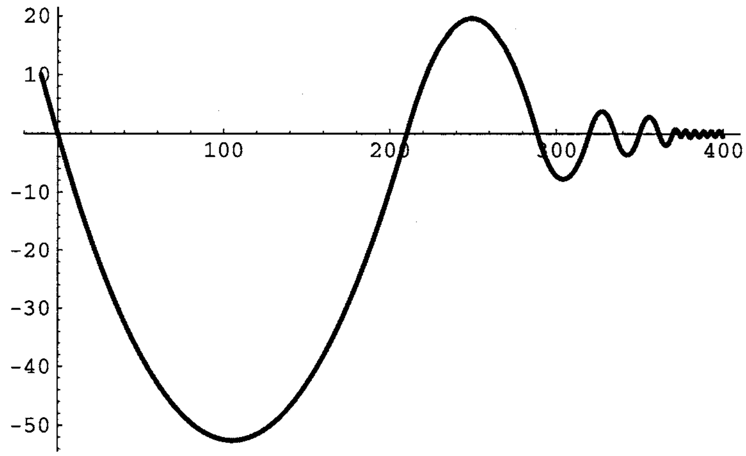

Fig. 5: This shows one-dimensional trapping, in which ϕ passes directly through the ESP $\phi = 0$. The vertical axis is the real part of ϕ, and the horizontal axis is time. The amplitude of the oscillations decreases exponentially as a result of parametric resonance, as we explain in §3.3.3.

In Figure 5 we plot the trajectory of a modulus which is aimed to pass directly through an ESP, with vanishing impact parameter. In this case the motion becomes effectively one-dimensional, and the field moves directly through the ESP $\phi = 0$. The trapping effect in this case is especially strong, and can be understood analytically to come from resonant production of χ particles, as we will now explain.

3.3.3 The Special Case of One-Dimensional Motion

In this section we will concentrate on the interesting and important special case of one-dimensional motion, i.e. vanishing impact parameter μ. Perhaps surprisingly, this is a good approximation to the general case. Indeed, the results of §3.2 demonstrate that trapping becomes exponentially suppressed when the impact parameter μ (the imaginary part of the moduli field) becomes greater than $\sqrt{\frac{v}{\pi g}}$. On the other hand, for $\mu \ll \sqrt{\frac{v}{\pi g}}$ the motion of the field ϕ stops at $\phi_* \sim \frac{4\pi^3 v^{1/2}}{g^{5/2}}$. The ratio of ϕ_* to μ in the regime where trapping is efficient (i.e. for $\mu < \sqrt{\frac{v}{\pi g}}$) is therefore

$$\frac{\phi_*}{\mu} > \frac{4\pi^{7/2}}{g^2} \, . \tag{3.3.13}$$

Thus, in the case of efficient trapping and weak coupling, the ellipticity of the moduli orbit is very high, so that the motion is effectively one-dimensional.

In the case $\mu = 0$ the number density of χ particles created when the field ϕ passes the ESP is

$$n_\chi = \frac{(gv)^{\frac{3}{2}}}{(2\pi)^3} \ . \tag{3.3.14}$$

At $|\phi| \gg \sqrt{\frac{v}{g}}$, when the χ particles are nonrelativistic, the mass of each particle is equal to $g|\phi|$, and their energy density is given by [67]

$$\rho_\chi(\phi) = g n_\chi |\phi| = \frac{g^{\frac{5}{2}} v^{3/2}}{(2\pi)^3} \ |\phi| \tag{3.3.15}$$

We have written $|\phi|$ because this energy does not depend on the sign of the field ϕ. This will be very important for us in what follows.

One should note that, strictly speaking, the χ particles have some kinetic energy even at $\phi = 0$, but for $g \ll 1$ this energy is much smaller than the kinetic energy of ϕ [67]:

$$\rho_\chi(\phi = 0) \sim \frac{g^2}{4\pi^{7/2}} \ \frac{v^2}{2} = \frac{g^2}{4\pi^{7/2}} \ \rho_\phi^{\rm kin}. \tag{3.3.16}$$

This means that the energy of ϕ decreases only slightly when it passes through the ESP $\phi = 0$. Although the initial energy in χ particles is small, this energy increases with $|\phi|$, $\rho_\chi \sim g n_\chi |\phi|$, and creates an effective potential for ϕ. The equation of motion for ϕ in this potential is [66]:

$$\ddot{\phi} + g n_\chi \frac{\phi}{|\phi|} = 0 \ . \tag{3.3.17}$$

The last term means that ϕ is attracted to the ESP $\phi = 0$ with a constant force proportional to n_χ.

At some location ϕ_1^* the χ energy density ρ_χ equals the initial kinetic energy density $\frac{1}{2}\dot{\phi}^2 \equiv \frac{1}{2}v^2$; at this point ϕ stops and then falls back toward $\phi = 0$.

On this second pass by the origin, the energy density of the χ particles again becomes much smaller than the kinetic energy of ϕ. Energy conservation implies that ϕ will pass the point $\phi = 0$ at almost exactly the initial velocity v. Since the conditions are almost the same as on the first pass, new χ particles will be created, i.e. n_χ will increase. The field ϕ will continue moving for a while, stop at some point ϕ_2^*, and then fall back once more to the ESP, creating more particles. Because each new collection of particles is created in the presence of previous generations of particles, the process occurs in the regime of parametric resonance, as in the theory of preheating.

A detailed theory of this process was considered in [66]; see in particular Eqs. (59),(60). By translating the problem into a one-dimensional quantum mechanics system (as in Appendix 3.A) with a particle scattering repeatedly across an inverted harmonic potential, [66] calculated the multiplicative increase of the Bogoliubov coefficients during each pass in terms of the reflection and transition amplitudes. In application to our problem, the equations describing the occupation numbers of χ particles with momentum k produced when the field passes through the ESP $j+1$ times look as follows:

$$n_k^{j+1} = n_k^j \exp(2\pi \mu_k^j), \tag{3.3.18}$$

where

$$\mu_k^j = \frac{1}{2\pi} \ln \left(1 + 2e^{-\pi\xi^2} - 2\sin\theta^j \; e^{-\frac{\pi}{2}\xi^2} \; \sqrt{1+e^{-\pi\xi^2}}\right). \tag{3.3.19}$$

Here $\xi^2 = \frac{k^2}{gv}$ and θ^j is a relative phase variable which takes values from 0 to 2π. In a cyclic particle creation process in which the parameters of the system change considerably during each oscillation (which is our case, as will become clear shortly), the phases θ^j change almost randomly. As a result, the coefficient μ_j for small k takes different values, from 0.28 to -0.28, but for 3/4 of all values of the angle θ^j the coefficient μ_j is positive. The average value of μ_j is approximately equal to 0.15. This means that, on average, the number density of χ particles grows by approximately a factor of two or three each time that ϕ passes through the ESP $\phi = 0$.

But this means that with each pass, the coefficient n_χ in (3.3.15) grows by a factor of two or three. It follows that the effective potential becomes two to three times more steep with each pass. Correspondingly, the maximal deviation $|\phi_i^*|$ from the point $\phi = 0$ exponentially decreases with each new oscillation. Since the velocity of the field at the point $\phi = 0$ remains almost unchanged until ϕ loses its energy to the created particles, the duration of each oscillation decreases exponentially as well. Therefore the whole process takes a time $\mathcal{O}(10)\phi_1^*/v$, after which the backreaction of the created particles becomes important, and the field falls to the ESP.

This process is very similar to the last stages of preheating, as studied in [66]. The main difference is that in the simplest models of preheating the field oscillates near the minimum of its classical potential. In our case the effective potential is initially absent, but a potential is generated due to the created particles. This is exactly what happens at the late stages of preheating, when the effective potential (with an account taken of the produced particles) becomes dominated by the rapidly-growing term proportional to $|\phi|$; see the discussion in Section VIII B of [66].

We would like to emphasize that until the very last stages of the process, the backreaction of the created particles can be studied by the simple methods described above. At this stage the total number of created particles is still very small, but their number grows exponentially with each new oscillation. This leads to an exponentially rapid increase of the steepness of the potential energy of the field ϕ (3.3.15) and, correspondingly, to an exponentially rapid decrease of the amplitude of its oscillations. This extremely fast trapping of ϕ happens despite the fact that at this first stage of oscillations the total energy of ϕ, including its potential energy, remains almost constant.

Once the amplitude of oscillations becomes smaller than the width of the nonadiabaticity region, $|\phi(t)| \lesssim \Delta\phi \sim \sqrt{v/g}$, one can no longer assume that the number of particles will continue to grow via a rapidly-developing parametric resonance. The amplitude of the oscillations is given by $\frac{v^2}{2g|\phi|n_\chi}$, so the amplitude becomes $\mathcal{O}(\sqrt{v/g})$ when the total number of the produced particles grows to

$$n_\chi \sim v^{3/2} g^{-1/2} \, . \tag{3.3.20}$$

Note that the typical energy of each χ particle at $|\phi(t)| \sim \sqrt{v/g}$ is of the same order as its kinetic energy $\mathcal{O}(\sqrt{gv})$. One can easily see that the total energy density of particles χ at that stage is roughly $\sqrt{gv}n_\chi \sim \mathcal{O}(v^2)$, i.e. it is comparable to the initial kinetic energy of ϕ.

Thus, our estimates indicate that the regime of the broad parametric resonance ends when a substantial part of the initial kinetic energy of ϕ is converted to the energy of the χ particles, and the amplitude of the oscillating field ϕ becomes comparable to the width of the nonadiabaticity region,

$$|\phi| \sim \Delta\phi = \sqrt{v/g} \, . \tag{3.3.21}$$

We will use these estimates in our discussion of the cosmological consequences of moduli trapping. In order to obtain a more complete and reliable description of the last stages of this process one should use lattice simulations, taking into account the rescattering of created particles [90,91]. An investigation of a similar situation in the theory of preheating has shown that rescattering makes the process of particle production more efficient. This speeds up the last stages of particle production and leads to a rapid decay of the field ϕ [93], which in our case corresponds to a rapid descent of ϕ toward the enhanced symmetry point.

3.4 Trapped Moduli in an Expanding Universe

3.4.1 Rapid Trapping

In this section we will study the conditions under which the trapping mechanism in quantum field theory survives the effects of coupling to gravity in an expanding universe.

First, we should point out one very beneficial effect of cosmological expansion. The field-theoretic mechanism presented above often leads to moduli being trapped in large-amplitude fluctuations (3.2.12) around an ESP when $\mu \neq 0$. On timescales where the expansion is noticeable, Hubble friction will naturally extract the energy from this motion, drawing the modulus inward and leading the modulus to come to rest at the ESP.

Let us now ask whether the expansion of the universe can impede moduli trapping. Consider a system of moduli coupled to gravity, with the fields arranged to roll near an ESP. For simplicity we will consider FRW solutions with flat spatial slices,

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2. \tag{3.4.1}$$

The Friedman equation determining $a(t)$ is

$$3H^2 = \frac{1}{M_p^2}\rho \tag{3.4.2}$$

where $H = \dot{a}/a$ and ρ is the energy density of the moduli.

The trapping effect will be robust against cosmological expansion if the timescale governing trapping is short compared to H^{-1}, i.e. if $H \ll v/\phi_*$, where ϕ_* is given by (3.2.12). Assuming that the potential energy of the moduli is non-negative, this implies that

$$\phi_* \ll \sqrt{6} M_p \rightarrow \frac{4\pi^3}{6^{1/2} g^{5/2}} \frac{v^{1/2}}{M_p} e^{\pi g \mu^2 / v} \ll 1. \tag{3.4.3}$$

This condition suffices to ensure that trapping is very rapid.

If this condition is satisfied, trapping occurs in much less than a Hubble time, in which case the analysis of §3.2 and §3.3 remains valid. We will show in §3.4.3 that even when (3.4.3) is not satisfied, trapping does still occur, although with somewhat different dynamics.

3.4.2 Scanning Range in an Expanding Universe

An important effect of the gravitational coupling is that during the expansion of the universe, the energy density in produced χ particles dilutes like $1/a^3$ if they are non-relativistic and like $1/a^4$ if they are relativistic. The energy in coherent motion of ϕ, however, has the equation of state $p = \rho$ and therefore dilutes much faster, as $1/a^6$.

This effect reduces the range of motion for the moduli even before they encounter any ESPs. Hubble friction slows the progress of any rolling scalar field, and if the distance between ESPs is sufficiently large then a typical rolling modulus will come to rest without ever passing near an ESP. In order to apply our results to the vacuum selection problem, we will need to know how large a range of ϕ we can scan over in the presence of Hubble friction. This can be obtained as follows [94].

If we are in an FRW phase,

$$a(t) = a_0 t^{\beta} \tag{3.4.4}$$

then the equation of motion for ϕ (ignoring any potential terms)

$$\ddot{\phi} + 3H\dot{\phi} = 0 \tag{3.4.5}$$

has solutions of the form

$$\dot{\phi}(t) = v \left(\frac{t_0}{t} \right)^{3\beta}. \tag{3.4.6}$$

We can integrate this to determine how far the field rolls before stopping.

Let us first consider the case $\beta = 1/3$, which corresponds to the equation of state $p = \rho$. This includes the case where the coherent, classical kinetic energy of ϕ drives the expansion. The value of ϕ,

$$\phi(t) = v t_0 \log \left(\frac{t}{t_0} \right) \tag{3.4.7}$$

diverges at large t. Thus ϕ can travel an arbitrarily large distance in moduli space.

In the more general case $\beta > 1/3$ the field will travel a distance

$$\phi(t) - \phi(t_0) = \frac{v}{H(t_0)} \frac{\beta}{3\beta - 1} \tag{3.4.8}$$

before stopping.

In order to be in a phase with $\beta > 1/3$, the kinetic energy of ϕ must not be totally dominant; that is, we must have $\frac{1}{2}\dot{\phi}^2 < \rho$, where $\rho \equiv 3M_p^2 H^2$ is the total energy density appearing on the right hand side of the Friedman equation. Plugging this into (3.4.8) we obtain the constraint

$$\phi(t) - \phi(t_0) < \sqrt{6} M_p \frac{\beta}{3\beta - 1}. \tag{3.4.9}$$

Let us consider a specific example. Suppose that we start at t_0 with kinetic energy domination: $K_0/\rho_0 = 1 - \epsilon$, $\epsilon \ll 1$, in some region of the universe that can be modelled as an expanding FRW cosmology. The kinetic energy drops like $K \sim \rho_0 (a_0/a)^6 \sim \rho_0 (t_0/t)^2$, while the other components of the energy dilute like

$$\rho(t) = \epsilon \rho_0 (t_0/t)^{1+w}, \tag{3.4.10}$$

with $w < 1$. The universe will stop being kinetic-energy dominated at the time $t_c = t_0 \epsilon^{-1/(1-w)}$, at which point, according to (3.4.7), the modulus has travelled a distance

$$\phi(t_c) - \phi(t_0) = -\frac{1}{1-w} v t_0 \log \epsilon. \tag{3.4.11}$$

After this the field keeps moving and covers an additional range

$$\phi(t_*) - \phi(t_c) = \sqrt{3} M_p \frac{2}{3(1-w)}. \tag{3.4.12}$$

To get a feel for the numbers, consider the case where $v t_0 \sim M_p$, $\epsilon \sim 10^{-2}$, and $w = 0$. Then ϕ will travel a total distance $\phi(t_*) - \phi(t_0) \sim 6 M_p$ in field space, which is not particularly far. However, as we will discuss in §3.6, certain moduli spaces of interest have a rich structure on sub-Planckian scales, so in these cases there is a good chance that the modulus will encounter an ESP and get trapped before Hubble friction brings the system to rest.

There is another natural possibility if we assume low-energy $\mathcal{N} = 1$ supersymmetry. If the moduli acquire their potentials from supersymmetry breaking then there is a large ratio between the Planck scale and the scale of these potentials, leading to significant scanning ranges. Specifically, consider a contribution to the energy density coming from a potential energy V at the supersymmetry-breaking scale. If the initial kinetic energy of the moduli is Planckian and the supersymmetry-breaking scale is TeV then there will be a prolonged phase in which kinetic energy dominates, since $\epsilon = V/M_p^4 \sim 10^{-64}$. This allows ϕ to scan a significantly super-Planckian range in field space.

3.4.3 Trapping in an Expanding Universe

We are now in a position to combine all the relevant effects and consider trapping during expansion of the universe. For simplicity, we will concentrate on the case of effectively one-dimensional motion, $\mu \ll \sqrt{v/g}$. Suppose that, taking into account Hubble friction, the modulus field passes in the vicinity of the ESP at some moment t_0, so that χ particles are produced, with $n_\chi(t_0) = \frac{(gv)^{3/2}}{(2\pi)^3}$. We will now determine the remaining evolution including both our trapping force and Hubble friction. After the particles have been produced, the field ϕ becomes attracted toward $\phi = 0$ by a force gn_χ, so taking into account the dilution of the produced particles, for $\phi > 0$ the equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} = -gn_\chi(t_0)\left(\frac{a(t_0)}{a(t)}\right)^3 \qquad (3.4.13)$$

For the general power law case, $a(t) \propto t^\beta$, this becomes

$$\ddot{\phi} + 3\frac{\beta}{t}\dot{\phi} = -gn_\chi(t_0)(t_0/t)^{3\beta} \; . \qquad (3.4.14)$$

The general solution of this equation is

$$\phi(t) = \phi(t_0) + c(t_0^{-3\beta+1} - t^{-3\beta+1}) + \frac{gn_\chi(t_0)t_0^2}{(2-3\beta)} - \frac{gn_\chi(t_0)}{(2-3\beta)}(t_0/t)^{3\beta}t^2 \qquad (3.4.15)$$

where c is some constant. In the important case $\beta = 2/3$, which corresponds to a universe dominated by pressureless cold matter, the general solution is

$$\phi = \phi(0) + c(t_0^{-1} - t^{-1}) - gn_\chi t_0^2 \log\frac{t}{t_0} \; . \qquad (3.4.16)$$

According to these solutions, in a universe dominated by matter with non-negative pressure (i.e. $\beta \leq 2/3$) the field ϕ moves to $-\infty$ as $t \rightarrow \infty$.

Of course, as soon as the field reaches the point $\phi = 0$, this solution is no longer applicable, since the attractive force changes its sign (the potential is proportional to $|\phi|$). The result above simply means that the attractive force is always strong enough to bring the field back to the point $\phi = 0$ within finite time. Then the field moves further, with ever decreasing speed, turns back again, and returns to $\phi = 0$ once again. The amplitude of each oscillation rapidly decreases due to the combined effect of the Hubble friction and of the (weak) parametric resonance. This means that once ϕ passes near the ESP, its fate is sealed: eventually it will be trapped there.

3.4.4 Efficiency of Trapping

It is useful to determine what fraction of all initial conditions for the moving moduli lead to trapping. There are several constraints to be satisfied. First of all, if the impact parameter μ is much larger than $\sqrt{v/g}$, the number of produced particles will be exponentially small, and the efficiency of trapping will be exponentially suppressed. Of course, eventually ϕ will fall to the enhanced symmetry point, but if this process takes an exponentially large time, the trapping effect will be of no practical significance. Thus one can roughly estimate the range of interesting impact parameters to be $\mathcal{O}(\sqrt{v/g})$.

Another constraint is related to the fact that even if initially the energy density of the universe was dominated by the moving moduli, as discussed in §3.4.2, these fields can only move the distance given by (3.4.11),(3.4.12). This distance depends on the initial ratio $1-\epsilon$ of kinetic energy to total energy, leading to a scanning range CM_p in field space, where the prefactor C is logarithmically related to ϵ.

Thus, the field becomes trapped only if there is an enhanced symmetry point inside a rectangle with sides of length CM_p along the direction of motion and width $\mathcal{O}(\sqrt{v/g})$ in the direction perpendicular to the motion.

Interestingly, the total area (phase space) of the moduli trap

$$S_{\rm trap} \sim CM_p\sqrt{\frac{v}{g}} \tag{3.4.17}$$

increases as the coupling decreases. This implies that the efficiency of trapping grows at weak coupling. Although this may seem paradoxical, it happens because the mass of the χ particles is proportional to the coupling constant and (fixing the other parameters) it is easier to produce lighter particles. On the other hand, if g becomes too small, the trapping force $gn_\chi \sim g^{5/2}v^{3/2}$ becomes smaller than the usual forces due to the effective potential, which we assumed subdominant in our investigation.

So far we have studied the simplest model where only one scalar field becomes massless at the enhanced symmetry point. Let us suppose, however, that N fields become massless at the point $\phi = 0$. If these fields interact with ϕ with the same coupling constant g, then particles of each of these fields are produced, and the trapping force becomes N times stronger. In other words, the trapping force is proportional to the degree of symmetry at the ESP.

3.5 String Theory Effects

It is interesting to ask if there is any controlled situation where string-theoretic effects become important for moduli trapping. Here we will simply list several circumstances in which stringy and/or quantum gravity effects might come into play, as well as some constraints on these effects. In Chapter 4 we will revisit this subtle and interesting situation.

3.5.1 Large χ Mass

One way stringy and quantum gravity effects could become important in the colliding D-brane case is if the χ mass at the turnaround point is greater than string scale, $g\phi_* > m_s$. This can happen even if the velocity is so small that during the non-adiabatic period near the origin only unexcited stretched strings are created. Then, as in our above field theory analysis, we have

$$g\phi_* = \frac{4\pi^3}{g^{3/2}} v^{1/2} e^{\pi g \mu^2 / v}. \tag{3.5.1}$$

In this case, the full system includes modes, namely the created χ strings, which are heavier than the string oscillator mode excitations on the individual branes. This means that the system as a whole cannot consistently be captured by pure effective field theory. However, it may still happen that the created stretched strings are relatively stable against annihilation or decay into the lighter stringy modes. Their annihilation cross section is suppressed by their large mass, as discussed in Appendix

3.B.[11] Furthermore, an individual stretched string will not directly decay if it is the lightest particle carrying a conserved charge.

This latter situation happens in the simplest version of a D-brane collision. The created stretched string cannot decay into lighter string or field theory modes because it is charged and they are not.

3.5.2 Large v and the Hagedorn Density of States

If we increase the field velocity $\dot{\phi} = v$, then we may obtain a situation in which excited string states are produced as ϕ passes the ESP. The number of string states produced in this process is enhanced by the Hagedorn density of states, so the Bogoliubov coefficients have the structure

$$|\beta_k|^2 = \sum_n e^{\frac{\sqrt{n}}{2\pi\sqrt{2}}} e^{-\pi(k^2+nm_s^2+g^2\mu^2)/(gv)} \tag{3.5.2}$$

where in the D-brane context, $g = \sqrt{g_s}$ is the Yang-Mills coupling on the D-branes. Because of the $e^{-\pi n m_s^2/(gv)}$ suppression in the second factor, this effect is only significant if $gv \gg m_s^2$.

However, in the case of colliding D-branes, and any situation dual to it, there is a fundamental bound on the field velocity from the relativistic speed limit of the branes. That is, for large velocity one must include the full Dirac-Born-Infeld Lagrangian for ϕ, which takes the form

$$S = -\frac{1}{(2\pi)^3 g_s \alpha'^2} \int d^4x \sqrt{1 - g^2 \frac{\dot{\phi}^2}{m_s^4}}. \tag{3.5.3}$$

This action governs the nontrivial dynamics of ϕ for velocities approaching the string scale, and in particular, it reflects the fact that the brane velocity $g\dot{\phi}\alpha'$ must be less than the speed of light in the ambient space. Applied to our situation, (3.5.3) implies that the D-brane velocity cannot be large enough for the Hagedorn enhancement (3.5.2) to substantially increase the trapping effect.

However, in the presence of a large velocity, the effective mass of the stretched string also has important velocity-dependent contributions [70]. As we will explain in Chapter 4, this will increase the non-adiabaticity near the origin and dramatically enhance the particle production effect.

---

[11] For stringy densities of stretched strings, there could be additional corrections to the annihilation rate, but we will not consider this possibility.

3.5.3 Light Field-Theoretic Strings

A further possibility is to formally reduce the tension of strings by considering strings in warped throats, strings from branes partially wrapped on shrinking cycles, and the like. In these situations, the strings are essentially field-theoretic, though string theory techniques such as AdS/CFT and "geometric engineering" of field theories may provide technical help in analyzing the situation.

3.6 The Vacuum Selection Problem

We can now apply the ideas of the previous five sections to the cosmology of theories with moduli.

A natural application of the moduli trapping effect is to the problem of vacuum selection. One mechanism of vacuum selection is based on the dynamics of light scalars during inflation. Moduli fields experience large quantum fluctuations during inflation and can easily jump from one minimum (or valley) of their effective potential to another. It was suggested long ago that such processes may be responsible, e.g., for the choice of the vacuum state in supersymmetric theories [95] and for the smallness of the cosmological constant [96]. The probability of such processes and the resulting field distribution depends on the details of the inflationary scenario and the structure of the effective potential [97].

The mechanism that we consider in this chapter is, in a certain sense, complementary to the inflationary mechanism discussed above. During inflation the average velocities of the fields are very small, but quantum fluctuations tend to take the light scalar fields away from their equilibrium positions. On the other hand, after inflation, the fields often find themselves not necessarily near the minima of their potentials or in the valleys corresponding to the flat directions, but on a hillside. As they roll down, they often acquire some speed along the valleys, see e.g. [68]. At this stage (as well as in a possible pre-inflationary epoch) the moduli trapping mechanism may operate.

This mechanism may reduce the question of how one vacuum configuration is selected dynamically out of the entire moduli space of vacua to the question of how one ESP is selected out of the set of all ESPs. This residual problem is much simpler because ESPs generically comprise a tiny subset of the moduli space.

3.6.1 Vacuum Selection in Quantum Field Theory

In pure quantum field theory, discussed in §3.2, we saw that if a scalar field ϕ is initially aimed to pass near an ESP, then ϕ gets drawn toward the ESP and is ultimately trapped there. This appears to be a basic phenomenon in time-dependent quantum field theory: moduli which begin in a coherent classical motion typically become trapped at an ESP. This leads to a dynamical preference for ESPs.

In many of the supersymmetric quantum field theories that have been studied rigorously [72], the moduli space contains singular points at which light degrees of freedom emerge. We have seen that moduli can become trapped near these points given suitable initial conditions.

3.6.2 Vacuum Selection in Supergravity and Superstring Cosmology

Compactifications of M/string theory which have a description as a low energy effective supersymmetric field theory can have a natural separation of scales: the string or Planck scale can be much larger than the energy scales in the effective field theory potential. Thus, the intrinsically stringy effects of §3.5 are unimportant in this limit. On the other hand, the effects of coupling to gravity given in §3.4 continue to provide a crucial constraint, as we will now discuss.

First of all, as in the case of pure quantum field theory, there exist very instructive toy models with extended supersymmetry, for which there is no potential at all on the moduli space. For these examples, in situations where higher-derivative corrections to the effective action are suppressed, a rolling scalar field has the equation of state $p = \rho$. This corresponds to the $\beta = 1/3$ case (3.4.7) of §3.4, for which one can scan an arbitrarily large distance in field space. Therefore, in this case, the trapping effect applies in a straightforward way to dynamically select the ESPs for regimes in which (3.4.3) is also satisfied.

More generally, however, one may wish to implement cosmological trapping in theories with some potential energy. In this case the requirement that the scanning range of ϕ (as constrained by Hubble friction in §3.4) should be large enough to cover multiple vacua is an important constraint. The absolute minimum requirement is that the scanning range is sufficient for the moduli to reach one ESP before stopping from Hubble friction; but to address the vacuum selection problem one should ideally scan a number of ESPs.

One context in which this can happen is in a phase in which the kinetic energy of the rolling scalar fields dominates the energy density of the universe so that the $\beta = 1/3$ result (3.4.7) applies. This may occur in a pre-inflationary phase in some patches of spacetime, though it is subject to the stringent limitation in duration given in (3.4.11). Given such a phase, the field will roll around until it gets trapped at an ESP.

During the ordinary radiation-dominated ($\beta = 1/2$) and matter-dominated ($\beta = 2/3$) eras, the more stringent constraint (3.4.9) applies. As we indicated in §3.4, this scanning range is not large in Planck units, so we can usefully apply moduli trapping to the problem of vacuum selection in these eras only if the vacuum has appropriately rich structure on sub-Planckian scales. In other words, the average distance in moduli space between ESPs should be sub-Planckian.

Gravitationally-coupled scalars ϕ generically have a potential energy $V(\phi/M_p)$ which has local minima separated by Planck-scale distances. In this cases, the limited scanning range during the $\beta \neq 1/3$ cosmological eras prevents our mechanism from addressing the vacuum selection problem. However, it is generic for compactification moduli to have special ESPs where the gravitationally-coupled system is enhanced to a system with light field theory degrees of freedom. Given a rich enough effective field theory in this ESP region, there will generically be interesting vacuum structure on sub-Planckian distances. In this sort of region moduli trapping will pick out the ESP vacua of the system.

3.6.3 Properties of the Resulting Vacua

Let us now consider the qualitative features of the vacua selected by moduli trapping, assuming that the constraint imposed by Hubble friction has been evaded in one of the ways described above.

First of all, it is important to recognize that what we have called ESPs may well be subspaces of various dimensions, not points. For example, in toroidal compactification of the heterotic string, there is one enhanced symmetry locus for each circle in the torus – new states appear when the circle is at the self-dual radius. Each of these loci is codimension one in the moduli space, but of course their intersections, where multiple radii are self-dual, have higher codimension.

When moduli trapping acts in such a system of intersecting enhanced symmetry loci, we expect that the moduli will first become trapped on the locus of lowest codimension, but retain some velocity parallel to this locus. Further trapping events can then localize the modulus to subspaces of progressively higher codimension. The final result is that the moduli come to rest on a locus of maximally enhanced symmetry.

The simplest examples of this phenomenon are toroidal compactification, in which all circles end up at the self-dual radius, and the system of N D-branes discussed in §3.2.3, in which the gauge symmetry is enhanced to $U(N)$.[12]

Quite generally, we expect that within the accessible range in field space, taking into account Hubble friction and the form of the potential, moduli trapping will select the ESPs with the largest number of light states, which often corresponds to the highest degree of symmetry.[13]

In some very early epoch the rolling moduli can have large velocities, so trapping can occur even at points where the "light" states χ have a relatively large mass, and the enhanced symmetry is strongly broken. However, Hubble friction inevitably slows the motion of the moduli. Thus, trapping at late times is possible only at ESPs with weakly-broken symmetries and very light particles. One could speculate about a possible relation of this fact to the mass hierarchy problem.

Note that even though we emphasized the natural role of enhanced symmetry in moduli trapping, in fact the only strict requirement was the appearance of new light particles at the trapping points. In some of the many vacua of string theory, particles may be light not because of symmetry but because of some miraculous cancellations. Invoking such unexplained cancellations to produce a small mass is highly undesirable. However, moduli trapping may ameliorate this problem, as those rare points in moduli space where the cancellation does happen are actually dynamical attractors.

---

[12] A toy model for this situation, in the case of three D-branes, has the potential $\frac{g^2}{2}\left[\chi_1^2|\phi_2-\phi_3|^2+\chi_2^2|\phi_1-\phi_3|^2+\chi_3^2|\phi_1-\phi_2|^2\right]$, where ϕ_i and χ_i are six different fields. Suppose that ϕ_2 moves through the point $\phi_2-\phi_3=0$. This creates χ_1 particles and traps the system at $\phi_2=\phi_3$, where χ_1 is massless. Subsequent motion of ϕ_1 can trap it at the point $\phi_1=\phi_2=\phi_3$, making the remaining fields χ_2 and χ_3 massless.

[13] Moreover, as we discuss in Appendix 3.B, the trapping effect is far more effective at ESPs for which the χ particles do not decay rapidly. We therefore expect moduli trapping to select ESPs which have relatively stable light states.

Thus, the attractive power of symmetry and of light particles may have implications for questions involving the distribution of vacua in string theory [3,98,5,99]. Given the strong preference we have seen for highly-enhanced symmetry, the distribution of all string vacua obtained by a naive counting, weighted only by multiplicity, may be quite different from the distribution of vacua produced by the dynamical populating process discussed in this chapter. It is therefore very tempting to speculate that some of the surprising properties of our world, which might seem to be due to pure chance or miraculous cancellations, in fact may result from dynamical evolution and natural selection.

3.7 The Moduli Problem

One aspect of the moduli problem is that reheating and nucleosynthesis can be corrupted by energy locked in oscillations of the moduli. The source of the problem is that the true minima of the low-temperature effective potential applicable after inflation do not coincide with the minima of the Hubble-temperature effective potential which is valid during inflation. It follows that moduli which sit in minima of the latter during inflation will find themselves displaced from their true, low-temperature minima once inflation is complete. The energy stored in this displacement, and in the resulting oscillations about the true minimum, poses problems for nucleosynthesis.

One way to address this problem is to permit initial displacements of the moduli, as described above, but somehow arrange that the oscillating moduli decay very rapidly to Standard Model particles. Alternatively, one could fix the moduli at a scale high enough that the Hubble temperature during inflation does not destabilize them. This may work in string models with stabilized moduli such as [98,100,5,20].

Another approach to this problem [75] is to posit that the moduli sit at an enhanced symmetry point minimum of the finite-temperature effective potential during inflation. Then, when inflation ends, the moduli are still guaranteed to be at an extremum of the effective potential. If this extremum is a minimum then the moduli have no problematic oscillations after inflation. Our trapping mechanism allies nicely with this idea by providing a preinflationary dynamical mechanism which explains the initial condition assumed in this scenario. That is, in parts of the universe where ϕ kinetic energy dominates well before inflation, the trapping effect can explain why the moduli find themselves in ESP minima at the onset of inflation.

3.8 Trapped Inflation and Acceleration of the Universe

The main motivation of our investigation was to study the behavior of moduli in quantum field theory and string theory. However, the results we have obtained have more general applicability. To give an example, in this section we will study the cosmological implications of the trapping of a scalar field ϕ with a relatively steep potential.

Consider the theory of a real scalar field ϕ with the effective potential $m^2\phi^2/2$. In the regime $\phi < M_p$ the curvature of the effective potential is greater than H^2, with H the Hubble parameter, so ϕ falls rapidly to its minimum, and inflation does not normally occur.

We will assume that ϕ gives some bosons χ a mass $g|\phi - \phi_1|$. Let us assume that ϕ falls from its initial value $\phi_0 = \phi_1(1+\alpha) < M_p$ with vanishing initial speed. If we take $\alpha \ll 1$ and neglect for the moment the expansion of the universe, then ϕ arrives at ϕ_1 with the velocity $v = \sqrt{2\alpha}\, m\phi_1$.

As ϕ passes ϕ_1, it creates χ particles with number density $n_\chi = (gv)^{3/2}/8\pi^3$. After a very short time these particles become nonrelativistic, and further motion of ϕ away from ϕ_1 requires an energy $g|\phi - \phi_1|n_\chi$. In other words, the effective potential becomes

$$V(\phi) \approx \frac{1}{2}m^2\phi^2 + gn_\chi|\phi - \phi_1| = \frac{1}{2}m^2\phi^2 + g^{5/2}\frac{v^{3/2}}{8\pi^3}|\phi - \phi_1|. \tag{3.8.1}$$

For $g^{5/2}\frac{v^{3/2}}{8\pi^3 m^2} > \phi_1$, the minimum of the effective potential is not at $\phi = 0$, but at the point ϕ_1, where the particle production takes place. The condition $g^{5/2}\frac{v^{3/2}}{8\pi^3 m^2} > \phi_1$ implies that

$$m < 2^{-9/2}\pi^{-6}g^5\alpha^{3/2}\phi_1. \tag{3.8.2}$$

Thus, if the mass of ϕ is sufficiently small, the field will be trapped near the point ϕ_1.

To give a particular example, take $\phi_1 \sim M_p/2$, $\alpha \sim 1/4$. Then ϕ is trapped near ϕ_1 if

$$m < 10^{-6}g^5 M_p. \tag{3.8.3}$$

For a very light field, such as a modulus with $m \sim 10^2$ GeV $\sim 10^{-16}M_p$, this condition is readily satisfied unless g is very small.

Once the field is trapped, it starts oscillating around ϕ_1 with ever-decreasing amplitude, creating new χ particles in the regime of parametric resonance. Eventually ϕ transfers a large fraction of its energy to χ particles. One can easily check that in this model the fall of ϕ to the point ϕ_1 and the subsequent process of creation of χ particles occurs within a time smaller than H^{-1}, so one can neglect expansion of the universe at this stage. This process is therefore governed by the theory described in §3.3. In particular, we may use the estimate (3.3.20) of the total number of χ particles produced in the process. At the end of the particle production, the correction to the effective potential becomes much larger than at the beginning of the process:

$$\Delta V = g|\phi - \phi_1| n_\chi \sim v^{3/2} g^{1/2} |\phi - \phi_1| \ , \tag{3.8.4}$$

Subsequent expansion of the universe dilutes the density of χ particles as a^{-3}, which eventually makes the correction to the effective potential small, so that ϕ starts moving down again. The field ϕ remains trapped at $\phi = \phi_1$ until the scale factor of the universe grows by a factor

$$a \sim \alpha^{1/4} \left(\frac{g\phi_1}{m} \right)^{1/6} \tag{3.8.5}$$

since the beginning of the trapping process.

In the beginning of the first e-folding, the kinetic energy of the χ particles and of the oscillations of ϕ is comparable to the potential energy of ϕ. However, the kinetic energy rapidly decreases, and during the remaining time the energy is dominated by the potential energy $V(\phi_1)$. This means that the trapping of ϕ may lead to a stage of inflation or acceleration of the universe, even if the original potential $V(\phi)$ is too steep to support inflation.

Let us consider various possibilities for the scales in the potential, to get some simple numerical estimates for the duration of inflation. For example, if we take $\alpha, g = \mathcal{O}(1)$, $\phi_1 \sim M_p$ and $m \sim 10^2$ GeV, then the scale factor during a single trapping event will grow by a factor of e^6. If one considers a model with $m \sim 10^{-30} M_p$, which can arise in a radiatively stable manner (as in the "new old inflation" model [81]), the scale factor during a single trapping event can grow by a factor of e^{11}. Finally, if the moduli mass is of the same order as a typical mass taken in theories of quintessence, $m \sim 10^{-60} M_p$, we can have an accelerated expansion of the universe by a factor e^{23}, in a sub-Planckian regime of field space, just from trapping. (In this last case, as in ordinary quintessence models, tuning is required.)

Thus, the stage of inflation in this simple model is shorter than the usual 60 e-folds, but it may nevertheless be very useful for initiating a first stage of inflation in theories where this would otherwise be impossible, or for diluting unwanted relics at the later stages of the evolution of the universe. Moreover, this scenario can easily describe the present stage of acceleration of the universe.

One can also make the effect more substantial by constructing a more complicated scenario, consisting of a chain of N particle production events at locations $\phi = \phi_i$, where some fields χ_i become light. The field ϕ may be trapped and enter the stage of parametric resonance near each of these points. Correspondingly, the universe enters the stage of inflation many times. One could arrange for 60 e-folds of inflation by taking, for example, $m \sim 10^2$ GeV, $N \sim 10$.

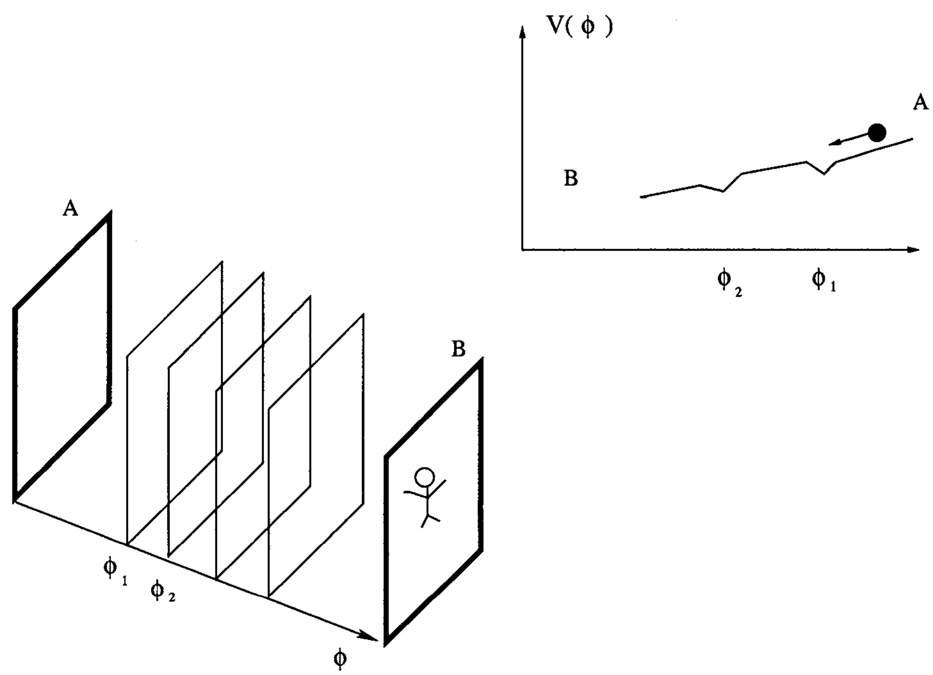

Fig. 6: The D-brane picture of a series of trapping events.

A D-brane example provides a useful geometrical model of this process. Suppose we have an observable brane B and another brane A approaching it. Suppose also that there are a number of other branes in between A and B. Each time the moving brane passes through one of the standing intermediate branes, stretched strings are created and slow the motion of A. The cumulative effect of a number of standing branes is perceived on the observable brane as a slowing-down of the motion of A due to the interactions.

One should note that inflation in our scenario is rather unusual: the inflaton ϕ rolls a short distance, then oscillates for a long time, but with period much smaller than H^{-1}, then rolls again, etc. This may lead to peculiar features in the spectrum of density perturbations. One can avoid these features if the points ϕ_i are very close to each other, and each of them does not stop the rolling of ϕ but only slows it down. In this case, particle production will not lead to parametric resonance, so it is not very important to us whether the fields χ_i are bosons or fermions, as long as their masses vanish at ϕ_i.

This scenario is similar to the string-inspired thermal inflation considered in [76] (see also [101]), but our proposal does not require thermal equilibrium. The main effect which supports inflation in our scenario is based on particle production and has a nonperturbative origin. (A closely-related mechanism uses the corrections to the kinetic terms in the strong coupling regime, where the particle production is suppressed [70].) We hope to return to a discussion of this possibility in a separate publication.

3.9 Conclusion

We have argued that the dynamics of rolling moduli is considerably modified due to quantum production of light fields. In flat space quantum field theory, moduli typically become trapped in orbits around loci which have extra light degrees of freedom. In the presence of gravity, Hubble friction limits the field range the system samples, but any trapping events which do occur are enhanced by Hubble friction, which rapidly brings the modulus to rest at an ESP. Moduli trapping may aid in solving the cosmological moduli problem by driving moduli to sit at points of enhanced symmetry. Furthermore, the trapping of a scalar field which has a potential can lead to a short period of accelerated expansion in situations with steeper potentials than would otherwise allow this. Finally, the trapping effect has important consequences for the problem of vacuum selection, as it can reduce the problem to that of selecting one point within the class of ESPs. An intriguing feature of this process is that the trapping is more efficient near points with a large number of unbroken symmetries.

3.A Particle Production Due to Motion on Moduli Space

In this section we will calculate the quantum production of χ particles, ignoring the effect of backreaction on the motion of ϕ.

A mode of χ with spatial momentum k obeys the wave equation

$$\Big(\partial_t^2 + k^2 + g^2(\mu^2 + v^2 t^2)\Big)u_k = 0. \tag{3.A.1}$$

There are two solutions to this equation, u_k^{in} and u_k^{out}, associated to vacuum states with no particles in the far past and no particles in the far future, respectively. These two sets of modes are related by a Bogoliubov transformation

$$u_k^{in} = \alpha_k u_k^{out} + \beta_k u_k^{out*}. \tag{3.A.2}$$

If we start in the state with no particles in the far past, then one can calculate the number density of particles in the far future to be

$$n_k = |\beta_k|^2 \tag{3.A.3}$$

in the k^{th} mode. This may be evaluated by solving equation (3.A.1) in terms of hypergeometric functions (see e.g. §3.5 of [77]), but we will present here a more physical argument.

One can view (3.A.1) as a one dimensional Schrödinger equation for particle scattering/penetration through an inverted parabolic potential. If we send in a wave ψ_k^{in} from the far right of the potential, part of it will penetrate to the far left, with an asymptotic amplitude $T_k\psi_k^{out}$, and part of it will be reflected back to the right, with an asymptotic amplitude $R_k\psi_k^{in*}$, where T_k and R_k are the transmission and reflection amplitudes.[14]

[14] The modes in the two problems are related by $u_k^{in}(t \to -\infty) = T_k^* \psi_k^{out*}$, etc.

The Bogoliubov coefficient in (3.A.2) is determined in terms of these transmission and reflection amplitudes via

$$\beta_k = \frac{R_k^*}{T_k^*}. \tag{3.A.4}$$

Now we use a trick from quantum mechanics to relate R and T using the WKB method. If we are moving along the real time coordinate, the WKB form of the solution $u_k^{in}(t)$ will be violated at small t, due to non-adiabaticity. However, if we take t to be complex then we can move from $t = -\infty$ to $t = +\infty$ along a complex contour in such a way that the WKB approximation

$$u_k^{in}(t) \sim \frac{1}{\sqrt{2\sqrt{k^2 + g^2(\mu^2 + v^2 t^2)}}} e^{-i\int^t \sqrt{k^2+g^2(\mu^2+v^2t'^2)}dt'} \tag{3.A.5}$$

is valid. Here the integral $\int^t dt'$ becomes a contour integral along a semicircle of large radius in the lower complex t plane. For large $|t|$, we can estimate the phase integral in (3.A.5) by expanding

$$\sqrt{k^2 + g^2(\mu^2 + v^2 t^2)} \sim gvt + \frac{k^2 + g^2\mu^2}{2gvt}. \tag{3.A.6}$$

As we go around half of the circle, this term generates a factor

$$(e^{-i\pi})^{-i(k^2+g^2\mu^2)/2gv-1/2} = ie^{-\pi(k^2+g^2\mu^2)/2gv}. \tag{3.A.7}$$

This is exactly the ratio between R^* and T^*, so we find

$$n_k = |\beta_k|^2 = e^{-\pi(k^2+g^2\mu^2)/gv}. \tag{3.A.8}$$

It is important to note that this result applies much more generally than for

$$\phi = i\mu + vt. \tag{3.A.9}$$

In many cases the nonadiabaticity is only appreciable near the origin $\phi = 0$, so that the near-origin trajectory can be approximated by (3.A.9) with some appropriate near-origin velocity v, even if the evolution away from the origin is very different from (3.2.2).

Moreover, in analogous circumstances (T-dual in the brane context) with a nontrivial electric field, we obtain a similar expression due to Schwinger pair production; a related point was made in [87]. In addition, formula (3.A.8) applies not only to scalar fields, but also to fields of arbitrary spin. From this universal behavior, it is tempting to speculate that (3.A.8) could provide an effective model for string theory effects, but we will not pursue this direction here.

The result (3.A.8) is nonperturbative in g (with g, not g^2, appearing in the denominator of the exponent); it is interesting to ask whether there is a simple interpretation of this nonanalytic, nonperturbative effect. Similarly, it is interesting to note that as discussed in §3.3, the potential for ϕ induced by particle production is linear, so that if extended to the origin it would have a nonanalytic cusp there.

Our results correspond to the low-velocity limit of the D-brane calculation by Bachas [87]. Bachas obtains an imaginary part to the action for moving D3-branes of the form

$$\mathrm{Im}\, S \propto \sum_{n=1}^{\infty} \frac{(-1)^{(n+1)}}{n} \left(\frac{gv}{\pi n}\right)^{3/2} \exp(-n\pi g\mu^2/v) \tag{3.A.10}$$

where we have translated his results into our variables. The first term in this expansion is proportional to the overlap $\int d^3\vec{k}|\beta_k|^2$ giving the number density (3.3.14) of produced particles; this agrees with what we expect from unitarity. More generally, backing away from this low-velocity limit, the calculation in [87] combined with unitarity provides a generalization of our results to the string case, as we will explain in Chapter 4.

3.B Annihilation of the χ Particles

In this section we study the effects of collisions and direct decays of the created χ particles, and demonstrate that for suitably chosen parameters the trapping effect receives only small corrections. More specifically, we place limits on the reduction of the χ number density through processes like $\chi\chi \rightarrow \phi\bar{\phi}$ and $\chi \rightarrow \eta\bar{\eta}$, where η is some light field.

Direct decays, if present, could easily ruin the trapping mechanism: if the χ particles decay too rapidly into light fields then the energy stored in created χ particles will not suffice to stop ϕ. In this case the modulus will roll past the ESP, feel a transient tug toward the ESP while the χ particles remain, and then gradually break free and glide off to infinity at a reduced speed.

We will therefore consider only models in which couplings of the form $\chi\bar{\psi}\psi$, with $\bar{\psi}, \psi$ very light, are negligible. As an example, one can easily exclude such decays in a supersymmetric model with a superpotential of the form $\mathcal{W} \sim g\Phi X^2$. Here X is a chiral superfield with scalar component χ and fermion ψ_χ, and Φ is a chiral superfield with scalar component ϕ and fermion ψ_ϕ. This generates Yukawa couplings of the form $\chi\psi_\chi\psi_\phi$ and $\phi\psi_\chi\psi_\chi$, which do not allow decays from a component of X to purely Φ particles. Thus, if all components of X are heavy, the X-particle energy density we produce cannot decrease by direct decays. In some of the simplest brane setups, exactly this situation is realized: a string which is heavy because it stretches between two branes separated in a purely closed string bulk space cannot decay perturbatively into two light, unstretched strings.

On the other hand, a priori we cannot ignore the coupling $\frac{g^2}{2}\chi^2\phi^2$ as it is this which gives rise to the desired trapping effect. This means that we must tolerate a certain rate of annihilation (as opposed to direct decay). We will now review the cross section for this process and determine its effect on the number density n_χ appearing in (3.2.10).

The Lorentz-invariant cross section for the annihilation process $\chi\chi \to \phi\bar{\phi}$ is, written in terms of center of mass variables,

$$\sigma = \frac{g^4 k'}{4\pi k E^2}, \tag{3.B.1}$$

where k and k' are the momenta of the ingoing and outgoing particles and E is the energy of the ingoing χ particles. The reverse process $\phi\bar{\phi} \to \chi\chi$ tends to enhance the trapping effect. As we are in search of a lower bound on the number of χ particles, we will simply omit this reverse process.

We now determine the annihilation rate to find the rate at which χ particles are lost. If we assume that all the χ's are produced at $t = 0$, we find

$$\frac{\dot{n}(\vec{k}_1, t)}{n(\vec{k}_1, t)} = -\int d\vec{k}_2 n(\vec{k}_2, t) \frac{\sqrt{(k_1 k_2)^2 - m_\chi^4}}{E_1 E_2} \sigma(\vec{k}_1, \vec{k}_2). \tag{3.B.2}$$

Here $u = \sqrt{(k_1 k_2)^2 - m_\chi^2} / E_1 E_2$ is the Lorentz-invariant relative velocity of the initial χ's and $\sigma(\vec{k_1}, \vec{k_2})$ is the cross section, to be calculated using (3.B.1) in the center of mass frame.

We can simplify (3.B.2) to get an upper bound on how fast χ decays. Ignoring the momentum dependence on the right hand side of (3.B.2), which amounts to taking the non-relativistic limit, and ignoring the mass of ϕ produced by the χ particles, we have

$$\frac{\dot{n}(\vec{k}_1,t)}{n(\vec{k}_1,t)} \geq -\frac{g^4}{2\pi m_\chi^2}\int d\vec{k}_2 n(\vec{k}_2,t). \tag{3.B.3}$$

We can bound the integral in the second term on the right hand side by n_χ, the total number of χ's produced, as given in (3.3.14). To approximate the time-dependence of the mass m_χ, we take $m_\chi^2 = \mu^2 + v^2t^2$, which is what the uncorrected motion for ϕ would give. So we have finally

$$\frac{\dot{n}(\vec{k}_1,t)}{n(\vec{k}_1,t)} \geq -\frac{g^4 n_\chi}{2\pi(\mu^2+v^2t^2)}, \tag{3.B.4}$$

which yields

$$\frac{n(\vec{k},t)}{n(\vec{k},0)} \geq \exp\left(-\frac{g^4 n_\chi}{2\pi\mu v}\arctan\frac{vt}{\mu}\right). \tag{3.B.5}$$

This is clearly bounded from below by

$$\exp\left(-\frac{g^4 n_\chi}{2\mu v}\right). \tag{3.B.6}$$

so that the number density is reduced over time by at worst the factor (3.B.6).

The total energy density in the χ particles at a given time is therefore

$$\begin{aligned} E &= \int d\vec{k} n(\vec{k},t)\sqrt{k^2+g^2(\mu^2+v^2t^2)} \\ &\geq n_\chi\sqrt{g^2(\mu^2+v^2t^2)}\exp\left(-\frac{g^4 n_\chi}{2\pi\mu v}\arctan\frac{vt}{\mu}\right). \end{aligned} \tag{3.B.7}$$

From this we see that the mass amplification effect of the χ particles inevitably prevails and stops ϕ from rolling arbitrarily far past the enhanced symmetry point.

This reduction of the number density softens, but does not ruin, the trapping effect. Using the energy density (3.B.7) in the simple estimate leading to (3.2.12), we find a new estimate for ϕ_*:

$$\phi_* = \frac{4\pi^3}{g^{5/2}} v^{1/2} e^{\pi g\mu^2/v} e^{g^4 n_\chi/4\mu v} \tag{3.B.8}$$

Thus, although collisions never lead to an escape, they do lead to a somewhat increased stopping length ϕ_*.

For suitably chosen parameters we can arrange that the effect of collisions is unimportant and the estimates (3.2.12),(3.B.8) approximately agree. For example, the final exponential factor, which encodes the consequences of annihilations, will be less important than the factor $e^{\pi g \mu^2/v}$ as long as $g^3 v \ll \mu^2$.

We conclude that direct decays can be forbidden using symmetry, whereas collisions increase the stopping length (3.B.8) but do not ruin the trapping effect.

3.C Classical Trapping Versus Quantum Trapping

In this section we will compare our quantum trapping mechanism with the purely classical trapping proposed in [102].

Consider for simplicity a theory of two real scalar fields, ϕ and χ, with the interaction $\frac{g^2}{2}\phi^2\chi^2$. In our discussion in the main text we assumed the initial conditions $\langle\chi\rangle = 0, \langle\phi\rangle \neq 0, \dot{\chi} = 0, \dot{\phi} = v$. Potentially interesting classical dynamics arises in the more general case in which the initial velocity of χ is nonzero [102].

Let us therefore consider the classical behavior of these fields, ignoring particle production entirely. If we define $v \equiv \sqrt{\dot{\chi}^2 + \dot{\phi}^2}$, then energy conservation implies that the trajectory of ϕ and χ is bounded by the surface $g^2\phi^2\chi^2 = v^2$. The fields will evidently start bouncing off the curved walls of the potential. This bouncing will be highly random.

Naively, one would expect that on average the fields become confined in the region

$$\langle\phi^2\rangle = \langle\chi^2\rangle \sim \frac{v}{g}. \tag{3.C.1}$$

This result would coincide with our estimate for the amplitude of the oscillations of ϕ at the end of the stage of parametric resonance, cf. (3.3.21).

However, the situation is more complicated. As we are going to show, the fields spend most of the time not at $|\phi| \sim |\chi| \sim \sqrt{\frac{v}{g}}$, but exponentially far away from this region, moving along one of the flat directions of the potential.

To see this, note that because of the chaotic nature of the bouncing, it will occasionally happen that the fields enter the valley $\chi \ll \phi$ at a small angle to the flat direction, i.e. with velocities obeying $|\dot{\chi}| \ll \dot{\phi}$. Defining $|\dot{\chi}| \equiv \alpha v$, we are interested in the case that the angle α happens to be small.

Energy conservation implies that the amplitude of the oscillations of χ at the initial stage of this process is approximately $\frac{\alpha v}{g\phi}$. Because of the interaction term $\frac{g^2}{2}\phi^2\chi^2$, these oscillations act on the field ϕ with an average returning force $\sim \frac{\alpha^2 v^2}{\phi}$, which corresponds to the logarithmic potential $V(\phi) \sim \alpha^2 v^2 \log\phi$ [102]. Clearly, this potential will eventually pull the field ϕ back to the ESP $\phi = 0$. However, this happens at exponentially large ϕ: the field starts moving back only after its value approaches

$$\phi^*_{\rm class} \sim \sqrt{\frac{v}{g}}\; e^{c/\alpha}, \tag{3.C.2}$$

where $c = \mathcal{O}(1)$.

Once again, because the bouncing process is highly random, we do not expect that the probability to enter the valley at a small angle α is exponentially suppressed. This means that after bouncing back and forth near the point $\phi = \chi = 0$, the fields ϕ and χ eventually enter one of the valleys at a small angle, and subsequently spend a very long time there. In general, the fields will spend an exponentially long time at an exponentially large distance from the origin. Thus, the classical trapping mechanism, unlike the particle production mechanism described in this chapter, does not lead to a permanent trapping of the fields in the vicinity of the point $\phi = \chi = 0$.

4. Relativistic D-brane Scattering

ABSTRACT OF ORIGINAL PAPER

We study the effects of quantum production of open strings on the relativistic scattering of D-branes. We find strong corrections to the brane trajectory from copious production of highly-excited open strings, whose typical oscillator level is proportional to the square of the rapidity. In the corrected trajectory, the branes rapidly coincide and remain trapped in a configuration with enhanced symmetry. This is a purely stringy effect which makes relativistic brane collisions exceptionally inelastic. We trace this effect to velocity-dependent corrections to the open string mass, which render open strings between relativistic D-branes surprisingly light. We observe that pair-creation of open strings could play an important role in cosmological scenarios in which branes approach each other at very high speeds.

4.1 Introduction

Thought experiments involving the scattering of strings or of D-branes provide the key to understanding certain essential phenomena in string theory. The discovery of strings in the theory is perhaps the most striking case, but other examples include the elucidation of the sizes of strings under various conditions and the appreciation of another length-scale in the dynamics of slow-moving D-branes.

Despite much early interest in the scattering of D-branes, certain important aspects of the dynamics have remained unexplored. In particular, the simplest treatments involve parameter regimes governed either by supergravity or by the

---

This chapter is reprinted from Liam McAllister and Indrajit Mitra, "Relativistic D-Brane Scattering is Extremely Inelastic," JHEP **0502** (2005) 019, by permission of the publisher.

effective worldvolume field theory of massless open strings. In the latter case, there can be significant quantum corrections arising from loops of light open strings or from pair-production of on-shell open strings.

A key consequence of the pair-production of open strings is the *trapping* of D-branes [18], which we now briefly review. Consider two Dp-branes, $p > 0$, moving with a small relative velocity. As the branes pass each other, the masses of stretched open strings vary with time. This leads to pair production, in a direct analogue of the Schwinger pair-creation process for charged particles [103] or strings [104] in an electric field. Because the velocities are low, the production of stretched strings with oscillator excitations is highly suppressed. The resulting unexcited stretched strings introduce an energy cost for the branes to separate; unless these strings can rapidly annihilate, the branes will be drawn close together. In collisions with a nonzero impact parameter, the brane pair carries angular momentum; in this case the branes spiral around their center of mass, radiating closed strings, until eventually they fall on top of each other. The final outcome is that the open strings trap the branes in a configuration with enhanced symmetry. Because this process involves the production of only unexcited open strings, it falls within the purview of effective field theory.

Our goal is to explore related processes which are not describable in the low-energy effective field theory but which instead involve intrinsically stringy physics. We will show that the ultrarelativistic scattering of D-branes is a suitable laboratory for such an investigation, as corrections from the massive string states turn out to be essential. In particular, we will demonstrate that production of highly-excited open strings generates crucial corrections to the brane dynamics and leads to spectacular trapping of the branes over distances which can be of order the string length. As we will show, these corrections are much stronger than a naive application of effective field theory would predict; hence this is a setting where the importance of purely stringy effects is a surprise. The explanation of such a huge production of highly excited strings is that these states effectively become quite light – the mass receives velocity dependent corrections. The fact that open string masses are in principle velocity-dependent is well-known, but we have not found any explicit computations of these masses in the literature. Our result leads to a formula for the masses of open strings between moving D-branes.

The intuition underlying this result is that in relativistic D-brane scattering, it should be possible to pair-produce highly-excited open strings. The density of string states at high excitation levels grows exponentially with energy; this is the well-known Hagedorn density of states. For this reason, even if the production of a given excited string state is exponentially suppressed compared to production of a massless string state, the competition of the growing and decaying exponentials will typically cause highly-excited strings to dominate the process, in terms of both their number and their share of the total energy. Thus, one expects pair production of a huge number of highly excited strings. This is indeed the case, as was first explained by Bachas in the important work [87]. Our further observation is that because the energy transferred into these open strings can easily be comparable to the initial kinetic energy of the D-brane pair, the massive open strings are absolutely central to the dynamics. This means that the backreaction arising from purely stringy effects is crucial.

We will study the effect on the dynamics of this explosive pair-production of massive modes. Our conclusion is that for a large range of velocities and impact parameters, almost all the initial kinetic energy of the branes is transferred to open strings and to closed string radiation. After the collision the branes are drawn together and come to rest. In near-miss scattering events with an impact parameter b, the branes revolve around their center of mass in a roughly circular orbit whose initial radius is of order b; this orbit swiftly decays via radiation of closed strings. This is to be contrasted to the much weaker trapping of nonrelativistic branes, which typically proceeds via very elliptical orbits, i.e. the stopping length is much greater than the impact parameter.

To recap, the dynamics of ultrarelativistic D-branes is strikingly inelastic: copious production of highly-excited stretched open strings rapidly drains the brane kinetic energy and traps the branes into a tight orbit, eventually leading the branes to coincide.

In this simple and controllable example it proves possible to understand aspects of the backreaction of open string production on the dynamics of colliding D-branes. The lessons of our analysis could be extended to cosmological models in which other sorts of fast-moving branes approach each other and collide. As we will discuss, these include the ekpyrotic/cyclic universe scenario, brane-antibrane scenarios, and the DBI model.

It is useful to indicate the various regions of parameter space that we will probe. We will outline this now to apprise the reader of our strategy; later, in §4.5.3, we will provide a more complete discussion.

The dimensionless quantities of interest are the impact parameter b measured in units of the string length; the string coupling g_s, which determines the mass of the D-branes in string units; and the initial relative velocity of the branes v. We will find it more convenient to convert this velocity into the rapidity, $\eta \equiv \text{arctanh}(v)$. We will usually set $\alpha' = 1$, except for a few cases where we will retain explicit factors of the string length for clarity.

Our goal in this work is to understand open string effects in *relativistic* dynamics; the nonrelativistic case is already well-understood [105,18]. We will therefore impose $1 - v \ll 1$ so that $\eta \gg 1$. Another important consideration is that the D-branes should have Compton wavelengths small compared to the impact parameter. Because the D-branes grow light at strong string coupling, this amounts to a requirement that the coupling should be sufficiently weak. Another obvious advantage of weak coupling is the suppression of string loop effects; our primary computation is a one-loop open string process. A further requirement is that the D-brane Schwarzschild radius should be much smaller than the impact parameter. This too can be achieved with a suitably small string coupling, as we will demonstrate in §4.5.3. Furthermore, although energy loss through closed string radiation can be an important effect in a system of moving branes, there is a wide range of string coupling, depending on η, for which this effect is subleading compared to open string production. Although all these considerations show that weak coupling is desirable for control, it is important to recognize that as the coupling decreases, the D-branes grow heavy and hence stretch the strings farther before coming to rest.

In summary, there is a range of values of the string coupling in which the backreaction of open strings is significant and competing effects are suppressed.

The organization of this chapter is as follows. First, in §4.2, we review the trapping of nonrelativistic branes, which provides the basic intuition for the more complicated, stringy process which we aim to study. Then, in §4.3, we study the interaction amplitude for moving branes. We compute the brane interaction via an annulus diagram and examine its imaginary part, which corresponds to open string pair production. This result is well-known, but we include it for logical completeness and to set our notation. Our primary result appears in §4.4, where we study the backreaction of open string production on the brane trajectory and estimate the stopping length on energetic grounds. In §4.5 we discuss potential corrections and additional effects, in particular the production of closed strings, and explain how they affect our considerations. We conclude with a few comments in §4.6. In Appendix 4.A we give a detailed check of our formula for the velocity-dependent string mass. Finally, we collect useful identities about the theta functions in Appendix 4.B.

4.2 Overview of the Trapping of Nonrelativistic Branes

We will now briefly review the trapping of D-branes in nonrelativistic motion, which was studied in [18]. (See also [106,107] for earlier work on related mechanisms in field theory and cosmology.) This process is governed by pair production of massless open strings and hence is describable in effective field theory. It provides the basic framework for understanding corrections to the brane dynamics, and so is a useful background for the stringy trapping which we will study in §4.3.

Because the field theory description is entirely sufficient, we can abstract the relevant properties of the worldvolume gauge theory and represent the system with a simplified model,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\bar{\phi} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{8\pi^2}|\phi|^2\chi^2 \tag{4.2.1}$$

in which a complex scalar field ϕ couples to a real scalar field χ. We have normalized the cross-coupling term so that the mass of χ is precisely the mass of a stretched string whose length is $|\phi|$ in string units. At the origin $\phi = 0$, χ becomes massless.

Let us consider the trajectory

$$\phi(t) = ib + vt \tag{4.2.2}$$

in which ϕ is separated from the origin by the impact parameter b. This is a solution to the classical equations of motion of (4.2.1) provided that $\chi = 0$. Along this trajectory, the mass of χ changes: in the limit where we impose (4.2.2) and ignore the effect of the coupling to χ, we may rewrite (4.2.1) as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\bar{\phi} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{8\pi^2}(b^2 + v^2t^2)\chi^2 \tag{4.2.3}$$

so that the effective mass of χ varies with time. This results in production of χ quanta.

This effect is easily understood in the quantum mechanics example of a harmonic oscillator whose frequency changes over time from ω_i to ω_f. If the oscillator begins in its ground state at frequency ω_i but the frequency changes nonadiabatically then the final state will not be the ground state of an oscillator of frequency ω_f.

One can readily compute the occupation numbers n_k of modes with momentum k. The result [18] is

$$n_k = \exp\Big(-\frac{4\pi^2 k^2 + b^2}{2v}\Big). \tag{4.2.4}$$

If instead we consider a model in which the mass of χ is nowhere zero,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \bar{\phi} + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \Big(\frac{m^2}{2} + \frac{1}{8\pi^2}|\phi|^2\Big)\chi^2 \tag{4.2.5}$$

the result is instead

$$n_k = \exp\Big(-\frac{4\pi^2 k^2 + 4\pi^2 m^2 + b^2}{2v}\Big). \tag{4.2.6}$$

The crucial, though intuitive, observation is that production of a massive species is exponetially suppressed. For this reason, production of massive string modes is entirely negligible when the velocity is small.

We may now apply the result of the simplified model to a pair of D-branes. Suppose that two Dp-branes, $p > 0$, are arranged to pass near each other. The brane motion changes the masses of stretched string states and induces pair production of unexcited stretched strings. As the branes begin to separate, these strings stretch and pull the branes back together.

This process can be followed in detail by numerically integrating the quantum-corrected equations of motion which follow from (4.2.1). Such an analysis was presented in [18]. However, analytical estimates are more readily generalized to the case of interest in this chapter, which is the stringy scattering of relativistic branes. We will therefore explain how one can use energetics to estimate the stopping length in the system (4.2.1). (It was shown in [18] that such estimates are in excellent agreement with the numerical results, although only the nonrelativistic case was studied there.)

After the branes have passed each other, the stretched open strings grow in mass. Even though pair production has ceased, the energy contained in open strings grows with time, because the strings are being stretched:

$$\rho_{open} \approx |\phi(t)| n_{open} \tag{4.2.7}$$

When the energy in open strings is of the same order as the initial brane kinetic energy, the backreaction of the open strings is of order one and the brane motion slows down significantly. We therefore define the 'stopping length' ϕ_* via $\rho_{open}(\phi_*) \approx \frac{1}{2}T_p v^2$ where T_p is the tension of a Dp-brane.

A few qualitative features of low-velocity trapping are worth mentioning. First, the greater the number density of produced strings, the shorter the stopping length. On the other hand, the stopping length increases if the brane velocity increases or the string coupling decreases (making the branes heavier in string units).

The behavior in the limit $v \to 1$ is not obvious *a priori*. To estimate the total number density ν_{total} of all string modes, we could take the nonrelativistic result (4.2.6) for the occupation numbers of a massive species and sum it over the levels n in the string spectrum, including a factor of the density of states $N(n)$. The result (which was also presented in [18]) is

$$\nu_{total} \propto \sum_{n=0}^{\infty} N(n) \exp\Big(-\frac{2\pi^2}{v}\Big(n + \frac{b^2}{4\pi^2}\Big)\Big). \tag{4.2.8}$$

As we explain in §4.4.1, the density of states at high levels n obeys

$$N(n) \sim n^{-11/4} \exp\Big(\sqrt{8\pi^2 n}\Big) \tag{4.2.9}$$

This does not grow rapidly enough to compete with the exponential suppression (4.2.6) of high levels, so the limit $v \to 1$ does not display strong production of excited strings.

However, we will show in detail in §4.3, following [87], that the actual number density of produced strings is very much larger than the nonrelativistic estimate (4.2.8) suggests. We will find instead

$$\nu_{total} \propto \sum_{n=0}^{\infty} N(n) \exp\Big(-\frac{2\pi^2}{\eta}\Big(n + \frac{b^2}{4\pi^2}\Big)\Big) \tag{4.2.10}$$

where $\eta = \text{arctanh}(v) \gg 1$. This result does not follow from special relativity alone; it is instead a stringy effect arising from velocity-dependent corrections to the stretched string masses, as we will show.

4.3 The Interaction Amplitude for Moving D-branes

We will now derive the interaction potential for two D-branes in relative motion with arbitrary velocity. Although this result is well-known [87], we include the calculation for completeness and to set notation.

4.3.1 Interaction Potential from the Annulus Diagram

We will derive the interaction potential by computing the open string one-loop vacuum energy diagram. This diagram is an annulus whose two boundaries correspond to the two D-branes. By the optical theorem, twice the imaginary part of this amplitude is the rate of pair production of on-shell open strings. Thus, our goal is to determine the imaginary part of the vacuum energy.

Several equivalent methods can be used to compute the vacuum energy. The original treatment [87] involves a direct computation of the spectrum of open strings between the moving branes; that is, it is possible to impose appropriate boundary conditions and solve for the mode expansion. The vacuum energy is then the sum of the zero-point energies of these oscillators.

We choose instead to review the perhaps more transparent computation given in [108]. Let us stress that in this subsection we follow the treatment of [108] in detail, with very minor modifications.

By double Wick rotation, a pair of branes in relative motion, separated by a transverse distance b, can be mapped to a stationary pair of branes at an imaginary relative angle, again separated by a distance b. We will make this precise below. Because the partition function for branes at angles is very well understood, the vacuum energy is easily computed in this approach.

Following [108], we begin with two D4-branes which are parallel to each other, extended along the directions $0, 1, 3, 5, 7$, and separated by a distance b (the impact parameter) along X^9. (To regulate the computation we compactify the spatial dimensions on a T^9 of radius R.) Now let one brane move towards the other along the direction X^8 with velocity v. That is, the coordinates of the moving brane are $X^8 = vX^0, X^9 = b$ while the other brane has $X^8 = X^9 = 0$. This is our actual problem.

We now perform the Wick rotation $X^0 \to -iX'^7$, $X^7 \to iX'^0$. This transforms the moving branes into static branes which are misaligned by an angle ϕ in the

$(7', 8)$ plane. The angle ϕ is given by $X'^7 \tan\phi = X^8$. The brane velocity v and rapidity η are related to this angle by $\phi = -i \text{ arctanh}(v) \equiv -i\eta$.

Next, it is useful to combine the coordinates into complex pairs Y_a, where $Y_1 = X^1 + iX^2, Y_2 = X^3 + iX^4, Y_3 = X^5 + iX^6, Y_4 = X'^7 + iX^8$. Define also the angles $\phi_1 = \phi_2 = \phi_3 = 0, \phi_4 = \phi$. The rotation then takes $Y_4 \to \exp(i\phi)Y_4$. It is now a simple matter to set up the boundary conditions satisfied by strings which stretch between the branes:

$$\begin{aligned} \sigma_1 = 0: &\qquad \partial_1 \text{Re}[Y_a] = \text{Im}[Y_a] = 0 \\ \sigma_1 = \pi: &\qquad \partial_1 \text{Re}[\exp(i\phi_a)Y_a] = \text{Im}[\exp(i\phi_a)Y_a] = 0\,. \end{aligned} \tag{4.3.1}$$

The solutions to the wave equation which satisfy these boundary conditions are:

$$Y_a(w, \bar{w}) = i\sqrt{\frac{\alpha'}{2}} \left(\sum_{r=\mathbf{Z}+\phi_a/\pi} \frac{\alpha_r^a}{r} \exp(irw - 2i\phi_a) - \sum_{r=\mathbf{Z}+\phi_a/\pi} \frac{\alpha_r^{a\,*}}{r} \exp(ir\bar{w}) \right), \tag{4.3.2}$$

where $w = \sigma_1 + i\sigma_2$. We can readily write down the partition function for these four scalars:

$$Z_{scalar}(\phi_a) = -i \frac{\exp(\phi_a^2 t/\pi)\eta(it)}{\theta_{11}(i\phi_a t/\pi, it)} \tag{4.3.3}$$

so that the resulting bosonic partition function is

$$Z_{boson} = \prod_{a=1}^{4} Z_{scalar}(\phi_a) \tag{4.3.4}$$

In a similar way, one can compute the fermionic partition function, keeping in mind the various spin structures:

$$Z_{ferm} = \prod_{a=1}^{4} Z_1^1(\phi_a/2, it)\,, \tag{4.3.5}$$

where

$$Z_1^1(\phi_a/2, it) \equiv \frac{\theta_{11}(i\phi_a t/2\pi, it)}{\exp(\phi_a^2 t/4\pi)\eta(it)} \tag{4.3.6}$$

We conclude that the one-loop potential is

$$V = -\int_0^\infty \frac{dt}{t} \frac{1}{\sqrt{8\pi^2\alpha' t}} \exp\left(-\frac{tb^2}{2\pi\alpha'}\right) \prod_{a=1}^{4} \frac{\theta_{11}(i\phi_a t/2\pi, it)}{\theta_{11}(i\phi_a t/\pi, it)}\,. \tag{4.3.7}$$

This potential governs D4-branes at a relative angle. To map into the case of interest, we T-dualize as many times as needed, each time introducing the replacement $\theta_{11}(i\phi_a t/\pi, it) \rightarrow i\sqrt{8\pi^2\alpha' t}\eta^3(it)/R$, where R is the size of the spatial torus.

This finally brings us to the potential for p-branes at an angle ϕ:

$$V = -iR^p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-p/2} \exp\left(-\frac{tb^2}{2\pi\alpha'}\right) \frac{\theta_{11}(i\phi t/2\pi, it)^4}{\theta_{11}(i\phi t/\pi, it)\eta(it)^9}. \qquad (4.3.8)$$

Our final interest is in the number density and energy density of open strings, so the spatial volume $R^p \equiv iV_p$ will eventually cancel.

To read off the desired result for moving branes, we set $\phi = -i\eta$ to get

$$V = V_p \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-p/2} \exp\left(-\frac{tb^2}{2\pi\alpha'}\right) \frac{\theta_{11}(\eta t/2\pi, it)^4}{\theta_{11}(\eta t/\pi, it)\eta(it)^9}. \qquad (4.3.9)$$

One can easily show that this agrees precisely with the result of [87], equation (11). To see this, use (4.B.2) and (4.B.8), define $t_{there} = 2t$, $\epsilon = \frac{\eta}{\pi}$, and set $\alpha' = \frac{1}{2}$.

A useful equivalent form for (4.3.9) is

$$\begin{aligned} V = V_p \int_{-\infty}^{\infty} d\tau \int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-p/2} \frac{\theta_{11}(\eta t/2\pi, it)^4}{\theta_{11}(\eta t/\pi, it)\eta(it)^9} \times \\ \exp\left(-\frac{t}{2\pi\alpha'}\left(b^2 + v^2\tau^2\right)\right) \frac{v}{\pi}\sqrt{\frac{t}{2\alpha'}} \end{aligned} \qquad (4.3.10)$$

In this form the time-dependence of the stretched string masses is manifest.

4.3.2 Imaginary Part and Pair-Production Rate

The above expression from the interaction potential is rich in information. The real part tells us about the velocity-dependent forces from closed string exchange, while twice the imaginary part is equal to the rate of production of open strings.

The potential (4.3.9) would be real if the integrand had no poles. However, $\theta_{11}(\eta t/\pi, it)$ has a zero for integral values of $\eta t/\pi \equiv k$, so we can compute the imaginary part of the integral by summing the residues at the corresponding poles.

$$\mathrm{Im}[V] = \frac{V_p}{2(2\pi)^p} \sum_{k=1}^{\infty} \frac{1}{k}\left(\frac{\eta}{\pi k}\right)^{p/2} \exp\Bigl(-\frac{b^2 k}{2\eta}\Bigr) Z(ik\pi/\eta)\Bigl(1 - (-1)^k\Bigr), \qquad (4.3.11)$$

where we have defined the partition function $Z(\tau) \equiv \frac{1}{2}\theta_{10}^4(0|\tau)\eta(\tau)^{-12}$. (The factor projecting out even values of k arises because of Jacobi's 'abstruse identity'.)

This expression, which was first derived in [87], will be essential to our investigation. By extracting its behavior in various limits we will be able to study the effect of open string production on the brane dynamics.

First of all, we can check the normalization of (4.3.11) by taking the low-velocity limit, in which $\eta \to v$. The result is

$$\mathrm{Im}[V] = \frac{8V_p}{(2\pi)^p} \sum_{k=1,3,5,\dots}^{\infty} \frac{1}{k}\left(\frac{v}{\pi k}\right)^{p/2} \exp\!\left(-\frac{b^2 k}{2v}\right). \tag{4.3.12}$$

This is identical to Schwinger's classic result (4.3.12) for the pair-production rate of electrons in a constant electric field. In the present case, the interpretation is of pair production of massless open strings between the branes, which was also obtained by the method of Bogoliubov coefficients in [18].

Our interest is in the case of velocities approaching the speed of light. We expect that the dominant contribution to pair production in this limit will come from highly-excited string states. Because the density of states grows exponentially (4.2.9) at high levels, we anticipate copious production of massive strings and, as a result, dramatic backreaction on the brane motion.

To investigate this, we begin with the high-velocity limit $\eta \gg 1$ of (4.3.11):

$$\mathrm{Im}[V] = \frac{V_p}{2(2\pi)^p} \sum_{k=1,3,5,\dots}^{\infty} \frac{1}{k}\left(\frac{\eta}{\pi k}\right)^{p/2-4} \times \exp\!\left(\frac{\eta}{k} - \frac{b^2 k}{2\eta}\right)\left(1 + \mathcal{O}(e^{-\eta/k})\right) \tag{4.3.13}$$

where we have used the asymptotics (2.B.5).

Keeping the dominant contribution, which comes from $k = 1$, and expressing the result as a number density ν_{open} of open strings stretching between the branes, we find

$$\nu_{open} \approx c_p \eta^{\frac{p}{2}-4} \times \exp\!\left(\eta - \frac{b^2}{2\eta}\right) \tag{4.3.14}$$

where $c_p = \left(2(2\pi)^p \pi^{p/2-4}\right)^{-1}$.

There are three important differences between the low-velocity effect in (4.3.12)and the high-velocity relation of (4.3.14). The first is that production of strings is exponentially suppressed at low velocities: this can be understood from the fact that the amount of strings produced at a given energy falls off exponentially with energy, while the density of states for such low energies is a simple power law. At high energies, however, the density of states grows exponentially and these two competing exponentials lead to copious string production if the initial velocity of the branes is sufficiently high.

The second important difference is that at low velocities, the efficacy of the trapping process is strongly dependent on the impact parameter. For large impact parameters, $b \gg 1$ (recall that b is measured in string units), the trapping is exponentially weak. For ultrarelativistic branes, however, the trapping weakens only when $b \gg \eta$. The effective range of strong trapping is evidently much increased in the ultrarelativistic limit.

Finally, in the low-velocity limit, the energy of produced open strings is a negligible fraction of the D-brane energy [18] until the branes separate far enough to stretch the open strings significantly. The associated distance, the 'stopping length', is generically much larger than the impact parameter. In the ultrarelativistic limit, in contrast, the energy carried by the open strings can be comparable to the brane kinetic energy even before any stretching. This occurs because high speeds make possible the production of highly-excited strings with significant oscillator energy. This consideration suggests that the backreaction of open strings is much more dramatic for relativistic branes than for nonrelativistic ones. We undertake a careful study of this in the following section.

4.4 Backreaction from Energetics

We have seen in the previous section that relativistic brane motion leads to the production of a tremendous number density (4.3.14) of stretched open strings. We would now like to estimate the effect of this process on the brane motion, and to do so we must estimate the energy density carried by the produced open strings.

4.4.1 Open String Energy

An open string stretched between two moving branes receives velocity-dependent corrections to its mass. The change in mass is understood to be due to a rescaling of the effective tension, and in the limit that the branes move towards each other at the speed of light, the strings must become massless. This rescaling can also be understood from the T-dual electric field perspective: as the electric field approaches a critical value, the strings can no longer hold themselves together, so their effective tension goes to zero [104]. We will now determine this rescaling in a simple way; in Appendix 4.A we will provide a detailed consistency check of this result.

The factor depending on b in (4.3.14) indicates that the effective area of a brane moving with rapidity η is [87]

$$r_{eff}^2 \approx \eta\alpha'. \tag{4.4.1}$$

This corresponds precisely to the logarithmic growth in cross-sectional area of a highly-boosted fundamental string, $r_{eff}^2 \sim \alpha' \ln(\alpha' s)$, where $\sqrt{s}$ is the center-of-mass energy. The explanation for this growth is that a Regge probe of an ultrarelativistic string is sensitive to rather high-frequency virtual strings, whose considerable length creates a large cloud of virtual strings [109]. We conclude that a D-brane with rapidity η has an apparent radius $r_{eff} = \sqrt{\eta\alpha'/v}$, where we have inserted the factor of v to produce the correct behavior in the zero-velocity limit.

We therefore propose that the effective string tension is:

$$T(\eta) = \frac{v}{2\pi\alpha'\eta}. \tag{4.4.2}$$

This rescaling of the effective tension of the string means that the energy of a string excited at level n is:

$$E(n)^2 = \frac{nv}{\alpha'\eta} + \frac{(b^2+r^2)v^2}{4\pi^2\alpha'^2\eta^2}. \tag{4.4.3}$$

In this expression b is the usual impact parameter, while r is the brane separation along the direction of motion.

In Appendix 4.A we demonstrate that precisely this dependence of mass on velocity explains the dramatic difference between the naive result (4.2.8) and the complete annulus computation (4.3.14) for the number density. Note also that the mass formula (4.4.3) does reduce to the usual formula for low speeds ($\eta \to v$).

We now proceed to calculate the energy density of the produced open strings. This energy is easily computed if we first rewrite the partition function Z as a sum over string states. This is conveniently parametrized in terms of the excitation level n and the number of states $N(n)$ at each level.

$$Z(ik\pi/\eta) \equiv \frac{1}{2}\theta_{10}(0, ik\pi/\eta)^4 \eta(ik\pi/\eta)^{-12} = \sum_{n=0}^{\infty} N(n) \exp\Big(-\frac{2\pi^2 nk}{\eta}\Big). \tag{4.4.4}$$

We would first like to determine the behavior of $N(n)$ at high excitation levels n. Taking the ansatz

$$N(n) \approx c_N n^a \exp\left(b\sqrt{n}\right), \tag{4.4.5}$$

approximating the sum by an integral, evaluating this integral by stationary phase, and demanding the asymptotics (2.B.5), we find

$$N(n) \approx (2n)^{-11/4} \exp\left(\pi\sqrt{8n}\right). \tag{4.4.6}$$

The numerical prefactor was chosen for convenience; strictly speaking, the approximate evaluation of the integral does not determine constant prefactors of order unity, but for our purposes it suffices to choose the factor now as in (4.4.6).

With this result in hand, we can rewrite (4.3.11) as

$$\mathrm{Im}[V] = \frac{V_p}{(2\pi)^p} \sum_{k=1,3,\ldots}^{\infty} \frac{1}{k} \left(\frac{\eta}{\pi k}\right)^{p/2} \exp\left(-\frac{b^2 k}{2\eta}\right) \sum_{n=0}^{\infty} N(n) \exp\left(-\frac{2\pi^2 nk}{\eta}\right). \tag{4.4.7}$$

An equivalent form for this relation is

$$\mathrm{Im}[V] = \frac{\sqrt{2}vV_p}{(2\pi)^{p+1}} \int_{-\infty}^{\infty} d\tau \sum_{k=1,3,\ldots}^{\infty} \frac{1}{k} \left(\frac{\eta}{\pi k}\right)^{\frac{p-1}{2}} \sum_{n=0}^{\infty} N(n) \exp\left(-\frac{2\pi^2 k\mu^2(\tau)}{\eta}\right). \tag{4.4.8}$$

where

$$\mu^2(\tau) \equiv \frac{n}{\alpha'} + \frac{b^2 + v^2\tau^2}{4\pi^2\alpha'^2}. \tag{4.4.9}$$

We can now express the energy density of produced open strings as

$$\rho_{open} = \frac{1}{(2\pi)^p} \sum_{k=1,3,\ldots}^{\infty} \frac{1}{k} \left(\frac{\eta}{\pi k}\right)^{p/2} \exp\left(-\frac{b^2 k}{2\eta}\right) \sum_{n=0}^{\infty} E(n) N(n) \exp\left(-\frac{2\pi^2 nk}{\eta}\right), \tag{4.4.10}$$

where (4.4.3) is used for $E(n)$. Because of the competition of the growing and decaying exponential factors, this sum is dominated by terms near some $n_{peak} \gg 1$. As indicated above, we approximate the sum on levels using the relation

$$\sum_{n=0}^{\infty} N(n) n^{\alpha} \exp\left(-\frac{2\pi^2 n}{\eta}\right) \approx 2^{-11/4} \int_{n_0}^{\infty} dn n^{\alpha-11/4} \exp\left(\pi\sqrt{8n} - \frac{2\pi^2 n}{\eta}\right) \tag{4.4.11}$$

where the lower bound $n_0 > 0$ is chosen so that the integral is dominated by $n \approx n_{peak}$, not $n \approx 0$. We have kept the leading term in the sum on k. By the method of stationary phase we find that the integral is dominated by $n \approx n_{peak} = \eta^2(2\pi^2)^{-1}$, leading to

$$2^{-11/4}\int_{n_0}^{\infty} dn n^{\alpha-11/4}\exp\Big(\pi\sqrt{8n} - \frac{2\pi^2 n}{\eta}\Big) \approx \frac{1}{2}e^{\eta}\Big(\frac{\pi}{\eta}\Big)^4\Big(\frac{\eta^2}{2\pi^2}\Big)^{\alpha}. \qquad (4.4.12)$$

For $\alpha = 0$ this reproduces the asymptotic behavior (4.B.5); we normalized (4.4.6) to arrange this.

This approximate result provides an important physical lesson: the primary contribution to the open string energy comes from strings at levels $2\pi^2 n \approx \eta^2$. For such a string,

$$E(n) = \sqrt{\frac{nv}{\eta} + \frac{(b^2+r^2)v^2}{4\pi^2\eta^2}} \approx \frac{1}{2\pi}\sqrt{2\eta + \frac{b^2+r^2}{\eta^2}}. \qquad (4.4.13)$$

Let us now examine this result in the parameter ranges of interest. If the stretched string length is large compared to $\eta^{3/2}$, $\sqrt{b^2+r^2} \gg \eta^{3/2}$, then the sum (4.4.10) is simply

$$\rho_{open} \approx \frac{\sqrt{b^2+r^2}}{2\pi\eta}\nu_{open}. \qquad (4.4.14)$$

On the other hand, when $\eta^{3/2} \gg \sqrt{b^2+r^2}$, we have instead

$$\rho_{open} \approx \frac{\sqrt{\eta}}{\pi\sqrt{2}}\nu_{open} \qquad (4.4.15)$$

where we have used (4.4.12) with $\alpha = 1/2$.

The key observation which follows from (4.4.15) is that the energy density carried by produced pairs of stretched open strings can be a significant fraction of the kinetic energy density of the Dp-brane. The backreaction from open string production is therefore an important contribution to the dynamics of relativistic D-branes. We will now examine this in detail.

4.4.2 Estimate of the Stopping Length

It will be very important to recognize three length-scales which arise in the problem: the effective size $r_{eff}(\eta) = \sqrt{\eta}\alpha'$ of a relativistic brane, the critical impact parameter $b_{crit}(\eta)$ beyond which the trapping rapidly weakens, and the size $r_{nad}(\eta)$ of the region in which the stretched open string masses change nonadiabatically.

To find the critical impact parameter, we note that the open string energy density obeys

$$\rho_{open} \propto \exp\left(\eta - \frac{b^2}{2\eta}\right), \qquad (4.4.16)$$

so that for $\eta \gg 1$, the critical distance is evidently $b_{crit} \sim \eta$. For impact parameters less than b_{crit}, the open string energy density is generically large. The trapping effect is therefore very strong for impact parameters of order b_{crit} and smaller. (Nevertheless, trapping still occurs for impact parameters much larger than b_{crit}.)

The nonadiabaticity is characterized by how rapidly the frequency changes with time. Quantitatively, it is measured by the dimensionless quantity $\xi \equiv \frac{\dot{\omega}}{\omega^2}$, where ω is the frequency. Using (4.4.3) with $r = vt$ we find

$$\xi = \frac{2\pi\eta v^4 t}{(4\pi^2 n v\eta + (b^2 + r^2)v^2)^{3/2}} \approx \frac{2\pi\eta r}{(4\pi^2 n\eta + b^2 + r^2)^{3/2}} \qquad (4.4.17)$$

which reaches its peak at $r^2 = \frac{1}{2}b^2 + 2\pi^2 n\eta$. For the energetically-dominant levels, $2\pi^2 n \approx \eta^2$, so that the effective region of nonadiabaticity has size $r_{nad} \sim \eta^{\frac{3}{2}}$ for $\eta \gg b$. Open strings are produced in large quantities when $-r_{nad} \lesssim r \lesssim r_{nad}$.

In summary, for relativistic speeds the critical impact parameter is $b_{crit} \sim \eta$, and is smaller than the size of the nonadiabatic region. The effective radius of a moving D-brane, i.e. the size of the stringy halo, is much smaller, $r_{eff} \sim \sqrt{\eta} \ll b_{crit}$. For any fixed, large η we can require

$$r_{eff} \ll b \ll b_{crit} \qquad (4.4.18)$$

so that the trapping is very strong but the stringy halos are small enough to be unimportant. The case of a head-on collision, $b \lesssim r_{eff}$, is also interesting, particularly for the question of string production in the cyclic universe models, but we will first explore the better-controlled regime (4.4.18).

With these estimates in hand we can at last compute the stopping length for a scattering event. Taking one brane to be at rest and the other to have velocity v, we define as before

$$\eta \equiv \text{arctanh}(v). \qquad (4.4.19)$$

Working instead in the center of mass frame, the branes approach each other with velocities

$$u = \tanh(\omega) = \tanh(\eta/2) \qquad (4.4.20)$$

so that the center-of-mass γ factor for either brane is

$$\gamma = \frac{1}{\sqrt{1-u^2}} \sim \frac{1}{2}e^{\omega} \tag{4.4.21}$$

when $\omega \gg 1$. The energy density of the brane pair is then

$$E_{tot} = 2T_{Dp}\gamma \sim T_{Dp}e^{\omega} = T_{Dp}e^{\eta/2}. \tag{4.4.22}$$

We therefore find that for $\eta \to \infty$,

$$\rho_{Dp} = T_{Dp}e^{\eta/2} = \frac{1}{g_s(2\pi)^p}e^{\eta/2}. \tag{4.4.23}$$

In the case of strong trapping, $b \ll b_{crit} \approx \eta$, the open string energy at the minimum brane separation is

$$\rho_{open} \approx \frac{c_p}{\pi\sqrt{2}}\eta^{\frac{p-7}{2}}\exp\Big(\eta\Big) \tag{4.4.24}$$

whereas for weak trapping, $b \gg b_{crit}$, the open string energy is instead

$$\rho_{open} \approx \frac{c_p}{2\pi}\eta^{\frac{p}{2}-5}\sqrt{b^2+r^2}\exp\Big(\eta - \frac{b^2}{2\eta}\Big), \tag{4.4.25}$$

where $c_p = \Big(2(2\pi)^p\pi^{p/2-4}\Big)^{-1}$. Of course, the open string energy depends on r even in the case of strong trapping, but this dependence is relatively unimportant until $r \sim \eta$.

Comparing (4.4.23),(4.4.24) we conclude that if an external force compels the branes to pass each other at constant, ultrarelativistic velocity, then, unless the string coupling is exponentially small, the energy stored in open strings at the point of closest approach is considerably larger than the initial kinetic energy of the branes. This means that without an artificial external force, the branes will *not* pass each other with undiminished speed, as this is energetically inconsistent.

We expect instead that as open strings are produced, the branes slow down gradually, leading to diminished further production of strings. The final result, of course, will be consistent with conservation of energy. (In §4.4.3 we will address the production of open strings between decelerating branes, and in §4.5.1 we will explain that the emission of closed string radiation also serves to reduce the rate of production of open strings.)

Although the open string energy in (4.4.24) is an overestimate for the reason just mentioned, we will nevertheless use it now to find an estimate of the stopping length. This will serve to illustrate our technique in a manageable setting; it will then be a simple matter to repeat the analysis including the corrections of §4.4.3 and §4.5.1, which will not alter the form of our result.

We define the stopping length r_* by $\rho_{open}(r_*) = \rho_{Dp}$, so that at $r = r_*$ all the initial energy has been stored in stretched open strings. Equating (4.4.23) and (4.4.25), we find the stopping length

$$r_* \approx \frac{4\pi^2}{g_s} \exp\left(-\frac{\eta}{2} + \frac{b^2}{2\eta}\right) \left(\frac{\eta}{\pi}\right)^{5-p/2}. \tag{4.4.26}$$

This is our main result. It manifests the surprising property that for sufficiently large rapidity, the stopping length *decreases* as the rapidity increases. (More precisely, for any fixed g_s, b there exists a rapidity η_{min} such that the stopping length decreases as η increases past η_{min}.) To understand this unusual property, it is useful to keep in mind the behavior of D-branes scattering at even greater speeds, so great that the stringy halos themselves collide. For any b there is an η such that $r_{eff} \gtrsim b$; the scattering of the branes is then described by the collision of absorptive disks of radius r_{eff} [87]. Moreover, for a suitable range of g_s the brane Schwarzschild radii are so large that black hole production is an important consideration. We have carefully chosen our parameter ranges to exclude these effects and focus instead on the more controllable regime of strong stringy trapping; however, the black disk collisions and black hole production serve to illustrate that the limit of arbitrarily high rapidity involves very hard scattering and high inelasticity, in good agreement with the large-η behavior of (4.4.26).

The stopping length (4.4.26) is large in string units only when

$$g_s \ll 4\pi^2 \exp\left(-\frac{\eta}{2} + \frac{b^2}{2\eta}\right) \left(\frac{\eta}{\pi}\right)^{5-p/2} \tag{4.4.27}$$

which is an exponentially small value of the coupling provided $\eta \gg b, \eta \gg 1$. Thus, although backreaction from open string production is a higher-order correction to the dynamics [110] which one might suppose is unimportant at moderately weak coupling, we have shown that for relativistic branes with $b \ll \eta$ the backreaction of open strings is crucial unless the string coupling is extraordinarily small.

4.4.3 Corrections from Deceleration

All of our computations so far have applied exclusively to a pair of branes approaching each other at constant velocity. On the other hand, we have demonstrated that the backreaction from open string production, as computed along this trajectory, necessarily causes the branes to decelerate. Clearly, the next step is to understand how the amount of string production changes when the branes follow a decelerating trajectory.

The analysis of string production during deceleration turns out to be tractable in the nonrelativistic limit. However, we have not found an exact answer for the relativistic case. Upon double Wick rotation the amount of string production between decelerating branes is mapped to the interaction between curved branes, which is not obviously solvable with conformal field theory techniques.

Even though we will not find an exact result for the string production, we will be able to place bounds on the resulting number density. This suffices to reveal the qualitative features of the trapping process: copious production of excited strings and very high inelasticity.

First, however, we will examine the limit of instantaneous deceleration. Take the branes to move with a velocity v_0 for all $t < 0$, but to come to rest for $t > 0$. This problem can be solved exactly by matching the parabolic cylinder functions (and their derivatives) to the plane wave solutions at $t = 0$. However, this setup clearly involves enormous non-adiabaticity and so there would be an extremely large amount of pair-production, far greater even than in the case of constant velocity. This is readily computed, but it is not useful; we would like a more conservative estimate.

A more realistic picture is one in which the relative velocity of the branes varies as a function of time, for example as $v(t) = v_0(1 - \tanh(t/f))$, where f measures how abruptly the brane slows down. (Note also that in this setup the initial velocity is $v(-\infty) = 2v_0$.) The wave equation governing the stretched strings is therefore

$$\left(\partial_t^2 + k^2 + \frac{b^2}{4\pi^2} + \frac{v_0^2}{4\pi^2}[t - \log(\cosh(t/f))]^2\right)\chi = 0\,. \tag{4.4.28}$$

It is instructive to consider the non-adiabaticity parameter $\xi \equiv \dot{\omega}/\omega^2$, where

$$4\pi^2\omega^2(t) = 4\pi^2 k^2 + b^2 + v_0^2[t - \log(\cosh(t/f))]^2. \tag{4.4.29}$$

Let us first take $f \ll 1$, which is the case of very rapid deceleration. In this limit the deceleration is concentrated at $t = 0$, so that for slightly later times, when the branes have come to a halt, we have $\xi = 0$ and hence no particle production. Comparing this scenario to that of branes moving with uniform velocity $2v$ and no deceleration, we see that an abrupt stop reduces the effective time available for particle production by a factor of two. Thus, for branes which come to a halt very rapidly, the total number of particles produced is approximately half the number produced when the branes move with uniform velocity.

We can analytically solve the problem in the opposite limit of very gentle deceleration, $f \gg \sqrt{4\pi^2k^2 + b^2}/v_0$. Using the steepest descent method to determine the Bogoliubov coefficients [111,39,112] and observing that in this limit there is a branch point very near the imaginary axis, at $-i\sqrt{4\pi^2k^2 + b^2}/v_0$, we find

$$|\beta_k|^2 = \exp\Big(-\frac{1}{2v_0}(4\pi^2k^2 + b^2)\Big). \tag{4.4.30}$$

This coincides with the exact result for the constant-velocity problem with velocity $v(t) = v_0$. However, as we already noted, in the present case the initial velocity is $v(-\infty) = 2v_0$. Our very simple conclusion is that this gradually decelerating trajectory leads to the same amount of string production as an unaccelerated trajectory in which the branes move at a uniform velocity which is smaller by a factor of two. The effective velocity, for purposes of particle production, is thus the average velocity $\frac{1}{2}(v(-\infty) + v(\infty))$.

We conclude that very gradual deceleration results in significantly reduced string production. In particular, comparing the limits of large and small f, we see that the reduction in number density is much greater for gradual than for rapid deceleration.

The above result applies to nonrelativistic motion. The string computation which would be analogous to the annulus partition function but incorporate deceleration is considerably more complicated. In particular, the acceleration of the branes breaks conformal invariance, so it is difficult to use conventional techniques to compute the string production in this case.

Fortunately, it is possible to estimate the stopping length without an exact result for the string production during deceleration. The simple argument relies only on energetics and on the constant-velocity result (4.3.14).

Suppose that open string production slows a moving brane, bringing it from an initial kinetic energy $E_i = \gamma_i T_p$ to an energy (at the point of closest approach) $E_f = \gamma_f T_p$, where γ_i, γ_f are the usual relativistic factors. The stopping length, defined again by $E_i = E_{open}(r_*)$, is easily seen to be

$$r_* \approx \frac{2\pi\eta E_i}{\nu_{open}} = \sqrt{2}\eta^{3/2}\frac{E_i}{E_i - E_f}, \tag{4.4.31}$$

where we have used (4.4.14),(4.4.15).

Consider first the case $\gamma_f \gg 1$. If the stopping length is large compared to the size r_{nad} of the nonadiabatic region, $r_* \gg \eta^{3/2}$, then the branes are moving quickly as they leave the region of nonadiabaticity. This means that the result (4.3.14) applies directly, and we return to an apparent inconsistency: the open string energy is large compared to the initial energy. This is a clear signal that the stopping length cannot be much larger than $r_{nad} \sim \eta^{3/2}$.

A stopping length of order $\eta^{3/2}$ or smaller is indicative of strong trapping: the branes come to rest around the time that the nonadiabaticity grows small, which means that a few strings are still being produced.

On the other hand, in the case $\gamma_f \sim 1$, we have $E_f \ll E_i$, so that (4.4.31) yields the stopping length $r_* \approx \sqrt{2}\eta^{3/2}$.

We conclude that no matter how the deceleration affects open string production, if the only process acting to slow the branes is loss of energy to open strings, then the stopping length is no more than of order $\eta^{3/2}$, i.e. the size of the nonadiabatic region. Thus, the trapping is very strong: very little stretching is required before the branes are brought to rest.

Given a good estimate of the open string production along a decelerating path, we could give a more accurate estimate of the stopping length. However, we have just demonstrated through energetics and the result (4.3.14) that in any event this stopping length is no larger than $\eta^{3/2}$. In fact, we expect that it is actually considerably smaller than this, as suggested by (4.4.26).

It remains a possibility that loss of energy through closed string radiation could modify this result. We now proceed to show that this is not the case.

4.5 Further Considerations

4.5.1 Production of Closed Strings

By incorporating the effects of open string production we have seen that relativistic D-branes decelerate abruptly as they pass each other. This deceleration will lead to radiation of closed strings, in a process analogous to bremsstrahlung. This drains energy from the brane motion, and, unlike the transfer of energy into stretched open strings, this energy is forever lost from the brane system. Closed string radiation therefore serves to increase the inelasticity of a brane collision. Now, the end state of a near-miss is a spinning 'remnant', i.e. two D-branes orbiting rapidly around each other, connected by a high density of strings. Loss of energy and angular momentum to closed string radiation will swiftly reduce the rotation of this remnant, at least until the velocities become nonrelativistic.

One potential worry is that the energy loss to radiation might be so large that the quantity of open strings produced during a near-miss is quite small, leading to weak trapping and a large stopping length. This is an example of the more general concern that string production could be highly suppressed if any other effect caused the branes to decelerate to nonrelativistic speeds before reaching each other. We will show that the radiation of massless closed strings can be energetically significant but, even so, does not alter our conclusion that the stopping length is not large in string units.

To estimate the energy emitted as massless closed strings, we will make use of the close analogy of this process to gravitational bremsstrahlung [113] and to gravitational synchrotron radiation [114]. Of course, one of the massless closed string modes is the graviton, but we also expect radiation of scalars, including the dilaton and, when present, the compactification moduli. Even so, it will not be at all difficult to convert results from general relativity to the case at hand, because in practice, relativists often use the far simpler scalar radiation to estimate the basic properties of gravitational radiation. We will do the same.

Consider a small mass m moving rapidly past a large mass M in a path which is, to first approximation, a straight line. A burst of gravitational radiation will be emitted in a very short time, at the moment of closest approach. This is called gravitational bremsstrahlung. The peak radiated power is approximately [113]

$$P \sim \frac{G^3 M^2 m^2}{b^4} \gamma^4 \qquad (4.5.1)$$

where G is the Newton constant, b is the impact parameter, and γ is the relativistic factor. For the remainder of this section we omit numerical prefactors: it will suffice to have the dimensional factors and the powers of γ.

The case of interest to us is extremely strong binding by open strings, for if the acceleration caused by the open strings is small then the closed string radiation should not play a key role, and the argument for trapping given in §4.4.3 suffices. Thus, we model the brane scattering by a gravitational scattering event in which the impact parameter is not much larger than the Schwarzschild radius of the larger mass. This gives

$$P \sim \frac{Gm^2}{b^2}\gamma^4. \tag{4.5.2}$$

Another useful case is that of gravitational synchrotron radiation from a mass m moving in a circular orbit with period ω_0. The power is [114]

$$P \sim Gm^2\omega_0^2\gamma^4 \sim \frac{Gm^2}{b^2}\gamma^4 \tag{4.5.3}$$

where we have identified the inverse frequency with the minimum expected orbital radius, which is of order the impact parameter. This result will be very useful for understanding the decay of the initial circular orbit.

Furthermore, one can directly compute, in the supergravity limit, the radiation from an accelerated D-brane. The result for circular motion with radius b is [115]

$$P = \frac{Gm^2}{b^2}\gamma^4 \tag{4.5.4}$$

The results (4.5.3),(4.5.2), and (4.5.4) are thus in good agreement.

Knowing now the power lost to closed strings for a given decelerating trajectory, we also wish to compute the quantity of open strings which would be required to produce this trajectory. Stated more generally, given an object being accelerated by an external force, we are interested in the ratio of the radiated power to the power associated with the driving force. For an accelerating electron this is a textbook problem; see e.g. [116], chapter 14.

The result is that there is a characteristic length $L_e = \frac{2}{3}\frac{e^2}{mc^2}$ governing radiation by electrons, and unless an electron's energy changes by of order its rest energy during acceleration over a distance of order L_e, the radiation is negligible compared to the external power. More specifically,

$$\frac{E_{radiated}}{E_{driving}} \equiv \Omega_e \approx \frac{\Delta E}{\Delta x}\frac{L_e}{mc^2} \tag{4.5.5}$$

where the total change in energy, from all causes, is ΔE over a distance Δx.

One can readily estimate the corresponding characteristic length L_D for massless closed string radiation from a D-brane by comparing to the power (4.5.4). The outcome is that $L_D \sim g_s l_s$.

Let us now consider a brane whose initial kinetic energy is $E_i = \gamma_i T_p$, where $\gamma_i \gg 1$. Suppose that the brane decelerates over a distance Δx to a new kinetic energy $E_f = \gamma_f T_p$, $\Delta\gamma \equiv \gamma_i - \gamma_f$. The 'driving force' here is loss of energy through open string production; we will now compare this to the energy lost to radiation.

$$\Omega_D \equiv \frac{E_{closed}}{E_{open}} \approx \frac{\Delta E}{T_p}\frac{L_D}{\Delta x} = g_s \Delta\gamma \frac{l_s}{\Delta x} \tag{4.5.6}$$

If $\Omega_D \ll 1$ then our previous conclusions hold automatically, as the closed strings are energetically negligible. If $\Omega_D \gg 1$, there are two cases to consider. First, if $\gamma_f \sim 1$, so that $\Delta\gamma \sim \gamma_i \gg 1$, the branes have slowed down to nonrelativistic motion. In this case the energy in open strings can be estimated to be

$$E_{open} \approx \frac{\Delta E}{\Omega_D} \approx \frac{T_p}{g_s}\frac{\Delta x}{l_s}. \tag{4.5.7}$$

To arrive at this rough estimate we did not need the Bogoliubov coefficients derived from the annulus amplitude; we have used instead the fact that the external driving force (open string production) can be determined based on the postulated trajectory. Proceeding to estimate the stopping length, we find

$$\frac{r_*}{l_s} \approx \frac{2\pi\eta E_f}{\nu_{open}} \approx \frac{\sqrt{2}\eta^{3/2} E_f}{E_{open}} \ll \frac{\eta^{3/2} T_p}{E_{open}} = \eta^{3/2} g_s \frac{l_s}{\Delta x}. \tag{4.5.8}$$

The distance Δx is roughly order $\eta^{3/2}$, because that is the size of the nonadiabatic region in which open strings are created. (Two branes approaching each other will begin to decelerate when they enter this region.) To make a very conservative estimate, however, we will use $\Delta x \gtrsim l_s$. Then, because we are working at weak string coupling, the stopping length is

$$\frac{r_*}{l_s} \ll \eta^{3/2} g_s \frac{l_s}{\Delta x} \ll \eta^{3/2} \tag{4.5.9}$$

so that the stopping length is *much* smaller than $\eta^{3/2} l_s$.

The second case is $\Omega_D \gg 1, \gamma_f \gg 1$, so that the brane is moving relativistically even after decelerating, and the relative velocity is large when the branes pass each other. Our general conclusion will be invalid only if the branes do not rapidly trap in this final case. However, if the branes separate to a considerable distance while moving rapidly, our annulus amplitude computation of open string production applies directly. In other words, by assuming that the branes can separate, we are arranging that they leave the region of nonadiabaticity, so that the number density of open strings is accurately given by (4.3.14), and the trapping length by (4.4.26). Thus, the assumption that the branes separate at high speed is not consistent.

We conclude that closed string emission can slow the motion of the brane pair, but it does not substantially increase the stopping length. In fact, radiation helps considerably to bring the branes to rest: once the branes are trapped and are spiraling around each other, rapid radiation losses will slow their rotation. This is enhanced by the familiar fact that, for relativistic objects, radiation losses are greater in circular motion than in rectilinear accelerated motion. Once the branes are trapped they slow down through this closed string synchrotron radiation. From the power (4.5.3) we conclude that the branes lose energy so rapidly that they would require only a few orbits to come to rest. In practice the spin-down process is prohibitively complicated, but this result suffices to show that the lifetime of the highly-excited, rapidly revolving remnant is in any case very short.

One important additional point is that the closed string radiation is strictly negligible only when the coupling is so small that the branes are rather heavy, and hence stretch the open strings farther before stopping. There is consequently a tradeoff between computability and control, which are best at extremely weak coupling, and the strength of the trapping, which is best for couplings above the bound (4.4.27). It is essential to recognize that for any nonzero coupling, the collision is inelastic and trapping eventually does occur; however, the stopping length increases when the coupling grows very small.

A further question which we have not addressed is the production of massive closed strings. In the case of very abrupt deceleration we would expect nonvanishing production of these modes. We will leave a precise computation of this effect within string theory as an interesting problem for future work.

For the present analysis, we can make a very crude estimate of massive string production by using a result on the spectrum of gravitational synchrotron radiation. For a mass in an orbit with period ω_0, the power per unit frequency is [114]

$$\frac{dP}{d\omega} \propto \exp\left(-\frac{\omega}{\omega_{crit}}\right) \tag{4.5.10}$$

where $\omega_{crit} = \frac{6}{\pi}\gamma^2\omega_0$. Thus, for $\omega_{crit} \ll l_s^{-1}$, massive closed strings should play a negligible role, but when gravitons of frequency l_s^{-1} are being produced, it is natural to expect massive modes as well. We therefore expect some emission of massive closed strings in processes where $\gamma^2 \gg \frac{b}{l_s}$. This will further increase the rate of energy loss from the revolving brane pair, speeding the trapping and increasing the effective inelasticity of the collision.

4.5.2 Summary of the Argument

For clarity, we will now briefly review our argument that the trapping of relativistic D-branes is powerful and abrupt.

The annulus partition function for open strings between moving D-branes indicates that the density of produced open strings is given, in the relativistic limit, by (4.3.14). The characteristic impact parameter below which the backreaction of these strings is strong can then be seen to be $b_{crit} \sim \eta l_s$. If the D-branes are assumed to separate to a distance larger than of order $\eta^{3/2}$, they have left the region of nonadiabaticity, so that (4.3.14) applies. The energy (4.4.15) in open strings then exceeds the initial brane energy, so that the assumption of significant separation was inconsistent.

The same argument applies when closed string radiation is taken into account. A straightforward estimate of the energy lost to radiation over a distance $\eta^{3/2}$ shows that the energy transferred to open strings is still sufficient to stop the branes before their separation exceeds $\eta^{3/2}$.

We expect that a detailed computation of the string production along a decelerating trajectory would show that the stopping length is at most of order b, which can be much smaller than $\eta^{3/2}$. In particular, we expect that in a head-on collision with negligible impact parameter the stopping length would be of order the string length. However, estimates involving (4.3.14) are strictly valid only when the branes eventually leave the window of nonadiabaticity, leading to the *very* conservative estimate $r_* \sim \eta^{3/2} l_s$.

A few potential objections remain. First of all, one might worry that the branes somehow slow down before reaching each other, so that at the moment of closest approach the velocities are nonrelativistic. In this case excited open strings would not be produced and we would simply have field theory trapping. We have already explained in §4.4.3 that if the branes slow down exclusively due to open string production, then they will still experience rapid trapping. Then, we showed in §4.5.1 that additional loss of energy through closed string radiation also does not ruin the trapping.

A final worry is that the branes could interact by creating string pairs at extremely high excitation levels. A vanishingly small number density of arbitrarily highly excited strings (with level much higher than η^2) could absorb all the initial kinetic energy and yet not generate a strong attractive force between the branes. However, we have seen that in fact string production peaks around level $n_{peak} \approx \eta^2(2\pi^2)^{-1}$, which is sufficiently small to ensure that the trapping is strong.

We therefore conclude that D-branes in relativistic motion generically trap each other through copious production of open strings, with a trapping length no larger than the size $\eta^{3/2} l_s$ of the nonadiabatic region. A sizeable fraction of the initial energy is eventually emitted in the form of massless closed string radiation.

The limitations to our argument which we have discussed above make it challenging to precisely and controllably compute the stopping length in an ultrarelativistic D-brane collision. However, these issues, and others – such as massless and massive closed string radiation, annihilation of the produced strings, and dilution of the produced strings in a cosmological background – do not in any way weaken our argument that the brane collision is inelastic. In fact, it is easy to see that radiation, annihilation, and dilution all extract energy from the brane system, slowing the brane motion. (See [18] for an analysis of these issues in the nonrelativistic context.) Happily, for applications to cosmological models, it is the inelasticity rather than the stopping length which is most immediately relevant.

4.5.3 Regime of Validity and Control

We will now examine the characteristics of the trapping process as a function of the dimensionless parameters g_s, b, η.

First of all, we will never work at strong string coupling ($g_s > 1$), since then we would have to include higher string loop effects. Furthermore, at strong coupling the D-branes become very light, and their Compton wavelength λ_D grows. We require $\lambda_D \ll b$ so that we can neglect these quantum effects.

Secondly, we should require that the Schwarzschild radius R_s of the D-brane is negligibly small compared to b. To estimate this, we treat the Dp-brane as a point source in $10-p$ dimensions. The black hole solution in $(10-p)$ dimensions for a p-dimensional extended object of tension T and zero charge is [117]

$$T = \left(\frac{8-p}{7-p}\right)\frac{R_s^{7-p}}{(2\pi)^7 d_p g_s^2} \tag{4.5.11}$$

where $d_p = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right)$.

We are interested in the limit of zero charge because the highly-boosted branes have far greater effective mass than the BPS bound requires. Note that in fact the metric for one of these moving branes is of a shock-wave form, not a static black hole. We are imagining that the branes collide inelastically and then asking whether the Schwarzschild radius of the excited remnant, seen in the center of mass frame, is comparable to the initial impact parameter.

In this scheme, the effective tension is the center-of-mass energy $2T_p\gamma \approx T_p e^{\eta/2}$. We therefore find, using the tension of a p-brane,

$$\left(\frac{R_s}{l_s}\right)^{7-p} = g_s\left(\frac{7-p}{8-p}\right)(2\pi)^{7-p} d_p e^{\eta/2} \tag{4.5.12}$$

from which we conclude that for $p < 7$, the Schwarzschild radius can be made parametrically less than any given impact parameter by reducing the string coupling.

Let us now fix b and η and take the string coupling to be small enough so that string loops, the brane Schwarzschild radius, and the brane Compton wavelength can be neglected. As we further decrease the coupling, the brane becomes heavier and the stopping length becomes greater. Now, recall that when we examined the open string production along a constant-velocity path, we found an energetic inconsistency: unless the coupling was exponentially small, the open string energy exceeded the initial kinetic energy of the system. Of course, deceleration reduces string production, so for any controllable coupling the energy in open strings will not exceed the initial energy. However, we can still define a value of g_s at which the energetics is consistent even before we incorporate the deceleration which arises from backreaction. Comparing (4.4.23) and (4.4.24), we find that the energetics are automatically consistent provided that

$$g_s < 2^{3/2}\pi^{p/2-3}\eta^{\frac{7-p}{2}}e^{-\eta/2}. \tag{4.5.13}$$

Thus, only for exponentially small string coupling are the branes so heavy that they stretch the open strings substantially before coming to rest.

4.6 Discussion

We have argued that the relativistic scattering of Dp-branes, $p > 0$, at small impact parameters is almost completely inelastic as a result of pair production of excited open strings. The time-dependence induces production of an extremely high density of highly-excited, stretched open strings, which rapidly draw the branes into a tight orbit. The resulting acceleration results in significant closed string radiation, which acts to further brake the motion.

Powerful stringy trapping of this sort occurs whenever the impact parameter, measured in string units, is small compared to the rapidity η. This is a much larger range of distances than that controlled by collision of the stringy halos of the two branes, whose radius grows as $\sqrt{\eta}$. Moreover, the strength of this stringy trapping was a surprise: it does not follow from summing the low-velocity result of [18] over the string spectrum. Instead, the velocity-dependence of stretched string masses enters in a crucial way to enhance the production effect.

Our result, which is essentially a simple observation about the quantum-corrected dynamics of D-branes, has obvious implications for scenarios involving branes in relativistic motion. One example[15] is the stage of reheating in cosmological models with fast-moving branes and antibranes. Brane-antibrane inflation models typically end with the condensation of the open string tachyon, leaving a dust of closed strings in the bulk as well as excited open strings on any remaining branes [118]. Despite much effort, this process is not fully understood [119]. Suppose, however, that the antibrane is moving relativistically toward the end of its evolution, and then passes by or collides with a stack of branes. (Ultrarelativistic brane motion is natural in the DBI models [120,121], for example, and could occur elsewhere.) In this case tachyon condensation governs only a small fraction of the energy released; most of the kinetic energy goes into open string pair production. Thus, reheating in such a model proceeds by stringy trapping (for related work, see [122]).

More speculatively, moduli trapping may be a useful mechanism for vacuum selection [18], as it gives a dynamical explanation for the presence of enhanced symmetry. (See also [123,124] for related work on moduli dynamics in string/M theory.) The stringy trapping presented here extends the trapping proposal not

[15] We are grateful to S. Kachru for suggesting this.

just to a new parameter range, but to a regime where the strength of the effect increases dramatically.

The inelasticity of D-brane scattering may be viewed as a calculable example of a more general question: to what extent do particle, string, and brane production affect motion toward or away from a given 'singular' configuration? Time-dependent orbifolds [125,126,127,128,129,130,131] (see also [132] and references therein) provide a relatively tractable setting for such a question. Berkooz and Pioline [130] and Berkooz, Pioline and Rozali [131] have emphasized the possibility of resolving a spacelike singularity through the pair production and condensation of winding strings. It would be very interesting to extend these results and repair more general spacelike singularities through the production of branes or strings; see [133] for work in this direction. Our analysis suggests that string production could be surprisingly important in such a setting.

Another interesting open question is whether the inelasticity of quantum-corrected D-brane collisions can be used to place bounds on the elasticity of other sorts of collisions. In the cyclic universe model [134,135], the orbifold boundaries of heterotic M-theory [136] approach each other and collide. An intrinsic assumption of these cyclic models is that the collision is very nearly elastic; this is essential to make possible a large number of collisions and the associated cyclic behavior. Our result makes it plain that D-brane collisions, which appear elastic classically, are highly inelastic when the quantum effects associated to fundamental strings are included.

In the cyclic model, the M2-branes stretched between the boundaries become tensionless at the instant of collision. In the weakly-coupled four-dimensional description these objects are heterotic strings whose tension, in four-dimensional Planck units, goes to zero at the moment of impact. Because the masses vary rapidly during the collision, the nonadiabaticity is large and we expect copious production of these strings. It would be extremely interesting to compute the energy loss through this string/membrane production and to understand the implications for the cyclic models [137].

We should point out that in the most realistic cyclic models, the brane velocities are required, for phenomenological reasons, to be nonrelativistic.[16] The results

[16] We are grateful to P. J. Steinhardt for helpful discussions on this point.

in this chapter appear to give an independent upper bound on the velocity of the branes before collision – this bound is one which is required for the self-consistency of the model, rather than one imposed by observational requirements. However, this argument is qualitative at present; an explicit extension of our results involving stretched fundamental strings to the case of stretched membranes would be nontrivial.

Another interesting application would be to investigate inelasticity in the relativistic dynamics of networks of cosmic strings [138].

4.A Masses of Strings Between Moving Branes

In this appendix we provide a consistency check of our result (4.4.3) for the mass of an open string stretched between moving branes. We motivated this result with several independent arguments for a rescaling of the string tension. We will now show that precisely for this value (4.4.3) of the mass, our WKB estimate of string production (4.2.6) is consistent with the complete string result (4.4.7) in their regime of common validity.

In order to compare these two results, we need to work in a regime where the WKB result is reliable. We therefore require that the occupation numbers of all string states are small, i.e. we work in the nearly-adiabatic limit. By examining the full string result (4.4.7) we see that this can be achieved by requiring that $b^2 \gg \eta$. Moreover, we are interested in the relativistic limit $\eta \gg 1$.

We can use the result of §4.2 to conclude that if the time-dependent χ mass is given by

$$m^2(t) = \frac{C^2(v)}{4\pi^2}(b^2 + v^2t^2) \tag{4.A.1}$$

then the number of produced χ particles is

$$\nu \propto \exp\left(-\frac{C(v)b^2}{2v}\right) . \tag{4.A.2}$$

Requiring consistency with (4.4.7) we see that

$$C(v) = v/\eta \, , \tag{4.A.3}$$

exactly matching our proposed formula (4.4.3).

This analysis allows us to determine the velocity-dependence of that part of the string energy which comes from stretching, i.e. the second term in (4.4.3). The necessity of working in the nearly-adiabatic limit prevents us from extracting the velocity-dependence of the string oscillator energy, which is the first term in (4.4.3). However, we view the exact agreement between (4.A.3) and the energy resulting from the rescaling of the string tension (4.4.2) as compelling evidence that the rescaled tension correctly encodes the properties of this system. We therefore propose that the oscillator energy is likewise given by the same rescaling of the tension as in (4.4.2). This leads to (4.4.3).

4.B Theta Function Identities

In this section we collect various identities about the elliptic theta functions. Because of the existence of several canonical notations for these functions, we define the functions as used in the chapter.

The theta functions are often expressed in terms of the variables ν and τ, or in terms of the nome $q = \exp(2\pi i\tau)$ and $z = \exp(2\pi i\nu)$. The four theta functions are written down below in both their series and product forms:

$$\begin{aligned}
\theta_{00}(\nu,\tau) = \theta_3(\nu|\tau) &= \sum_{n=-\infty}^{n=\infty} q^{n^2/2} z^n = \prod_{m=1}^{\infty} (1-q^m)(1+zq^{m-1/2})(1+z^{-1}q^{m-1/2}) \\
\theta_{01}(\nu,\tau) = \theta_4(\nu|\tau) &= \sum_{n=-\infty}^{n=\infty} (-1)^n q^{n^2/2} z^n \\
&= \prod_{m=1}^{\infty} (1-q^m)(1-zq^{m-1/2})(1-z^{-1}q^{m-1/2}) \\
\theta_{10}(\nu,\tau) = \theta_2(\nu|\tau) &= \sum_{n=-\infty}^{n=\infty} q^{(n-1/2)^2/2} z^{n-1/2} \\
&= 2e^{\pi i\tau/4}\cos(\pi\nu) \prod_{m=1}^{\infty} (1-q^m)(1+zq^m)(1+z^{-1}q^m) \\
\theta_{11}(\nu,\tau) = -\theta_1(\nu|\tau) &= -i \sum_{n=-\infty}^{n=\infty} (-1)^n q^{(n-1/2)^2/2} z^{n-1/2} \\
&= -2e^{\pi i\tau/4}\sin(\pi\nu) \prod_{m=1}^{\infty} (1-q^m)(1-zq^m)(1-z^{-1}q^m) \,.
\end{aligned} \tag{4.B.1}$$

In addition to the theta functions, we shall also need the Dedekind eta function:

$$\eta(\tau) = q^{1/24} \prod_{m=1}^{\infty} (1 - q^m) = \left[\frac{\partial_\nu \theta_{11}(0,\tau)}{-2\pi}\right]^{1/3} . \tag{4.B.2}$$

These functions have the following modular transformation properties:

$$\begin{aligned}
\theta_{00}(\nu/\tau, -1/\tau) &= (-i\tau)^{1/2} \exp(\pi i \nu^2/\tau)\theta_{00}(\nu,\tau) \\
\theta_{01}(\nu/\tau, -1/\tau) &= (-i\tau)^{1/2} \exp(\pi i \nu^2/\tau)\theta_{10}(\nu,\tau) \\
\theta_{10}(\nu/\tau, -1/\tau) &= (-i\tau)^{1/2} \exp(\pi i \nu^2/\tau)\theta_{01}(\nu,\tau) \\
\theta_{11}(\nu/\tau, -1/\tau) &= -(-i\tau)^{1/2} \exp(\pi i \nu^2/\tau)\theta_{11}(\nu,\tau) \\
\eta(-1/\tau) &= (-i\tau)^{1/2}\eta(\tau) .
\end{aligned} \tag{4.B.3}$$

We will often need the asymptotic behavior of the theta and eta functions. When $q \ll 1$ we can immediately find the asymptotics using the above expansions, whereas for $q \to 1$ we must first perform a modular transformation.

The asymptotic behavior of a particular combination will be especially helpful. Define the fermionic partition function $Z(\tau) \equiv \frac{1}{2}\theta_{10}^4(0|\tau)\eta(\tau)^{-12}$. Then for $-i\tau \equiv s \gg 1$ we have

$$Z(is) = 8 + \mathcal{O}(e^{-2\pi s}) \tag{4.B.4}$$

whereas for $s \ll 1$ we find, using the modular transformations above,

$$Z(is) = \frac{1}{2} s^4 \exp\Big(\frac{\pi}{s}\Big)\Big(1 + \mathcal{O}(e^{-\frac{\pi}{s}})\Big) \tag{4.B.5}$$

We will also need a few identities involving the theta functions:

$$\begin{aligned}
&\theta_{00}^4(0,\tau) - \theta_{01}^4(0,\tau) - \theta_{10}^4(0,\tau) = 0 \qquad \theta_{11}(0,\tau) = 0 \\
&\prod_{a=1}^4 Z_0^0(\phi_a, it) - \prod_{a=1}^4 Z_1^0(\phi_a, it) - \prod_{a=1}^4 Z_0^1(\phi_a, it) - \prod_{a=1}^4 Z_1^1(\phi_a, it) = 2\prod_{a=1}^4 Z_1^1(\phi'_a, it) ,
\end{aligned} \tag{4.B.6}$$

where

$$\begin{aligned}
Z_\beta^\alpha(\phi, it) &= \frac{\theta_{\alpha\beta}(i\phi t/\pi, it)}{\exp(\phi^2 t/\pi)\eta(it)} \\
\phi'_1 &= \frac{1}{2}(\phi_1 + \phi_2 + \phi_3 + \phi_4) \qquad \phi'_2 = \frac{1}{2}(\phi_1 + \phi_2 - \phi_3 - \phi_4) \\
\phi'_3 &= \frac{1}{2}(\phi_1 - \phi_2 + \phi_3 - \phi_4) \qquad \phi'_4 = \frac{1}{2}(\phi_1 - \phi_2 - \phi_3 + \phi_4) .
\end{aligned} \tag{4.B.7}$$

The identity (4.B.6) leads in the case $\phi_2 = \phi_3 = \phi_4 = 0$ to

$$2\theta_{11}^4(\nu/2,\tau) = \theta_{00}(\nu,\tau)\theta_{00}^3(0,\tau) - \theta_{01}(\nu,\tau)\theta_{01}^3(0,\tau) - \theta_{10}(\nu,\tau)\theta_{10}^3(0,\tau). \tag{4.B.8}$$

5. Heterotic Moduli Stabilization

ABSTRACT OF ORIGINAL PAPER

We show that fractional flux from Wilson lines can stabilize the moduli of heterotic string compactifications on Calabi-Yau threefolds. We observe that the Wilson lines used in GUT symmetry breaking naturally induce a fractional flux. When combined with a hidden-sector gaugino condensate, this generates a potential for the complex structure moduli, Kähler moduli, and dilaton. This potential has a supersymmetric AdS minimum at moderately weak coupling and large volume. Notably, the necessary ingredients for this construction are often present in realistic models. We explore the type IIA dual phenomenon, which involves Wilson lines in D6-branes wrapping a three-cycle in a Calabi-Yau, and comment on the nature of the fractional instantons which change the Chern-Simons invariant.

5.1 Introduction

When string theory is compactified on a Calabi-Yau manifold [139], the resulting low-energy field theory typically contains some number of massless scalar fields, or moduli. Gravitational experiments and the requirement of consistency with nucleosynthesis place rather strong constraints on the existence of such fields (see e.g. [140]). If moduli were an essential feature of all string compactifications then model building would be very difficult. Fortunately, moduli are only endemic in the simplest, most symmetric constructions. General backgrounds involving fluxes, as well as nonperturbative effects, tend to create potentials for some or all moduli. Even

---

This chapter is reprinted from Sergei Gukov, Shamit Kachru, Xiao Liu, and Liam McAllister, "Heterotic Moduli Stabilization with Fractional Chern-Simons Invariants," Phys.Rev. D**69** (2004) 086008, by permission of the publisher.

so, although compactifications with reduced moduli spaces are easy to construct, it remains challenging to eliminate all of the moduli in a given model.

Two fields which have proven particularly difficult to stabilize are the Calabi-Yau volume and, in heterotic compactifications, the dilaton. The problem is especially acute in these cases because the dilaton and volume directly influence the gauge and gravitational couplings in our world, making rolling values unacceptable. Moreover, as these parameters govern the string and sigma-model perturbation expansions, a controllable compactification requires that the dilaton and volume be stabilized at weak coupling and large radius.

We will demonstrate that this can be achieved in a certain class of heterotic compactifications on Calabi-Yau spaces with a large fundamental group. The context for this proposal is the original work [21] of Dine, Rohm, Seiberg, and Witten, who observed that the combination of a gaugino condensate (in the hidden-sector of the $E_8 \times E_8$ heterotic string) and a background three-form flux generates a potential for the dilaton but leaves the cosmological constant zero at tree level.[17] As was understood there and in more detail in subsequent work, because of the quantization condition for the three-form of the heterotic theory, the dilaton cannot be fixed at weak coupling. The essential difficulty is that the gaugino condensate term is nonperturbatively small when the coupling is weak, whereas quantization forces the flux term to be of order one. The resulting potential drives the dilaton to strong coupling.

It is important, however, that the Chern-Simons contribution to the heterotic three-form flux does not obey the same quantization condition as the contribution from the field strength of the antisymmetric tensor. In fact, as we will explain below, the Chern-Simons contribution of a flat gauge bundle can take fractional values of order $1/N$, where N is related to the order of the fundamental group. On Calabi-Yau manifolds with sufficiently large fundamental group this provides a natural mechanism to stabilize the dilaton at weak coupling. The same effect stabilizes all Kähler moduli once the dependence of the gauge coupling on these moduli is correctly incorporated. For related earlier work see [142,143,144,145,146,147,148].

The requirements that the Calabi-Yau manifold should have non-trivial fundamental group and that the gauge bundle should have nonzero Wilson lines are

[17] Closely related simultaneous work appears in [141].

actually well motivated by other model-building considerations. In fact, most models of particle physics based on Calabi-Yau compactifications of the heterotic string involve manifolds with non-trivial fundamental group and associated gauge bundles with Wilson lines.

A standard way to construct such manifolds is to quotient a simply-connected Calabi-Yau space by a freely-acting discrete symmetry group G. The resulting string GUT model solves a number of important problems. For instance, in simple constructions the number of generations is divided by $|G|$, leading to models with realistically low numbers of generations [139]. Moreover, one can naturally solve the doublet-triplet splitting problem [149,150] in this setting.

More importantly, the non-trivial fundamental group allows us to introduce Wilson lines. In addition to being an attractive method of GUT symmetry breaking, Wilson lines are actually indispensable, as standard heterotic string models do not admit adjoint Higgses of the GUT group [149].

We will add the stabilization of moduli to this list of problems which admit natural solutions on Calabi-Yau manifolds with non-trivial fundamental group and non-trivial gauge connection. The dilaton, Kähler moduli, and complex structure moduli can all be stabilized by incorporating the effects of gaugino condensation and the flux induced by the Wilson lines.

We would like to underscore the happy coincidence that the necessary ingredients for our construction are automatically present in certain realistic models. Wilson lines typically lead to Chern-Simons flux, as we will explain in §5.3.3. Thus, heterotic string GUT models with Wilson-line symmetry breaking often have a background flux and an associated constant term in the superpotential. To the best of our knowledge the consequences of this term have not been well explored in the literature. In a restricted subset of models, namely those with hidden-sector gaugino condensation and very small Chern-Simons flux, the effect is dramatic: the moduli can be fixed, in a controllable regime, by the mechanism we are proposing.

The organization of this chapter is as follows. In §5.2 we review basic facts about the relevant supergravity Lagrangians in ten and four dimensions, and about the superpotential generated by gaugino condensation in the hidden E_8. In §5.3 we review the quantization conditions on three-form flux and describe how fractional flux can arise in the presence of flat connections with fractional Chern-Simons invariant. In §5.4 we describe how the fractional flux of §5.3 can be combined with gaugino condensation to stabilize the dilaton at weak coupling, along with the complex structure moduli. In §5.5 we include loop corrections and show that it becomes possible to simultaneously stabilize the Kähler moduli as well as the dilaton; this requires more restrictive assumptions about the choices of gauge bundles. We observe that a strong coupling transition naturally arises in this setting and we provide a toy model which illustrates the smoothness of this transition. In §5.6 we discuss some basic aspects of the dual descriptions of our story, including the dual type IIA theories with wrapped D6-branes. In §5.7 we explore the nature of the domain walls which interpolate between configurations with distinct fractional Chern-Simons invariants. We conclude with a discussion of possible extensions and broader issues in §5.8.

As this work was being finalized, three papers which have some overlap with our results appeared [151,152,153].

5.2 Gaugino Condensation in the Heterotic String

In this section we review the structure of the heterotic string low-energy effective Lagrangian, with particular attention to terms coupling the heterotic three-form flux, H, to the gauginos. In §5.2.1 we fix notation by presenting the low-energy action for the heterotic string in ten dimensions. We dimensionally reduce this action on a Calabi-Yau threefold and describe the potential appearance of a gaugino condensate in the resulting $\mathcal{N}=1, d=4$ configuration. In §5.2.2 we show how to derive the four-dimensional action of §5.2.1 from a simple superpotential induced by the flux and the gaugino condensate. In §5.2.3 we explain that the dilaton potential does not have a minimum at finite coupling unless the background flux is fractional.

5.2.1 Effective Lagrangian for the Heterotic Theory

The low-energy effective action for the heterotic string in ten-dimensional Einstein frame is [154]

$$S = \frac{1}{2\alpha'^4}\int d^{10}x\sqrt{-g_{10}}\left(\mathcal{R}_{10} - \frac{1}{2}\partial_A\phi\partial^A\phi - \frac{1}{12}e^{-\phi}\left(H_{ABC} - \frac{\alpha'}{16}e^{\frac{\phi}{2}}\overline{\chi}\Gamma_{ABC}\chi\right)^2 \right.$$
$$\left. - \frac{\alpha'}{4}e^{-\frac{\phi}{2}}\mathrm{tr}(F_{AB}F^{AB}) - \alpha'\mathrm{tr}(\overline{\chi}\Gamma^A D_A\chi)\right) \tag{5.2.1}$$

Indices A, B run from 0...9, and μ, ν are four-dimensional spacetime indices. The internal space has real indices m, n and (anti)holomorphic indices $i, j, \bar{\imath}, \bar{\jmath}$. The Einstein-frame metric g_{10} has Ricci scalar $\mathcal{R}_{10}$, while ω is the spin connection and ϕ is the dilaton. The heterotic string has gauge field strength $F_{\mu\nu}$ and gaugino field χ; all traces are taken in the fundamental representation. The three-form flux H_{ABC} is defined by

$$H = dB - \frac{\alpha'}{4}\Big(\Omega_3(A) - \Omega_3(\omega)\Big) \tag{5.2.2}$$

where Ω_3 is the Chern-Simons three-form,

$$\Omega_3(A) \equiv \mathrm{tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right) \tag{5.2.3}$$

with a similar formula for $\Omega_3(\omega)$.

To reduce to four-dimensional Einstein frame, we use the ansatz

$$ds_{10}^2 = e^{-6\sigma} ds_4^2 + e^{2\sigma} g_{mn}^0 dy^m dy^n \tag{5.2.4}$$

where g_{mn}^0 is a fixed fiducial metric normalized to have volume $4\alpha'^3$. Although this differs from the usual convention

$$ds_{10}^2 = e^{-6(\sigma-\sigma_0)} ds_4^2 + e^{2\sigma} g_{mn}^0 dy^m dy^n \tag{5.2.5}$$

by a constant rescaling, (5.2.4) is nevertheless appropriate for a discussion of moduli stabilization, as we do not know what the vev σ_0 will be until we stabilize σ. For a similar reason, we go between ten-dimensional string and Einstein frame with the unconventional scaling $g_{MN}^S = g_{MN}^E e^{\frac{\phi}{2}}$, while one usually sees $g_{MN}^S = g_{MN}^E e^{\frac{\phi-\phi_0}{2}}$ [108]. The resulting Minkowski metric differs from the conventional $diag(-1, 1, 1, 1)$ by a constant scaling depending on the vevs of the dilaton and volume modulus. To relate dimensionful quantities here to those directly measured from experiments, one must perform an inverse rescaling. Finally, note that the gamma matrices built from the metric scale with e^σ.

Let us decompose the ten-dimensional Majorana-Weyl gaugino χ as

$$\chi = \chi_6^* \otimes \chi_4 + \chi_6 \otimes \chi_4^* \tag{5.2.6}$$

where χ_6 and χ_4 are six and four-dimensional Weyl spinors with positive chirality and χ_6 is the zero mode of the internal Dirac operator for the gaugino, with the normalization

$$\chi_6{}^{\dagger}\chi_6 = 1. \tag{5.2.7}$$

We will choose to express the action in terms of a rescaled four-dimensional gaugino λ

$$\lambda \equiv \chi_4 e^{-\frac{9}{2}\sigma + \frac{\phi}{4}} \tag{5.2.8}$$

which will give the standard kinetic term after dimensional reduction.

Coupling Constants

The four-dimensional gauge coupling is

$$g^2_{YM} \equiv e^{\varphi}. \tag{5.2.9}$$

where the four-dimensional dilaton φ is related to the ten-dimensional dilaton and volume modulus via

$$\varphi = \frac{\phi}{2} - 6\sigma \tag{5.2.10}$$

Another important scalar field of the four-dimensional theory is the volume scalar[18] ρ,

$$\rho = \frac{\phi}{2} + 2\sigma \tag{5.2.11}$$

The fields φ,ρ are related to the scalar components of two $\mathcal{N} = 1$ chiral superfields S, T:

$$\begin{aligned} S &= e^{-\varphi} + ia \\ T &= e^{\rho} + ib \end{aligned} \tag{5.2.12}$$

where a and b are the axions which arise from the spacetime and internal components of B_{AB}, respectively. In particular,

$$(*da)_{\mu\nu\rho} = e^{-2\varphi} H_{\mu\nu\rho} \tag{5.2.13}$$

with an analogous relation for b.

[18] For the moment we assume that the Calabi-Yau has only one volume modulus. We will present the more general case in §5.5.2.

The holomorphic Wilsonian gauge coupling functions f_i^W (where $i = 1, 2$ runs over the two E_8 gauge groups) can be expressed in terms of S and T by

$$f_i^W = S + \beta_i T + \mathcal{O}(e^{-S}) + \mathcal{O}(e^{-T}) \tag{5.2.14}$$

where the coefficient β_i represents the one-loop correction to the gauge coupling function, and the last two terms represent nonperturbative corrections. Higher loop corrections vanish by standard holomorphy arguments, since the dilaton and radion are partnered in chiral multiplets with axions. The physical effective coupling differs from the Wilsonian coupling by wave-function renormalization and integration over the low momentum modes.

Four-dimensional Action

Combining the relations given above, we reach the dimensionally-reduced action [19]

$$S_{4d} = S_{gravity} + S_{gauge} + S_{CY} \tag{5.2.15}$$

$$S_{gravity} = \frac{2}{\alpha'} \int d^4x\sqrt{-g_4}\left(\mathcal{R}_4 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3}{2}\partial_\mu\rho\partial^\mu\rho\right) \tag{5.2.16}$$

$$S_{gauge} = \int d^4x\sqrt{-g_4}\left(-\frac{1}{2g_{YM}^2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{2}{g_{YM}^2}\mathrm{tr}(\bar{\lambda}\Gamma^\mu D_\mu\lambda)\right) \tag{5.2.17}$$

$$S_{CY} = -\frac{1}{24\alpha'^4}\int d^4x\sqrt{-g_4}e^{\varphi-3\rho}\int_X d^6y\sqrt{-g^0}\left(H_{lmn} - \frac{\alpha'}{16}e^{12\sigma}T_{lmn}\right)^2 \tag{5.2.18}$$

where we have defined

$$T_{lmn} = \mathrm{tr}\left((\chi_6{}^\dagger\bar{\lambda}_D^* + \chi_6{}^T\bar{\lambda}_D)\Gamma^0_{lmn}(\chi_6{}^*\lambda_D + \chi_6\lambda_D{}^*)\right) \tag{5.2.19}$$

and λ_D is the Dirac spinor corresponding to λ. The perfect-square interaction term (5.2.18) couples the background flux to the gauginos and therefore gives rise, as we will see in detail, to a potential for the dilaton.

---

[19] The unusual gravitational coupling $\kappa_4^2 = \frac{\alpha'}{4}$ is an artifact of our ansatz (5.2.4). The physical gravitational coupling differs from this by the constant rescaling mentioned previously.

Gaugino Condensation

Recall that in a pure $\mathcal{N}=1$ supersymmetric Yang-Mills theory in four dimensions with gauge group H, the gaugino condensate which develops at low energies is given by [155,156,30,157]:

$$\left\langle \mathrm{tr}\left(\frac{1}{2}\bar{\lambda}_D(1-\gamma_5)\lambda_D\right)\right\rangle = \langle \mathrm{tr}(\lambda_\alpha\lambda^\alpha)\rangle = 16\pi^2 M^3 \exp\left(-\frac{8\pi^2 f^W}{C_H}\right). \quad (5.2.20)$$

Here M is the ultraviolet cutoff for the gauge theory, f^W is given by (5.2.14), and C_H denotes the dual Coxeter number of H. We are interested in studying a gaugino condensate in some subgroup H of the hidden sector E_8 gauge group which arises in compactification of the $E_8 \times E_8$ heterotic string on a Calabi-Yau manifold. The appropriate ultraviolet cutoff M for a string compactification is the mass scale of Kaluza-Klein excitations,

$$M^3 = c\left(\frac{e^{-12\sigma}}{2\alpha'^{3/2}}\right) \quad (5.2.21)$$

where c is a constant of order one. Combining (5.2.20) and (5.2.21), we find that the gaugino condensate in $H \subset E_8$ satisfies

$$\langle \mathrm{tr}(\lambda\lambda)\rangle = 8\pi^2 c\left(\frac{e^{-12\sigma}}{\alpha'^{3/2}}\right)\exp\left(-\frac{8\pi^2 f^W}{C_H}\right) \quad (5.2.22)$$

5.2.2 Superpotential from Flux and a Gaugino Condensate

For a variety of reasons it will prove useful to work with a superpotential and Kähler potential from which one can reproduce the interaction (5.2.18).

One can derive the kinetic terms in (5.2.16) using the Kähler potential

$$\mathcal{K} = -\log(S+\bar{S}) - 3\log(T+\bar{T}) - \log\left(-\frac{i}{4\alpha'^3}\int \Omega\wedge\bar{\Omega}\right). \quad (5.2.23)$$

The superpotential for this system takes the form

$$W = W_{flux} + W_{condensate} \quad (5.2.24)$$

where the first term is induced by the background flux and the second term is a nonperturbative contribution arising from the gaugino condensate.

The flux-induced superpotential can be written as an integral over the Calabi-Yau space [158,159,160,161]

$$W_{flux} = \frac{2\sqrt{2}}{\alpha'^4}\int H\wedge\Omega \quad (5.2.25)$$

This superpotential leads to the following term in the scalar potential

$$V_{flux} = \frac{1}{24\alpha'^4} e^{\varphi - 3\rho} \int_X d^6 y \sqrt{-g^0} H_{lmn} H^{lmn} \qquad (5.2.26)$$

which is precisely the first term in (5.2.18). As we will explain in §5.3, the number of quanta of H-flux is roughly given by

$$h = \frac{1}{4\pi^2 \alpha'^4} \int H \wedge \Omega \qquad (5.2.27)$$

so that we may define a mass parameter μ,

$$\mu^3 = \frac{4\sqrt{2} c \pi^2}{\alpha'^{3/2}} \qquad (5.2.28)$$

in terms of which

$$W_{flux} = \left(\frac{2\mu^3}{c}\right) h \qquad (5.2.29)$$

The nonperturbative contribution is conveniently expressed in terms of the Wilsonian coupling [162]

$$W_{condensate} = -C_H \mu^3 \exp\left(-\frac{8\pi^2 f^W}{C_H}\right) \qquad (5.2.30)$$

where the normalization was obtained by comparing to (5.2.18). Putting these two pieces together, the total superpotential is

$$W = \left(\frac{2\mu^3}{c}\right) h - C_H \mu^3 \exp\left(-\frac{8\pi^2 f^W}{C_H}\right). \qquad (5.2.31)$$

5.2.3 Conditions for a Stabilized Dilaton

A potential for the dilaton arises from the perfect-square interaction term (5.2.18), which couples the background flux to the gauginos. To analyze this expression we first observe that the gaugino bilinear appearing in (5.2.18) is proportional to the covariantly constant holomorphic three-form. This follows from the fact that χ_6 is a gaugino zero mode on the Calabi-Yau manifold [21]:

$$\mathrm{tr}\left(({\chi_6}^{\dagger} \bar{\chi}^* + {\chi_6}^T \bar{\chi}) \Gamma^0_{lmn} (\chi_6^* \chi + \chi_6 \chi^*) \right) = 2 \langle \mathrm{tr}(\lambda\lambda) \rangle \Omega_{lmn} + c.c. \qquad (5.2.32)$$

Here Ω is the holomorphic $(3,0)$ form on the Calabi-Yau, with the normalization $\frac{1}{3!}\Omega_{ijk}\bar{\Omega}^{ijk} = 1$.

Minimizing the perfect square (5.2.18) forces $\langle\lambda\lambda\rangle\Omega + \langle\lambda\lambda\rangle^*\bar{\Omega}$ to align itself along the same direction in $H^3(M, R)$ as the three-form flux H. This uniquely fixes the complex structure moduli and the four-dimensional gaugino condensate. Because the gaugino condensate depends on the four-dimensional dilaton, it follows that the interaction (5.2.18) generates a potential for the dilaton.

However, the minimum of this potential is generically at infinite coupling. In the absence of Chern-Simons contributions, the three-form H obeys the quantization condition

$$\frac{1}{2\pi^2\alpha'}\int_Q dB = n \tag{5.2.33}$$

for any Q in $H_3(X, \mathbb{Z})$. The second term inside the perfect square of (5.2.18), on the other hand, integrates over three-cycles to

$$\begin{aligned} &\int_Q \frac{\alpha' e^{12\sigma}}{8}\Big(\langle \mathrm{tr}(\lambda\lambda)\rangle\Omega_{ijk} + c.c.\Big) \\ &= \frac{c\pi^2}{\alpha'^{1/2}}\exp\left(-\frac{8\pi^2}{C_H g_{YM}^2}\right)\left(e^{-i\theta}\int_Q \Omega + e^{+i\theta}\int_Q \bar{\Omega}\right) \\ &\simeq c\pi^2\alpha' \exp\left(-\frac{8\pi^2}{C_H g_{YM}^2}\right) \end{aligned} \tag{5.2.34}$$

These two terms cancel only if

$$\frac{c}{2}\exp\left(-\frac{8\pi^2}{C_H g_{YM}^2}\right) \simeq n. \tag{5.2.35}$$

This has no solution because the left hand side is almost always[20] less than one. This means that instead of stabilizing the four-dimensional dilaton at a finite value, turning on an integral flux dB actually drives the system to infinitely strong coupling. Our proposal is to use *fractional* fluxes to overcome this problem and stabilize g_{YM} at finite coupling. We therefore turn to an investigation of the conditions under which fractional flux can arise in the heterotic string.

[20] We are assuming that the constant c in (5.2.21) is of order one. If c takes a larger value in a particular model then integral flux might possibly stabilize the dilaton, albeit at relatively strong coupling. We will not investigate this possibility here.

5.3 Fractional Flux Induced by Gauge Fields

In §5.3.1 we review the quantization condition for three-form flux and explain its relation to the Chern-Simons invariant. In §5.3.2 we briefly discuss the class of three-manifolds used in our models and construct a simple example. In §5.3.3 we provide expressions for the Chern-Simons invariants of these manifolds. In §5.3.4 we discuss the conditions under which fractional Chern-Simons flux leads to a world-sheet anomaly, and we explain how this can be avoided in our setup.

5.3.1 Quantization Conditions for Three-Form Flux

Consider a compactification of the $E_8 \times E_8$ heterotic string on a Calabi-Yau manifold X. The two-form $B_{\mu\nu}$ is required to satisfy

$$\frac{1}{2\pi^2\alpha'}\int_Q dB = n \tag{5.3.1}$$

for any three-cycle Q in $H_3(X,\mathbb{Z})$ in order for the action of worldsheet instantons to be single-valued [142]. However, the gauge invariant field strength is

$$H = dB - \frac{\alpha'}{4}\Omega_3(A) + \frac{\alpha'}{4}\Omega_3(\omega). \tag{5.3.2}$$

This does not need to obey the same quantization law, due to the presence of the Chern-Simons term. To see this let us assume for simplicity that the background B-field is trivial, and that the contribution of the spin connection ω can be ignored. Then only the remaining factor of the gauge connection contributes. So instead of (5.3.1) we find the quantization rule

$$\frac{1}{2\pi^2\alpha'}\int_Q H = -CS(A,Q) \tag{5.3.3}$$

where we introduced a standard notation

$$\begin{aligned} CS(A,Q) &= \frac{1}{8\pi^2}\int_Q \Omega_3(A) \\ &= \frac{1}{8\pi^2}\int_Q \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) \end{aligned} \tag{5.3.4}$$

for the Chern-Simons invariant associated with a three-manifold Q and a connection one-form A.

The invariant $CS(A,Q)$ plays an important role in the theory of three-manifolds. In particular, if V' is a gauge bundle over Q and if A is a flat gauge connection on V', then $CS(A,Q)$ is a topological invariant, in the sense that $CS(A,Q)$ takes a fixed value on each component of the moduli space of flat connections on Q. Moreover, it is well known that $CS(A,Q)$ is well defined only modulo integers and can take fractional values. If we further assume that the bundle V' pulls back to a gauge bundle V over the Calabi-Yau manifold X, then we obtain the desired situation where the three-form flux takes fractional values. In the following sections we will use this as a mechanism to produce small quanta of the H-flux, which can then be used to stabilize the various moduli.

5.3.2 Three-cycles with Fractional Flux

Certain classes of three-cycles in Calabi-Yau manifolds admit connections with fractional Chern-Simons invariant. We now turn to a discussion of the properties of such three-cycles.

Since only holomorphic and antiholomorphic components of the three-form flux contribute to the superpotential (5.2.25), the only fractional fluxes we need to consider are those of Hodge type $(3,0)+(0,3)$. These can be viewed as fluxes through special Lagrangian cycles Q. Typically these are compact three-manifolds with non-negative curvature which support gauge fields suitable for our purposes. According to McLean [163], the deformations of a special Lagrangian submanifold Q can be identified with the harmonic one-forms on Q. Specifically, the deformation space has real dimension $b_1(Q)$. Therefore, rigid special Lagrangian three-cycles are precisely rational homology three-spheres, i.e. three-manifolds with $b_1(Q)=0$. We shall henceforth restrict our attention to rigid special Lagrangian three-cycles. The local Calabi-Yau geometry near such cycles is always of the form,

$$T^*Q$$

For example, we can choose Q to be the base of the special Lagrangian torus fibration [164],

$$f\colon X \to Q \tag{5.3.5}$$

Indeed, following Strominger, Yau, and Zaslow [164], consider a BPS state in the effective four-dimensional theory represented by N D6-branes wrapped over the entire mirror manifold $\tilde{X}$. These D6-branes are rigid and, because the fundamental group of $\tilde{X}$ is finite, there is only a discrete set of Wilson lines. In fact, the latter account for the degeneracy of D-brane bound states [165]. Namely, the number of bound states of N D-branes is given by the number of N-dimensional irreducible representations of $\pi_1(\tilde{X})$. Under mirror symmetry (realized as T-duality on T^3 fibers) these D6-branes become D3-branes wrapped around the base Q. In order for the D3-branes to have no continuous moduli the base manifold Q must be a rational homology three-sphere. Also, by looking at the degeneracy of D-brane bound states for different values of N, we conclude that $\pi_1(Q)$ and $\pi_1(\tilde{X})$ should be related. Notice that since both X and its mirror $\tilde{X}$ are fibered over the same base Q, the above arguments imply that their homotopy groups should be related as well. In particular, in a large class of examples one finds that the abelian parts of $\pi_1(X)$ and $\pi_1(\tilde{X})$ are isomorphic, cf. [166].

Let us study a simple example that will be relevant in the following. Consider a quintic hypersurface in $\mathbf{CP}^4$,

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 + \text{ deformations } = 0 \tag{5.3.6}$$

This hypersurface represents a Calabi-Yau variety X_0 with $h^{1,1} = 1$, $h^{2,1} = 101$. Unfortunately, $\pi_1(X_0)$ is trivial, so X_0 does not admit a fractional flux induced by non-trivial gauge fields. Moreover, since the number of generations in a heterotic compactification on a Calabi-Yau threefold X is related, in the case of the standard embedding, to the Euler number of X [139], in the present case with the standard embedding we find an unrealistically large number, $N = \frac{1}{2}|\chi(X_0)| = 100$. A model with a more realistic spectrum that does not suffer from these problems can be obtained by considering a quotient of X_0,

$$X = X_0/\Gamma$$

by a discrete symmetry group $\Gamma = \mathbb{Z}_5 \times \mathbb{Z}_5$, generated by two elements

$$\begin{aligned} g_1&: (z_1, z_2, z_3, z_4, z_5) \to (z_5, z_1, z_2, z_3, z_4) \\ g_2&: (z_1, z_2, z_3, z_4, z_5) \to (\zeta z_1, \zeta^2 z_2, \zeta^3 z_3, \zeta^4 z_4, z_5) \end{aligned} \tag{5.3.7}$$

where $\zeta = \exp(2\pi i/5)$. Since Γ acts freely on X_0, we have $\chi(X) = \chi(X_0)/25 = 8$ and $\pi_1(X) = \mathbb{Z}_5 \times \mathbb{Z}_5$. Therefore, compactification of the heterotic string on the resulting manifold X with the standard embedding provides a model with only four generations, and there is a possibility to turn on non-trivial Wilson lines on X. Also, it is easy to see that the base, Q, of the special Lagrangian torus fibration in this case is a rational homology three-sphere with non-trivial fundamental group.

For the quintic hypersurface (5.3.6), the base Q_0 of the special Lagrangian torus fibration can be represented by the image of the moment map, $z_i \to |z_i|^2$. The topology of Q_0 can easily be understood in the large complex structure limit, where it is close to the boundary of the toric polytope. Hence, $Q_0 \cong \mathbf{S}^3$. Now let us consider the action of the discrete group Γ. From (5.3.7) it follows that the generator g_2 acts trivially on Q_0, whereas g_1 acts freely. Therefore, we find that the base of the special Lagrangian torus fibration $X \to Q$ is a Lens space,

$$Q = \mathbf{S}^3/\mathbb{Z}_5 \tag{5.3.8}$$

In particular, we have $\pi_1(Q) = \mathbb{Z}_5$ and, as we will show below, there are many choices for the gauge bundle V' and for the gauge connection A over this three-manifold, such that $CS(A, Q)$ has fractional values. If V' is such a bundle, we can define its pullback $V = f^{-1}V'$ under the projection map (5.3.5). The resulting gauge bundle V over X has the desired properties and, according to the quantization rule (5.3.3), the three-form flux in heterotic string theory on this background can take fractional values.

This construction can easily be generalized to an arbitrary special Lagrangian three-cycle Q which is rigid inside X. As was explained above, the condition of rigidity implies that Q is a rational homology three-sphere. Examples of rational homology three-spheres that can occur as special Lagrangian cycles in Calabi-Yau threefolds include Lens spaces, Brieskorn homology three-spheres, and, more generally, Seifert fibered three-manifolds. Recall that the Seifert three-manifold, $\Sigma(a_1, \ldots, a_n)$, is a circle fibration over a two-sphere, with n multiple fibers. This includes Brieskorn spheres and Lens spaces as a special case, $n = 3$. For instance, the Lens space $L(p, 1) = \mathbf{S}^3/\mathbb{Z}_p$ is a Seifert three-manifold with $(a_1, a_2, a_3) = (p, 2, 2)$. Many of these three-manifolds support non-trivial gauge connections with fractional Chern-Simons functional [167,168].

5.3.3 Formulas for the Chern-Simons Invariant

In order to determine the set of values of $CS(A,Q)$ for a given three-manifold Q, one has to study the space of representations of the fundamental group, $\pi_1(Q)$, into the gauge group. A familiar example of a reducible[21] gauge connection on a manifold with $\pi_1 = \mathbb{Z}_p$ corresponds to a discrete Wilson line of the form

$$U = \mathrm{diag}(e^{2\pi i k_1/p}, \ldots, e^{2\pi i k_8/p}) \tag{5.3.9}$$

variations of which are often used to break the GUT gauge group to a smaller subgroup, such as the Standard Model gauge group [149]. The Chern-Simons invariant of such a connection is [170] (see also [171])

$$CS(A,Q) = \sum_i \frac{k_i^2}{2p} \qquad \mathrm{mod} \;\; \mathbb{Z} \tag{5.3.10}$$

where the sum is over all eight complex worldsheet fermions. For appropriate choices of p and of the k_i the result is a fractional Chern-Simons invariant.[22]

This has the surprising consequence mentioned in the introduction: in many cases the Wilson lines which are used to break the GUT gauge group to the Standard Model introduce a fractional Chern-Simons invariant, and hence a fractional flux.

We now turn to the more general question of the fractional Chern-Simons invariants of Seifert three-manifolds; this choice covers a fairly large class of models relevant to the physical problem at hand. Without loss of generality, we can take the gauge group to be $SU(2)$ (which can be realized as a subgroup in one of the two E_8's). Let $Q = \Sigma(a_1, \ldots, a_n)$ be a Seifert three-manifold. In this case, the irreducible representations,

$$\rho: \quad \pi_1(Q) \to SU(2)$$

[21] A connection A is called reducible if its isotropy subgroup, that is a maximal subgroup that commutes with all the holonomies of A, is a continuous group. Otherwise, A is called irreducible. For example, an $SU(2)$ gauge connection is reducible if its isotropy subgroup is $U(1)$. Notice that reducible gauge connections may have non-zero Chern-Simons invariant, see e.g. [169].

[22] In §5.3.4 we review the existence and cancellation of a potential worldsheet global anomaly in such backgrounds.

are characterized by what are called "rotation numbers", $(\pm m_1, \ldots, \pm m_n)$, where each m_i is defined modulo a_i,

$$m_i \sim m_i + a_i$$

Furthermore, there exists at most one component of the representation variety realizing a given set of rotation numbers $(m_1, \ldots, m_n)$. If A is the corresponding connection one-form, the value of the Chern-Simons functional, $CS(A, Q)$, is given by the simple formula

$$CS(A,Q) = -\sum_{i=1}^{3} \frac{1}{a_i} (m_i + \lambda)^2 \tag{5.3.11}$$

where

$$\lambda = 0 \, , \; \frac{1}{2} \tag{5.3.12}$$

In particular, if $Q = \mathbf{S}^3/\mathbb{Z}_p$ is a Lens space, from the general formula (5.3.11) we find

$$CS(A,Q) = -\frac{1}{p} (m_1 + \lambda)^2 - \frac{\lambda^2}{2} \qquad \text{mod } \mathbb{Z} \tag{5.3.13}$$

where for simplicity we set $m_2 = m_3 = 0$. This expression gives two sets of values of the Chern-Simons functional (listed in [167]) corresponding to $\lambda = 0$ and $\lambda = 1/2$, respectively. It is convenient to introduce a new integer parameter

$$m = 2m_1 + 2\lambda \qquad \text{mod } 2p$$

and rewrite (5.3.13) in the form

$$CS(A,Q) = -\frac{m^2}{4p} - \frac{\lambda^2}{2} \qquad \text{mod } \mathbb{Z}. \tag{5.3.14}$$

In general, it follows from (5.3.11) that $CS(A, Q)$ is a rational number whose denominator can be as large as the order of the fundamental group, $\pi_1(Q)$.

5.3.4 A Global Worldsheet Anomaly from Fractional Chern-Simons Invariants

For completeness, we now discuss a technical issue related to modular invariance in a fractional flux background. Specifically, we present a sufficient condition for cancellation of the worldsheet anomaly induced by fractional Chern-Simons flux.[23]

[23] We are indebted to E. Witten for explaining to us much of the content in this subsection.

When the heterotic string propagates on a nontrivial geometry M with nontrivial Wilson lines, there is a global worldsheet anomaly in addition to the one-loop anomaly seen in the ten-dimensional supergravity [170]. This signals that the worldsheet instanton path integral is not necessarily single-valued in such a background.

To compute the anomaly, consider a one-parameter (t) family of maps from a one-parameter family of worldsheets into the target space, with the worldsheets at $t = 0$ and $t = 1$ identified by a large diffeomorphism h preserving the spin structure: $\varphi : (\Sigma \times [0,1]_t)_h \to M$. The change of the fermion determinant can be calculated using an index theorem [170],

$$\ln Z(\phi^i, t=1) - \ln Z(\phi^i, t=0) = -2\pi i \int_{\varphi(\Sigma\times[0,1])_h} \Omega_3(A), \tag{5.3.15}$$

where

$$Z(\phi^i, t; g_{ij}, B_{ij}, A_i{}^A{}_B) = (\det{}^+_T)(\det{}^-_{V_1})(\det{}^-_{V_2})(\det{}^+ R). \tag{5.3.16}$$

Here the first three terms inside the logarithm are Dirac determinants for the right- and left-moving fermions coupled to the pull-back of the spin connection and gauge connection, and the fourth term comes from the right-moving Rarita-Schwinger ghost. If we were unable to find other sources to cancel the factor on the right hand side, we would have to set the Chern-Simons invariant to an integer to maintain the single-valuedness of the determinants.

Fortunately the Wess-Zumino term on the worldsheet can help us. For the heterotic string on a Calabi-Yau with flat B field and with no Wilson lines, the worldsheet action looks like

$$\begin{aligned} S = \int d^2x \Big(&\big(g_{ij}(\phi) + B_{ij}(\phi)\big)\partial_+\phi^i\partial_-\phi^j + i g_{ij}\psi^i\big(\partial_-\psi^j + \Gamma^i_{jk}\partial_-\phi^k\psi^l\big) \\ &+ iG_{AB}(\phi)\lambda^A\Big(\partial_+\lambda^B + A_i{}^B{}_C(\partial_+\phi^i)\lambda^C\Big) + \frac{1}{2}\widetilde{F}_{ijAB}\psi^i\psi^j\lambda^A\lambda^B\Big) \end{aligned} \tag{5.3.17}$$

where ψ^i and λ^A are the right- and left-moving fermions, Γ^i_{jk} is the Levi-Civita connection of the target space, and G_{AB} is the metric on the gauge bundle. This action has manifest (0,2) supersymmetry. The question is, if we now turn on flat Wilson lines supporting fractional Chern-Simons invariant, resulting in multi-valued fermion determinants, can we find cancelling effects from the bosonic worldsheet action? The answer is *yes, provided there is no torsion in* $H^4(M, \mathbb{Z})$.

To see this, consider the following exact sequence:

$$\cdots \longrightarrow H^3(M,\mathbf{R}) \xrightarrow{e} H^3(M,U(1)) \xrightarrow{d} H^4(M,\mathbb{Z}) \longrightarrow H^4(M,\mathbf{R}) \longrightarrow \cdots. \qquad (5.3.18)$$

The Chern-Simons invariant $\exp(i\int \Omega_3(A))$ for a flat bundle takes values in $H^3(M,U(1))$ and is mapped into the torsion part of $H^4(M,\mathbb{Z})$. If $H^4(M,\mathbb{Z})$ is torsion-free, the Chern-Simons invariant lives in the kernel of d and therefore $\Omega_3(A)$ lives in $H^3(M,\mathbf{R})$. So there exists, locally, a two-form $\widetilde{B}$:

$$d\widetilde{B} = \Omega_3(A).$$

It is crucial that $\widetilde{B}$ is not globally defined when $\int \Omega_3(A)$ is fractional. The change in phase from the coupling of $\widetilde{B}$ to the worldsheets cancels the change in the fermion determinants in equation (5.3.15)[172]. On the other hand, if $H^4(M,\mathbb{Z})$ has a torsion piece, $\widetilde{B}$ does not exist for bundles supporting fractional Chern-Simons invariant and we cannot cancel the global worldsheet anomaly. The only consistent Wilson lines are then those that give integer Chern-Simons fluxes.

The reader will have noticed that if we modify the Wess-Zumino term into

$$\int_\Sigma B + \widetilde{B},$$

we no longer have (0,2) worldsheet supersymmetry. We can preserve (0,1) supersymmetry by modifying the connection to

$$\widetilde{\Gamma}^i_{jk} = \Gamma^i_{jk} + g^{il}(d\tilde{B})_{jkl} = \Gamma^i_{jk} + g^{il}\Omega(A)_{jkl}. \qquad (5.3.19)$$

However, the complex structure $J^i{}_j$ is no longer covariantly constant. Thus, just as we expected, turning on a flat bundle with Chern-Simons gauge flux generates a spacetime superpotential $W = \int \Omega_3(A) \wedge \Omega$ and breaks $\mathcal{N}=1$ spacetime supersymmetry and (0,2) worldsheet supersymmetry. It is obvious from the supergravity effective action that with the addition of a gaugino condensate, spacetime supersymmetry can be restored. However, we do not expect a useful worldsheet description after including such spacetime effects.[24]

[24] Alternatively, to preserve (0,2) worldsheet supersymmetry, one could modify $J^i{}_j$ so that $\widetilde{\nabla}_i J^j{}_k = J^j{}_{k,i} + \widetilde{\Gamma}^j_{il} J^l{}_k - \widetilde{\Gamma}^l_{ik} J^j{}_l = 0$ with respect to the modified connection. This typically cannot be achieved by a local modification (i.e. a continuous deformation) and requires starting with a non-Kähler manifold. This is closely related to [173] and to more recent literature on non-Kähler compactifications. The difference is that here we would consider non-Kählerity due to $\Omega_3(A)$ instead of the more conventional non-flat dB.

We have seen, then, that a sufficient condition for cancellation of the worldsheet anomaly in the presence of fractional flux is absence of torsion in $H^4(M, \mathbb{Z})$. More specifically, it is enough that no three-cycle Q on which the Chern-Simons form integrates to a fraction is a torsion cycle in $H_3(M, \mathbb{Z})$. We will henceforth assume that this condition is satisfied.

5.4 Dilaton Stabilization

We will now demonstrate that the combination of a gaugino condensate and a fractional flux induced by the Chern-Simons term of the $E_8 \times E_8$ gauge connection can lead to stabilization of the dilaton at finite (and, with sufficient tuning, weak) coupling.

We denote the two gauge groups $E_8^{(i)}$, $i = 1, 2$. Let us henceforth adopt the convention that $E_8^{(1)}$ is the observable E_8 and $E_8^{(2)}$ is the hidden sector. We imagine that there is a suitable visible-sector bundle which breaks $E_8^{(1)}$ to an attractive GUT group. If a realistic model is desired, we may also require that the observable $E_8^{(1)}$ has a gauge bundle with $|\int c_3| = 6$ to give three generations of quarks and leptons.[25] In the remaining visible-sector group we then turn on Wilson lines which have fractional Chern-Simons invariant on some three-cycle. The resulting fractional flux generates a superpotential via (5.2.25).[26]

For the purposes of this section we could take the hidden-sector bundle to be trivial, so that $E_8^{(2)}$ is unbroken. However, it will prove useful in §5.5 to include a non-trivial gauge bundle in each of the E_8s. We therefore embed an $SU(2)$ bundle into $E_8^{(2)}$, breaking $E_8 \to E_7$. There is no index theorem protecting charged matter in E_7 (as it has only real representations), so we can safely assume that the low-energy E_7 gauge theory in the hidden sector has no light **56**s. The gauge group then confines at low energies, providing a gaugino condensate to balance the fractional flux, as in §5.2.2.

---

[25] Examples of Calabi-Yau models with three generations and nontrivial π_1 have appeared in [174], and undoubtedly many more could be constructed in a systematic search.

[26] The fractional flux could instead come from hidden-sector Wilson lines. We focus on visible-sector Wilson lines for simplicity.

The overall result is the superpotential (5.2.31):

$$\frac{W}{\mu^3} = \frac{2h}{c} - 18 \exp\left(-\frac{8\pi^2 S}{18}\right) \qquad (5.4.1)$$

where $h = (2\pi^2 \alpha'^{5/2})^{-1} \int H \wedge \Omega$ is the flux contribution and the second term is the result of gaugino condensation (the dual Coxeter number of E_7 is 18).[27]

To look for a supersymmetric vacuum, we solve the equation $D_S W = 0$, with the result

$$h = \left(9c + 8c\pi^2 Re(S)\right) \exp\left(-\frac{8\pi^2 S}{18}\right) . \qquad (5.4.2)$$

Modest values of the Chern-Simons invariant lead to a solution at weak coupling. For example, if h is approximately $\frac{1}{10}$, which is easily attainable using the constructions of §5.3, then (5.4.2) can be solved with $Re(S) \sim 1.6$, which corresponds to $\alpha_{GUT} \sim \frac{1}{20}$. To achieve instead the often-quoted value $\alpha_{GUT} \sim \frac{1}{25}$ one needs h of order $\frac{1}{40}$. Of course the requirements are weaker if we take the pure hidden sector gauge group to be E_8 instead of E_7.

There are many variations of this mechanism which involve slightly different choices of bundles. It seems to us that the most elegant models are those in which one set of Wilson lines breaks the observable-sector GUT group to the Standard Model and also provides the needed fractional Chern-Simons invariant.

We have already solved the dilaton equation $D_S W = 0$. We can likewise solve the equations for the complex structure moduli by making H of type $(3,0) + (0,3)$. In this way the H-flux from the Chern-Simons invariant generically stabilizes all complex structure moduli. The Kähler moduli of the Calabi-Yau, however, are not yet fixed. In particular, there is a flat direction for the volume modulus T.[28]

In fact, this flat direction is a general property of "no-scale" models. From the form (5.2.23) of the Kähler potential, combined with the fact that W is independent of the volume modulus T at this order, we see that the supergravity potential undergoes a simplification

$$V = e^K \left(g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right) \to e^K \left(g^{a\bar{b}} D_a W \overline{D_b W}\right) \qquad (5.4.3)$$

[27] This superpotential is of the same form as the one appearing in, for instance, equation (12) of [5]. There, the small constant term comes from the $(0,3)$ part of the type IIB G_3 flux, while the exponential arises from nonperturbative gauge dynamics as in our system.

[28] If there are vector bundle moduli then these are also unfixed. However, in §5.8 we explain why bundle moduli could be absent in generic situations.

where i, j run over all fields, but a, b run over all fields *except* T. As a result, we are left with a flat direction, T. Generically $D_T W \neq 0$, so supersymmetry is broken. Nevertheless, the vacuum energy vanishes at this order of approximation, since we have solved $D_a W = 0$ for all a. Loop corrections will plausibly destabilize T, resulting in a runaway problem for the overall volume.

We will suggest a solution to this problem, in the context of Calabi-Yau compactification, in the next section. However, we should point out that investigation of supersymmetric non-Kähler compactifications of string theory has recently been renewed (see e.g. [175,176,152,177]). In such compactifications the overall volume modulus can be stabilized at tree level by balancing fluxes against the non-Kähler nature of the geometry. The combination of this tree-level T stabilization with our results on dilaton stabilization could plausibly yield weakly-coupled models with all moduli stabilized. This would require a compactification manifold which admits moderately small Chern-Simons invariants.

5.5 Dilaton *and* Volume Stabilization in Calabi-Yau Models

In §5.5.1 we show that it is possible, with appropriate choices of bundles, to stabilize both the dilaton and the overall volume by incorporating the one-loop correction to the gauge coupling. In §5.5.2 we extend this mechanism to stabilize all the Kähler moduli of a threefold. In §5.5.3 we investigate the strong-coupling transition which occurs in these models. We present a toy model to illustrate the physical smoothness of this transition. In §5.5.4 we discuss the conditions under which the resulting theory is weakly coupled. In §5.5.5 we summarize our assumptions concerning the Calabi-Yau and the E_8 gauge bundles.

5.5.1 One-loop Correction

We first consider, for simplicity, the case of a Calabi-Yau threefold which has $h^{1,1} = 1$ and hence a single Kähler modulus. When one-loop corrections are incorporated, the Wilsonian gauge kinetic functions have the form (5.2.14):

$$f^W_{(i)} = S + \beta_i T, \tag{5.5.1}$$

where $i = 1, 2$ labels the gauge groups $E_8^{(1)}$, $E_8^{(2)}$. In the case without space-filling heterotic five-branes, it is a simple matter to derive the linear terms in T by dimensional reduction of the $B \wedge X_8(F_1, F_2, R)$ term in the ten-dimensional $E_8 \times E_8$ theory. The result is

$$\beta_1 = \frac{1}{8\pi^2} \int_X J \wedge \Big(c_2(V_1) - c_2(V_2) \Big) \,, \tag{5.5.2}$$

$$\beta_2 = \frac{1}{8\pi^2} \int_X J \wedge \Big(c_2(V_2) - c_2(V_1) \Big) \,. \tag{5.5.3}$$

Here J is the generator of $H^{1,1}(X, \mathbb{Z})$. Notice that

$$\beta_1 + \beta_2 = 0 \tag{5.5.4}$$

while in the case of the standard embedding

$$\beta_1 - \beta_2 = \frac{1}{4\pi^2} \int_X J \wedge c_2(TX) \,. \tag{5.5.5}$$

This fact that the difference of the gauge coupling functions is given by a topological invariant (in the case of the standard embedding) was observed in e.g. [178]. One can easily calculate β for a few simple examples. We present the calculation below for $J \wedge c_2(TX)$; one can imagine partitioning this into $c_2(V_{1,2})$ in various ways.

$$\int_{[4\|5]} J \wedge c_2 = 10 \int_{[4\|5]} J \wedge J \wedge J = 50,$$

$$\int_{[5\|3\ 3]} J \wedge c_2 = 6 \int_{[5\|3\ 3]} J \wedge J \wedge J = 54,$$

$$\int_{[6\|3\ 2\ 2]} J \wedge c_2 = 5 \int_{[6\|3\ 2\ 2]} J \wedge J \wedge J = 60.$$

From these examples it is plausible that β can be reasonably large, at least of order one.

We will choose the gauge bundle V_2 so that $E_8^{(2)}$ is broken to a subgroup H (say E_7) without any light charged matter. The resulting four-dimensional theory therefore has a sector which is pure $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group H, which undergoes gaugino condensation at low energies. Let us furthermore choose the bundle V_1 so that $E_8^{(1)}$ is broken to a low-energy group and matter content which can contain the Standard Model. Finally, we take $\beta_2 = -\beta_1 = \beta > 0$, so that $E_8^{(1)}$ is more strongly coupled than $E_8^{(2)}$.[29]

[29] Notice that we are putting more instantons in the hidden sector than in the observable sector, which is a somewhat unusual situation compared to the bulk of the literature.

The complete superpotential is then

$$\frac{W}{\mu^3} = \frac{2h}{c} - C_H \exp\left(-\frac{8\pi^2}{C_H}(S+\beta T)\right). \tag{5.5.6}$$

This superpotential depends nontrivially on both of the chiral multiplets S and T. The condition for a supersymmetric vacuum is

$$W_{;S} = W_{;T} = 0 \tag{5.5.7}$$

where the Kähler covariant derivatives are determined using (5.2.23).

A solution of (5.5.7) necessarily satisfies

$$3S = \beta T, \tag{5.5.8}$$

$$h = \left(\frac{C_H c}{2} + 8c\pi^2 Re(S)\right) \exp\left(-\frac{32\pi^2 S}{C_H}\right) \tag{5.5.9}$$

The resulting solution is a supersymmetric AdS vacuum in which both the four-dimensional dilaton φ and the four-dimensional volume modulus ρ have been stabilized. We will defer our discussion of the physics in $E_8^{(1)}$ to §5.5.3.

5.5.2 Stabilization of Multiple Kähler Moduli

On a threefold X with $h^{1,1} > 1$ Kähler moduli, the formulas of the previous section can be generalized:

$$f_{(i)}^W = S + \beta_i^\alpha T_\alpha, \tag{5.5.10}$$

where $i = 1, 2$ labels the gauge groups $E_8^{(1)}$, $E_8^{(2)}$ and $\alpha = 1, \ldots, h^{1,1}$ indexes the independent Kähler moduli.

We will need to define a few quantities related to the generators J^α of $H^{1,1}(X, \mathbb{Z})$:

$$\beta_1^\alpha \equiv \frac{1}{8\pi^2} \int_X J^\alpha \wedge \left(c_2(V_1) - c_2(V_2)\right), \tag{5.5.11}$$

$$\beta_2^\alpha \equiv \frac{1}{8\pi^2} \int_X J^\alpha \wedge \left(c_2(V_2) - c_2(V_1)\right). \tag{5.5.12}$$

$$c_{\alpha\beta\gamma} \equiv \int_X J^\alpha \wedge J^\beta \wedge J^\gamma \tag{5.5.13}$$

The $c_{\alpha\beta\gamma}$ are the intersection numbers of X.

The Kähler potential (5.2.23) now takes the form

$$\mathcal{K} = -\log(S+\bar{S}) - \log\left(c_{\alpha\beta\gamma}\mathcal{T}^\alpha\mathcal{T}^\beta\mathcal{T}^\gamma\right) - \log\left(-\frac{i}{4\alpha'^3}\int\Omega\wedge\bar{\Omega}\right) \tag{5.5.14}$$

with $2\mathcal{T}^\alpha \equiv T^\alpha + \bar{T}^\alpha$, while the complete superpotential, including hidden-sector gaugino condensation, is

$$\frac{W}{\mu^3} = \frac{2h}{c} - C_H \exp\left(-\frac{8\pi^2}{C_H}(S+\beta^\alpha T_\alpha)\right). \tag{5.5.15}$$

This superpotential depends nontrivially on the dilaton and on all the Kähler moduli.

In order to find a supersymmetric solution we will assume that all the β^α are nonzero. Combining (5.5.15) and (5.5.14) and imposing $W_{;S} = W_{;T_\alpha} = 0$, we find

$$S\frac{\partial}{\partial T_\delta}\left(c_{\alpha\beta\gamma}\mathcal{T}^\alpha\mathcal{T}^\beta\mathcal{T}^\gamma\right) = \beta_2^\delta\left(c_{\alpha\beta\gamma}\mathcal{T}^\alpha\mathcal{T}^\beta\mathcal{T}^\gamma\right) \tag{5.5.16}$$

$$h = \left(\frac{C_H c}{2} + 8c\pi^2 Re(S)\right)\exp\left(-\frac{32\pi^2 S}{C_H}\right) \tag{5.5.17}$$

where the second relation is identical to (5.5.9).

The result is a supersymmetric AdS vacuum without moduli. To recapitulate, we have now seen that the combination of fractional flux with a gaugino condensate can stabilize the complex structure moduli, the Kähler moduli, and the dilaton.

5.5.3 A Strong Coupling Problem

We have just seen that the potential for the dilaton and Kähler moduli has a supersymmetric AdS minimum whose location is given, in the case of one Kähler modulus, by (5.5.8),(5.5.9). However, there is an evident problem with this minimum. Suppose that some subgroup of $E_8^{(1)}$ remains unbroken at low energies. The naive $E_8^{(1)}$ gauge coupling function, $f_1 = S - \beta T$, appears to be *negative*, $f_1 = -2S$.

Moreover, one might think that before becoming negative, f_1 must pass through zero, at which point one encounters a singularity where the gauge coupling diverges.

It is clear a priori that such a problem cannot exist in the full theory. Moduli (and parameter) spaces of four-dimensional supersymmetric theories are complex and hence can only have singularities at complex codimension one. It follows that one can always continue around any point of naively singular gauge coupling, obtaining a unitary theory with positive g^2 on the "other side". Numerous examples of such phenomena have been explored in various four-dimensional supersymmetric gauge theories over the past several years, most recently in interpreting the G_2 flop in [179].

In fact, what we are encountering here is (at least in those cases which are most easily understood) a close relative of the well-studied strong coupling transitions in six-dimensional string vacua with (0,1) supersymmetry [180]. The observable sector gauge coupling diverges precisely when the ratio S/T reaches a fixed value; this is in fact a point in moduli space where an effective *six*-dimensional coupling is becoming strong. As explained in [180], in dual type II or F-theory descriptions, this phenomenon can be modeled locally in terms of a geometric transition which affects the D-branes or local geometry responsible for $E_8^{(1)}$. On the other side of the geometric transition, the $E_8^{(1)}$ physics remains sensible, and there is a new effective description of the low energy gauge theory.

In the remainder of this subsection we investigate this strong coupling singularity. The resolution is necessarily model-dependent, so we simply review some dual descriptions which shed light on the phenomenon, and give an explicit example where the physics on the "other side" of the transition is fully understood. Of course in as much as one wishes to embed the standard model in $E_8^{(1)}$, it would be crucial to have a good dual description of this new phase. For readers who find this too daunting a challenge, we can only suggest that the special case $\beta_{1,2} = 0$ neatly sidesteps the issue, leaving a no-scale model with an unfixed volume modulus. However we emphasize that more generally, the only assumption we really need to make is that the physics of the transition does not introduce new terms in the superpotential. For models where $E_8^{(1)}$ is broken to a low-energy field theory that does not dynamically generate a superpotential, this is quite plausible.

Dual Descriptions of the Strong Coupling Singularity

The appearance of strong gauge coupling in heterotic models with nonzero β is well known. The problem is easily seen in compactifications of heterotic M-theory to four dimensions, where it manifests as a linear shrinking [181] of the Calabi-Yau volume as a function of location on the M-theory interval. For some critical size of the interval, the Calabi-Yau has zero volume at one boundary, rendering the supergravity approximation invalid.

A closely related problem arises in compactifications of the $E_8 \times E_8$ heterotic string on $K3 \times T^2$. The gauge bundle in such a model is specified in part by a choice of instanton numbers $(12-n, 12+n)$ in the two E_8s. If n is positive then the first E_8 is more strongly coupled than the second; this is analogous to positive β in our models. At a finite value of the heterotic dilaton the first E_8 has infinite gauge coupling.

This configuration is dual to compactification of type IIA string theory on a Calabi-Yau threefold which is an elliptic fibration over the Hirzebruch surface F_n. Recall that F_n has a single curve of self-intersection $-n$. The volume of this curve is dual to the heterotic dilaton in such a way that shrinking the $(-n)$ curve to zero volume coincides with infinite gauge coupling in the first E_8. This suggests that one could use the type II geometry to understand the nature of the strong coupling singularity. While this approach is rather complicated for general n (see e.g. [180] for work in this direction), we will see that the case $n = 1$ is relatively straightforward.

It is important to remember that type II strings on such a Calabi-Yau threefold yield $\mathcal{N} = 2$ supersymmetry in four dimensions, twice as much as the models we have considered in this chapter. This greatly facilitates analysis of the singularity, in particular because the geometry can be described via a prepotential. A direct study of the $\mathcal{N} = 1$ system would be more challenging, but we expect the generic features, including the positive gauge coupling function, to be similar in the two cases. One would simply have to study the geometry of a dual F-theory compactification on a Calabi-Yau fourfold, instead of type II strings on a Calabi-Yau threefold.

A Simple Flop Model of the Strong Coupling Singularity

We will now construct a simple model in which, in a sense which we will make precise, the gauge kinetic term f_1 undergoes a flop.

Recall that in the flop of a curve, the volume of the curve vanishes on a wall of the Kähler cone.[30] However, instead of continuing to negative values on the far

[30] In the full physical theory the volume is complexified, and one can go "around" the wall of the Kähler cone by turning on a nonzero θ angle [182,183].

side, the volume is actually positive in the new Kähler cone. In certain $\mathcal{N}=2$ heterotic/type IIA dual pairs [184], the singularity in the Calabi-Yau prepotential when a curve in the type IIA geometry undergoes a flop (and an effective gauge coupling becomes singular) is dual to a heterotic strong coupling singularity. We describe one such example below. It is important to stress that as expected on completely general grounds, the effective g^2 remains positive everywhere in the properly-interpreted type II moduli space.

The examples we have in mind, and their heterotic duals, are well known. Our presentation of a specific example will closely follow [185], which mapped out in detail several heterotic/type II dual pairs.

Let X be the Calabi-Yau threefold which is an elliptic fibration over F_1. The prepotential for the Kähler moduli space of X is [185]:

$$\mathcal{F}_{II} = \frac{4}{3}t_1^3 + \frac{3}{2}t_1^2 t_2 + \frac{1}{2}t_1 t_2^2 + t_1^2 t_3 + t_1 t_2 t_3 \tag{5.5.18}$$

where t_i are the Kähler moduli. The volume of the (-1) curve is controlled by t_3. One can find a set of dual heterotic variables S, T, U, which are related to the type II variables by

$$t_1 = U, \qquad t_2 = T - U, \qquad t_3 = S - \frac{T}{2} - \frac{U}{2} \tag{5.5.19}$$

In heterotic variables, the prepotential reads

$$\mathcal{F}_h = STU + \frac{1}{3}U^3 \tag{5.5.20}$$

We know that the type II operation of shrinking the (-1) curve corresponds to strong gauge coupling in the heterotic picture. This instructs us to identify $S-\frac{T}{2}-\frac{U}{2}$ with the visible-sector gauge coupling.[31]

Now, to study the effect of the strong gauge coupling, we flop the curve corresponding to t_3. The fields transform as

$$(t_1, t_2, t_3) \to (t_1 + t_3,\ t_2 + t_3,\ -t_3) \tag{5.5.21}$$

[31] To make contact with our earlier notation, T and U are the two T^α, and $\beta_2 = \frac{1}{2}$ for $\alpha = 1, 2$.

leading to the prepotential for $\tilde{X}$, the image of X under the flop. It turns out that $\tilde{X}$ is not a $K3$ fibration, and furthermore it is not dual to a perturbative heterotic model.

Given this linear implementation of the flop in type II variables, we can apply this transformation to the heterotic variables (3.9). This yields

$$(U,\ T-U,\ S-\frac{T}{2}-\frac{U}{2}) \to (S+\frac{U}{2}-\frac{T}{2},\ S+\frac{T}{2}-\frac{3U}{2},\ \frac{T}{2}+\frac{U}{2}-S) \quad (5.5.22)$$

The key result is that the visible-sector gauge coupling has changed sign,

$$S-\frac{T}{2}-\frac{U}{2} \to -S+\frac{T}{2}+\frac{U}{2} \quad (5.5.23)$$

In this new Kähler cone, the visible-sector coupling is sensible provided $T+U > 2S$, which is complementary to the initial restriction $T+U < 2S$.

We have therefore seen that in this very simple example, the gauge coupling function for the visible sector is sensible and positive on both sides of the strong coupling transition. We expect this result to hold in all of the cases of interest, simply from macroscopic arguments about supersymmetric theories. It would be interesting to generalize the simple illustration above to $\mathcal{N}=1$ heterotic vacua by studying the dual geometric transitions in F-theory compactifications on Calabi-Yau fourfolds.

5.5.4 Fractional Invariants and Weak Coupling

Let us now determine the conditions under which the stable vacuum exists at modestly large values of S and T. Note that this does not mean that all of the physics is weakly coupled, since as we just discussed, we have undergone a strong coupling transition in $E_8^{(1)}$! However, some other sectors of the theory may remain perturbative at large S and T, so it is still of interest to know that stabilization at large S and T is possible.

The goal is to arrange that the volume of the Calabi-Yau is large in string units, while the string coupling is small:[32]

$$(ST)^{\frac{1}{8}} = e^{\sigma} > 1, \quad (5.5.24)$$

[32] For simplicity we now present the formulae for the case of one Kähler modulus, the overall volume; the generalization is straightforward.

$$\left(\frac{T^3}{S}\right)^{\frac{1}{2}} = e^{\phi} < 1. \tag{5.5.25}$$

Recall that ϕ is the ten-dimensional dilaton; we denote the four-dimensional dilaton by φ. Using the relation (5.5.8), we have

$$\left(\frac{3}{\beta}\right) S^2 = e^{8\sigma} \tag{5.5.26}$$

$$\left(\frac{3}{\beta}\right)^3 S^2 = e^{\phi} \tag{5.5.27}$$

Clearly $\beta > 3$ is a necessary condition for perturbative validity. It follows from (5.5.3) that this condition can only be met if the bundle V_2 is nontrivial; hence gaugino condensation in an *unbroken* hidden-sector E_8 is not compatible with this method of volume stabilization. To see explicitly that large β is possible within known constructions we refer to the plots of [186].

From the form of the solution (5.5.9) it is clear that the values of S and T at the stable minimum increase as the Chern-Simons invariant becomes smaller. We are therefore interested in finding three-cycles admitting extremely small Chern-Simons invariant.

Small values of the Chern-Simons invariant are distasteful but not unattainable. We saw in §5.3 that it is possible to get a small Chern-Simons invariant h by working on a Calabi-Yau which has a three-cycle Q satisfying

$$\pi_1(Q) = \mathbb{Z}_p$$

for $p \gg 1$. The simplest example of this is a Lens space. One way to generate even smaller h is to take Q to be a general Seifert manifold $\Sigma(a_1, \ldots, a_n)$, since the minimal value of h would scale like

$$h^{-1} \sim \prod_{i=1}^{n} a_i \tag{5.5.28}$$

With several a_i one could then generate very small fractional fluxes.

5.5.5 Summary of Requirements

Let us briefly review the conditions on the Calabi-Yau X and the gauge bundles V_i which ensure the existence of the supersymmetric vacuum (5.5.9) with both dilaton and Kähler moduli stabilized. Conditions essential to the mechanism are listed first, while those related to detailed model-building come last.

(1) In order to achieve a small value of the three-form flux, the Calabi-Yau manifold X must have a nontrivial fundamental group and must admit gauge connections which have fractional Chern-Simons invariant on a three-cycle Q which is not torsion. One of the bundles V_1,V_2 must then be chosen to have such a gauge connection, i.e. suitable Wilson lines. These conditions are *automatically* met in a large class of realistic string models.

(2) For gaugino condensation to be possible in $H \subset E_8^{(2)}$, the bundle V_2 must break $E_8 \to H$ without introducing any light charged matter, leaving a pure gauge group. For example, if $H = E_7$ then there is no index theorem protecting charged matter **56**s, so we expect that this condition is generically satisfied. If instead $H = E_6$ the number of chiral generations is $|\frac{1}{2}\int_X c_3(V^{(2)})|$. The bundle V_2 should be chosen so that this vanishes.[33]

(3) In order to stabilize the overall volume we must choose bundles for which the quantity β_2 defined in (5.5.3) is nonzero. To stabilize multiple Kähler moduli we must take all of the β_2^α to be nonzero. To ensure stabilization of the volume above the string scale, we should also have $\beta_2 > 3$, with an analogous condition for the case of many moduli.

(4) If the Kähler moduli are to be stabilized, the initial configuration and the final stable minimum are on opposite sides of a transition in which the visible sector becomes strongly coupled. It follows that the visible-sector gauge theory can only be properly understood in models where this strong coupling transition can be followed in detail. Better understanding of this transition is a necessary prelude to the building of realistic models. Readers uncomfortable with the transition are advised to set $\beta_1 = \beta_2 = 0$, in which case one is left with a no-scale model with fixed dilaton and an unfixed volume modulus.

---

[33] One could imagine other possibilities in which charged matter in the hidden sector generates a nonperturbative superpotential which can be used for stabilization. See e.g. [187] for a discussion of this possibility in the context of racetrack models.

(5) Further constraints will be necessary to obtain realistic low-energy physics. For example, V_1 should contain appropriate Wilson lines which break the visible-sector GUT to the Standard Model gauge group. (It is sometimes possible to arrange that these same Wilson lines also provide the fractional Chern-Simons invariant.) The vacua we have constructed have negative cosmological constant, with an energy density not far below the string scale. This must certainly be modified to lead to a sensible cosmological model! Finally, if we wish to stabilize at very weak coupling then the fundamental group of the Calabi-Yau must be unusually large.

Clearly, the greatest obstacle to calculability in this scenario is the strong coupling transition in the observable sector. It is conceivable that one could avoid this difficulty by combining fractional Chern-Simons invariants and gaugino condensation with a non-Kähler compactification geometry, for in this case the volume modulus can be stabilized at tree level. However, for the bulk of our analysis, the only real assumption we have made is that the unknown physics of the visible sector does not modify the superpotential. This seems believable provided that the low-energy $\mathcal{N} = 1$ gauge theory which emerges from $E_8^{(1)}$ is not one which dynamically generates a superpotential.

5.6 Duality to Type IIA and M-theory

The models studied in this chapter are related by various dualities to a particular class of $\mathcal{N} = 1$ compactifications of M-theory and Type IIA string theory. These models have recently received some attention due in part to phenomenological applications, see e.g. [188,189,150,190,191,192,193,194]. After appropriate duality transformations our mechanism for moduli stabilization can be applied to these models as well. In this section we briefly discuss various aspects of these dualities, as well as their implications.

5.6.1 Heterotic/Type IIA Duality

Our considerations have thus far been limited to the $E_8 \times E_8$ heterotic string, but the discussion can be repeated almost verbatim for the $Spin(32)/\mathbb{Z}_2$ heterotic string compactified on a Calabi-Yau manifold. The latter theory is related to an $\mathcal{N}=1$ compactification of type IIA string theory by the following chain of dualities:

$$Spin(32)/\mathbb{Z}_2 \text{ Het } \stackrel{S}{\longleftrightarrow} \text{ Type I } \stackrel{\simeq}{\longleftrightarrow} \text{ Type IIB } /\Omega \stackrel{T}{\longleftrightarrow} \text{ Type IIA } /(\Omega\cdot\mathcal{I}) \quad (5.6.1)$$

Let us now explain each step in this duality in more detail and, in particular, find the relation between the parameters and the coupling constants. The first relation is the standard strong-weak coupling duality between the $Spin(32)/\mathbb{Z}_2$ heterotic string theory and type I string theory. The effective supergravity action in the latter theory is similar to the heterotic supergravity action, with the type I and heterotic variables related by

$$\phi_I = -\phi_H \quad (5.6.2)$$

$$g^I_{MN} = g^H_{MN} e^{-\phi_H} \quad (5.6.3)$$

At the next step in the chain of dualities (5.6.1) we identify type I string theory with an orientifold of type IIB closed string theory, where Ω denotes the world-sheet parity symmetry. The parameters and the coupling constants in the supergravity action do not change under this identification, although some terms acquire a different interpretation. In particular, in the type IIB theory the gauge degrees of freedom arise as open string states on the world-volume of 32 space-filling D9-branes. Thus, the Wilson lines of the original heterotic string theory become Wilson lines on D9-branes, and the ten-dimensional gauge coupling is simply

$$\frac{g_{10}^2}{\alpha'^3} = e^{\phi_I} = e^{\phi_{IIB}} \quad (5.6.4)$$

From (5.6.2) and (5.6.4) we find

$$\phi_{IIB} = \phi_I = -\phi_H \quad (5.6.5)$$

The last step in (5.6.1) is the T-duality — mirror symmetry, to be more precise — between type IIB string theory on a Calabi-Yau manifold X and type IIA theory on the mirror manifold $\tilde{X}$. Strictly speaking, the dual background is an orientifold of $\tilde{X}$, where the involution changes the orientation of the T^3 fibers. Under T-duality, the space-filling D9-branes transform into D6-branes wrapped over the base, Q, of the special Lagrangian torus fibration [164]. The parameters of the resulting type IIA background can be obtained from the usual T-duality rules:

$$\phi_{IIA} = \phi_{IIB} - \log\left(\frac{V_X^{IIB}}{V_Q^{IIB}\alpha'^{3/2}}\right) = \frac{1}{2}\phi_H - \log\left(\frac{V_X^H}{V_Q^H\alpha'^{3/2}}\right) \tag{5.6.6}$$

Here V_X and V_Q denote, respectively, the volume of the Calabi-Yau space X and the volume of the base three-manifold Q in the string theory given by the superscript.

To summarize, after a chain of dualities (5.6.1) we found that our heterotic string models are dual to IIA string theory on a mirror Calabi-Yau manifold $\tilde{X}$, with D6-branes wrapped over the special Lagrangian three-cycle Q. This is precisely the configuration studied in [189,150,190,191]. In these papers, Q is usually taken to be a Lens space, $Q = \mathbf{S}^3/\mathbb{Z}_p$, and the Calabi-Yau manifold $\tilde{X}$ is usually assumed to be non-compact. If $\tilde{X}$ is compact, as described above, then the presence of orientifold 6-planes is crucial to cancel the D6-brane charge.

Observe that on the D6-brane world-volume there is a topological coupling between the gauge field, $F = dA + A \wedge A$, and the Ramond-Ramond tensor fields $C = C_1 + C_3 + \ldots$,

$$\mathrm{tr} \int_{\mathbf{R}^4 \times Q} C \wedge e^F \tag{5.6.7}$$

Among other terms, this expression contains a coupling

$$CS(A,Q) \int_{\mathbf{R}^4} G \tag{5.6.8}$$

which we obtained by expanding (5.6.7) and integrating by parts. It follows that D6-branes wrapped over Q with a non-zero value of the Chern-Simons invariant act as an effective source for the Ramond-Ramond four-form field strength in the four uncompactified directions.

Comments on Proton Decay

Using the chain of dualities (5.6.1) we have now related our setup to compactifications of type IIA string theory, where the GUT gauge theory is realized on the world-volume of D6-branes wrapped over a compact three-manifold Q. Similar configurations have been discussed in a recent work of Klebanov and Witten [191] (see also [195]), where it was shown that the proton decay rate from dimension six operators is given by[34]

$$A_{IIA} \sim \frac{g_{YM}^{4/3} L(Q)^{2/3} e^{\frac{1}{3}\phi_{IIA}}}{M_{GUT}^2} \tag{5.6.9}$$

where g_{YM} is the GUT gauge coupling, and M_{GUT} is the unification scale. This scale is determined by the size of the three-manifold Q,

$$M_{GUT} = \left(\frac{L(Q)}{V_Q}\right)^{1/3} \tag{5.6.10}$$

where the extra factor $L(Q)$ accounts for the one-loop threshold corrections from Kaluza-Klein harmonics on Q [190,191]. Specifically, $L(Q)$ is a topological invariant of Q, known as the Ray-Singer torsion.

Let us now compute the proton decay rate in our heterotic models. In contrast to the result of [191], we expect in our case the conventional amplitude

$$A_h \sim \frac{g_{YM}^2}{M_{GUT}^2} \sim \alpha' e^{\frac{\phi}{2}+2\sigma} \tag{5.6.11}$$

where the unification scale and the gauge coupling are given by (5.2.9) and (5.2.21), respectively. By tracing the chain of dualities (5.6.1) in reverse, being careful to include the constant rescaling of the Einstein-frame metric mentioned in §5.2, one can verify that (5.6.9) and (5.6.11) differ by the factor $\alpha_{GUT}^{1/3} e^{-\frac{1}{3}\phi_{IIA}}$, which exhibits the anomalous scaling with α_{GUT} explained in [191].

5.6.2 Lift to M-theory

Now let us consider the M-theory lift of the type IIA configuration considered above. Since D6-branes wrapped over a special Lagrangian submanifold $Q \subset \tilde{X}$ preserve $\mathcal{N} = 1$ supersymmetry in four dimensions, their lift to M-theory must be described by a seven-dimensional manifold, X_{G_2}, with G_2 holonomy. Topologically, X_{G_2} can be viewed as a $K3$ fibration over Q [196],

$$\begin{array}{ccc} K3 & \rightarrow & X_{G_2} \\ & & \downarrow \\ & & Q \end{array} \tag{5.6.12}$$

34 For simplicity, we omit numerical factors of order one.

such that each $K3$ fiber has an ADE singularity, which corresponds to the type of the gauge group on the D6-branes. For example, $SU(5)$ gauge theory would lift to a G_2-manifold with A_4 singularities in the fiber. The dual M-theory geometry (5.6.12) can be obtained directly from the heterotic string theory on a Calabi-Yau manifold X by using the familiar duality between M-theory on $K3$ and heterotic string theory on T^3. Applying this duality to each fiber in the special Lagrangian torus fibration, $X \to Q$, we end up with M-theory on a seven-manifold X_{G_2} with G_2 holonomy and topology (5.6.12). Various aspects of M-theory on G_2-manifolds of this kind have been studied in [188,189,197,198,150,190].

Now let us consider a D6-brane configuration with non-trivial gauge fields characterized by $CS(A,Q) \neq 0$. According to (5.6.8), such gauge fields act as a source (localized on the three-cycle Q) for the space-time component of the four-form flux, G_{0123}. In the effective four-dimensional field theory, this means there is a non-zero superpotential induced by $CS(A,Q)$. In M-theory, the relevant interaction term (5.6.8) appears due to anomaly inflow at the location of ADE singularities [199], while the effective superpotential is generated by topologically non-trivial gauge fields supported at the singularities [192].

The models studied in this chapter have real values of the Chern-Simons invariant $CS(A,Q)$. However, Acharya has argued [192] that, in a more general setting, the superpotential induced by gauge fields should be given by a *complex* Chern-Simons invariant. A deeper understanding of the connection between these ideas would be quite interesting.

5.7 Domain Walls

In order to obtain an expression for the effective superpotential of an $\mathcal{N} = 1$ supersymmetric gauge theory, it is often useful to study the spectrum of BPS domain walls. Moreover, in a theory with gaugino condensation, the domain walls provide information about the breaking of chiral symmetry and about other phenomena of interest.

With this motivation in mind, let us consider domain walls in our models[35], where different vacua are characterized by the values of the Chern-Simons functional, $CS(A,Q)$. Hence, the BPS domain walls are represented by self-dual field

[35] For a related discussion see also [200,201].

configurations (instantons) supported on $Q \times \mathbf{R}$, where $\mathbf{R}$ represents a spatial direction orthogonal to the domain wall. Since $CS(A,Q)$ takes fractional values, such instantons carry fractional charge,

$$c_2 = -\frac{1}{8\pi^2} \int \mathrm{tr}\,(F \wedge F) = CS(A,Q)|_{-\infty} - CS(A,Q)|_{+\infty} \qquad (5.7.1)$$

The instanton action is given by $\int_{Q\times\mathbf{R}} \mathrm{tr}\,(F \wedge *F)$, which, using the self-duality of the gauge field F, can be written as

$$\int_{Q\times\mathbf{R}} \mathrm{tr}\,(F \wedge F)$$

Furthermore, using (5.7.1) one can rewrite the instanton action as the difference of the values of the Chern-Simons functional, $\Delta CS(A,Q)$. Comparing this formula with the standard expression for the tension of a domain wall in $\mathcal{N}=1$ supersymmetric theory, $T = |\Delta W|$, we come to our previous result (5.2.25) for the effective superpotential induced by non-trivial gauge fields [158,159]

$$W_{flux} = \int_Q \Omega_3(A) \qquad (5.7.2)$$

Now let us consider the degeneracy of domain walls interpolating between two vacua with fractional Chern-Simons functional, $CS(A,Q)$, for some three-cycle $Q \subset X$. At least in the classical theory, the BPS domain walls come in continuous families. Specifically, the moduli space of domain walls with fractional charge c_2 is isomorphic to the moduli space of charge-c_2 instantons on $Q \times \mathbf{R}$,

$$\mathcal{M}\,(Q \times \mathbf{R}; c_2) \qquad (5.7.3)$$

Without loss of generality, we can study $SU(2)$ instantons and, for concreteness, take Q to be a Lens space,

$$Q = \mathbf{S}^3/\mathbb{Z}_p$$

Then, according to (5.3.14), the Chern-Simons functional on Q can take the fractional values:

$$CS(A,Q) = -\frac{m^2}{4p} - \frac{\lambda^2}{2} \qquad (5.7.4)$$

Here we follow the notations of [202], introduced at the end of §5.3, where m is an integer defined modulo $2p$.

Consider an instanton on $Q \times \mathbf{R}$ which interpolates between different values of the Chern-Simons invariant, $CS(A,Q)$. According to (5.7.1) and (5.7.4), such an instanton connects two states characterized by different rotation numbers m and $m' = m$ mod 2, and carries a fractional instanton charge, $c_2 = k/p$. Put differently, it is described by a triplet of integers, (k, m, m'). Following [202], let us express $(m, m') \sim (a-b, a+b)$ in terms of a and b, such that

$$\begin{aligned} a &= (m'+m)/2 \qquad \text{mod } p \\ b &= (m'-m)/2 \qquad \text{mod } p \end{aligned} \tag{5.7.5}$$

Using the above expression (5.7.4) for the value of the Chern-Simons functional, we find the corresponding instanton number:

$$\begin{aligned} c_2 &= CS(A,Q)|_{-\infty} - CS(A,Q)|_{+\infty} = \\ &= -\frac{(a-b)^2}{4p} + \frac{(a+b)^2}{4p} = \frac{ab}{p} \end{aligned}$$

Therefore, we have

$$k = ab \qquad \text{mod } p \tag{5.7.6}$$

Now we are in a position to describe the moduli space, $\mathcal{M}$, of instantons on $Q \times \mathbf{R}$ that interpolate between gauge connections with rotation numbers $m = a - b$ and $m' = a + b$. Since instanton configurations always have a modulus that represents their position in $\mathbf{R}$, it makes sense to divide by translations and consider the reduced moduli space,

$$\mathcal{M}' = \mathcal{M}/\mathbf{R}$$

Using index theorems one can compute the virtual dimension of the reduced moduli space [202],

$$\mathrm{Dim}(\mathcal{M}') = \frac{8k}{p} - 4 + n + \frac{2}{p}\sum_{j=1}^{p-1} \cot^2\frac{\pi j}{p}\left(\sin^2\frac{\pi j m}{p} - \sin^2\frac{\pi j m'}{p}\right) \tag{5.7.7}$$

where $n \in \{0,1,2\}$ is the number of m, $m' \neq 0, p$. It turns out that this virtual dimension is always even. In order to illustrate this general formula, in the table below we list the dimensions of the moduli spaces of fractional charge instantons on $\mathbf{S}^3/\mathbb{Z}_5 \times \mathbf{R}$. In terms of a and b, $m = a - b$, $m' = a + b$, and the instanton number $k = ab$ mod 5.

$a\backslash b$	0	1	2	3	4
0	–	–	–	–	–
1	–	0	2	2	2
2	–	2	4	6	10
3	–	2	6	12	18
4	–	2	10	18	24

Table 1: Dim($\mathcal{M}'$) *for the Lens space,* $Q = \mathbf{S}^3/\mathbb{Z}_5$.

The dimension of the moduli space tends to grow with the instanton number, $k = ab$. For low values of the dimension, one can describe $\mathcal{M}'$ rather explicitly using general topological properties [202](see also [203,204]). When Dim($\mathcal{M}'$) = 0, the reduced moduli space must be just a point. In this case, we have only one domain wall interpolating between two vacua. Furthermore, the Euler number of $\mathcal{M}'$ is given by the number of solutions (a, b) to the equations (5.7.5), such that $ab = k$. In particular, this implies that

$$\chi(\mathcal{M}') \geq 0 \tag{5.7.8}$$

Hence, when Dim($\mathcal{M}'$) = 2, the reduced moduli space must be of the form,

$$\mathcal{M}' = \mathbf{S}^2 \setminus F$$

where F is a set of 0, 1, or 2 points.

For example, let us take $p = 5$, $a = 2$, and $b = 1$. This implies $k = 2$, $m = 1$, and $m' = 3$. Then, from Table 1 we find that $\mathcal{M}'$ must be of real dimension 2, and by looking at the Euler number $\chi(\mathcal{M}') = 2$ one concludes that in this example the moduli space is simply a two-sphere,

$$\mathcal{M}' = \mathbf{S}^2$$

Since this space is compact, we expect that the degeneracy of domain walls of charge $c_2 = 2/5$ interpolating between vacua with $m = 1$ and $m' = 3$ is given by the cohomology of $\mathcal{M}'$. Therefore, in this example we find

$$\#(\text{ domain walls }) = 2$$

The above results suggest the following conjecture for the degeneracy of domain walls with small fractional charge, $c_2 = k/p$,

$$\#(\text{ domain walls }) = \begin{cases} 1 & \text{if } k = 1 \\ 2 & \text{if } k = 2 \end{cases} \tag{5.7.9}$$

In other words, we expect that there is always only one domain wall of the minimal fractional charge, whereas the degeneracy of domain walls with twice the minimal charge is equal to 2. It would be interesting to pursue this analysis further.

5.8 Discussion

We have argued that it is possible to stabilize the complex structure moduli, Kähler moduli, and dilaton of heterotic Calabi-Yau compactifications. Our ingredients are hidden-sector gaugino condensation combined with a flux-generated superpotential arising from a flat connection with fractional Chern-Simons invariant. For the non-Kähler compactifications of [175,176] our result looks even more promising, since there the volume is stabilized at tree level, and the only concern is the dilaton.

One omission from our list of stabilized moduli is the vector bundle moduli. Following the analysis in [205], it seems likely that the very existence of bundle moduli is not generic. Massless modes arising from the moduli of a vector bundle V are associated with elements of the group $H^1(X, End(V))$. Typically there is no index theorem which allows one to argue that this group should be nontrivial. Even if the group were nontrivial, a generic infinitesimal deformation of the vector bundle is obstructed at some finite order and so does not constitute a modulus.[36]

[36] Nevertheless, simple bundles constructed by mere humans often have moduli. In many such simple cases, even nonperturbative sigma model effects do not suffice to lift them [206]. Examples of superpotentials arising for the bundle moduli associated with small instantons in heterotic M-theory are described in e.g. [207].

In addition to the omission of a detailed discussion of bundle moduli, we have used standard approximations in describing the hidden sector gaugino condensation. For instance, in real string models, the hidden sector would have massive fields charged under the hidden E_8. This would lead to corrections to the form of the superpotential used here, which presumably arise as more highly damped exponentials in S. While for reasonable values of $Re(S)$ this should not be a large correction, it would be nice to have exact results. These are not yet available for $\mathcal{N}=1$ supersymmetric compactifications of heterotic strings.

The solutions we have constructed are supersymmetric AdS vacua. It is natural to ask whether one can add a source of supersymmetry-breaking energy which lifts these models to de Sitter vacua, along the lines of [5]. In fact, there are significant similarities between the type IIB constructions of [5] and the heterotic models discussed here. As noted in §5.4, the superpotential in each case consists of a small, constant term from flux and an exponential term from nonperturbative gauge dynamics. To continue this analogy and include supersymmetry breaking, one would have to introduce the heterotic dual of the anti-D3-brane introduced in [5]. In heterotic M-theory this would correspond to a non-supersymmetric wrapped M5-brane. To achieve control over the construction, one would need to introduce such an object in a heterotic background with significant warping.

Burgess, Kallosh, and Quevedo [208] have recently proposed that a Fayet-Iliopoulos D-term potential could serve as another useful source of energy for uplifting heterotic models. The stable AdS vacua we have discussed would appear to be a suitable setting for such a mechanism, but we leave the construction of explicit models as a subject for future exploration. Again, one would have to arrange for a suitably small D-term to justify the analysis.

The present proposal for manufacturing vacua without moduli, combined with the constructions in [5,192,208,209,210], is a small step towards filling out our picture of the "discretuum" [3] of string/M-theory vacua. This is the full space of vacua of string theory, including all of the possibilities for the background fluxes, wrapped branes, and other discrete data. Interesting general aspects of this landscape of string theory vacua have recently been discussed in e.g [99,211,212], while statistical arguments relying on the existence of the discretuum have been used in e.g. [3,213,5] in tuning the cosmological constant.

Although this is a bit far from the concrete goal of this chapter, it is worth discussing how this discretuum may be expected to arise in the heterotic theory. In type II theories, as in M-theory, the discretuum is populated by vacua with various quantized values of the RR and NS fluxes, and with different wrapped branes, consistent with the tadpole conditions arising from the Gauss' law constraints on the various p-form field strengths. In the heterotic theory, there are a few quantum numbers which contribute to the large number of vacua. In addition to the large number of choices of vector bundles on a fixed manifold (characterized by the topological numbers $c_2(V_i)$, $c_3(V_i)$, for instance), there are also background NS fluxes. Finally, there is the possibility of non-Kählerity, which is roughly dual to the possibilities of different fluxes in type II theories [175].

As described at length in [99], to get a good handle on this large set of possibilities, it will probably be necessary to find auxiliary ensembles which accurately model the space of vacua. We have little to say about this at present but leave it as an ambitious goal for future research.

6. Towards Inflation in String Theory

ABSTRACT OF ORIGINAL PAPER

We investigate the embedding of brane inflation into stable compactifications of string theory. At first sight a warped compactification geometry seems to produce a naturally flat inflaton potential, evading one well-known difficulty of brane-antibrane scenarios. Careful consideration of the closed string moduli reveals a further obstacle: superpotential stabilization of the compactification volume typically modifies the inflaton potential and renders it too steep for inflation. We discuss the non-generic conditions under which this problem does not arise. We conclude that brane inflation models can only work if restrictive assumptions about the method of volume stabilization, the warping of the internal space, and the source of inflationary energy are satisfied. We argue that this may not be a real problem, given the large range of available fluxes and background geometries in string theory.

6.1 Introduction

Inflation provides a compelling explanation for the homogeneity and isotropy of the universe and for the observed spectrum of density perturbations [214,215]. For this reason, we would hope for inflation to emerge naturally from any fundamental theory of microphysics. String theory is a promising candidate for a fundamental theory, but there are significant obstacles to deriving convincing models of inflation from string theory.

This chapter is reprinted, with changes, from Shamit Kachru, Renata Kallosh, Andrei Linde, Juan Maldacena, Liam McAllister, and Sandip Trivedi, "Towards Inflation in String Theory," JCAP **0310** (2003) 013, by permission of the publisher.

One problem is that string compactifications come with moduli fields which control the shape and size of the compactification manifold as well as the string coupling. Inflation is possible only if these fields are either stable or else have relatively flat potentials which do not cause fast, non-inflationary rolling in field space. Controlling the moduli in this way is a difficult problem. In particular, the potential for the dilaton and for the compactification volume tends to be a rather steep function [216].

A second problem is that the inflaton potential itself must be exceptionally flat to ensure prolonged slow-roll inflation. A successful microphysical theory would naturally produce such a flat potential. Since the flatness condition for the potential involves the Planck scale one should ensure that quantum gravity corrections do not spoil it. Hence, the problem should be analyzed in a theory of quantum gravity, such as string theory. The hope of brane-antibrane inflation scenarios is that the brane-antibrane interaction potential can play the role of the inflaton potential (see [217] for a nice review), but it is well known that this potential is *not* naturally flat. Since in string theory one cannot fine-tune by hand, but only by varying background data (like the compactification manifold or the choice of flux), one concludes that in generic compactifications, brane inflation will not work. However, the many choices of flux and compactification make possible a considerable degree of discrete fine-tuning, so for very special choices of the background one would expect to find potentials which are sufficiently flat for inflation.

In this chapter we discuss these problems in the concrete context of the warped type IIB compactifications described in e.g. [4,218]. One reason for working in this setting is that one can sometimes stabilize all the moduli in a geometry of this type, avoiding the first problem mentioned above. In addition, the constructions of [4] naturally admit D3-branes and anti-D3-branes transverse to the six compact dimensions. Furthermore, one could wish for a model which accommodates both inflation and the present-day cosmic acceleration. This might be possible if one could construct inflationary models which asymptote at late times to the de Sitter vacua of [5] (or variants on that construction, as described in e.g. [219,220]; earlier constructions in non-critical string theory appeared in [98]). As these vacua included one or more anti-D3-branes in a warped type IIB background, it is quite natural to consider brane-antibrane inflation in this context.

Our idea, then, is to begin with the de Sitter vacua constructed in [5], add a mobile D3-brane, and determine whether the resulting potential is suitable for inflation. For the impatient reader, we summarize our findings here. We find that modest warping of the compactification geometry produces an extremely flat brane-antibrane interaction potential, provided that we neglect moduli stabilization. This solves the second problem listed above. However, a new problem appears when we incorporate those terms in the potential which led, in the construction of [5], to the stabilization of the volume modulus. We show that generic volume-stabilizing superpotentials also impart an unacceptably large mass to the inflaton, halting inflation.

While these conclusions are "generic," it is very important to emphasize that the problem of the inflaton mass might be circumvented in at least two different ways. First, the stabilization mechanism for the moduli might be different from that in [5]. For example, the volume modulus could be stabilized by corrections to the Kähler potential, which, as we will see, can naturally circumvent this problem. Second, the mobile brane might be located not at a generic point in the compact manifold but close to some preferred point. If the location of the D3-brane is appropriately chosen then there could be significant corrections to the superpotential. In general models, the superpotential may be a rather complicated function of both the brane positions and the volume modulus. Little is known about the form of these nonperturbative superpotentials in string compactifications. Our arguments show that if the functional form of the superpotential is generic then inflation does not occur. Nevertheless, it seems quite likely, given the range of available fluxes and background geometries, that cases exist which are sufficiently non-generic to permit inflation, although with predictions which are altered from those of naive brane inflation.[37]

Our conclusions should be viewed as a first pass through the class of brane inflation models, in the context of the moduli stabilization mechanism which has recently been developed in [4,5]. Once the non-perturbative superpotentials involved in such constructions are better understood, and/or as soon as other mechanisms for moduli stabilization become available, one could re-examine brane inflation in light

[37] This point is made more quantitative in Appendix 6.F, where we explain that the degree of non-genericity required corresponds roughly to a fine-tune of one part in 100.

of this further concrete knowledge. This may well lead to a precise determination of the non-generic cases where working models of brane inflation in string theory can be realized.

Our analysis clearly indicates that any viable inflation scenario in string theory has to address the moduli stabilization problem. Since essentially all papers on the subject, to the best of our knowledge, have ignored the problem of moduli stabilization, their conclusions are questionable in view of our results. In particular, should a more detailed analysis reveal the possibility of inflation in various non-generic situations, as suggested above, we expect that the resulting inflationary parameters will typically be quite different from those calculated in the existing literature by neglecting moduli stabilization.

This chapter is organized as follows. In §6.2 we review basic facts about brane-antibrane inflation [221,222], with special attention to the case of D3-branes, and discuss some generic problems for such models. In §6.3 we show that warping of the geometry can help with some of these problems. In §6.4 we explain one method of embedding the warped inflation scenario into string theory, using the warped compactifications of [4]. In §6.5 we describe further problems that arise in the string theory constructions when one tries to stabilize the overall volume modulus. Generic methods of stabilization (e.g. via a nonperturbative superpotential) modify the inflaton potential and make inflation difficult to achieve. We discuss several ways to overcome this problem. We conclude with some general remarks in §6.6.

Appendix 6.A contains a general discussion of the gravitational interaction of an (unwarped) brane-antibrane pair, and demonstrates that the potentials which arise are typically not flat enough to lead to prolonged inflation. In Appendix 6.B we specialize to a warped background and derive the interaction potential. In Appendix 6.C we explore the detailed properties of inflation in warped brane-antibrane models, assuming that a solution to the challenges of §6.5 has been found. In Appendix 6.D we explain that eternal inflation may be possible in this scenario. In Appendix 6.E we discuss the exit from inflation and point out that the production of undesirable metric perturbations due to cosmic strings, which are typically created during brane-antibrane annihilation, is highly suppressed in warped models. Finally, in Appendix 6.F we discuss the possibility of fine-tuning of the inflaton potential in order to achieve an inflationary regime.

After completing this work, we became aware of the papers [223], in which related issues are addressed.

6.2 Brief Review of $D3/\overline{D3}$ Inflation

In brane-antibrane inflation one studies the relative motion of a brane and an antibrane which are initially separated by a distance r on the compactification manifold M. One should assume $r \gg l_s$, so that the force is well approximated by the Coulomb attraction due to gravity and RR fields. Then the potential takes the form

$$V(r) = 2T_3 \left(1 - \frac{1}{2\pi^3} \frac{T_3}{M_{10,Pl}^8 r^4}\right) . \tag{6.2.1}$$

where $M_{10,Pl}$ is the ten-dimensional Planck scale, defined by $8\pi G_{10,N} = M_{10,Pl}^{-8}$, and T_3 is the tension of a D3-brane. In terms of a canonically normalized scalar field ϕ, one can rewrite this as

$$V(\phi) = 2T_3 \left(1 - \frac{1}{2\pi^3} \frac{T_3^3}{M_{10,Pl}^8 \phi^4}\right) . \tag{6.2.2}$$

It was suggested in [221] that for large fields (large r), one may obtain inflation from this potential.

A basic (and well known [217]) problem with this scenario is the following. The standard inflationary slow-roll parameters ϵ and η are defined via

$$\epsilon \equiv \frac{M_{Pl}^2}{2} (\frac{V'}{V})^2 \tag{6.2.3}$$

$$\eta \equiv M_{Pl}^2 \frac{V''}{V} . \tag{6.2.4}$$

One generally wants $\epsilon, \eta \ll 1$ to get slow-roll inflation with sufficient e-foldings. Is this possible in the model (6.2.2)? The four-dimensional Planck mass appearing in (6.2.4) is $M_{Pl}^2 = M_{10,Pl}^8 L^6$ where L^6 is the volume of M. This implies that η is

$$\eta = -\frac{10}{\pi^3} (L/r)^6 \sim -0.3 (L/r)^6 \tag{6.2.5}$$

Hence, $\eta \ll 1$ is possible only for $r > L$ – but two branes cannot be separated by a distance greater than L in a manifold M of size L!

One can try to evade this constraint by considering anisotropic extra dimensions or non-generic initial conditions which yield flatter potentials than (6.2.2). We argue in Appendix 6.A that this is not possible. There are always some tachyonic directions in the potential with $\eta \leq -2/3$. This implies that the slow-roll approximation cannot be maintained for a large number of e-foldings.

In §6.3 we will explore another possibility that successfully evades this problem – we will modify the potential (6.2.2) by considering branes and antibranes in a warped geometry. We should mention that there are other proposals which might evade the above problem, such as branes at angles or branes with fluxes, see [224,225,226,42,227].

However, all of these models have an unsolved problem: moduli stabilization. For an internal manifold of size L, the correct four-dimensional Einstein-frame potential is not quite (6.2.2). If one assumes that the main contribution to the inflationary energy comes from the D3-brane tension then one finds, for $r \gg l_s$, that

$$V(\phi, L) \sim \frac{2T_3}{L^{12}} \tag{6.2.6}$$

The energy in the brane tensions sources a steep potential for the radial modulus L of the internal manifold. Therefore, in the absence of a stabilization mechanism which fixes L with sufficient mass so that the variation of L in (6.2.6) is negligible, one will find fast-roll in the direction of large L rather than slow-roll in the direction of decreasing r. This means that it is important to study concrete scenarios where the volume modulus has already been stabilized. However, we will show that not every means of volume stabilization is compatible with inflation, even when the naive inter-brane potential is flat enough to inflate. We will return to the issue of volume stabilization in §6.5 , where we will discuss a new and generic problem which appears when one considers the issue in detail.

6.3 Inflation in a Warped Background: Essential Features

Our modified brane-antibrane proposal is that inflation might arise from the interaction potential between a D3-brane and an anti-D3-brane which are parallel and widely separated in five-dimensional anti de Sitter space (AdS_5).[38]

The anti-D3-brane is held fixed at one location in the infrared end of the geometry (this is naturally enforced by the dynamics, as we shall explain). The D3-brane is mobile; it experiences a small attractive force towards the anti-D3-brane. The distance between the branes plays the role of the inflaton field.

[38] This is a slight simplification; in §6.4 we will construct compact models which deviate from AdS_5 both in the infrared and in the ultraviolet. It is nevertheless convenient to work out the essential features of the model in this simpler geometry.

The forces on the brane and antibrane arise as follows. A single D3-brane experiences no force in an AdS background: electrostatic repulsion from the five-form background exactly cancels gravitational attraction. The addition of a distant anti-D3-brane results in a relatively weak interaction potential arising from the attraction between the brane and the antibrane. We interpret this as a slowly varying potential for the inflaton. We will demonstrate in §6.3.2 and in Appendix 6.B that this potential is much flatter than the interaction potential for a brane-antibrane pair in flat space.

In the remainder of this section we explain this key idea in more detail. §6.3.1 is a review of gravity in a warped background. §6.3.2 deals with the motion of a brane probe in such a background.

It is important to point out that throughout this discussion, we will ignore the possibility that other moduli (or the effects which stabilize them) interfere with inflation. In the context of the string constructions of §6.4, the relevant other modulus is the compactification volume, and the generic problems associated with its stabilization are the subject of §6.5. In fact we will see that this modulus problem will generically stop inflation.

6.3.1 Gravity in an AdS Background

We first consider a compactification of string theory on $AdS_5 \times X_5$ where X_5 is a five-dimensional Einstein manifold.[39] This arises in string theory as a solution of ten-dimensional supergravity coupled to the five-form field strength F_5. The AdS_5 solution is given in Poincaré coordinates by the metric

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}dr^2 \tag{6.3.1}$$

There is, in addition, a five-form flux: if the geometry (6.3.1) arises as the near-horizon limit of a stack of N D3-branes, then the five-form charge (in units of the charge of a single D3-brane) is N. R, the characteristic length scale of the AdS_5 geometry, is related to the five-form charge by

$$R^4 = 4\pi a g_s N \alpha'^2, \tag{6.3.2}$$

[39] The detailed form of X_5 will not matter for the moment. For concreteness the reader may imagine that $X_5 = S^5$.

where the constant a depends on X_5. It will be useful to recall that AdS is a maximally symmetric, constant curvature spacetime. Its curvature scales like $\frac{1}{R^2}$ and is independent of the radial location r. As long as $N \gg 1$ this curvature is small and supergravity analysis is reliable. We will choose to truncate AdS_5 to the region $r_0 < r < r_{max}$.

The reader will notice that, apart from the additional manifold X_5, this background is identical to that considered by Randall and Sundrum in [43]. Two physical insights from [43] will be crucial for our model. First, one can see from the warped metric (6.3.1) that the region of small r is the bottom of a gravitational well. Energies along the t, x^i coordinates therefore get increasingly redshifted as r decreases. (The region of significant redshift is consequently referred to as the infrared end of the geometry.) Second, as a result of truncating the AdS region, the four-dimensional effective theory which governs low-energy dynamics will have a finite gravitational constant, and will include four-dimensional gravity described by the Einstein-Hilbert action: [40]

$$S_{grav} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \mathcal{R}. \tag{6.3.3}$$

Recall also that in [43], the truncation of AdS space was achieved in a brute force manner by placing two branes, conventionally called the Planck brane and the Standard Model brane, at r_{max} and r_0, respectively. In the string theory constructions of [4], the truncation arises because the compactification geometry departs significantly from that of $\mathrm{AdS}_5 \times \mathrm{X}_5$ away from the region $r_0 < r < r_{max}$. In the ultraviolet, in the vicinity of $r \geq r_{max}$, the AdS geometry smoothly glues into a warped Calabi-Yau compactification. In the infrared, near $r = r_0$, the AdS region often terminates smoothly (as in the example of [13]). The infrared smoothing prevents the redshift factor r/R from decreasing beyond a certain minimum whose value will be very important for our model.

[40] The graviton zero modes have polarizations parallel to t, x_i, are constant on X_5, and have a profile identical to the warped background.

6.3.2 Brane Dynamics

We mentioned above that the warped nature of the geometry gives rise to a redshift dependent on the radial location. It will be important in the discussion below that the redshift results in a very significant suppression of energies at the location of the antibrane; that is, the ratio r_0/R is very small. Also note that within the truncated AdS geometry, $r_0 < r < r_{max}$, we have chosen to place the anti-D3-brane at the infrared cutoff $r = r_0$, where it has minimum energy due to the redshift effect.

The five-form background is given by

$$(F_5)_{rtx^1x^2x^3} = \frac{4r^3}{R^4}. \tag{6.3.4}$$

In a suitable gauge the corresponding four-form gauge potential C_4 takes the form

$$(C_4)_{tx^1x^2x^3} = \frac{r^4}{R^4}. \tag{6.3.5}$$

The D3-brane stretches along the directions t, x^1, x^2, x^3. Its location in the radial direction of AdS space will be denoted by r_1. In the discussion below we will assume (self-consistently) that the D3-brane has a fixed location along the angular coordinates of the X_5 space. The motion of the D3-brane is then described by the Born-Infeld plus Chern-Simons action

$$S = -T_3 \int \sqrt{-g} d^4x \left(\frac{r_1^4}{R^4}\right) \sqrt{1 - \frac{R^4}{r_1^4} g^{\mu\nu} \partial_\mu r_1 \partial_\nu r_1} + T_3 \int (C_4)_{tx^1x^2x^3} dt dx^1 dx^2 dx^3. \tag{6.3.6}$$

The indices μ, ν denote directions parallel to the D3-brane along the t, x^1, x^2, x^3 coordinates, and $g^{\mu\nu}$ is the metric along these directions. The D3-brane tension, T_3, is

$$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}. \tag{6.3.7}$$

For future purposes we note here that since an anti-D3-brane has the same tension as a D3-brane but opposite five-form charge, it is described by a similar action where the sign of the second term is reversed.

Now consider a D3-brane slowly moving in the background given by (6.3.1) and (6.3.4), with no antibranes present. It is easy to see that because of a cancellation between the Born-Infeld and Chern-Simons terms, the D3-brane action at low energies is just that of a free field,

$$S = T_3 \int d^4x \sqrt{-g} \frac{1}{2} g^{\mu\nu} \partial_\mu r_1 \partial_\nu r_1. \tag{6.3.8}$$

This in accord with our comment above that the net force for a D3-brane in the background (6.3.1),(6.3.4) vanishes due to gravitational and five-form cancellations.

We are now ready to consider the effect of an antibrane on the D3-brane. Physically this arises as follows. The anti-D3-brane has a tension and a five-form charge and perturbs both the metric and the five-form field. This in turn results in a potential energy dependent on the location of the D3-brane.

The potential between a brane located at r_1 and an antibrane located at r_0, in the limit when $r_1 \gg r_0$, is given by:

$$V = 2T_3 \frac{r_0^4}{R^4} \left(1 - \frac{1}{N} \frac{r_0^4}{r_1^4}\right). \tag{6.3.9}$$

For a derivation see Appendix 6.B.

The first term in the potential is independent of the location of the D3-brane and can be thought of as a constant potential energy associated with the anti-D3-brane. It is proportional to the tension T_3. For the antibrane the force exerted by gravity and the five-form field are of the same sign and add, so we have a factor of 2. In addition, the warped geometry gives rise to a redshift, which reduces the effective tension of the antibrane by a factor r_0^4/R^4.

The second term in (6.3.9) depends on the location of the D3-brane; its negative sign indicates mutual attraction between the pair. Two features of this term will be important in the subsequent discussion. First, the term varies slowly, as the inverse fourth power of the radial location of the D3-brane. Second, due to the warping of the background, the coefficient of this second term is highly suppressed, by a redshift factor r_0^8/R^4.

Two more comments are in order at this stage. We have assumed that the antibrane is fixed at r_0. From (6.3.9), we see that this is in fact a good approximation to make. In the $r_1 \gg r_0$ limit the first term in (6.3.9) is much bigger than the second, and most of the energy of the anti-D3-brane arises due to interaction with the background. This is minimized when the anti-D3-brane is located at r_0 in the truncated AdS spacetime. Second, in our analysis above, we are working in the approximation $r_1 \gg r_0$. We will see below that the D3-brane is far away from the anti-D3-brane while the approximately sixty e-foldings of inflation occur, so this condition is met during the inflationary epoch. Eventually the D3-brane approaches the antibrane, $r_1 \sim r_0$, and this approximation breaks down. The potential then becomes quite complicated and more model dependent (e.g. it depends on the separation between the brane and antibrane along X_5). The resulting dynamics is important for reheating.

A summary of the discussion so far is as follows. We have considered a D3-brane moving in an $AdS_5 \times X_5$ background with five-form flux, in the presence of a fixed anti-D3-brane. This system is described by an action:

$$S = \int d^4x \left(\frac{1}{2} T_3 g^{\mu\nu} \partial_\mu r_1 \partial_\nu r_1 - 2T_3 \frac{r_0^4}{R^4} \left(1 - \frac{1}{N} \frac{r_0^4}{r_1^4} \right) \right) \tag{6.3.10}$$

The reader will notice in particular that r_1, the location of the D3-brane, is a scalar field in the effective four-dimensional theory.

Once we cut off the AdS_5 space as in the Randall-Sundrum models we will find that we can add to (6.3.10) the four-dimensional Einstein action. However, we should also add an extra coupling of the form $\frac{T_3}{12} r_1^2 \mathcal{R}$ coming from the fact that the scalar field r_1 describing the position of the D3-brane is a conformally coupled scalar [228]. This unfortunately leads to a large contribution to η. We will discuss this phenomenon in more generality (from the perspective of the effective low-energy four-dimensional supergravity) in §6.5.

The model described above has several appealing features in addition to the flatness of the potential. We study these properties in Appendices 6.C,6.D, and 6.E, with the assumption that one can somehow overcome the problems of §6.5 (which must be tantamount to cancelling the conformal coupling). In Appendix 6.C we compute the inflationary parameters and show that observational constraints are easily met. In Appendix 6.D we argue that eternal inflation can be embedded into this model, and in Appendix 6.E we point out that the warped geometry suppresses the production of metric perturbations due to cosmic strings (which naturally form during the brane/anti-brane annihilation).

6.4 A Concrete Example in String Theory

We now show how to realize our proposal in a specific class of string compactifications. In §6.4.1 we present the compactifications and explain why they contain warped throat regions. As the warped throat is well-described by the Klebanov-Strassler (KS) solution [13], we dedicate §6.4.2 to a very brief review of the KS geometry. In §6.4.3 we show that a brane moving in the KS background might give rise to inflation, realizing the general idea presented in §6.3. Throughout this discussion, we ignore the problem of stabilizing the overall volume modulus, which is unfixed in the constructions of [4]. We consider the problem of volume stabilization in §6.5, where we will find that generic methods of volume stabilization can perturb the inflaton enough to stop inflation.

6.4.1 The Compactification

Our starting point is type IIB string theory compactified on a six-dimensional Calabi-Yau orientifold. More generally one could use F-theory on an elliptically-fibered Calabi-Yau fourfold. We choose to turn on background fluxes: the three-form fluxes F_3, H_3 present in the theory are placed along cycles in the internal space (and F_5 is fixed as in [4]). These fluxes induce warping of the background. One can show that the resulting space is a warped product of Minkowski space and the Calabi-Yau:

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n \qquad (6.4.1)$$

where y_i are coordinates on the compactification manifold and g_{mn} is the Calabi-Yau metric. As was discussed in [4], one expects that with a generic choice of flux, all the complex structure moduli of the Calabi-Yau, as well as the dilaton-axion, will be fixed. We will assume that the compactification has only one Kähler modulus, the overall volume of the internal space.

As described in [4], one can use the above construction to compactify the warped deformed conifold solution of Klebanov and Strassler (KS). We spend the next section reviewing a few facts about this geometry, as certain details will be important for inflation.

6.4.2 The Klebanov-Strassler Geometry

The Klebanov-Strassler geometry [13] is a noncompact ten-dimensional solution to type IIB supergravity in the presence of background fluxes. The spacetime naturally decomposes into a warped product of a Minkowski factor and a six-dimensional internal space. The six-dimensional space has a tip which is smoothed into an S^3 of finite size. Far from this tip the geometry can be approximated by a cone over the Einstein manifold $T^{1,1}$, which is topologically $S^2 \times S^3$. Our coordinates will be five angles on $T^{1,1}$, which we can consistently neglect in the following, and a radial coordinate r which measures distance from the tip. The background fluxes are given by

$$\frac{1}{(2\pi)^2\alpha'}\int_A F = M, \qquad \frac{1}{(2\pi)^2\alpha'}\int_B H = -K \tag{6.4.2}$$

where A is the S^3 at the tip and B is its Poincaré-dual three-cycle. We will require that $M \gg 1$ and $K \gg 1$; these conditions are important in deriving the solution. The exact metric is known, but for our purposes a simpler form, valid far from the tip, will be more useful. For large r we may express the complete ten-dimensional solution as

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}\left(dr^2 + r^2 ds^2_{T^{1,1}}\right) \tag{6.4.3}$$

where now

$$h(r) = \frac{27\pi}{4r^4}\alpha'^2 g_s M\left(K + g_s M\left(\frac{3}{8\pi} + \frac{3}{2\pi}\ln(\frac{\mathrm{r}}{\mathrm{r_{max}}})\right)\right). \tag{6.4.4}$$

Neglecting the logarithmic corrections and the second term on the right, this takes the form [41]

$$R^4 = \frac{27}{4}\pi g_s N\alpha'^2 \tag{6.4.5}$$

$$N \equiv MK \tag{6.4.6}$$

When the KS geometry is embedded in a compactification then at some location $r = r_{max}$ the warped throat geometry is smoothly joined to the remainder of the warped Calabi-Yau orientifold. Near this gluing region, departures from the $\mathrm{AdS}_5 \times \mathrm{T}^{1,1}$ geometry are noticeable; eventually the AdS must end. In terms of redshift this

[41] The second term on the r.h.s. of (6.4.4) can easily be included. For the numerical values discussed in Appendix 6.C, this gives a three percent correction.

location corresponds to the deep ultraviolet, and so the gluing region plays the role of the ultraviolet cutoff (Planck brane) in the AdS of §6.3.

The exact solution likewise shows departures from (6.4.3) in the far infrared, near the tip, although the geometry remains smooth.[42] The details of the deviation from (6.4.3), although known, are unimportant here; it will suffice to know the redshift at the tip. This can be modeled by cutting off the radial coordinate at some minimum value r_0, which is the location of the tip. It was shown in [4] that the minimal redshift satisfies

$$\frac{r_0}{R} = e^{-\frac{2\pi K}{3g_s M}} \tag{6.4.7}$$

This can be extremely small given a suitable choice of fluxes.

6.4.3 Inflation from Motion in the KS Region

In [5] additional anti-D3-branes were introduced in the KS region. These anti-D3-branes minimize their energy by sitting at the location where the redshift suppression is maximum, i.e. at the very tip of the deformed conifold, where $r \sim r_0$ (the dynamics of anti-D3-branes in the KS geometry was studied in [229]).

Thus we see that the string construction outlined above has all the features of the general model of §6.3: a truncated AdS_5 geometry, an associated five-form flux of the correct strength, and anti-D3-branes fixed at the location of maximum redshift. In addition most of the moduli associated with the compactification, including the dilaton, are stabilized. The one exception is the volume modulus; we will discuss the complications its stabilization introduces separately, in §6.5.

No mobile D3-branes were included in the construction of [5], but it is easy to incorporate them. One needs to turn on somewhat different values of three-form flux, which allow the four-form tadpole to cancel in the presence of the additional D3-branes. This is straightforward to do and does not change any of the features discussed above.

We will take one such D3-brane to be present in the KS region of the compactification. The general discussion of §6.3 applies to this brane. Since the D3-brane is described by the action (6.3.6), with R now given by (6.4.5), the calculation of the brane-antibrane potential follows the discussion in Appendix 6.C, which we outline here.

[42] The radius of curvature is $\sqrt{g_s M \alpha'}$, so the tip is smooth provided $g_s M \gg 1$.

In the KS model the warp factor (6.4.1) is given in terms of a function $h \equiv e^{-4A}$ which obeys a Laplace equation, with the fluxes and branes acting as sources. In particular, a single D3-brane located at $r = r_1$ will correct the background according to

$$h_{new}(r) = h(r) + \Delta h(r, r_1). \tag{6.4.8}$$

Here $h(r)$ is the background given in (6.4.4) and $\Delta h(r, r_1)$ is the correction due to the D3-brane. In a region where the original warp factor is very small we see that $h(r_0) \gg 1$, so that the total warp factor can be expanded as

$$e^{4A} \sim h(r_0)^{-1} \left(1 - \frac{\Delta h(r, r_1)}{h(r_0)}\right). \tag{6.4.9}$$

This warp factor yields the contribution to the energy due to the presence of an antibrane. If $h(r_0) \gg 1$ this typically gives a very flat potential.

The small warp factor and the consequent exponential flatness are the heart of our proposal, so an alternative explanation of the origin of these small numbers may be helpful. Recall that there is a holographic dual gauge theory which describes the geometry of the KS model. This gauge theory is approximately scale invariant in the deep ultraviolet, with slowly running gauge couplings. It undergoes K duality cascades before leading in the infrared to a confining gauge theory with a mass gap. Then the smallness of the redshift factor,

$$\left(\frac{r_0}{R}\right)^4 = e^{-\frac{8\pi K}{3 g_s M}} \tag{6.4.10}$$

can be ascribed to the exponential smallness of the confinement scale in such a gauge theory.

In summary, we have seen that one can construct concrete examples of string compactifications which lead to the general behavior described in §6.3. One of their virtues is that they automatically lead to very flat inflaton potentials, without the need for large brane separation or excessive fine-tuning of initial conditions. The primary source of this flatness is the redshift suppression (6.4.7) which is exponentially sensitive to the (integer) choice of fluxes K and M. However, all of these virtues must be re-examined in the light of concrete ideas about how to stabilize the closed string moduli. In this general class of flux compactifications, the fluxes stabilize many moduli but not e.g. the overall volume. We now turn to the discussion of volume stabilization.

6.5 Volume Stabilization: New Difficulties for D-brane Inflation

The results of §6.3,6.4 indicate that warped geometries provide a promising setting for making models of inflation with naturally small ϵ and η. However, as emphasized in §6.2, one must ensure that the compactification volume is stabilized in order to avoid rapid decompactification instead of inflation. We will now demonstrate that in the concrete models of [4] this is far from a trivial constraint.

In these models the four-dimensional $\mathcal{N} = 1$ supergravity at low energies is of the no-scale type. The Kähler potential for the volume modulus ρ and the D-brane fields ϕ takes the form [218][43]

$$K(\rho, \bar{\rho}, \phi, \bar{\phi}) = -3 \log \left(\rho + \bar{\rho} - k(\phi, \bar{\phi})\right) \tag{6.5.1}$$

Let us pause for a moment to explain how this is obtained. In the tree level compactification the massless fields are the volume, the axion and the position ϕ of the branes. The axion comes from a four-form potential proportional to a harmonic four-form in the internal manifold [4]. At first sight one would think that the moduli space is simply a product of the moduli space for ϕ, which is just the internal Calabi-Yau manifold, and the space spanned by the volume and the axion. This is not correct; the axion describes a circle which is non-trivially fibered over the ϕ moduli space. This structure arises from the coupling of the four-form potential to the worldvolume of the moving D3-brane. The moduli space has a metric of the form

$$ds^2 = \frac{3}{2r^2}\left(dr^2 + (d\chi + \frac{1}{2}ik_{,j}d\phi^j - \frac{1}{2}ik_{,\bar{j}}d\bar{\phi}^{\bar{j}})^2\right) + \frac{3}{r}k_{,i\bar{j}}d\phi^i d\bar{\phi}^{\bar{j}} \tag{6.5.2}$$

where r is proportional to the volume of the Calabi Yau (in the notation of [4], $r \sim e^{4u}$). If we tried to work with a complex variable $r + i\chi$ then (6.5.2) would not follow from a Kähler potential. It turns out that the good complex variable is ρ, which is defined as follows. The imaginary part of ρ is the axion, while the real part of ρ is defined by

$$2r = \rho + \bar{\rho} - k(\phi, \bar{\phi}) \ . \tag{6.5.3}$$

It is then possible to see that (6.5.1) gives rise to (6.5.2). This type of definition of ρ arises when we Kaluza-Klein compactify supergravity theories; see for example [230].

[43] The variable ρ is called $-i\rho$ in [5].

The superpotential is of the form

$$W = W_0 \tag{6.5.4}$$

where W_0 is a constant (we assume the D-branes are on their moduli space, so we do not write down the standard commutator term). This arises from the (0,3) part of the three-form flux in the full theory including the complex structure moduli and the dilaton. We have not yet included the anti-D3-branes used in §6.3,6.4; these will be incorporated at the end of the discussion.

It is important that with the Kähler potential (6.5.1), one obtains the no-scale cancellation in the potential

$$V = e^K (g^{a\bar{b}} K_{,a} K_{,\bar{b}} |W|^2 - 3|W|^2) = 0 \ . \tag{6.5.5}$$

since

$$g^{a\bar{b}} \partial_a K \partial_{\bar{b}} K = 3 \tag{6.5.6}$$

where a, b run over ρ and ϕ. [44]

Using (6.5.5), it is clear that a generic $W(\phi)$ will yield a potential for the D-brane fields ϕ, but that the potential for the ρ modulus will vanish if the solution for the ϕ fields has $\partial_\phi W = 0$. It is also clear that a *constant* superpotential, as in (6.5.4), gives no potential to the ϕ fields. This is consistent with the analysis in [4], where the pseudo-BPS nature of the flux background leaves the D3-brane moduli unfixed.

We are interested in finding a situation where the D-branes can move freely in the Calabi-Yau (so the ϕ fields are *unfixed*), but the volume is stabilized. Before we discuss various scenarios for such a stabilization, it is important to distinguish carefully between the ρ chiral superfield, and the actual volume modulus, r, which controls the α' expansion.

The Kähler potential (6.5.1) has the following peculiar feature. Let us imagine that there is one D-brane, and hence a triplet of fields ϕ describing its position on the Calabi-Yau space. Then $k(\phi, \bar{\phi})$ should be the Kähler potential for the Calabi-Yau metric itself, at least at large volume. However, under Kähler transformations

[44] The easiest way to check (6.5.6) is to note that in expression (6.5.6) we can switch back to the variables r, a, ϕ in (6.5.2). In these variables K is only a function of r.

of k, the expression (6.5.1) is not well behaved. This can be fixed by assigning the transformation laws

$$k(\phi, \bar{\phi}) \to k + f(\phi) + \overline{f(\phi)}, \quad \rho \to \rho + f, \quad \bar{\rho} \to \bar{\rho} + \bar{f} \; . \tag{6.5.7}$$

This is a manifestation of the fact that the circle described by the axion is non-trivially fibered over the ϕ moduli space. Note that the physical volume of the internal dimensions, which is given by r, (6.5.3), is invariant under (6.5.7).

Armed with this knowledge, and given (6.5.1) and (6.5.4) as our starting point, we can now explore various scenarios for volume stabilization.

6.5.1 Scenario I: Superpotential Stabilization

Perhaps the most straightforward method of stabilizing the volume involves a nonperturbative contribution to the superpotential. Various sources of nonperturbative superpotentials for the ρ modulus are known; one instructive example described in [5] involves a superpotential

$$W(\rho) = W_0 + Ae^{-a\rho} \tag{6.5.8}$$

where A and a are constants and W_0 is the contribution (6.5.4) of the three-form flux. For the remainder of this section we will consider $W = W(\rho)$ to be a general holomorphic function of ρ.

In the presence of D3-branes the superpotential must in addition develop some dependence on ϕ, as it should be invariant under (6.5.7). For instance, as argued in [231], the superpotential due to Euclidean brane instantons or gauge dynamics on D7-branes has to vanish when a D3-brane hits the relevant cycle. This can be understood directly by examining and integrating out the massive D3-D7 strings in the latter case. This subtlety must be accounted for to get a globally well-defined W, and we will see in a moment that this actually changes the inflaton mass term. Nevertheless, we will first study the simpler case $W = W(\rho)$, both because it reflects the essential features of the problem and because the full dependence of W on ϕ is not known.

Let us start by presenting a general argument which highlights a problem faced by any inflationary model involving a moving D3-brane in the models of [5]. The main point is that one will choose some configuration with a positive energy V.

When the compact manifold is large then this energy will go to zero rather quickly, as a power of the volume modulus r:

$$V(r,\phi) = \frac{X(\rho)}{r^\alpha} = \frac{X(\rho)}{(\rho - \phi\bar{\phi}/2)^\alpha} \tag{6.5.9}$$

where α is a number of order one and the form of $X(\rho)$ depends on the source of energy. This follows because in existing proposals the inflationary energy arises either from brane tensions or from fluxes, and all known brane and flux energies vanish as some power of r. On the other hand the stabilization mechanism would fix ρ (or else some combination of ρ and ϕ) rather than r. This implies that as the brane moves and ϕ changes there will be a change in the potential,

$$V = V_0\left(1 + \alpha\frac{\phi\bar{\phi}}{2r} + \ldots\right). \tag{6.5.10}$$

This will lead to a contribution to η of order one, unless there is a compensating contribution to the mass term from some other source.

One possible source of such a cancellation is a dependence of the superpotential on ϕ, not just ρ. If $V(r,\phi) = X(\rho,\phi)r^{-\alpha}$ then we would get an additional contribution to the mass term,

$$V(\rho,\phi) = \frac{X(\rho,\phi)}{(\rho - \phi\bar{\phi}/2)^\alpha} = \frac{X(\rho)}{\rho^\alpha}\left(1 + \alpha\frac{\phi\bar{\phi}}{2r} + \ldots\right) + \frac{\Delta(\rho)}{r^\alpha}\phi\bar{\phi}$$

where

$$X(\rho,0) \equiv X(\rho) \qquad \Delta(\rho) \equiv \partial_\phi\partial_{\bar{\phi}}X(\rho,\phi)|_{\phi=0}$$

so that at the minimum $\rho = \rho_c$ we find

$$V(\rho_c,\phi) = V_0(\rho_c) + \left(\frac{\alpha V_0(\rho_c)}{2\rho_c} + \frac{\Delta(\rho_c)}{\rho_c^\alpha}\right)\phi\bar{\phi} + \ldots$$

In principle the second contribution to the mass term might substantially cancel the first, alleviating the problem of the inflaton mass. This would certainly require fine-tuning at the level of one percent (in order to make η sufficiently small to allow sixty e-foldings). More importantly, the dependence of W on ϕ is not known, so the question of which models admit such fine-tuning cannot be answered at present. We should emphasize that the problem we are discussing is quite general, but one might well be able to find non-generic configurations in which the problem is absent.

Let us discuss these issues more concretely for the case of a brane-antibrane pair transverse to a stabilized Calabi-Yau. In principle one should be able to compute the inflaton potential directly, by substituting the complete superpotential into the supergravity F-term potential

$$V^F = e^K(g^{i\bar{j}}D_iW\overline{D_jW} - 3|W|^2) \tag{6.5.11}$$

and possibly including the effects of D-term contributions. This turns out to be a rather subtle problem, essentially because of the breaking of supersymmetry in the brane-antibrane system.

We will begin instead by understanding the (supersymmetric) system of a single D3-brane transverse to a Calabi-Yau. We will find that superpotential stabilization of the volume necessarily generates mass terms for the scalars ϕ which describe the motion of the D3-brane. An implicit assumption in brane-antibrane inflation scenarios is that the brane and antibrane are free, in the absence of interactions, to move around the Calabi-Yau; the gentle force from their Coulomb interaction is then expected to lead to a relatively flat inflaton potential. Significant mass terms for the D3-brane (or any external forces on the D3-brane) invalidate this assumption and make inflation impossible.

Let us therefore consider the effective potential governing a D3-brane transverse to a Calabi-Yau manifold. We substitute the superpotential $W(\rho)$ and the Kähler potential (6.5.1) into (6.5.11), where the physical volume modulus r is given by (6.5.3). The resulting four-dimensional effective potential is

$$V^F = \frac{1}{6r}\left(\partial_\rho W\overline{\partial_\rho W}(1+\frac{1}{2r}\frac{k_{,\phi}k_{,\bar{\phi}}}{k_{,\phi\bar{\phi}}}) - \frac{3}{2r}(\overline{W}\partial_\rho W + W\overline{\partial_\rho W})\right) . \tag{6.5.12}$$

In the vicinity of a point in moduli space where $k(\phi,\bar{\phi}) = \phi\bar{\phi}$, this can be simplified to

$$V^F = \frac{1}{6r}\left(|\partial_\rho W|^2 - \frac{3}{2r}(\overline{W}\partial_\rho W + W\overline{\partial_\rho W})\right) + \left(\frac{|\partial_\rho W|^2}{12r^2}\right)\phi\bar{\phi} . \tag{6.5.13}$$

We must now incorporate the effects of an anti-D3-brane. In the scenario of [5] the superpotential (6.5.8) stabilized the compactification volume and generated a negative cosmological term V_0. The positive, warped tension of an anti-D3-brane was added to this to produce a small positive cosmological constant. In our notation, the anti-D3-brane induces an additional term in the effective potential (6.5.12),

$$V = \frac{1}{6r}\left(\partial_\rho W \overline{\partial_\rho W}(1 + \frac{1}{2r}\frac{k_{,\phi}k_{,\bar{\phi}}}{k_{,\phi\bar{\phi}}}) - \frac{3}{2r}(\overline{W}\partial_\rho W + W\overline{\partial_\rho W})\right) + \frac{D}{(2r)^2} \quad (6.5.14)$$

where D is a positive constant. Notice that this induced term differs from the one in [5] by a factor of r. This arises because the anti-D3 tension in the warped compactifications of [4] scales like $\frac{1}{r^3}e^{4A}$, and in the highly warped regime, $e^{4A} \sim r\ exp(-\frac{8\pi K}{3g_s M})$. This does not alter the conclusions of [5], though it changes the numerology.

Suppose that the potential (6.5.14) has a de Sitter minimum V_{dS} at $\rho = \rho_c, \phi, \bar{\phi} = 0$. We will now compute the mass of the D3-brane moduli in an expansion about this minimum. To simplify the analysis we assume that at the minimum ρ is real, and also that for real ρ, $W(\rho)$ is real. The canonically normalized scalar which governs the motion of the D3-brane is not ϕ but is instead a rescaled field $\varphi = \phi\sqrt{3/(\rho+\bar{\rho})}$; it is the mass of φ which we will compute.

First, we rewrite (6.5.14) as

$$V = \left(W'(\rho)^2\rho - 3W(\rho)W'(\rho) + \frac{D}{4}\right)(\rho - \phi\bar{\phi}/2)^{-2} \quad (6.5.15)$$

where primes denote derivatives with respect to ρ, and define V_0 by

$$V_0(\rho_c) = \frac{1}{\rho_c^2}\left(W'(\rho_c)^2\rho_c - 3W(\rho_c)W'(\rho_c) + \frac{D}{4}\right). \quad (6.5.16)$$

Then

$$V = \frac{V_0(\rho_c)}{(1-\varphi\bar{\varphi}/3)^2} \approx V_0(\rho_c)\left(1 + \frac{2}{3}\varphi\bar{\varphi}\right). \quad (6.5.17)$$

This means that the field φ acquires the mass

$$m_\varphi^2 = \frac{2}{3}V_{dS} = 2H^2 \quad (6.5.18)$$

This is in fact precisely the result one would obtain for a conformally coupled scalar in a spacetime with cosmological constant V_{dS}. This is most easily understood by setting $D = 0$ and studying the resulting AdS_4. The four-dimensional AdS curvature is $R_{AdS} = 4V_0$, so that (6.5.18) corresponds to a coupling

$$\delta V = \left(\frac{1}{6}R_{AdS}\right)\varphi\bar{\varphi}. \quad (6.5.19)$$

If the D3-brane is in a highly warped region this result could have been anticipated, since this highly warped region is dual to an almost conformal four-dimensional field theory [232] and the scalar field describing the motion of the brane is conformally coupled (see [228])[45]. The derivation of (6.5.19) is also valid even when the D3-brane is far from the near horizon region.

We now see that the D3-brane moduli masses are necessarily of the same scale as the inflationary energy density V_0, since during inflation, the extra antibrane(s) simply sit at the end of the throat and provide an energy density well-modeled by (6.5.14). It is straightforward to verify that such masses lead to a slow-roll parameter $\eta = 2/3$, incompatible with sustained slow-roll inflation.

It is instructive to compare this result with the well-known η-problem, which bedevils most models of F-term inflation in $\mathcal{N} = 1$ supergravity. One begins by asking whether slow-roll inflation is possible in a model of a single field ϕ with any type of Kähler potential and any superpotential $W(\phi)$. For a minimal Kähler potential and a generic superpotential $W(\phi)$ one typically has a inflaton mass $m_\phi^2 = \mathcal{O}(H^2)$, and hence no inflation, just as in the generic case considered in the present chapter. But this does not mean that inflation in $\mathcal{N} = 1$ supergravity is impossible. Various superpotentials with non-generic dependence on ϕ have been found, some of which permit inflation. For example, in supergravity with the canonical Kähler potential and a linear superpotential for the inflaton, the mass term contribution to the potential cancels:

$$K = \bar{\phi}\phi\ , \qquad W = \phi \qquad \Rightarrow \quad V = e^{\bar{\phi}\phi}\left((1+\bar{\phi}\phi)^2 - 3\bar{\phi}\phi\right) = 1 + \frac{1}{2}(\bar{\phi}\phi)^2 + \cdots \tag{6.5.20}$$

A similar effect occurs for the superpotential $W = \phi(\sigma_1\sigma_2 - M^2)$, which leads to a simple realization of F-term hybrid inflation [233]. Moreover, the dangerous mass terms for the inflaton do not appear at all in D-term inflation [234].

It is quite possible, therefore, that one could find a consistent inflation scenario in string theory by studying superpotentials which depend on the inflaton field. As mentioned above, this would undoubtedly require a fine-tuned configuration in which two contributions to the mass cancel to high precision. We treat this question in detail in Appendix 6.F, where we show that the introduction of a superpotential

[45] Note that the kinetic term for φ is of the form $\int d^4x \nabla\varphi\nabla\bar{\varphi}$.

depending on the inflaton field ϕ leads to a modification of the mass-squared m_ϕ^2 of the inflaton field which could make it much smaller (or much greater) than $2H^2$. This issue merits further investigation, which should become possible as we learn more about the detailed dependence of $W(\rho, \phi)$ on the background geometry and on the fluxes in string compactifications.

6.5.2 Scenario II: Kähler Stabilization

One model of stabilization that would be compatible with the inflationary scenario of §6.3,6.4 is the following. We have seen that the true Kähler-invariant expansion parameter which controls the α' expansion in these models, is r. Furthermore, r and ϕ have independent kinetic terms.

A method of directly stabilizing r could freeze the volume directly, without stopping inflation. Since r is not a chiral superfield itself, stabilization via effects in the superpotential cannot accomplish this. However, given that $W_0 \neq 0$, one *can* imagine that corrections to the Kähler potential could directly stabilize r.

In fact, Kähler stabilization has been proposed earlier for rather different reasons (see e.g. [235], which discusses Kähler stabilization of the heterotic string dilaton). Here we would need the α' corrections to (6.5.1) to break the no-scale structure and fix r. Some of these corrections have been calculated (see e.g. [236]). The subset of terms presented in [236] does not lead to this kind of stabilization, though there are likely to be other terms at the same orders which could change this conclusion. However, Kähler stabilization would be very difficult to find in a controlled calculation, so one might simply have to state it as a model-building assumption.

If one does assume that r is stabilized by corrections to the Kähler potential, then the models of §6.3,6.4 could be realized in the framework of [4]. In Appendix 6.C we show that in these models one can easily satisfy observational constraints such as the number of e-foldings and the size of the density perturbations.

6.6 Conclusion

One of the most promising ideas for obtaining inflation in string theory is based on brane cosmology. However, brane-antibrane inflation [221] suffers from various difficulties when one tries to embed it in full string compactifications with moduli stabilization, such as the (metastable) de Sitter vacua of [5].

We have argued here that some of these difficulties can be resolved by introducing highly warped compactifications. The warped brane-antibrane models introduced in general form in §6.3 and in a compact string theory example in §6.4 give rise to slow-roll inflation with an exponentially flat potential. In the compact example, the slow-roll parameters and the density perturbations can be fixed at suitable values by an appropriate choice of discrete fluxes in the warped region.

The above discussion assumes a suitable stabilization mechanism for the volume modulus of the compactification manifold. As described in §6.5, this is a highly nontrivial issue. Indeed, we have found that if one stabilizes the moduli as in [5] then this field acquires an effective mass-squared $m_\phi^2 = \mathcal{O}(H^2)$, making inflation impossible. As discussed in §6.5.1, fine-tuned dependence of the superpotential on ϕ could reduce this mass. With *generic* dependence on ϕ the problem persists.

The arguments leading to our conclusion that generic methods of stabilization stop inflation are rather general, and should apply to any system where the energy density depends on the volume modulus as $r^{-\alpha}$ with $\alpha > 0$. There are general arguments that this should always be the case, for the sources of energy we know about in string theory [237]. Thus, it appears very difficult to achieve slow-roll brane inflation in a manner compatible with stabilization of the compactified space in string theory. At the very least, it is challenging to find a model which works for *generic* forms of the stabilizing superpotential, which itself varies in a way that depends on all of the microscopic details of the compactification. In those non-generic cases where inflation is possible, the inflationary predictions will depend on the details of the moduli stabilization.

One should note that the degree of fine-tuning required for slow-roll inflation in these models is not extraordinary (see Appendix 6.F), and may well be attainable within the large class of known models. Moreover, even though fine-tuning is certainly undesirable, it may not be a grave problem. Indeed, if there exist many realizations of string theory, then one might argue that all realizations not leading to inflation can be discarded, because they do not describe a universe in which we could live. Meanwhile, those non-generic realizations which lead to eternal inflation (see Appendix 6.D) describe inflationary universes with an indefinitely large and ever-growing volume of inflationary domains. This makes the issue of fine-tuning less problematic. It will not escape the reader's notice that this argument is anthropic in nature [3,238,211]. It is worth pointing out that it is an independent, presumably well-defined mathematical question, whether or not string theory has solutions which are consistent with present experiments (e.g. which contain the standard model of particle physics, have sufficiently small cosmological term, and allow early inflation). This question can of course be studied directly (see e.g. [239] for recent work in this direction), and is an important one for string theorists to answer. Only if string theory does admit such solutions, does anthropic reasoning in this context become tenable. The large diversity of string vacua makes it reasonable to be optimistic on this score.

We have primarily focused on the implications of superpotential stabilization of the moduli for D3-brane/anti-D3-brane inflation. Our analysis has implications for other models of brane inflation as well. These include $Dp - \overline{Dp}$ systems and Dp-branes at angles with $p = 5, 7$. In these cases, Chern-Simons couplings will generically induce a $D3$-brane charge on the branes due to the presence of a non-trivial B_{NS} field. Such a charge will also be generated due to the curvature couplings for generic topologies of the cycles the branes wrap. If the induced charge is of order unity or more, the discussion of the previous section will apply. The volume modulus and the inflaton field will mix non-trivially in the Kähler potential and as a result a superpotential of the kind considered in §6.5.1, or in fact any source of energy which scales like $1/r^{\alpha}$, will generically impart an unacceptably big mass to the inflaton. It would be interesting to explore the special cases where such a charge is not induced, to see if one can make simple working models of brane inflation.

Other existing proposals for brane inflation depend on Fayet-Iliopoulos terms in the low-energy field theory [234]. The status of these FI terms in the effective $\mathcal{N} = 1$ supergravity arising from compactified string theory therefore merits careful investigation. String theory models with D-terms were realized in brane constructions [224,42] without consideration of volume stabilization. A consistent embedding of this model into compactified string theory is under investigation [240].

6.A General Discussion of Brane-Antibrane Potentials

Here we compute the gravitational force between a D3-brane and an anti-D3-brane which are transverse to a general compact six-dimensional space. We assume that there is no warping before we add the D-branes. Our objective is to compute the expression for the slow-roll parameter η (6.2.4) in this setup. For this purpose we note that the brane tension as well as the ten-dimensional Planck mass drop out from the expression for η if we express it in terms of the physical distance. We can therefore set $M_{Pl,10} = 1$, $T_{D3} = 1$, to avoid clutter in the equations.

The action for the system has the form

$$S = \int d^6x \frac{1}{2}(\nabla\varphi)^2 + \sum_i (1 + \gamma\varphi(x_i)) \tag{6.A.1}$$

where γ is a constant we will determine below. Here φ is the gravitational potential on the internal space. The equation of motion from (6.A.1) is

$$-\nabla^2\varphi + \gamma \sum_i \delta(x - x_i) = 0 \tag{6.A.2}$$

Treating one brane as the source and the other as a probe and comparing with (6.2.1) we see that $\gamma^2 = 2$.[46] The expression for the energy of N branes is thus

$$V \sim N + \frac{1}{2}\sum_i \gamma\varphi_{others}(x_i) \tag{6.A.3}$$

where the subscript in φ indicates that we compute the potential due to the other branes, with $j \neq i$, and evaluate it at x_i. There is also a self-energy correction. We assume that the latter is independent of position. This is true in homogeneous spaces, such as tori.

The equation of motion (6.A.2) is not consistent since all the charges on the left hand side of (6.A.2) have the same sign. A minimal modification that makes the equation consistent is to write it as

$$-\nabla^2\varphi + \gamma \sum_i \left(\delta(x - x_i) - \frac{2}{v_6}\right) = 0 \tag{6.A.4}$$

where v_6 is the volume of the compact manifold. This term comes naturally from the curvature of the four-dimensional spacetime, which, in the approximation that we neglect the potential, is de Sitter space. This positive curvature gives rise to a negative contribution to the effective potential in the six internal dimensions. It

[46] Note that (6.2.1) contains a contribution both from gravity and from the Ramond fields, so the gravity contribution is half of that in (6.2.1).

is reasonable to assume that the negative term is smeared over the compact space as in this minimal modification, as long as the transverse space is approximately homogeneous.[47]

Note that this term does not arise for the Ramond fields since the total charge is zero.

Let us now consider, for simplicity, the case of a single brane and a single antibrane. In order to compute η we compute the Laplacian of the potential V with respect to x_1. We get

$$\nabla^2_{x_1}\varphi_{x_2}(x_1) = -\frac{2}{v_6}\gamma \tag{6.A.5}$$

The subscript in φ indicates that this is the potential due to the brane at x_2. For a pair of branes the potential is $V = 2 + \gamma\varphi_{x_2}(x_1)$. The Laplacian has a constant negative value (6.A.5). We see that this implies that there exists at least one direction in which the second derivative has a value $V'' \sim \gamma\varphi'' \leq -\frac{\gamma^2}{3}\frac{1}{v_6}$, since there are six transverse dimensions. When we compute the contribution to η the factor v_6 cancels out.

When there are many fields, one should consider η as a matrix. In order to have slow roll inflation we need to demand that the matrix has no negative eigenvalue that is too large. If we have a large negative eigenvalue, then even if the scalar field is not initially rolling in that direction, it will typically start moving in this direction after a few e-foldings. The discussion above implies that η, viewed as a matrix, has an eigenvalue more negative than

$$\eta|_{eigenvalue} \leq -\frac{2}{3} \, . \tag{6.A.6}$$

This implies that at least one of the moduli acquires a tachyonic mass $m^2 \leq -2H^2$, which typically prevents a prolonged stage of inflation.

A similar analysis can be carried out for the general case of a Dp-brane/anti-Dp-brane system. It is easy to see that the only change is that the coefficient $\frac{2}{3}$ in (6.A.6) is replaced by $\frac{4}{(9-p)}$. More interestingly, the above analysis can also be applied to the

[47] In compactifications with orientifold planes, there would also be localized negative terms. However, these would be cancelled by the tensions of the branes which are present even after brane/antibrane annihilation. The extra energy of the inflationary brane/antibrane pair can be expected to induce a smeared negative contribution over and above the orientifold plane contribution.

case of Dp-branes at angles. By this we mean a system of slightly misaligned branes and orientifold planes, [225]. The supersymmetry breaking scale in such a system is controlled parametrically by an angle which measures the relative orientation of the branes. For small values of this angle, the vacuum energy, $V \sim \sum_i T_i$, obtained by summing over all the branes and planes, can be much smaller than the tension of any individual brane or plane. The force on a brane in such a system arises due to graviton-dilaton and RR exchange. In these systems there can be a cancellation between the graviton-dilaton and the RR force in such a way that the resulting force, computed with non-compact "internal" dimensions, is parametrically smaller than the value of the cosmological constant. Once the internal dimensions are compact, we have to make some modification of the gravitational equation in order to make it consistent. The simplest modification is to add a constant term on the right hand side of the corresponding Laplace equation. In this case the constant term will be proportional to the effective four dimensional cosmological constant. Then, repeating the analysis above, one finds that the resulting potential satisfies the inequality

$$V'' \leq -\frac{\gamma^2}{(9-p)} \frac{1}{v_6} T_p \sum_i T_i.$$

As a result, once again one obtains a value of η, (6.A.6), with the coefficient $\frac{2}{3}$ replaced by $\frac{4}{(9-p)}$. In other words, both the potential and its second derivative scale in the same way with the small angle which supresses supersymmetry breaking, making η independent of this angle.

6.B The $D3/\overline{D3}$ Potential in Warped Geometries

To calculate the potential it is actually easier to turn things around and view the D3-brane as perturbing the background and then calculate the resulting energy of the anti-D3-brane in this perturbed geometry. This of course gives the same answer for the potential energy of the brane-antibrane pair.

The coupling of the metric and the five-form to the D3-brane is given by (6.3.6). On general grounds one expects that the changes in the metric and F_5 caused by the D3-brane will vary in the directions transverse to the brane. These directions are spanned by the radial coordinate r and the directions along X_5. It is useful to observe that the background can be written as follows:

$$ds^2 = h^{-\frac{1}{2}}\left(-dt^2 + d\vec{x}^2\right) + h^{\frac{1}{2}}\left(dr^2 + \frac{r^2}{R^2}\tilde{g}_{ab}dy^a dy^b\right) \tag{6.B.1}$$

$$(F_5)_{rtx^1x^2x3} = \partial_r h^{-1}, \tag{6.B.2}$$

where $\tilde{g}_{ab}dy^a dy^b$ is the line element on X_5, and $h(r)$ is given by

$$h(r) = \frac{R^4}{r^4}. \tag{6.B.3}$$

It is easy to check that $h(r)$ is a harmonic function in a six-dimensional space spanned by r and the directions along X_5, with metric

$$ds_6^2 = dr^2 + \frac{r^2}{R^2}\tilde{g}_{ab}dy^a dy^b. \tag{6.B.4}$$

Adding one additional D3-brane at a radial location r_1 results in a perturbed background which is of the form (6.B.1), but with a harmonic function now given by

$$h(r) = \frac{R^4}{r^4} + \delta h(r). \tag{6.B.5}$$

δh solves the equation $\nabla_6^2 \delta h(r) = C\delta^6(\vec{r} - \vec{r_1})$ in the six-dimensional space (6.B.4).[48] For $r \ll r_1$ a simple calculation shows that

$$\delta h(r) = \frac{R^4}{N}\frac{1}{r_1^4} \tag{6.B.6}$$

independent of r and the detailed metric on X_5. In (6.B.6) the coefficient N arises because the ambient background is supported by N units of charge, whereas the perturbation we are interested in arises due to a single D3-brane. From (6.B.5) the resulting harmonic function is

$$h(r) = R^4\left(\frac{1}{r^4} + \frac{1}{N}\frac{1}{r_1^4}\right). \tag{6.B.7}$$

To determine the potential we now couple this new background to the anti-D3-brane. The anti-D3-brane is described by an action of the form (6.3.6), except that, as was mentioned before, the sign of the Chern-Simons term is reversed relative to the case of a D3-brane. We also remind the reader that the antibrane is located at $r = r_0$; we will assume that $r_1 \gg r_0$. Combining all these results, after a simple calculation one recovers the desired potential (6.3.9).

This calculation of the potential is valid for one brane-antibrane pair. For one brane and p antibranes, to leading order, (6.3.9) is simply multiplied by p. Corrections to this leading-order potential are suppressed for small p.

[48] The constant C is determined by the tension of the D3-brane.

6.C Warped Inflation

In this appendix we discuss how inflation would look if one managed to fix the overall volume modulus without giving a mass to the brane motion. We argued above that the low energy dynamics of the system is described by the action (6.3.10). The radial position of the D3-brane, r_1, will play the role of the inflaton below. We define a canonically normalized field

$$\phi = \sqrt{T_3} r_1 \tag{6.C.1}$$

and $\phi_0 = \sqrt{T_3} r_0$. The effective action is then given by

$$S = \int d^4x \sqrt{-g} \left(\frac{\mathcal{R}}{16\pi G_N} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{4\pi^2 \phi_0^4}{N} \left(1 - \frac{1}{N} \frac{\phi_0^4}{\phi^4} \right) \right) \tag{6.C.2}$$

We have assumed that there are no significant additional terms in the effective action (6.C.2).

This inflaton potential is extremely flat: the first term in the potential, which is independent of the inflaton, is larger than the second term by a factor proportional to $(\frac{r_1}{r_0})^4$. This factor can be interpreted as the relative redshift between the brane location r_1 and the antibrane location r_0; as we explained in §6.4, this redshift is exponentially sensitive to the parameters of the model:

$$r_0/R = e^{-\frac{2\pi K}{3 g_s M}} \tag{6.C.3}$$

where g_s is the string coupling and K, M are integers that specify fluxes turned on in the compactification.

The slow-roll parameters can now be calculated in standard fashion. We will use conventions where $8\pi G_N = M_{Pl}^2$. One finds that

$$\begin{aligned} \epsilon &\equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{8}{N^2} M_{Pl}^2 \frac{\phi_0^8}{\phi^{10}} \\ \eta &\equiv M_{Pl}^2 \frac{V''}{V} \simeq -\frac{20}{N} M_{Pl}^2 \frac{\phi_0^4}{\phi^6}. \end{aligned} \tag{6.C.4}$$

Slow-roll requires that $|\eta| \ll 1, |\epsilon| \ll 1$. Of these the condition on η is more restrictive. It can be met by taking

$$\phi \gg \left(\frac{20}{N} M_{Pl}^2 \phi_0^4 \right)^{1/6}. \tag{6.C.5}$$

The number of e-foldings is given by

$$N_e = \frac{1}{M_{Pl}^2}\int \frac{V}{V'}d\phi \simeq \frac{N}{24}\frac{1}{M_{Pl}^2}\frac{\phi^6}{\phi_0^4}. \tag{6.C.6}$$

Requiring $N_e \sim 60$ can be achieved by taking ϕ to be sufficiently large and is compatible with the bound (6.C.5).

Finally, the adiabatic density perturbations are given by ([215], page 186)

$$\delta_H = \frac{1}{\sqrt{75}\pi}\frac{1}{M_{Pl}^3}\frac{V^{3/2}}{V'} = \frac{\sqrt{N_e}}{2\sqrt{75}}\frac{\phi^5}{\phi_0^2 M_{Pl}^3}. \tag{6.C.7}$$

This quantity should be equal to $1.9 \cdot 10^{-5}$ at $N_e \sim 60$, when the perturbations responsible for the large scale structure of the observable part of the universe are produced.

After some algebra, δ_H can be expressed in terms of N_e as follows:

$$\delta_H = C_1 N_e^{5/6}\left(\frac{T_3}{M_{Pl}^4}\right)^{1/3}\left(\frac{r_0}{R}\right)^{4/3}. \tag{6.C.8}$$

C_1 is a constant which is somewhat model dependent; using (6.C.7) and (6.C.2), one has

$$C_1 = \frac{3^{1/3}2^{3/2}}{5\pi}\left(\frac{N}{T_3 R^4}\right)^{1/6} \tag{6.C.9}$$

and after using (6.4.5), (6.3.7) one finds that $C_1 = 0.39$ for the model of §6.4.[49]

The four-dimensional Planck scale ($M_{Pl}^{-2} \equiv 8\pi G_N$) is given by

$$M_{Pl}^2 = \frac{2V_6}{(2\pi)^7 \alpha'^4 g_s^2} \tag{6.C.10}$$

where V_6 is the volume of the Calabi-Yau. This formula is strictly applicable only to a Kaluza-Klein compactification, not a warped compactification of the kind considered here. However, the approximation is a good one since the graviton zero mode has most of its support away from the regions with large warping (where its wave function is exponentially damped.) We may express the brane tension as

$$\frac{T_3}{M_{Pl}^4} = \frac{(2\pi)^{11}}{4} g_s^3 \frac{\alpha'^6}{V_6^2} \tag{6.C.11}$$

[49] C_1 increases by a factor $\sqrt{p}$ when there are p antibranes. While making numerical estimates we set $p = 2$.

This dimensionless ratio evidently depends on the string coupling constant and the volume of the six compact dimensions. The value $T_3/M_{Pl}^4 \sim 10^{-3}$ is quite reasonable: it corresponds to $g_s \sim 0.1$ and a Calabi-Yau volume of a characteristic size $(V_6)^{1/6} \sim 5\sqrt{\alpha'}$. Larger values of V_6 lead to smaller values for T_3/M_{Pl}^4, which make it easier to meet the density perturbation constraints.

More important, for present purposes, is the factor $(r_0/R)^{4/3}$, which has its origins in the redshift suppression of the potential that was emphasized in the discussion above. By taking this factor to be small enough we see that the constraint on δ_H, (6.C.7), can be met. As an example, taking $T_3/M_{Pl}^4 \sim 10^{-3}$ and $N_e = 60$, we find that $\delta_H \approx 1.91 \cdot 10^{-5}$ for $r_0/R = 2.5 \cdot 10^{-4}$. This condition on r_0/R can easily be met for reasonable values of the flux integers K, M. Taking $g_s = 0.1$, we get $r_0/R = 2.5 \cdot 10^{-4}$, with $K/M \simeq 0.4$. The latter condition can be achieved using moderate values of flux, e.g. $K = 8, M = 20$.

Now that we have ensured that the various constraints can be met in our model, it is worth exploring the resulting inflationary scenario a little more. The energy scale during inflation can be expressed in terms of δ_H. One finds from (6.C.8), and using the fact that the potential is well approximated by the first term in (6.3.9), that

$$\frac{V}{M_{Pl}^4} = \frac{2\delta_H^3}{C_1^3 N_e^{5/2}}. \tag{6.C.12}$$

Taking $\delta_H = 1.91 \cdot 10^{-5}$, $N_e = 60$, $C_1 = 0.39$ and $M_{Pl} = 2.4 \cdot 10^{18}$ GeV one finds that the energy scale is

$$\Lambda \equiv V^{1/4} = 1.3 \cdot 10^{14}\,\text{GeV}. \tag{6.C.13}$$

This is considerably lower than the GUT scale $\sim 10^{16}$ GeV. This low scale of inflation is a generic feature of the scenario.

Next, it is straightforward to see that δ_H is given in terms of V and ϵ by

$$\delta_H = \frac{1}{5\pi\sqrt{6\epsilon}} \left(\frac{V}{M_{Pl}^4}\right)^{1/2}. \tag{6.C.14}$$

Solving for V from (6.C.12) gives

$$\epsilon = \frac{\delta_H}{75\pi^2 C_1^3 N_e^{5/2}}. \tag{6.C.15}$$

Taking $\delta_H = 1.91 \cdot 10^{-5}$, $C_1 = 0.39$,$N_e = 60$ gives

$$\epsilon = 1.54 \cdot 10^{-11}, \tag{6.C.16}$$

a very small number. The ratio of the anisotropy in the microwave background generated by gravitational waves to that generated by adiabatic density perturbations is given (at large l) by

$$r \simeq 12.4\epsilon. \tag{6.C.17}$$

In our model this is very small, so the anisotropy is almost entirely due to density perturbations.

Finally, η can be related to N_e, and is given by

$$\eta = -\frac{5}{6}\frac{1}{N_e}. \tag{6.C.18}$$

Setting $N_e = 60$ gives

$$\eta = -0.014. \tag{6.C.19}$$

Clearly, as we mentioned above, $|\eta| \gg \epsilon$. The tilt parameter is given by

$$n = 1 - 6\epsilon + 2\eta \simeq 1 + 2\eta \approx 0.97, \tag{6.C.20}$$

in excellent agreement with observational data from WMAP.

In summary, in our model the scale of inflation Λ (6.C.13) is generically low. Most of the anisotropy originates from adiabatic density perturbations, since ϵ is extremely small, and the tilt in the spectrum, (6.C.20), is determined by η. The values for these parameters are nearly model-independent: they are almost entirely determined by the observed value for δ_H and by the number of e-foldings, N_e.

6.D Eternal Inflation

At large ϕ, the potential $V(\phi)$ in (6.C.2) becomes extremely flat. For flat potentials, the force pushing the field ϕ down becomes very small, whereas the amplitude of inflationary fluctuations remains practically constant. As a result, the motion of the field ϕ at large ϕ is mainly governed by quantum jumps. This effect is known to lead to eternal inflation [241,242].

Eternal inflation leads to formation of a fractal structure of the universe on a very large scale. It occurs for those values of the field ϕ for which the post-inflationary amplitude of perturbations of the metric δ_H would exceed unity [214]. In our case δ_H is proportional to ϕ^5, cf. (6.C.7). Since the amplitude of the density perturbations is $\delta_H \sim 1.9 \cdot 10^{-5}$ in the observable part of the universe, eternal inflation should occur for all values of the field ϕ that are greater than $10 \cdot \phi_{60}$. Here ϕ_{60} is the value of the field at the moment starting from which the universe inflated $e^{N_e} \sim e^{60}$ times. In other words, if r_{60} is the brane separation corresponding to the moment when the large-scale structure of the observable part of the universe was produced, then the regime of eternal inflation occurred when the brane separation was ten times greater than r_{60}. The possibility of eternal inflation in our model is very interesting since this regime makes the existence of inflation much more plausible: even if the probability of initial conditions for eternal inflation is small, the universes (or the parts of the universe) where these conditions are satisfied rapidly acquire indefinitely large (and ever growing) volume [243].

6.E Exit from Inflation

In this appendix we comment on the exit from inflation through brane-antibrane annihilation.

The brane-antibrane potential used in our analysis of inflation is no longer valid when the brane separation is comparable to the string length. At that stage a tachyon appears and then condenses. (In this sense, our model, like all the brane inflation models described in [217], is a particular version of the hybrid inflation scenario [244].) One may attempt to use the properties of this brane-antibrane tachyon [118,245] to describe the exit from inflation. Here we will show that one of the possible problems of this scenario, the overproduction of cosmic strings [246,14], is ameliorated by the warped geometry.

In the case of a merging brane-antibrane pair, the tachyon is a complex field and there is a $U(1)$ symmetry. Formation of cosmic strings associated with the $U(1)$ symmetry breaking leads to large-scale perturbations of the metric which are compatible with the current observations of the cosmic microwave anisotropy [247] only if $G_N T_1 = \frac{T_1}{8\pi M_{Pl}^2} \lesssim 10^{-7}$, where T_1 is the cosmic string tension [248]. This tension can be evaluated either by the methods of [249], or by identifying cosmic strings with D1-branes. In the usual case (i.e. ignoring warping) one has

$$T_1 = \frac{1}{2\pi g_s \alpha'}. \tag{6.E.1}$$

The requirement $G_N T_1 = \frac{T_1}{8\pi M_{Pl}^2} \lesssim 10^{-7}$ reads

$$G_N T_1 = \frac{g_s}{16\pi} \frac{(2\pi l_s)^6}{V_6} \lesssim 10^{-7}, \tag{6.E.2}$$

i.e.

$$V_6 \gtrsim 2 \times 10^5 \; g_s \; (2\pi l_s)^6. \tag{6.E.3}$$

This shows that the cosmic string contribution to the perturbations of the metric produced after inflation is unacceptably large unless the volume of the compactified space V_6 is at least five orders of magnitude greater than $g_s(2\pi l_s)^6$.

In the brane inflation models of §6.3, §6.4, however, the relevant tension is redshifted by the warped geometry, which leads to exponential suppression of T_1:

$$T_1 = \frac{1}{2\pi g_s \alpha'} e^{-\frac{4\pi K}{3 g_s M}}. \tag{6.E.4}$$

As a result, the undesirable contribution of cosmic strings (D1-branes) to perturbations of the metric becomes exponentially suppressed.

6.F Fine-tuning of the Superpotential

In this appendix we study a toy model in order to make more precise our statements concerning the degree of fine-tuning which is required for slow-roll brane inflation. We should note here that we will be discussing the degree to which the inflaton potential itself must be tuned. In a given string model, one cannot directly tune the potential, but only vary choices of the background data like fluxes, compactification manifold, or brane positions. It could be that the tuning required in terms of this data is more or less severe than our estimate below, but explicit string calculations of the relevant superpotentials will be necessary to determine this.

Before studying the example, let us mention how small the inflaton mass term must be for a given model of slow-roll inflation to be compatible with experiment. The goal is to have a long stage of inflation producing metric fluctuations with a fairly flat spectrum. Recent observations suggest that, modulo some uncertainties, the tilt is $n_s \approx 1 + \frac{2m_\phi^2}{3H^2} = 0.97 \pm 0.03$ [247,250]. This is compatible with an inflaton mass $|m_\phi^2|/H^2 \sim 10^{-1} - 10^{-2}$.

This could be achieved through fine-tuning of m_ϕ^2 by only about one part in 100. Thus, the fine-tuning that we need to perform is not extraordinary. Given the large number of possible compactifications, the existence of some configurations which allow inflation seems quite likely.

We now turn to an example which illustrates this point. Consider a D3-brane transverse to a warped compactification; we would like to know how the (brane-antibrane) inflaton mass terms vary as the inflaton-dependence of the superpotential varies.

The Kähler potential for the volume modulus and the D3-brane field ϕ takes the form $K(\rho, \bar{\rho}, \phi, \bar{\phi}) = -3\log\left(\rho + \bar{\rho} - k(\phi, \bar{\phi})\right)$. We will work in the vicinity of the point $\phi = \bar{\phi} = 0$ in moduli space, where $k(\phi, \bar{\phi}) = \phi\bar{\phi}$. We choose a superpotential of the form

$$W(\rho, \phi) = W_0 + g(\rho) f(\phi) \tag{6.F.1}$$

where $g(\rho)$ is an arbitrary function of ρ, $f(\phi) = (1 + \delta\,\phi^2)$, and W_0 and δ are constants. This is a slight generalization of the superpotential in [5], which corresponds to $\delta = 0$ and $g(\rho) = Ae^{-a\rho}$.

One can now calculate the supergravity potential $V^F = e^K(g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2)$ for the two complex fields ρ, ϕ. The exact potential has a simple dependence on $\mathrm{Im}\,\rho$ and $\mathrm{Im}\,\phi$ which shows that the point $\mathrm{Im}\,\rho = \mathrm{Im}\,\phi = 0$ is an extremum of the potential (it is a minimum, at least for small ϕ). Therefore we will present here the exact potential $V^F(\sigma, \psi)$ as a function of $\mathrm{Re}\,\rho = \sigma$ and $\mathrm{Re}\,\phi = \psi$ at $\mathrm{Im}\,\rho = \mathrm{Im}\,\phi = 0$.

$$V^F(\sigma, \psi) = \frac{g(\sigma)^2}{6(\sigma - \psi^2/2)^2}\left(2\delta^2\psi^2 + f(\psi)\frac{g'(\sigma)}{g(\sigma)}\left((\sigma - 1)f(\psi) - \frac{3W_0}{g(\sigma)} - 2\right)\right) \tag{6.F.2}$$

We are interested in the total potential $V^F(\sigma, \psi) + V_{\overline{D}3}$ at small ψ, where $V_{\overline{D}3}$ is the potential due to the antibrane (cf. (6.5.14)). We may therefore use the stabilization of the volume in the first approximation at $\psi^2 = 0$ and calculate the potential at the AdS critical point $\sigma_c = r_c$, where, using $D_\rho W|_{\phi=0} = 0$, one finds

$$W_0 = -g(\sigma_c) + \frac{2}{3}\sigma_c g'(\sigma_c)\ , \qquad V_{AdS} = -\frac{(g'(\sigma_c))^2}{6\sigma_c} \tag{6.F.3}$$

We now change variables to $\psi^2 = \frac{2}{3}\sigma_c\varphi^2$, where φ is a field with the canonical kinetic term $(\partial\varphi)^2$. We find

$$V^F(\sigma_c,\varphi) = \frac{1}{6\sigma_c(1-\varphi^2/3)^2}\Big(-(g')^2 + \frac{4\delta^2 g^2}{3}\varphi^2 - \frac{2\delta g g'}{3}\varphi^2 \\ -(\frac{2\delta}{3})^2 g g'\sigma_c\varphi^4 + \frac{4}{9}\delta^2\varphi^4\sigma_c^2\Big) \tag{6.F.4}$$

From the antibrane we get the additional contribution mentioned above. Keeping terms up to those quadratic in ϕ, we finally arrive at

$$V^F(\sigma_c,\varphi) + V_{\bar{D}3}(\sigma_c,\varphi) \approx V_{dS} + \frac{2\varphi^2}{3}\left(V_{dS} + \frac{1}{6\sigma_c}(2\delta^2 g^2 - \delta g' g)\right) \tag{6.F.5}$$

Here V_{dS} is the value of the potential at the de Sitter minimum,

$$V_{dS} = V_{AdS} + \frac{D}{4\sigma_c^2} = -\frac{(g'(\sigma_c))^2}{6\sigma_c} + \frac{D}{4\sigma_c^2} \equiv 3H^2 \tag{6.F.6}$$

The mass-squared of the field ϕ is

$$m_\phi^2 = 2H^2 + \frac{2|V_{AdS}|}{3}\left[2\left(\delta\frac{g}{g'}\right)^2 - \delta\frac{g}{g'}\right] \tag{6.F.7}$$

To make m_ϕ^2 small, we need $\delta\frac{g}{g'} > 0$ as well as $\frac{|V_{AdS}|}{3}\left[2\left(\delta\frac{g}{g'}\right)^2 - \delta\frac{g}{g'}\right] \approx -H^2$. If the parameters of the model were arbitrary then this would certainly be possible.

We will express our results in terms of a parameter $\beta = \delta\frac{g}{g'}$:

$$m_\phi^2 = 2H^2 - \frac{2}{3}|V_{AdS}|(\beta - 2\beta^2) = 2H^2\left(1 - \frac{|V_{AdS}|}{V_{dS}}(\beta - 2\beta^2)\right) \tag{6.F.8}$$

For $\beta = 0$ we recover the "conformal" result

$$m_\phi^2 = 2H^2 \tag{6.F.9}$$

As a simple example, if $g(\rho) = Ae^{-a\rho}$, as in [5], we find $\beta = -\frac{\delta}{a}$. However, let us assume, as in [5], that $|V_{AdS}| \gg V_{dS}$. Then for the simple value $\beta = 1$ (i.e. $\delta = -a$) we have

$$m_\phi^2 = 2H^2\left(1 + \frac{|V_{AdS}|}{V_{dS}}\right) \approx \frac{2}{3}|V_{AdS}| \gg 2H^2 \qquad (6.F.10)$$

Thus, whereas it is true that our knowledge of $W(\rho,\phi)$ is not particularly good, our absence of knowledge does not allow us to say much about m_ϕ^2. The only thing we can say is that in our particular example, for $|V_{AdS}| \gg V_{dS}$, this mass can be fine-tuned to take almost any value.[50] In particular, one has a flat potential with $m_\phi^2 = 0$ for

$$\beta = \frac{1}{4}\left(1 \pm \sqrt{1 - \frac{8V_{dS}}{|V_{AdS}|}}\right) \qquad (6.F.11)$$

This equation always has solutions for $|V_{AdS}| \geq 8V_{dS}$. For $|V_{AdS}| \gg 8V_{dS}$, the solutions are:

$$\beta_1 = \frac{\delta_1}{a} = \frac{1}{2} - \frac{V_{dS}}{|V_{AdS}|} \approx \frac{1}{2} \qquad (6.F.12)$$

and

$$\beta_2 = \frac{\delta_2}{a} = \frac{V_{dS}}{|V_{AdS}|} \ll 1 \qquad (6.F.13)$$

In order to satisfy one of these two conditions and have $m_\phi^2 = 0$ one can fine-tune either the ratio $\frac{V_{dS}}{|V_{AdS}|}$ (as was done in [5]) or the coefficient δ in the superpotential. In order to prove that inflation in this scenario is impossible, one would need to prove that neither of these types of fine-tuning is possible.

It is instructive to compare this situation with the problem of realizing the chaotic inflation scenario in $\mathcal{N} = 1$ supergravity. Let us consider a canonical Kähler potential $K = \bar{\phi}\phi + \bar{\sigma}_i\sigma_i$, where ϕ is the inflaton field and σ_i are some other fields. If the superpotential is a function of the fields σ_i but not of the field ϕ, then the potential of the scalar fields has the general structure as a function of the real part of the field ϕ, $V = e^{\phi^2}\, V(\sigma_i)$, which implies that $m_\phi^2 = 3H^2$, i.e. $\eta = 1$.

50 Incidentally, Eq. (6.F.10) implies that if one does not make any fine-tuning, then for the model described in [5], with $V_{dS} \sim 10^{-120}$ in Planck units, the typical mass squared of the D3 brane moduli fields is expected to be $\mathcal{O}(|V_{AdS}|)$, which can be extremely large. This result may have interesting phenomenological implications.

One can resolve this problem by introducing a superpotential depending on the inflaton field, just as we did in this appendix. However, in the simplest version of chaotic inflation one needs the inflaton field to be at $\phi \gg 1$, in Planck mass units, and to change significantly, by $\Delta\phi = \mathcal{O}(1)$, during the last 60 e-folds. It is this last part that causes substantial difficulties for inflation in $\mathcal{N} = 1$ supergravity. It is always possible to find a superpotential which depends on the inflaton field ϕ in such a way that the potential becomes flat in the vicinity of one particular point. However, one must do this for all ϕ in a large interval $\Delta\phi = \mathcal{O}(1)$. One needs enormous *functional fine-tuning* in a large interval at $\phi \gg 1$, where the term $\sim e^{\phi^2}$ grows very fast.

Meanwhile, in our case the situation is much better. Instead of a functional fine-tuning in a large interval of ϕ we need to make a fine-tuning at a single point $\sigma = \sigma_c$, $\phi = 0$. In order to estimate the required degree of fine-tuning, let us e.g. fix $\beta = 1/2$ and change the ratio $|V_{AdS}|/V_{dS}$ in Eq. (6.F.8) in the interval $0 < |V_{AdS}|/V_{dS} < 4$. As one can easily see, in this case the mass squared of the inflaton field changes from $2H^2$ to $-2H^2$. In approximately 1% of this interval the condition $n_s \approx 1 + \frac{2m_\phi^2}{3H^2} = 0.97 \pm 0.03$ is satisfied. On the other hand, if this condition is substantially violated, which happens in the main part of this interval, then inflation becomes either too short or impossible, and the universe most probably becomes unsuitable for life.

Finally, if inflation can be eternal (and it can be eternal in the models of §6.3,6.4; see Appendix 6.D), then the parts of the universe where eternal inflation is possible have an indefinitely large and ever-increasing volume. For this reason, regions of the universe where eternal inflation does occur, however improbable that may have been, are in some sense favored. One could therefore argue that the problem of fine-tuning in inflationary cosmology is not as dangerous as one could expect, and sometimes it may not even be particularly relevant.

7. An Inflaton Mass Problem from Threshold Corrections

ABSTRACT OF ORIGINAL PAPER

Inflationary models whose vacuum energy arises from a D-term are believed not to suffer from the supergravity eta problem of F-term inflation. That is, D-term models have the desirable property that the inflaton mass can naturally remain much smaller than the Hubble scale. We observe that this advantage is lost in models based on string compactifications whose volume is stabilized by a nonperturbative superpotential: the F-term energy associated with volume stabilization causes the eta problem to reappear. Moreover, any shift symmetries introduced to protect the inflaton mass will typically be lifted by threshold corrections to the volume-stabilizing superpotential. Using threshold corrections computed by Berg, Haack, and Körs, we illustrate this point in the example of the D3-D7 inflationary model, and conclude that inflation is possible, but only for fine-tuned values of the stabilized moduli. More generally, we conclude that inflationary models in stable string compactifications, even D-term models with shift symmetries, will require a certain amount of fine-tuning to avoid this new contribution to the eta problem.

7.1 Introduction

In any model of slow-roll inflation [6], one needs the inflaton potential $V(\phi)$ to be rather flat, as measured by the slow-roll parameters:

$$\epsilon \equiv \frac{M_{\rm p}^2}{2}\left(\frac{V'}{V}\right)^2 \tag{7.1.1}$$

$$\eta \equiv M_{\rm p}^2\left(\frac{V''}{V}\right) \tag{7.1.2}$$

This chapter is reprinted, with changes, from Liam McAllister, "An Inflaton Mass Problem in String Inflation from Threshold Corrections to Volume Stabilization," hep-th/0502001.

where $M_{\rm p}$ is the four-dimensional reduced Planck mass and primes denote derivatives with respect to the inflaton ϕ. It is convenient to rewrite (7.1.2) as

$$\eta = \frac{V''}{3H^2} \tag{7.1.3}$$

so that η measures the inflaton mass in units of the Hubble scale H. Observations require that $\eta \leq 10^{-2}$. A key issue in inflationary model-building is the solution of this constraint.

Inflationary models in supergravity can be divided into F-term models and D-term models according to the source of the supersymmetry-breaking energy which drives inflation. F-term models suffer from what is known as the eta problem, or the inflaton mass problem [251]. The F-term energy

$$V^{\rm F} \equiv e^K \Big(K^{,\alpha\bar{\beta}} D_\alpha W \overline{D_\beta W} - 3|W|^2 \Big) \tag{7.1.4}$$

depends on the inflaton ϕ because ϕ necessarily appears in the Kähler potential. Even if the superpotential depends weakly or not at all on ϕ, the total energy does vary with ϕ. Thus, restoring factors of the Planck mass, we have

$$V_{\rm F}'' = \frac{K''}{M_{\rm p}^2} V_{\rm F} + ... \tag{7.1.5}$$

and so a canonically-normalized scalar has $\eta \sim 1$. The only general solution to this problem in F-term models is fine-tuning the contributions in (7.1.5) to cancel each other to reasonable accuracy, leaving a small net η.

D-term models [234], however, are well-known to be immune to the eta problem, as the Kähler potential does not appear in the D-term energy.[51] This is argued to imply that the inflaton mass need not obey $m_\phi \sim H$, as is generically true in F-term models, but can instead be much smaller. This is a fairly strong argument in favor of D-term inflation.

The goal of this chapter is to demonstrate that this statement no longer holds in string compactifications whose volume is stabilized by a nonperturbative superpotential: both D-term and F-term models, including shift-symmetric constructions, receive inflaton mass corrections from threshold corrections to the nonperturbative

[51] For a discussion of important updates to the D-term inflation scenario, see [252].

superpotential. We will see that these mass corrections are generically of order the Hubble scale, so that $\eta \sim 1$.

The source of the problem is readily understood. Superpotential stabilization of Kähler moduli proceeds by introducing an F-term potential whose minimum determines[52] the compactification volume. Just as in the eta problem of F-term inflation, this energy depends on the inflaton through the Kähler potential. Although the inflationary dynamics may be designed to proceed according to a weak interaction, e.g. of widely-separated branes [221,222], the inflaton-dependence of the volume-stabilizing F-term energy typically introduces a *stronger* interaction and renders the total potential too steep for inflation.[53]

A solution to this problem that has received considerable attention [254,257,258] is the introduction of continuous geometric symmetries to protect the inflaton mass. In this approach, one posits the existence of an approximate shift symmetry along the inflationary trajectory.

One purpose of the present chapter is to point out that one-loop threshold corrections to the volume-stabilizing nonperturbative superpotentials will typically lift any such shift symmetry and introduce an inflaton mass of order H. Thus, shift symmetries do not suffice to protect the inflaton mass, because quantum corrections will lift these symmetries and change the inflaton potential. Specifically, threshold corrections to the nonperturbative superpotential introduce a dependence of the F-term energy on the various moduli in the system, including both open-string and closed-string fields. The inflaton is usually constructed as one of these moduli, so the F-term potential depends on the inflaton. If this dependence is generic then $\eta \sim 1$. This implies the existence of a rather general eta problem for inflation in nonperturbatively-stabilized string compactifications.

Volume stabilization is indispensable for a consistent model, and at present the best-understood methods of volume stabilization use nonperturbative superpotentials, along the lines suggested by Kachru, Kallosh, Linde, and Trivedi (KKLT) [5].[54] Thus, the presence of an eta problem in the context of nonperturbative volume

[52] In some cases, as we will review, the physical volume and the stabilized Kähler modulus are closely related but not identical. For simplicity we will nevertheless refer to this situation as 'volume stabilization'.

[53] This conflict between F-term stabilization and slow-roll inflation was recognized in a concrete form in the brane inflation [221,222] scenario of [22] and has been addressed in e.g. [24,25,253,254,255,256].

[54] For very interesting examples of perturbative volume stabilization, see [259,260].

stabilization is an important aspect of inflation in string theory.

We will be able to observe this effect in detail. Berg, Haack, and Körs (BHK) [24,25] computed the one-loop threshold corrections to the nonperturbative superpotential for the case of type IIB string theory on certain toroidal orientifolds. They observed that the loop corrections introduce a moduli-dependent mass for a mobile D3-brane in this background. (They further showed that this mass correction may be used to fine-tune a brane-antibrane potential to render it flat enough for inflation.) Their result clearly demonstrates, for the case that the inflaton is a D3-brane position and the compactification is a toroidal orientifold, that the inflaton-dependence of the threshold corrections is indeed sufficiently strong to affect inflation.

In §7.5.2 we will apply the result of BHK to compute the inflaton mass in a stabilization of the D3-D7 inflationary model [42]. A key point is that the D3-D7 model is a D-term model that has been constructed to enjoy a shift symmetry [254], so it might be expected not to be subject to an eta problem. As we will see, even though D-term inflation and shift symmetries do sometimes remove the usual eta problem, neither one suffices to remove the eta problem explored in this chapter.

An important qualification of this result, discussed in §7.5.3, is that the choice of a moduli-stabilizing configuration for the D3-D7 model can affect the inflaton mass. We will illustrate our points using a minimal implementation of the proposal of KKLT, i.e. a gaugino condensate from a single stack of D7-branes. More complicated configurations are possible, and in some special configurations with approximate symmetries, the inflaton mass can be fine-tuned to be small.

These statements should not be taken as criticisms of the D3-D7 model in particular. We would expect similar results for nearly any model of moving branes in a stabilized string compactification. More generally, the inflaton need not be a brane coordinate; closed string moduli can certainly appear in the threshold corrections, giving a mass to a closed string inflaton. Moreover, although nonperturbative superpotentials play an essential role in our concrete discussion, any F-term moduli-stabilizing energy could in principle lead to the same result.

7.2 The Eta Problem in Supergravity

In this section we will briefly review the supergravity eta problem and mention how D-term models avoid the problem. In later sections we will argue that this success of D-term models does *not* extend to superpotential-stabilized string compactifications.

7.2.1 F-term Inflation and the Eta Problem

In F-term models, inflation proceeds by slowly reducing the F-term energy,

$$V^{\mathrm{F}} \equiv e^{K}\Big(K^{,\alpha\bar{\beta}}D_{\alpha}W\overline{D_{\beta}W} - 3|W|^{2}\Big). \tag{7.2.1}$$

We are interested in computing the slow-roll parameter η (7.1.2).

Let us work with a canonically normalized inflaton ϕ, which we take to be complex for convenience. Then $\partial_{\phi}\partial_{\bar{\phi}}K = 1$, so that as a function of ϕ,

$$V^{\mathrm{F}}(\phi) = V^{\mathrm{F}}(0)\Big(1 + \phi\bar{\phi} + \ldots\Big) \tag{7.2.2}$$

We may therefore organize the contributions to (7.1.3) as

$$\eta = 1 + \frac{e^{K}}{V^{\mathrm{F}}(0)}\partial_{\phi}\partial_{\bar{\phi}}\Big(K^{,\alpha\bar{\beta}}D_{\alpha}W\overline{D_{\beta}W} - 3|W|^{2}\Big). \tag{7.2.3}$$

A successful model requires that the two terms on the right hand side of (7.2.3) are arranged to cancel to reasonable accuracy, leaving a small net inflaton mass. This sort of fine-tuning is the only *general* solution to the η problem in F-term models.

In particular, if the inflaton does not mix in the Kähler potential with any other fields, so that $K_{,\alpha\bar{\phi}} = 0$ unless $\alpha = \phi$, then the second term in (7.2.3) depends on the inflaton only through the superpotential, and the necessary fine-tuning must be achieved by adjusting the inflaton-dependence of the superpotential.

7.2.2 D-term Inflation

D-term models [234,261] are those in which the inflationary trajectory follows a direction which is not D-flat, so that inflation proceeds by slowly reducing a D-term energy. The particular advantage of this approach is that the Kähler potential does not appear in the D-term energy, so the argument of §7.2.1 does not apply. Thus, the inflaton mass does not receive the corrections of order H that plague F-term models.

At first sight, this conclusion appears surprisingly strong. The mass terms given in §7.2.1 are merely a concrete example of a general expectation: because the inflationary energy V breaks supersymmetry, we expect soft scalar masses to be induced by gravitational mediation, even if no more direct coupling is present. The resulting masses will be of order $V/M_{\rm p}^2 = 3H^2$.

This problematic coupling of the inflaton to the supersymmetry-breaking energy arises from the tree-level Kähler potential for the case of F-term models. D-term inflation sidesteps the problem by providing an inflationary energy which is insensitive to the Kähler potential [262].[55]

We will find that this statement requires careful reexamination in the context of stabilized string compactifications. The reason is that moduli stabilization typically introduces an F-term energy, reviving the problem of §7.2.1.

7.3 Nonperturbative Superpotentials and Volume Stabilization

The remainder of our discussion will rely on the details of moduli stabilization, so in this section we will first outline the logic of moduli stabilization and then explain how nonperturbative superpotentials can be used to fix Kähler moduli.

7.3.1 The Necessity of Volume Stabilization

String compactifications on Calabi-Yau manifolds typically have a large number of massless scalar fields, or moduli. For our purposes the most interesting moduli are the complex structure moduli, the positions of D-branes, and the Kähler parameters, including the overall volume.

Moduli can ruin cosmological models in various ways. They can store energy during inflation and then interfere with nucleosynthesis, or they could have time-dependent vevs at the present epoch, leading to changes in various physical constants. Finally, the presence of these light, gravitationally-coupled fields would typically lead to unobserved fifth-force interactions. Cosmological models which

[55] S. Thomas has emphasized that Planck-suppressed couplings of the inflaton in the Kähler potential can sometimes produce an inflaton mass even in D-term models [263,264].

aim to be successful in detail should somehow remove most or all of these light fields.

One modulus in particular presents a grave problem. The overall compactification volume does not have a flat potential, but is in fact unstable: it has a runaway direction toward decompactification. The reason is that the various sources of inflationary energy in string theory will necessarily appear, in the four-dimensional (Einstein-frame) description, multiplied by inverse powers of the volume:

$$V_{4\mathrm{d}} = \frac{C}{\rho^{\alpha}}. \tag{7.3.1}$$

Here $V_{4\mathrm{d}}$ is the inflationary potential, ρ is the volume modulus (taken to be real), C is a volume-independent factor, and α is positive. This result is easily obtained by dimensional reduction of ten-dimensional sources of energy, such as branes, strings, and fluxes.

If the volume were held fixed by hand, then a mild inflaton-dependence in C could lead to an inflating model. However, in reality we expect that a fast roll in the ρ direction, toward decompactification, will remove the possibility of slow roll in the ϕ direction.

It is therefore absolutely essential to introduce some form of volume-stabilizing potential $U(\rho)$, so that

$$V = \frac{C}{\rho^{\alpha}} + U(\rho) \tag{7.3.2}$$

has a minimum at a finite value of ρ.

The proposal of KKLT, which we will now review, is that a nonperturbative superpotential could lead to the necessary volume-dependence.

7.3.2 Nonperturbative Superpotentials and Volume Stabilization

Let us work in the concrete and well-studied example of the type IIB string on a six-dimensional orientifold, which we view as a limit of a compactification of F-theory on a fourfold. For simplicity we assume that the threefold has exactly one Kähler modulus, ρ. Three-form fluxes H_3, F_3 in the internal space lead to a superpotential [158]

$$W_0 = \int_{\mathrm{CY}} (F_3 - \tau H_3) \wedge \Omega \tag{7.3.3}$$

which depends on the complex structure moduli $\chi_i, i = 0, \dots h^{2,1}$ and the dilaton τ.

An additional contribution $W(\rho)$ to the superpotential would allow simultaneous solution of

$$D_\rho W = D_\tau W = D_{\chi_i} W = 0. \tag{7.3.4}$$

In this supersymmetric solution the dilaton, the complex structure moduli, and the volume are stabilized. (For more details on the stabilization of the complex structure moduli and the dilaton in this scenario, see e.g. [5,4,265].)

KKLT proposed that a nonperturbative superpotential $W_{\rm np}(\rho)$ from either of two sources could provide the necessary effect:

(1) Euclidean D3-branes wrapping a divisor in the Calabi-Yau [12].

(2) Gaugino condensation on a stack of $N > 1$ D7-branes wrapping a divisor in the Calabi-Yau, and filling spacetime.

In either case, the resulting superpotential takes the form

$$W_{\rm np} = \Sigma(\zeta, \phi) e^{-a\rho}. \tag{7.3.5}$$

In this formula a is a numerical constant and Σ is a holomorphic function of the various moduli ζ (such as the complex structure moduli χ_i and the positions of any D-branes) and of the inflaton ϕ.

In the absence of background flux, such a superpotential is possible only when the divisor D satisfies a rather stringent topological condition: the arithmetic genus $\chi(D, \mathcal{O}_D)$ of the divisor must obey $\chi = 1$ [12].

As explained in [266], the effect of fluxes is to permit gaugino condensation to occur somewhat more generally, so that divisors with $\chi > 1$ can contribute to the superpotential. There are reasons to believe that the same conclusion applies to the Euclidean D3-brane superpotential [267,268].

A special feature of the gaugino condensate superpotential is that $a = 4\pi^2/N$ for the condensate of a pure $SU(N)$ gauge group, whereas $a \sim 1$ for the case of Euclidean D3-branes.[56]

We will now turn our attention to the holomorphic prefactor $\Sigma(\zeta, \phi)$.

---

[56] Our conventions for a and ρ differ by a factor of (2π) from those of KKLT: $a_{KKLT} = 2\pi/N$.

7.3.3 Threshold Corrections to Nonperturbative Superpotentials

Recall that in $\mathcal{N}=1$ Yang-Mills, the Wilsonian gauge coupling is given by the real part of a holomorphic function f:

$$\frac{1}{g^2} = \mathrm{Re}\Big(f(\zeta,\phi)\Big) \tag{7.3.6}$$

This holomorphic coupling receives one-loop (and nonperturbative) corrections, but no higher-loop corrections [269,270], so that f is the sum of a tree-level piece and a one-loop correction: $f = f_0 + f_1$.

The one-loop correction f_1 is known as a "threshold correction" because it encodes the effect on the Wilsonian gauge coupling of heavy particles at the threshold, i.e. at the ultraviolet cutoff [185]. This correction is a holomorphic function of the moduli, including, in general, the inflaton.

The gaugino condensate superpotential in pure $SU(N)$ Yang-Mills with ultraviolet cutoff M_{UV} and gauge kinetic function f is given by [269]

$$W = 16\pi^2 M_{\mathrm{UV}}^3 \exp\Big(-\frac{8\pi^2}{N}f\Big) \equiv \Sigma(\zeta,\phi)e^{-a\rho}. \tag{7.3.7}$$

We have absorbed the constants in the exponent into a, we have omitted the dimensionful prefactor, and we have used the fact that dimensional reduction of the $7+1$ dimensional theory on the D7-brane relates the tree-level gauge coupling to the volume ρ of the divisor. All further moduli dependence arising from f_1 has been encoded in $\Sigma(\zeta,\phi)$.

In the remainder of the chapter we will analyze the physical consequences of the prefactor $\Sigma(\zeta,\phi)$, viewed as a threshold correction to a gaugino condensate superpotential. This means that we are focusing our attention on gaugino condensation instead of Euclidean D3-branes as the source of the superpotential.

The motivation for this choice is that $\Sigma(\zeta,\phi)$ is more readily computed in the gaugino condensate case. For a Euclidean D3-brane superpotential, $\Sigma(\zeta,\phi)$ represents a one-loop determinant of fluctuations around the instanton. In the M-theory description of this effect, this depends on the worldvolume theory of an M5-brane, which is rather subtle [12]. Although explicit results for Σ are unavailable in the Euclidean brane case, we do still expect to find nontrivial inflaton-dependence, leading, as we will see for the gaugino condensate case, to an eta problem.

7.4 The Eta Problem in String Compactifications

We will now examine the relation between moduli stabilization and the eta problem. In §7.4.1 we recall a problem which can be thought of as the incarnation of the (usual) supergravity eta problem in a very specific string context. Then, in §7.4.2 we explain how shift symmetries have been used to address this problem, and we indicate a few important obstacles to the construction of shift-symmetric models.

7.4.1 Inflaton-Volume Mixing and the Eta Problem

In the context of brane inflation in type IIB string theory, the eta problem takes a novel form [22]. We will examine this now because it presents a concrete setting in which shift symmetries may be used to solve the usual eta problem. Our eventual goal is to understand a new and different eta problem which these symmetries do *not* eliminate, but to achieve this it will be very useful to review the shift symmetry idea in a simpler setting.

D-brane inflation [221] requires mobile, space-filling D-branes, and in a type IIB compactification this is most simply achieved with D3-branes. It will be important for our considerations that the coordinates of D3-branes (i.e., their center-of-mass position moduli) $\phi_i, i = 1, 2, 3$ appear in the Kähler potential as [271][57]

$$K = -3\log\Big(\rho + \bar{\rho} - k(\phi_i, \bar{\phi}_i)\Big) \tag{7.4.1}$$

where $k(\phi_i, \bar{\phi}_i)$ is the (unknown) Kähler potential for the Calabi-Yau manifold itself, which is closely related to the D3-brane moduli space. Singling out one direction as the inflaton and denoting it by ϕ, we have $k(\phi, \bar{\phi}) = \phi\bar{\phi} + \ldots$, where the expansion is performed around a point in the D3-brane moduli space where the kinetic term is canonical.

This mixing of the brane coordinates with the geometric modulus ρ has important implications. The physical volume r in this setting is no longer simply $Re(\rho)$, but is instead

$$2r = \rho + \bar{\rho} - \phi\bar{\phi}. \tag{7.4.2}$$

[57] For additional explanation of this point, see [22] and especially [24].

This implies a revision of (7.3.1), namely

$$V_{4d} = \frac{C}{r^{\alpha}} = \frac{C}{(\rho - \phi\bar{\phi}/2)^{\alpha}} \tag{7.4.3}$$

so that [22]

$$V_{4d}(\phi) = V_{4d}(0)\Big(1 + \frac{\alpha}{2r}\phi\bar{\phi}\Big) \tag{7.4.4}$$

This introduces a contribution of order one to η. Because this effect arises from a term in the Kähler potential, it is reasonable to view it as the manifestation, in this specific model, of the usual eta problem. (The new problem we will discuss shortly does not have this property.)

7.4.2 Solving the Eta Problem with Geometric Shift Symmetries

Shift symmetries [254] are a promising approach to solving the eta problem reviewed in the previous section. The idea is to consider a special compactification which happens to have a particular continuous geometric symmetry.

The proposed symmetry is that the tree-level Kähler potential is independent of one particular (real) field, such as the real part of ϕ. There are strong arguments [258,257] from $\mathcal{N} = 2$ gauged supergravity that this is indeed the case in certain examples, at least before supersymmetry is broken.

The resulting Kähler potential, for example for D3-branes moving along the torus directions of $K3 \times T^2$, takes the form

$$K = -3\log\Big(\rho + \bar{\rho} - (\phi - \bar{\phi})^2\Big) \tag{7.4.5}$$

so that $Re(\phi)$ receives no mass from the term analogous to (7.4.3). This solves the eta problem expressed in (7.4.4).

Various corrections[58] will alter this result and lift the shift symmetry of (7.4.5). In particular, Berg, Haack, and Körs have very clearly demonstrated that threshold corrections to the D7-brane gauge coupling lift the shift symmetry of a certain toroidal orientifold model.

We would like to observe that this conclusion is both generic and problematic, and is in fact a symptom of a new eta problem for string inflation.

[58] Perturbative corrections to the Kähler potential will almost certainly lift this symmetry, although we will not address this [272,273].

Before moving to our main point, we pause to consider some of the obstacles to implementing the shift symmetry argument. (This is an aside because in §7.5 we will ignore these difficulties and grant the presence of such a symmetry, in the absence of threshold corrections, and then demonstrate that the inclusion of threshold corrections still causes an eta problem.)

The first difficulty is that requiring a geometric shift symmetry places severe constraints on the compactification manifold. It is well-known (cf. [274], p.484) that ordinary Calabi-Yau threefolds, i.e. Calabi-Yau threefolds whose holonomy is $SU(3)$ and not a subgroup, do not have any continuous isometries. Thus, orientifolds of tori and of $K3 \times T^2$ are the only suitable candidates for shift-symmetric models. This implies a tremendous reduction in the number of compactifications available for model-building.

Furthermore, the strategy of guessing general results based on detailed study of toroidal orientifold examples is not always reliable. In particular, even if most such simple examples have continuous symmetries, we know for certain that ordinary Calabi-Yau manifolds do not. Hence, any conclusions about shift symmetries that are inferred from toroidal orientifold examples apply only to that context, and not to the general case. This is one of the reasons that our conclusions are different from those of [275].

Moreover, some important aspects of model-building are actually more difficult in the nominally simplified setting of toroidal orientifolds. Although partial stabilization of Kähler moduli has been achieved in this context [276,277], complete stabilization remains challenging. At present it is not clear that known methods will suffice to stabilize all the Kähler moduli in an order-one fraction of toroidal orientifold models. In this regard, Calabi-Yau threefolds with unreduced holonomy can be much more tractable [278]. This is a fairly serious objection to toroidal constructions, given the importance of moduli stabilization for an inflationary model. Even so, it is possible that complete moduli stabilization will eventually be achieved for a toroidal orientifold with properties appropriate for inflation.

7.5 Threshold Corrections Change the Inflaton Mass

We now present the key observation of this chapter, which is that threshold corrections induce an entirely new eta problem which D-term and shift-symmetry techniques do *not* solve. That is, we explain how threshold corrections lead to an inflaton mass that is generically of order H, even in the special case that a shift symmetry was present before the inclusion of these corrections.

In §7.5.1 we discuss the potential sources of an inflaton mass, and in §7.5.2 we illustrate our considerations with the D3-D7 model [42], in which the problem is particularly clear. In §7.5.3 we explore potential solutions to this problem.

7.5.1 General Results

The total potential in a stabilized inflationary model is the sum of several contributions:

$$V = V_{\rm F} + V_{\rm pos} + V_{\rm int}. \tag{7.5.1}$$

The first contribution, $V_{\rm F}$, is the F-term moduli-stabilizing energy. In the KKLT scenario, $V_{\rm F} = V_{\rm AdS} < 0$ is also the vacuum energy of a supersymmetric AdS_4 solution. A supersymmetry-breaking effect then adds an energy $V_{\rm pos}$ which 'uplifts' the total vacuum energy to a positive value, creating a metastable de Sitter vacuum. The prototypical source of positive energy is an anti-D3-brane [5], though there are various alternatives [279,280].

The final and most model-dependent ingredient is an interaction potential $V_{\rm int}$ designed to produce the dynamics of slow-roll inflation. Simple examples include the weak interactions between a widely-separated brane-antibrane pair [222,22] or between a D3-brane and a D7-brane [42].

The η condition for slow-roll inflation (where primes denote derivative with respect to the canonically-normalized inflaton) is

$$V_{\rm F}'' + V_{\rm pos}'' + V_{\rm int}'' \ll 3H^2 \tag{7.5.2}$$

By far the simplest case has $V_{\rm F}$ and $V_{\rm pos}$ independent of ϕ, so that η is determined by $V_{\rm int}''$ alone. Then, if the interaction potential is reasonably flat, the slow-roll condition can be satisfied. The only remaining challenge is to design an interaction $V_{\rm int}(\phi)$ that is sufficiently weak.

Of course, this simple case is hard to achieve. Let us now repeat the potential problems:

(1) If $V_{\rm int}$ is an F-term energy then the e^K prefactor leads to an inflaton mass of order H. This is the classic supergravity eta problem [251].

(2) If the inflaton and compactification volume mix, as in (7.4.2), and the energy is proportional to the volume, as in (7.4.3), then this produces an eta problem as in (7.4.4). This was the problem in [22].

(3) If the volume-stabilizing V_F has inflaton dependence, e.g. from threshold corrections, then this leads to yet another eta problem. The inflaton mass depends on the detailed form of these threshold corrections, but is not expected to be parametrically small.

D-term inflationary energy avoids the first problem, as we recalled in §7.2.2; shift symmetries [254,257] avoid the second problem, as we explained in §7.4.2; but it appears that some more clever mechanism, or an explicit fine-tuning, will be necessary to overcome the third problem. That is the point of the present chapter.

7.5.2 The Example of the D3-D7 Model

It will be worthwhile to illustrate the assertions of the previous section in a specific example. We will focus on the D3-D7 model of [42]. This model is particularly interesting for our purposes because it is a D-term model which can moreover be constructed to take advantage of a shift symmetry, so that the first and second problems of §7.5.1 are not present. This leaves the inflaton mass from threshold corrections as the final obstacle to a working model.[59]

We will now briefly review the aspects of the D3-D7 model [42][60] that are relevant for our considerations. The general proposal is that the weak interaction between a mobile D3-brane and a D7-brane whose worldvolume flux $\mathcal{F}$ is not self-dual can give rise to inflation. The D3-brane moves toward the D7-brane and then, at a critical distance, dissolves.

The flux in question is $\mathcal{F} \equiv dA - B$, where A is the gauge potential on the D7-brane worldvolume and B is the pullback of the NS-NS two-form potential. If this flux is not self-dual in the four-dimensional space described by the divisor which

[59] Berg, Haack, and Körs have done a careful study [24,25] of the inflaton mass corrections in the brane-antibrane model of [22]. Because the second and third effects listed in §7.5.1 are *both* present in that example, it is possible to balance these effects against each other and fine-tune away the eta problem. In contrast, our present point is that the third effect, from threshold corrections, is problematic in general, and particularly so in shift-symmetric models.

[60] For a more recent generalization, see [281,282].

the D7-brane wraps, then supersymmetry is broken and there is a force between the D7-brane and the D3-brane [42].

This model can be compactified on $K3 \times T^2/\mathbb{Z}_2$, with the orientifold action explained in [42]. The maximal gauge symmetry is $SO(8)^4$, which arises when four D7-branes and one O7-plane sit at each of the four fixed points of the orientifold action. In more general configurations, the branes will be dispersed around the torus. However, at least one stack of D7-branes is necessary for the KKLT method of volume stabilization. When gaugino condensation occurs on two or more independent stacks, the superpotential takes on a more complicated 'racetrack' form. We will make the minimal assumption that fixes the K3 volume: gaugino condensation on a single stack of N D7-branes that wrap the $K3$ and sit at a point on the torus, which we take to be the origin.

We will now see that this stack of branes exerts a force on a probe D3-brane elsewhere on the torus. In the special case that the motion of the D3-brane towards the origin corresponds to the inflaton direction, this immediately implies a violation of the slow-roll condition. Much more generally, the effect of the stack of D7-branes is to deflect the D3-brane from an otherwise suitably flat inflaton trajectory.

Note that the translational symmetry along the torus may be thought of as the origin of the shift symmetry [254]. Correspondingly, the Kähler potential for this model is given by the shift-symmetric form (7.4.5)[254,258,257]. Without loss of generality, we will take the shift-symmetric direction to correspond to the *real* part of the field ϕ that parametrizes the D3-brane coordinate on the torus.

The holomorphic gauge coupling on the stack of D7-branes, including the string loop correction, is [25]

$$2f = \rho - \frac{1}{4\pi^2}\log\vartheta_1(\phi, U) + \dots \tag{7.5.3}$$

where ϑ_1 is a Jacobi theta function, U is the complex structure of the T^2, ϕ is the inflaton, and the omitted terms are independent of ϕ.

For generic values of ϕ, even the epsilon slow-roll condition will not be satisfied. However, when the D7-brane stack and the probe D3-brane are antipodal, i.e. when $\phi = 1/2$, the term in the potential linear in ϕ vanishes. Expanding the superpotential around $\phi = 1/2$, BHK find

$$W_{\rm np}(1/2+\phi) = W_{\rm np}(1/2)\Big(1+\delta(U)\phi^2\Big) \tag{7.5.4}$$

where

$$\delta(U) = \frac{a}{24}\Big(E_2(U) + \vartheta_3(0,U)^4 + \vartheta_4(0,U)^4\Big). \tag{7.5.5}$$

Here E_2 is the second Eisenstein series, related to derivatives of the ϑ-functions, and a is the numerical constant appearing in (7.3.5). In these expressions ϕ is dimensionless; the canonically-normalized inflaton, with mass dimension one, is

$$\varphi = M_{\rm p}\phi\sqrt{\frac{3}{\rho+\bar{\rho}}} \tag{7.5.6}$$

We can now compute the mass term for a D3-brane probe of this compactification by using (7.5.4) to expand the F-term energy. For motion along $Im(\phi) = 0$[61], the mass is conveniently expressed as

$$\eta = \frac{4}{3}\left|\frac{V_{\rm AdS}}{V}\right|\Big(2\frac{\delta(U)^2}{a^2} + 3\frac{\delta(U)}{a}\Big) \tag{7.5.7}$$

where, as in KKLT, $V_{\rm AdS}$ is the vacuum energy at the AdS_4 minimum which is uplifted to create a de Sitter vacuum.

This result is slightly different from the result of [25] for the mass of a $D3-\overline{D3}$ inflaton. The reason is that the Kähler potential relevant for brane-antibrane inflation is (7.4.1), but for the present example of D3-D7 inflation the Kähler potential takes the shift-symmetric form (7.4.5).

Let us now assess whether η (7.5.7) can satisfy the slow-roll condition $\eta \leq 10^{-2}$. Each of the factors in (7.5.7), except for $\delta(U)$, is roughly of order one or larger. The ratio $|V_{\rm AdS}|/V$ cannot be parametrically small, because $V_{\rm AdS}$ determines the height of the potential barrier that prevents decompactification, and the energy density V should not exceed this.[62] The constant $1/a$ is likewise not parametrically small; in the concrete example given in KKLT, a was taken to be $2\pi/10$ (where we have included a factor of $(2\pi)^{-1}$ which converts their result to our notation), and more generally, $a = 4\pi^2/N$ for a stack of N coincident D7-branes.

[61] For other choices of inflaton trajectory, i.e. different locations in $Im(\phi)$ of the D7-brane bearing anti-self-dual flux, the results are more difficult to express in closed form. However, for any such model the inflaton mass can still be found by using (7.5.3) to determine the F-term energy, and it remains true that for generic values of U, η is not small.

[62] This statement is model-dependent; our present discussion assumes moduli stabilization by the method of KKLT [5].

The only factor which might be small is $\delta(U)$. As explained in [25], δ is not automatically small, but there does exist a small range of values of U, the torus complex structure, for which $\delta(U) \ll 1$. A small inflaton mass can therefore be arranged by a choice of fluxes that fixes U in this window. This amounts to an explicit fine-tuning of the inflaton mass.

We conclude that with KKLT volume stabilization, the D3-D7 model requires a modest fine-tuning which can be achieved by a judicious choice of fluxes.[63]

7.5.3 Discussion

The result of the previous sections accords with the general expectations discussed in §7.2. An inflaton mass which is much smaller than H does not arise automatically, nor even with the imposition of a shift symmetry; in the end, a fine-tuning at the percent level is necessary to make the model work. In the scheme of inflationary fine-tuning, this is not a serious problem; in particular, it should be contrasted to the functional fine-tuning required for certain models in which $\phi \gg M_{\rm p}$. Even so, the necessity of fine-tuning in the present case cannot be ignored.

This result should not be interpreted as a stroke against the D3-D7 model (or any other model) in particular. In fact, we would expect almost any complete and fully-realized model to require some fine-tuning of parameters. Omission or simplification of certain physical ingredients, especially moduli stabilization, may obscure the eta problem and make a model appear to work automatically, but sufficient inspection can be expected to reveal one or more problems of detail that require fine-tuning.

It would be extremely interesting to find a solution to this eta problem that does not amount to a fine-tuning of parameters. A slightly modified mechanism of volume stabilization, such as the proposal of [283], does alter the mass formula (7.5.7), but does not naturally produce a small mass. However, it may be possible to invent a method of volume stabilization which does not affect the inflaton mass. Volume dependence through a D-term energy would be a promising candidate.

A further possibility is to circumvent this problem through a suitably symmetric configuration of D7-brane stacks. Although such arrangements are presumably less

[63] We should again emphasize that corrections to the Kähler potential may introduce further changes in the inflaton mass.

generic, i.e. arise from a more limited choice of F-theory fluxes, any such explicit model would be quite interesting.

Another interesting possibility [262] is that an inflaton charged under a symmetry G can sometimes be excluded from the holomorphic correction term f_1, so that $\frac{\partial f_1}{\partial \phi} = 0$. However, in simple examples, such as the D3-D7 model, no such symmetry is present. Moreover, D-term inflation requires [234] that ϕ is neutral under the $U(1)$ gauge group G_D whose D-term energy drives inflation, so in particular G cannot coincide with G_D. It is reasonable to expect, however, that discrete symmetries of the appropriate form can sometimes be arranged.

7.6 Conclusion

We have seen that threshold corrections to volume-stabilizing nonperturbative superpotentials create an eta problem for inflationary models in string theory. These threshold corrections cause the volume-stabilizing F-term energy to depend, generically, on the values of the open-string and compactification moduli. Because the inflaton is expected to consist of one of these moduli, the threshold correction changes the dependence of the inflationary energy on the inflaton vev, altering the slow-roll parameters and creating an eta problem.

This conclusion applies to models which satisfy several assumptions, which we now repeat for clarity. Our general considerations were limited to models of inflation which can be realized in a string compactification. In any such model it is essential that the instability to decompactification has been removed by moduli stabilization; it is also desirable that all other moduli have also been stabilized. We have explicitly assumed that the volume stabilization arises from a nonperturbative contribution to the superpotential, as in KKLT [5]. (For interesting alternatives, see [259,260].) We have also assumed that the inflaton is a modulus whose flat direction is slightly lifted by a further supersymmetry-breaking effect. This could correspond, for example, to a brane interaction.

Thus, our result applies to any model of inflation in string theory which uses a compactification stabilized by methods analogous to those of KKLT. Every aspect of the discussion is simplest in the case of D-brane inflation in a type IIB compactification, but the result applies much more broadly. For example, current techniques for moduli stabilization in the heterotic string [284] and in M-theory on G_2 manifolds

[285] also use a combination of flux and nonperturbative superpotentials. Any inflationary model[64] which is elaborated on one of these foundations would be subject to an eta problem from threshold corrections to these superpotentials.

Moreover, although we have seen that the threshold corrections of Berg, Haack and Körs [24] lead to an explicit result for the inflaton mass in a particularly simple D3-D7 model, generic moduli dependence will lead to an eta problem even in more complicated cases. For example, the threshold corrections are not known for generic Calabi-Yau threefolds, so no complete and explicit computation of the slow-roll parameters is possible at present for an inflationary model arising in a compactification on such a space. Progress in this direction appears to be important for inflationary model-building in string compactifications.

It is essential to recognize that although the conclusions of this chapter are somewhat general, the actual computation of the inflaton mass is only strictly applicable to a supersymmetric AdS_4 configuration that can be uplifted to produce an inflationary scenario. In particular, the one-loop exactness of threshold corrections in supersymmetric theories permits us to be somewhat precise about the inflaton mass in a supersymmetric vacuum, but, as we have emphasized throughout, supersymmetry-breaking effects will typically produce substantial corrections to these mass terms.

Nevertheless, the strategy of understanding the lifting of (inflaton) flat directions in a supersymmetric vacuum is a sensible one.[65] If no suitably flat direction exists in the supersymmetric configuration, it is very hard to believe that the addition of gravitationally-mediated soft terms will remedy this problem. Moreover, it is usually not possible to compute these corrected masses in detail.

Thus, it is usually impossible to prove that a given string model has a small inflaton mass, including all quantum corrections. On the other hand, it is possible to establish that a given model has an eta problem, because if a problem arises from one set of quantum corrections, such as threshold corrections to the gauge coupling, then further quantum corrections will generically not undo this problem. In this chapter we have focused on establishing a problem using the one-loop-exact results

[64] For interesting examples in this category, see [286].

[65] This perspective was the one used to expose and address the problem of a brane-antibrane inflaton mass in [22,253,254].

for the superpotential, with the understanding that additional corrections, e.g. to the Kähler potential, should not conspire to flatten the inflaton potential.

There are several interesting directions for future work. First of all, it is the threshold corrections from closed string moduli that are relevant when the inflaton itself is a closed-string field, for example a geometric modulus [287]. The mass of such a closed string inflaton depends on these corrections, and it would be useful to understand their form.

Furthermore, we have only examined the nonperturbative superpotentials resulting from gaugino condensation, but Euclidean D3-branes are known to play an important role in stabilizing certain classes of Kähler moduli [266,278]. In this context the inflaton dependence of the instanton superpotential arises through a moduli-dependent one-loop determinant $\Sigma(\zeta,\phi)$ of fluctuations around the instanton. It would be extremely interesting, although challenging [12], to compute prefactors of this sort, not only for the considerations of this chapter, but for rather general moduli stabilization.

In addition, corrections to the Kähler potential can further adjust the dependence of the total inflationary energy on the inflaton vev. A complete and consistent model requires inclusion of these effects, which have also not yet been calcuated.

Looking forward, we can hope that a thorough understanding of the effect of threshold corrections on shift-symmetric brane configurations will guide us to models in which the threshold corrections, and all other quantum corrections, are indeed small, so that the shift symmetry is an approximate symmetry of the full quantum theory. If this could be achieved, it would be a significant step toward a controllable model of inflation.

7.A A Field-Theory Model of the Brane Interaction

In this appendix we will point out a counterintuitive aspect of our conclusion. We will then use a field-theory model to expose the flaw in this intuition, and to further demonstrate that our results are correct.

The inflaton mass term from threshold corrections is the result of an interaction induced by massive strings stretched between the D3-brane and the D7-branes, which we refer to as 3-7 strings. In the field theory description, these 3-7 strings correspond to a massive flavor whose mass m_{37} is controlled by the modulus ϕ. From this perspective, one might expect this massive flavor to decouple when its mass is very large, and to give rise to a negligible interaction in that limit. It is therefore somewhat surprising that the inflaton mass (7.5.7) does not diminish when $|\phi|$ is large. Should we not expect the BHK result to vanish for widely-separated branes?

To resolve this puzzle, we first note that as soon as m_{37} approaches the mass of a string winding the torus, the D3-D7 interaction induced by the superpotential is correctly described by the full string threshold correction of BHK, and not by its field theory limit. Thus, we can place an upper limit $\Lambda_{\rm UV} < m_{\rm W}$ on the ultraviolet cutoff of our field-theory description, where $m_{\rm W}$ denotes the mass of the lightest wound string. In other words, the field theory that provided the decoupling intuition applies only to situations in which the brane separation is much less than the smallest radius of the torus.[66] At greater separations, wound strings can appear in the theory and contribute an additional interaction between the D3-brane and the D7-branes.

We should therefore ask whether decoupling sufficient for slow roll is possible within this limit imposed by the radius of the compact space. To do this, we will examine a simple field theory that models the D3-D7 interaction induced by stretched (but not wound) 3-7 strings. (We will check our model by verifying that it coincides with the small-separation limit of the full BHK result.)

The model is a supersymmetric $SU(N)$ Yang-Mills theory with a single chiral superfield Q whose mass is controlled by a parameter ϕ. Here we will take ϕ to be non-dynamical, and will examine the gaugino condensate superpotential as a function of ϕ.[67]

The gaugino condensate superpotential below the scale m_{37}, i.e. after integrating out Q, can be matched to the superpotential above this scale. For $N > 2$ the result is simply [288]

$$W_{\rm low} \propto \Lambda_{\rm high}^{3-1/N} m_{37}^{1/N} \tag{7.A.1}$$

with $\Lambda_{\rm high}$ the dynamically-generated scale of the high-energy theory. Thus, in the low-energy theory, $W = C\phi^{1/N}$ with C independent of ϕ. We can precisely

[66] I am grateful to M. Berg and M. Haack for discussions on this point.

[67] For simplicity we are studying the supersymmetric configuration; the supersymmetry-breaking effects used in the model of [42] would generate additional corrections to the inflaton mass, in addition to introducing a tachyon.

reproduce (7.A.1) by expanding the superpotential (7.3.7), including the full string threshold correction (7.5.3) of BHK, in the limit $\phi \ll 1$, after using the relation $a = 4\pi^2/N$.

Let us now compute η in this model. From the supergravity formula for the F-term energy, we have

$$V = -3e^K C^2 \phi^{2/N} \tag{7.A.2}$$

so that[68]

$$\eta = -\frac{2}{N}\left(1 - \frac{2}{N}\right)\left(\frac{M_{\rm p}}{\varphi}\right)^2 . \tag{7.A.3}$$

Taking ρ real and using $\varphi = M_{\rm p}\phi\sqrt{\frac{3}{2\rho}}$, we have

$$\eta \sim -\frac{2}{N}\left(\frac{2\rho}{3\phi^2}\right) \tag{7.A.4}$$

so that applying $a = 4\pi^2/N$, we finally come to

$$\eta \sim -\frac{a\rho}{3\pi^2\phi^2}. \tag{7.A.5}$$

However, $a\rho \gg 1$ was a condition for the validity of the nonperturbative superpotential used by KKLT: (7.3.5) is the leading approximation, analogous to a single-instanton effect, and there will be corrections suppressed by further powers of $e^{-a\rho}$. Furthermore, $|\phi| \leq \frac{1}{2}$ measures the distance from the origin on a unit torus, so $|\phi| \ll \frac{1}{2}$ is necessary in order for the brane separation to be small compared to the size of the torus (and hence for the field theory model to be a good approximation to the true result, which incorporates wound strings.) Thus, there is no controllable parameter regime in which (7.A.5) is small.

Indeed, even at the extreme boundary of the region of control, $a\rho \sim 1, |\phi| \sim \frac{1}{2}$, we have at best $\eta \sim \frac{1}{7}$. If we were to extend the toy model to $\varphi > M_{\rm p}$ then the interaction would no longer be strong enough to affect slow-roll. However, this is not an allowed range in the full model, because of the UV cutoff of the effective field theory, which corresponds to the limit imposed by the radius of the compact space.

68 We have again replaced the dimensionless ϕ with the canonically-normalized φ, cf. (7.5.6).

We conclude that one cannot arrange suitable decoupling simply by separating the branes; a somewhat more complicated fine-tuning will be necessary to remove the inflaton mass terms under consideration. This could be achieved, for example, by introducing a suitably symmetric configuration of several D7-brane stacks bearing gaugino condensates.

We have certainly not demonstrated that slow-roll is impossible for D3-D7 systems in the regime in which separations are small compared to the size of the torus. We have simply shown that the interaction captured by threshold corrections produces, on its own, an unsuitably large inflaton mass in this range, so that some fine-tuning against other effects would be needed to make a phenomenologically acceptable model. Thus, one cannot evade the arguments of this chapter by separating the D3-brane from the D7-branes and invoking decoupling.

References

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," Astron. J. **116**, 1009 (1998), astro-ph/9805201.

[2] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," Astrophys. J. **517**, 565 (1999), astro-ph/9812133.

[3] R. Bousso and J. Polchinski, "Quantization of Four-form Fluxes and Dynamical Neutralization of the Cosmological Constant," JHEP **0006**, (2000) 006, hep-th/0004134.

[4] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002), hep-th/0105097.

[5] S. Kachru, R. Kallosh, A. Linde, and S. Trivedi, "de Sitter Vacua in String Theory," Phys. Rev. D **68**, (2003) 046005, hep-th/0301240.

[6] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic, 1990;
E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley, 1990;
A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, 2000.

[7] D. N. Spergel *et al.* [WMAP Collaboration], "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters," Astrophys. J. Suppl. **148**, 175 (2003), astro-ph/0302209.

[8] S. Degl'Innocenti, G. Fiorentini, G.G. Raffelt, B. Ricci and A. Weiss, "Time variation of Newton's constant and the age of globular clusters," Astron. Astrophys. **312**, 345 (1996), astro-ph/9509090.

[9] A.I. Shlyakhter, "Direct test of the constancy of fundamental nuclear constants," Nature **264** (1976) 340 ;
T. Damour and F. Dyson, "The Oklo bound on the time variation of the fine-structure constant revisited," Nucl. Phys. B **480**, 37 (1996), hep-ph/9606486.

[10] T. Damour, "Gravitation, experiment and cosmology," gr-qc/9606079.

[11] M. Dine and N. Seiberg, "Nonrenormalization Theorems In Superstring Theory," Phys. Rev. Lett. **57**, 2625 (1986).

[12] E. Witten, "Non-Perturbative Superpotentials In String Theory," Nucl. Phys. B **474**, 343 (1996), hep-th/9604030.

[13] I. Klebanov and M.J. Strassler, "Supergravity and a Confining Gauge Theory: Duality Cascades and χSB Resolution of Naked Singularities," JHEP **0008**, (2000) 052, hep-th/0007191.

[14] S. Sarangi and S. H. Tye, "Cosmic String Production Towards the End of Brane Inflation," Phys. Lett. B **536**, (2002) 185, hep-th/0204074; N. T. Jones, H. Stoica and S. H. Tye, "The Production, Spectrum and Evolution of Cosmic Strings in Brane Inflation," Phys. Lett. B **563**, (2003) 6, hep-th/0303269; L. Pogosian, S. H. Tye, I. Wasserman and M. Wyman, "Observational Constraints on Cosmic String Production During Brane Inflation," Phys. Rev. D **68**, (2003) 023506, hep-th/0304188.

[15] E. J. Copeland, R. C. Myers and J. Polchinski, "Cosmic F- and D-strings," JHEP **0406**, 013 (2004), hep-th/0312067.

[16] D. B. Kaplan and A. V. Manohar, "Current Mass Ratios Of The Light Quarks," Phys. Rev. Lett. **56** (1986) 2004.

[17] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Spacetime*, Cambridge University Press, 1973.

[18] L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, "Beauty is attractive: Moduli trapping at enhanced symmetry points," JHEP **0405**, 030 (2004), hep-th/0403001.

[19] L. McAllister and I. Mitra, "Relativistic D-brane scattering is extremely inelastic," JHEP **0502**, 019 (2005), hep-th/0408085.

[20] S. Gukov, S. Kachru, X. Liu and L. McAllister, "Heterotic Moduli Stabilization with Fractional Chern-Simons Invariants," Phys. Rev. D **69**, 086008 (2004), hep-th/0310159.

[21] M. Dine, R. Rohm, N. Seiberg, and E. Witten, "Gluino Condensation in Superstring Models," Phys. Lett. B **156** (1985) 55.

[22] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, "Towards inflation in string theory," JCAP **0310**, 013 (2003), hep-th/0308055.

[23] L. McAllister, "An inflaton mass problem in string inflation from threshold corrections to volume stabilization," hep-th/0502001.

[24] M. Berg, M. Haack and B. Körs, "Loop Corrections to Volume Moduli and Inflation in String Theory," Phys. Rev. D **71**, 026005 (2005) hep-th/0404087.

[25] M. Berg, M. Haack and B. Körs, "On the Moduli Dependence of Nonperturbative Superpotentials in Brane Inflation," hep-th/0409282.

[26] R. Brandenberger and C. Vafa, "Superstrings in the Early Universe," Nucl. Phys. B **316** 391 (1989).

[27] M. Gasperini and G. Veneziano, "Pre-Big Bang in String Cosmology," Astropart. Phys. **1**, 317 (1993), hep-th/9211021.

[28] D. Lüst, "Cosmological String Backgrounds," hep-th/9303175;
C. Kounnas and D. Lüst, "Cosmological String Backgrounds from Gauged WZW Models," Phys. Lett. B **289**, 56 (1992), hep-th/9205046.

[29] R. Brustein and R. Madden, "Graceful Exit and Energy Conditions in String Cosmology," Phys. Lett. B **410**, 110 (1997), hep-th/9702043.

[30] N. Seiberg, "From Big Crunch to Big Bang: Is it Possible?," hep-th/0201039;
J. Khoury, B. Ovrut, N. Seiberg, P. Steinhardt and N. Turok, "From Big Crunch to Big Bang," Phys. Rev. D **65**, 086007 (2002), hep-th/0108187.

[31] V. Balasubramanian, S. Hassan, E. Keski-Vakkuri and A. Naqvi, "A Space-Time Orbifold: A Toy Model for a Cosmological Singularity," hep-th/0202187.

[32] N. Nekrasov, "Milne Universe, Tachyons, and Quantum Group," hep-th/0203112.

[33] L. Cornalba and M. Costa, "A New Cosmological Scenario in String Theory," hep-th/0203031.

[34] H. Liu, G. Moore and N. Seiberg, "Strings in a Time-dependent Orbifold," hep-th/0204168.

[35] S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, "From Big Bang to Big Crunch and Beyond," hep-th/0204189.

[36] L. Cornalba, M.S. Costa and C. Kounnas, "A Resolution of the Cosmological Singularity with Orientifolds," hep-th/0204261.

[37] B. Craps, D. Kutasov and G. Rajesh, "String Propagation in the Presence of Cosmological Singularities," hep-th/0205101.

[38] A. Kehagias and E. Kiritsis, "Mirage Cosmology," JHEP **9911**, 022 (1999), hep-th/9910174.

[39] S. Gubser, "AdS/CFT and Gravity," Phys. Rev. D **63**, 084017 (2001), hep-th/9912001.

[40] C. Csaki, M. Graesser, L. Randall and J. Terning, "Cosmology of Brane Models with Radion Stabilization," Phys. Rev. D **62**, 045015 (2000), hep-th/9911406.

[41] J. Khoury, B. Ovrut, P. Steinhardt and N. Turok, "The Ekpyrotic Universe: Colliding Branes and the Origin of the Hot Big Bang," Phys. Rev. D **64**, 123522 (2001), hep-th/0103239;
R. Kallosh, L. Kofman and A. Linde, "Pyrotechnic Universe," Phys. Rev. D **64**, 123523 (2001), hep-th/0104073.

[42] K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, "D3/D7 Inflationary Model and M-theory," Phys. Rev. D **65**, 126002 (2002), hep-th/0203019.

[43] L. Randall and R. Sundrum, "A Large Mass Hierarchy from a Small Extra Dimension," Phys. Rev. Lett. **93**, 3370 (1999), hep-th/9905221.

[44] A. Buchel, "Gauge/gravity Correspondence in an Accelerating Universe," hep-th/0203041.

[45] I. Klebanov and A. Tseytlin, "Gravity Duals of Supersymmetric $SU(N) \times SU(N+M)$ Gauge Theories," Nucl. Phys. B **578**, 123 (2000), hep-th/0002159.

[46] J. Bekenstein, "Exact Solutions of Einstein-Conformal Scalar Equations," Ann. Phys. **82** 535 (1974);
J. Bekenstein, "Nonsingular General-relativistic Cosmologies," Phys. Rev. D **11**, 2072 (1975).

[47] C. Molina-Paris and M. Visser, "Minimal Conditions for the Creation of a Friedmann-Robertson-Walker Universe from a 'Bounce'," Phys. Lett. B **455**, 90 (1999), gr-qc/9810023.

[48] R. Fakir, "General Relativistic Cosmology with No Beginning of Time," gr-qc/9810054.

[49] M. Visser and C. Barcelo, "Energy Conditions and their Cosmological Implications," gr-qc/0001099.

[50] A.J.M. Medved, "Big Bangs and Bounces on the Brane," hep-th/0205037.

[51] S. Mukherji and M. Peloso, "Bouncing and Cyclic Universes from Brane Models," hep-th/0205180.

[52] H. Verlinde, "Holography and Compactification," Nucl. Phys. B **580**, 264 (2000), hep-th/9906182;
C. Chan, P. Paul and H. Verlinde, "A Note on Warped String Compactification," Nucl. Phys. B **581**, 156 (2000), hep-th/0003236.

[53] K. Dasgupta, G. Rajesh and S. Sethi, "M-theory, Orientifolds and G-flux," JHEP **9908** 023 (1999), hep-th/9908088;
B. Greene, K. Schalm and G. Shiu, "Warped Compactifications in M and F theory," Nucl. Phys. B **584**, 480 (2000), hep-th/0004103;
P. Mayr, "Stringy World Branes and Exponential Hierarchies," JHEP **0011** 013 (2000), hep-th/0006204.

[54] C. Herzog, I. Klebanov and P. Ouyang, "Remarks on the Warped Deformed Conifold," hep-th/0108101.

[55] I. Klebanov and E. Witten, "Superconformal Field Theory on Three-branes at a Calabi-Yau Singularity," Nucl. Phys. B **536**, 199 (1998), hep-th/9807080.

[56] B. Grinstein, D.R. Nolte and W. Skiba, "On a Covariant Determination of Mass Scales in Warped Backgrounds," hep-th/0012074.

[57] P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, "Brane cosmological evolution in a bulk with cosmological constant," Phys. Lett. B **477**, 285 (2000), hep-th/9910219.

[58] S. Mukohyama, "Brane-world solutions, standard cosmology, and dark radiation," Phys. Lett. B **473**, 241 (2000), hep-th/9911165.

[59] J. Khoury and R.-J. Zhang, "On the Friedmann Equation in Brane-World Scenarios," hep-th/0203274.

[60] S. Kachru, J. Pearson and H. Verlinde, "Brane/Flux Annihilation and the String Dual of a Nonsupersymmetric Field Theory," hep-th/0112197.

[61] J. Audretsch and G. Schäfer, "Thermal Particle Production in a Contracting and Expanding Universe Without Singularity," Phys. Lett. A **66** 459 (1978).

[62] O. Aharony, M. Fabinger, G. Horowitz and E. Silverstein, "Clean Time-dependent String Backgrounds from Bubble Baths," hep-th/0204158.

[63] For example see: S. Bhattacharya, D. Choudhury, D. Jatkar and A.A. Sen, "Brane Dynamics in the Randall-Sundrum Model, Inflation and Graceful Exit," hep-th/0103248.

[64] D. Youm, "Closed Universe in Mirage Cosmology," Phys. Rev. D **63**, 085010 (2001), hep-th/0011290.

[65] E. Papantonopoulos and I. Pappa, "Type 0 Brane Inflation from Mirage Cosmology," Mod. Phys. Lett. A **15**, 2145 (2000), hep-th/0001183;
P. Brax and D.A. Steer, "Non-BPS Brane Cosmology," hep-th/0204120.

[66] L. Kofman, A. D. Linde and A. A. Starobinsky, "Towards the theory of reheating after inflation," Phys. Rev. D **56**, 3258 (1997), hep-ph/9704452.

[67] G. N. Felder, L. Kofman and A. D. Linde, "Instant preheating," Phys. Rev. D **59**, 123523 (1999), hep-ph/9812289.

[68] G. Felder, L. Kofman, and A. Linde, "Inflation and Preheating in NO Models," Phys.Rev. D **60**, (1999) 103505, hep-ph/9903350.

[69] D.J.H. Chung, E.W. Kolb, A. Riotto, and I.I. Tkachev, "Probing Planckian Physics: Resonant Production of Particles During Inflation and Features in the Primordial Power Spectrum," Phys.Rev. D **62**, (2000) 043508, hep-ph/9910437.

[70] E. Silverstein and D. Tong, "Scalar speed limits and cosmology: Acceleration from D-cceleration," hep-th/0310221; M. Alishahiha, E. Silverstein, and D. Tong, in progress.

[71] E. Witten, "Bound states of strings and p-branes," Nucl. Phys. B **460**, (1996) 335, hep-th/9510135.

[72] N. Seiberg and E. Witten, "Electric-Magnetic Duality, Monopole Condensation, and Confinement in N=2 Supersymmetric Yang-Mills Theory," Nucl.Phys. B **426**, (1994) 19, hep-th/9407087 ;

N. Seiberg and E. Witten, " Monopoles, Duality and Chiral Symmetry Breaking in N=2 Supersymmetric QCD," Nucl.Phys. B **431**, (1994) 484, hep-th/9408099 ;
K. Intriligator and N. Seiberg, "Lectures on Supersymmetric Gauge Theories and Electric-Magnetic Duality," Nucl.Phys.Proc.Suppl. **45** BC (1996) 1, hep-th/9509066.

[73] E. Witten, "String Theory Dynamics In Various Dimensions," Nucl.Phys. B **443** (1995) 85, hep-th/9503124 ;
S. Katz, D. R. Morrison, and M. R. Plesser, "Enhanced Gauge Symmetry in Type II String Theory," Nucl.Phys. B **477**, (1996) 105, hep-th/9601108 ;
M. Bershadsky, K. Intriligator, S. Kachru, D.R. Morrison, V. Sadov, and C. Vafa, "Geometric Singularities and Enhanced Gauge Symmetries," Nucl.Phys. B **481**, (1996) 215, hep-th/9605200.

[74] E. Witten, "Small Instantons in String Theory," Nucl.Phys. B **443**, (1995) 85, hep-th/9511030.

[75] M. Dine, "Towards a Solution of the Moduli Problems of String Cosmology," Phys.Lett. B **482**, (2000) 213, hep-th/0002047 ;
M. Dine, Y. Nir, and Y. Shadmi, "Enhanced Symmetries and the Ground State of String Theory," Phys.Lett. B **438**, (1998) 61, hep-th/9806124.

[76] A. Berera and T. W. Kephart, "Ubiquitous inflaton in string-inspired models," Phys. Rev. Lett. **83**, (199) 1084, hep-ph/9904410.

[77] N. D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England (1982).

[78] A. Linde, "Hybrid Inflation," Phys.Rev. D **49**, (1994) 748, astro-ph/9307002.

[79] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, "Dynamics of symmetry breaking and tachyonic preheating," Phys. Rev. Lett. **87**, (2001) 011601, hep-ph/0012142.

[80] L. Kofman, A. D. Linde and A. A. Starobinsky, "Non-Thermal Phase Transitions After Inflation," Phys. Rev. Lett. **76** (1996) 1011, hep-th/9510119;
G. N. Felder, L. Kofman, A. D. Linde and I. Tkachev, "Inflation after preheating," JHEP **0008** (2000) 10, hep-ph/0004024.

[81] G. Dvali and S. Kachru, "New Old Inflation," hep-th/0309095.

[82] R. Easther, J. Khoury and K. Schalm, "Tuning Locked Inflation: Supergravity versus Phenomenology," hep-th/0402218.

[83] L. Jarv, T. Mohaupt, and F. Saueressig, "Effective Supergravity Actions for Flop Transitions," JHEP **0312**, (2003) 047, hep-th/0310173 ;
L. Jarv, T. Mohaupt, and F. Saueressig, "M-theory Cosmologies from Singular Calabi-Yau Compactifications," hep-th/0310174.

[84] M. Brandle and A. Lukas, "Flop transitions in M-theory cosmology," Phys. Rev. D **68**, (2003) 24030, hep-th/0212263.

[85] T. Banks, W. Fischler, and L. Motl, "Dualities versus Singularities," JHEP **9901** (1999) 019, hep-th/9811194 ;
L. Motl and T. Banks, "On the hyperbolic structure of moduli spaces with 16 SUSYs," JHEP **9905** (1999) 015, hep-th/9904008.

[86] M. R. Douglas, D. Kabat, P. Pouliot and S. H. Shenker, "D-branes and short distances in string theory," Nucl. Phys. B **485** (1997) 85, hep-th/9608024.

[87] C. Bachas, "D-brane dynamics," Phys. Lett. B **374**, (1996) 37, hep-th/9511043.

[88] J. Horne and G. Moore, "Chaotic Coupling Constants," Nucl.Phys. B **432**, (1994) 109, hep-th/9403058.

[89] T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore and P. J. Steinhardt, Phys. Rev. D **52**, (1995) 3548, hep-th/9503114.

[90] S. Y. Khlebnikov and I. I. Tkachev, "Classical decay of inflaton," Phys. Rev. Lett. **77**, 219 (1996), hep-ph/9603378.

[91] G. N. Felder and I. Tkachev, "LATTICEEASY: A program for lattice simulations of scalar fields in an expanding universe," hep-ph/0011159.

[92] L. Randall and R. Sundrum, "A Large Mass Hierarchy from a Small Extra Dimension," Phys.Rev.Lett. **83**, (1999) 3370, hep-ph/9905221.

[93] G. N. Felder and L. Kofman, "The development of equilibrium after preheating," Phys. Rev. D **63**, 103503 (2001), hep-ph/0011160.

[94] G. N. Felder, A. V. Frolov, L. Kofman and A. D. Linde, "Cosmology with negative potentials," Phys. Rev. **D** 66 (2002) 023507, hep-th/0202017.

[95] A. D. Linde, "Inflation Can Break Symmetry In SUSY," Phys. Lett. B **131**, (1983) 330.

[96] A. D. Linde, "Inflation And Quantum Cosmology," Print-86-0888 (June 1986), in *300 Years of Gravitation*, S.W. Hawking and W. Israel, eds., Cambridge University Press, Cambridge (1987).

[97] A. D. Linde, D. A. Linde and A. Mezhlumian, "From the Big Bang theory to the theory of a stationary universe," Phys. Rev. D **49**, (1994) 1783, gr-qc/9306035;
J. Garcia-Bellido, A. D. Linde and D. A. Linde, "Fluctuations of the gravitational constant in the inflationary Brans-Dicke cosmology," Phys. Rev. D **50**, (1994) 730, astro-ph/9312039;
J. Garriga and A. Vilenkin, "A prescription for probabilities in eternal inflation," Phys. Rev. D **64**, (2001) 23507, gr-qc/0102090;
A. Linde, "Inflation, quantum cosmology and the anthropic principle," in *Science and Ultimate Reality: From Quantum to Cosmos*," J.D. Barrow,

P.C.W. Davies, and C.L. Harper, eds., Cambridge University Press, Cambridge (2003), hep-th/0211048;
J. Garriga, A. Linde and A. Vilenkin, "Dark energy equation of state and anthropic selection," hep-th/0310034.

[98] A. Maloney, E. Silverstein and A. Strominger, "de Sitter Space in Non-Critical String Theory," hep-th/0205316.

[99] M. R. Douglas, "The Statistics of String/M Theory Vacua," JHEP **0305**, (2003) 046, hep-th/0303194; S. Ashok and M. R. Douglas, "Counting Flux Vacua," hep-th/0307049.

[100] B. S. Acharya, "A moduli fixing mechanism in M theory," hep-th/0212294.

[101] R. Fardon, A.E. Nelson, and N. Weiner, "Dark Energy from Mass Varying Neutrinos," astro-ph/0309800.

[102] R. Helling, "Beyond eikonal scattering in M(atrix)-theory," hep-th/0009134.

[103] J. S. Schwinger, "On gauge invariance and vacuum polarization," Phys. Rev. **82** (1951) 664.

[104] C. Bachas and M. Porrati, "Pair creation of open strings in an electric field," Phys. Lett. B **296**, (1992) 77, hep-th/9209032.

[105] M. R. Douglas, D. Kabat, P. Pouliot, and S. H. Shenker, "D-branes and short distances in string theory," Nucl. Phys. B **485**, (1997) 85, hep-th/9608024.

[106] L. Kofman, A. D. Linde, and A. A. Starobinsky, "Towards the theory of reheating after inflation," Phys. Rev. D **56**, (1997) 3258, hep-ph/9704452.

[107] G. N. Felder, L. Kofman, and A. D. Linde, "Inflation and preheating in NO models," Phys. Rev. D **60**, (1999) 103505, hep-ph/9903350.

[108] J. Polchinski, *String Theory. Vol. 2: Superstring Theory and Beyond*, Cambridge, UK: Univ. Pr. (1998) 531 pp.

[109] J. Polchinski, *String Theory. Vol. 1: An Introduction to the Bosonic String*, Cambridge, UK: Univ. Pr. (1998) 402 pp.

[110] C. P. Bachas, "Lectures on D-branes," hep-th/9806199.

[111] D. J. H. Chung, "Classical inflaton field induced creation of superheavy dark matter," Phys. Rev. D **67**, (2003) 083514, hep-ph/9809489.

[112] S. S. Gubser, "String creation and cosmology," hep-th/0312321.

[113] P. C. Peters, "Relativistic gravitational bremsstrahlung," Phys. Rev. D **1** (1970) 1559.

[114] R. Breuer, "Gravitational perturbation theory and synchrotron radiation," Berlin: Springer-Verlag (1975) 196 pp.

[115] M. Abou-Zeid and M. S. Costa, "Radiation from accelerated branes," Phys. Rev. D **61**, (2000) 106007, hep-th/9909148.

[116] J. D. Jackson, *Classical Electrodynamics*, New York: Wiley (1999) 808 pp.

[117] G. T. Horowitz and A. Strominger, "Black strings and P-branes," Nucl. Phys. B **360** (1991) 197.

[118] A. Sen, "Tachyon Condensation on the Brane Anti-Brane System," JHEP **9808**, (1998) 012, hep-th/9805170.

[119] N. Lambert, H. Liu, and J. Maldacena, "Closed strings from decaying D-branes," hep-th/0303139.

[120] E. Silverstein and D. Tong, "Scalar speed limits and cosmology: Acceleration from D- cceleration," hep-th/0310221.

[121] M. Alishahiha, E. Silverstein, and D. Tong, "DBI in the sky," hep-th/0404084.

[122] X. Chen, "Cosmological rescaling through warped space," hep-th/0406198.

[123] S. Watson, "Moduli stabilization with the string Higgs effect," hep-th/0404177.

[124] L. Jarv, T. Mohaupt, and F. Saueressig, "M-theory cosmologies from singular Calabi-Yau compactifications," JCAP **0402**, (2004) 012, hep-th/0310174.

[125] H. Liu, G. Moore, and N. Seiberg, "Strings in a time-dependent orbifold," JHEP **06**, (2002) 045, hep-th/0204168.

[126] H. Liu, G. Moore, and N. Seiberg, "Strings in time-dependent orbifolds," JHEP **10**, (2002) 031, hep-th/0206182.

[127] G. T. Horowitz and J. Polchinski, "Instability of spacelike and null orbifold singularities," Phys. Rev. D **66**, (2002) 103512, hep-th/0206228.

[128] M. Fabinger and J. McGreevy, "On smooth time-dependent orbifolds and null singularities," JHEP **06**, (2003) 042, hep-th/0206196.

[129] M. Fabinger and S. Hellerman, "Stringy resolutions of null singularities," hep-th/0212223.

[130] B. Pioline and M. Berkooz, "Strings in an electric field, and the Milne universe," JCAP **0311**, (2003) 007, hep-th/0307280.

[131] M. Berkooz, B. Pioline, and M. Rozali, "Closed strings in Misner space: Cosmological production of winding strings," hep-th/0405126.

[132] L. Cornalba and M. S. Costa, "Time-dependent orbifolds and string cosmology," Fortsch. Phys. **52**, (2004) 145, hep-th/0310099.

[133] J. J. Friess, S. S. Gubser, and I. Mitra, "String creation in cosmologies with a varying dilaton," Nucl. Phys. B **689**, (2004) 243, hep-th/0402156.

[134] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt, and N. Turok, "From big crunch to big bang," Phys. Rev. D **65**, (2002) 086007, hep-th/0108187.

[135] P. J. Steinhardt and N. Turok, "A cyclic model of the universe," hep-th/0111030.

[136] P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. B **475**, (1996) 94, hep-th/9603142.

[137] M. Perry, P. J. Steinhardt, and N. Turok, "M theory model of a big crunch / big bang transition," hep-th/0408083.

[138] M. G. Jackson, N. T. Jones and J. Polchinski, "Collisions of cosmic F- and D-strings," hep-th/0405229.

[139] P. Candelas, G. Horowitz, A. Strominger, and E. Witten, "Vacuum Configurations for Superstrings," Nucl. Phys. B **258** (1985) 46.

[140] G. Coughlan, W. Fischler, E. Kolb, S. Raby and G. Ross, "Cosmological Problems for the Polonyi Potential," Phys. Lett. B **131** (1983) 59;
B. de Carlos, J.A. Casas, F. Quevedo, and E. Roulet, "Model Independent Properties and Cosmological Implications of the Dilaton and Moduli Sectors of 4-D Strings," Phys. Lett. B **318** (1993) 447, hep-ph/9308235;
T. Banks, D.B. Kaplan and A.E. Nelson, "Cosmological Implications of Dynamical Supersymmetry Breaking," Phys. Rev. D **49** (1994) 779, hep-ph/9308292.

[141] J.P. Derendinger, L. E. Ibanez, and H.P. Nilles, "On the Low Energy D=4, N=1 Supergravity Theory Extracted from the D=10, N=1 Superstring," Phys. Lett. B **155** (1985) 65.

[142] R. Rohm and E. Witten, "The Antisymmetric Tensor Field in Superstring Theory," Annals Phys. **170** (1986) 454.

[143] A. Niemeyer and H.P. Nilles, "Gaugino Condensation and the Vacuum Expectation Value of the Dilaton," hep-th/9508173.

[144] H.P. Nilles, "Dynamical Gauge Coupling Constants," hep-ph/9601241.

[145] P. Binétruy, M.K. Gaillard, and Y.-Y. Wu, "Dilaton Stabilization in the Context of Dynamical Supersymmetry Breaking through Gaugino Condensation," Nucl.Phys. B **481**, (1996) 109, hep-th/9605170.

[146] T. Barreiro, B. de Carlos, and E.J. Copeland, "Stabilizing the Dilaton in Superstring Cosmology," Phys.Rev. D **58** , (1998) 083513, hep-th/9805005.

[147] Y.-Y. Wu, "Dilaton Stabilization and Supersymmetry Breaking by Dynamical Gaugino Condensation in the Linear Multiplet Formalism of String Effective Theory," Nucl.Phys. B **481**, (1996) 109, hep-th/9610089.

[148] R. Xiu, "Supersymmetry Breaking Scheme and The Derivation of $M_{GUT} = 10^{16} GeV$ from A String Model," hep-ph/9412262.

[149] E. Witten, "Symmetry Breaking Patterns In Superstring Models," Nucl. Phys. B **258** (1985) 75.

[150] E. Witten, "Deconstruction, G_2 holonomy, and Doublet-Triplet Splitting," hep-ph/0201018.

[151] G. Cardoso, G. Curio, G. Dall'Agata and D. Lüst, "Heterotic String Theory on non-Kaehler Manifolds with H-Flux and Gaugino Condensate," hep-th/0310021.

[152] K. Becker, M. Becker, K. Dasgupta, P. Green and E. Sharpe, "Compactifications of Heterotic Strings on Non-Kähler Complex Manifolds: II," hep-th/0310058.

[153] E.I. Buchbinder and B.A. Ovrut, "Vacuum Stability in Heterotic M-Theory," hep-th/0310112.

[154] E. Bergshoeff, M. de Roo, B. de Wit, and P. van Nieuwenhuizen, "Ten-Dimensional Maxwell-Einstein Supergravity, its Currents, and the Issue of its Auxiliary Fields," Nucl.Phys. B **195**, (1982) 97.

[155] M.A. Shifman and A.I. Vainshtein, "On Gluino Condensation in Supersymmetric Gauge Theories with SU(N) and O(N) Groups," Nucl.Phys. B **296** (1988) 445.

[156] M.A. Shifman and A.I. Vainshtein, "On Holomorphic Dependence and Infrared Effects in Supersymmetric Gauge Theories," Nucl.Phys. B **359** (1991) 571.

[157] D.Finnell and P.Pouliot, "Instanton Calculations versus Exact Results in 4 Dimensional SUSY Gauge Theories," hep-th/9503115.

[158] S. Gukov, C. Vafa and E. Witten, "CFT's from Calabi-Yau four-folds," Nucl. Phys. B **584**, 69 (2000), hep-th/9906070.

[159] S. Gukov, "Solitons, Superpotentials and Calibrations", Nucl. Phys. B **574**, (2000) 169, hep-th/9911011.

[160] M. Becker and D. Constantin, "A Note on Flux Induced Superpotentials in String Theory," hep-th/0210131.

[161] K. Behrndt and S. Gukov, "Domain Walls and Superpotentials from M-theory on Calabi-Yau Three-Folds," Nucl. Phys. B **580** (2000) 225, hep-th/0001082.

[162] V. Kaplunovsky and J. Louis, "Field Dependent Gauge Couplings in Locally Supersymmetric Effective Quantum Field Theories," Nucl.Phys. B **422**, (1994) 57, hep-th/9402005.

[163] R. McLean, "Deformations of Calibrated Submanifolds," Comm. Anal. Geom. **6** (1998) 705.

[164] A. Strominger, S. T. Yau, and E. Zaslow, "Mirror Symmetry is T-duality," Nucl. Phys. B **479**, (1996) 243, hep-th/9606040.

[165] R. Gopakumar and C. Vafa, "Branes and Fundamental Groups," Adv. Theor. Math. Phys. **2** (1998) 399, hep-th/9712048.

[166] P. S. Aspinwall and C. A. Lutken, "Geometry Of Mirror Manifolds," Nucl. Phys. B **353** (1991) 427; "Quantum Algebraic Geometry Of Superstring Compactifications," Nucl. Phys. B **355** (1991) 482.

[167] H. Nishi, "$SU(n)$-Chern-Simons Invariants of Seifert Fibered 3-Manifolds," Int. J. Math **9** (1998) 295.

[168] D. Auckly, "Topological Methods to Compute Chern-Simons Invariants," Math. Proc. Camb. Phil. Soc. **115** (1994) 229.

[169] L. Rozansky, "A Large k Asymptotics of Witten's Invariant of Seifert Manifolds," Commun. Math. Phys. **171** (1995) 279.

[170] E. Witten, "Global Anomalies in String Theory," in Symposium on Anomalies, Geometry, Topology; Bardeen and White, eds., World Scientific, 1985.

[171] J.O. Conrad, "On Fractional Instanton Numbers in Six Dimensional Heterotic E8 x E8 Orbifolds," JHEP **0011** (2000) 022, hep-th/0009251.

[172] E. Witten,"World-Sheet Corrections Via D-Instantons," JHEP **0002**, (2000) 030, hep-th/9907041.

[173] A. Strominger, "Superstrings with Torsion," Nucl. Phys. B **274** (1986) 253.

[174] See e.g.: B.R. Greene, K. Kirklin, P. Miron and G.G. Ross, "A Three Generation Superstring Model I: Compactification and Discrete Symmetries," Nucl. Phys. B **278** (1986) 667;
D. Gepner, "String Theory on Calabi-Yau Manifolds: The Three Generations Case," hep-th/9301089;
S. Kachru, "Some Three Generation (0,2) Calabi-Yau Models," Phys. Lett. B **349**, (1995) 76, hep-th/9501131;
B. Ovrut, T. Pantev and R. Reinbacher, "Torus Fibered Calabi-Yau Threefolds with Nontrivial Fundamental Group," JHEP **0305**, (2003) 040, hep-th/0212221.

[175] K. Dasgupta, G. Rajesh, and S. Sethi, "M-theory, Orientifolds and G-flux," JHEP **9908**, (1999) 023, hep-th/9908088;
K. Becker and K. Dasgupta, "Heterotic Strings with Torsion," JHEP **0211**, (2002) 006, hep-th/0209077;
S. Gurrieri, J. Louis, and A. Micu, "Mirror Symmetry in Generalized Calabi-Yau Compactifications," Nucl. Phys. B **654**, (2003) 61, hep-th/0211102;
G.L. Cardoso, G. Curio, G. Dall'Agata, D. Lüst, P. Manousselis, and G. Zoupanos, "Non-Kaehler String Backgrounds and their Five Torsion Classes," Nucl. Phys. B **652**, (2003) 5, hep-th/0211118;
S. Kachru, M. Schulz, P. Tripathy, and S. Trivedi, "New Supersymmetric String Compactifications," JHEP **0303**, (2003) 061, hep-th/0211182;
K. Becker, M. Becker, K. Dasgupta, and P. Green, "Compactifications of Heterotic Theory on Non-Kähler Complex Manifolds I, " JHEP **0304**, (2003) 007, hep-th/0301161.

[176] K. Becker, M. Becker, K. Dasgupta, and S. Prokushkin, "Properties of Heterotic Vacua from Superpotentials," hep-th/0304001.

[177] G. L. Cardoso, G. Curio, G. Dall'Agata and D. Lust, "BPS Action and Superpotential for Heterotic String Compactifications with Fluxes," JHEP **0310**, (2003) 004, hep-th/0306088.

[178] M. Bershadsky, S. Cecotti, H. Ooguri, and C. Vafa, "Kodaira-Spencer Theory of Gravity and Exact Results for Quantum String Amplitudes," Commun. Math. Phys. **165**, (1994) 311, hep-th/9309140.

[179] M. Atiyah, J. Maldacena and C. Vafa, "An M-theory Flop as a Large N Duality," J. Math. Phys. **42**, (2001) 3209, hep-th/0011256.

[180] See e.g.: D. Morrison and C. Vafa, "Compactifications of F-theory on Calabi-Yau Threefolds I," Nucl.Phys. B **473** (1996) 74, hep-th/9602114; "Compactifications of F-theory on Calabi-Yau Threefolds II," Nucl.Phys. B **476** (1996) 437, hep-th/9603161;
N. Seiberg and E. Witten, "Comments on String Dynamics in Six Dimensions," hep-th/9603003.

[181] E. Witten, "Strong Coupling Expansion of Calabi-Yau Compactification", hep-th/9602070.

[182] E. Witten, "Phases of $N = 2$ Theories in Two Dimensions," Nucl. Phys. B **403**, (1993) 159, hep-th/9301042.

[183] P. Aspinwall, B. Greene and D. Morrison, "Calabi-Yau Moduli Space, Mirror Manifolds, and Space-Time Topology Change in String Theory," Nucl. Phys. B **416**, (1994) 414, hep-th/9309097.

[184] S. Kachru and C. Vafa, "Exact Results for N=2 Compactifications of Heterotic Strings," Nucl. Phys. B **450** (1995) 69, hep-th/9505105;
S. Ferrara, J. Harvey, A. Strominger and C. Vafa, "Second-Quantized Mirror Symmetry," Phys. Lett. B **361** (1995) 59, hep-th/9505162.

[185] V. Kaplunovsky and J. Louis, "On gauge couplings in string theory," Nucl. Phys. B **444**, 191 (1995), hep-th/9502077;
J. Louis and K. Foerger, "Holomorphic couplings in string theory," Nucl. Phys. Proc. Suppl. **55** B, 33 (1997), hep-th/9611184.

[186] R. Schimmrigk, "Scaling Behavior in String Theory," Phys. Lett. B **388**, (1996) 60, hep-th/9412077.

[187] B. de Carlos, J.A. Casas, and C. Munoz, "Supersymmetry Breaking and Determination of the Unification Gauge Coupling Constants in String Theories," Nucl. Phys. B **399**, (1993) 623, hep-th/9204012.

[188] M. Atiyah and E. Witten, "M-Theory Dynamics On A Manifold Of G_2 Holonomy," hep-th/0107177.

[189] B. Acharya and E. Witten, "Chiral Fermions from Manifolds of G_2 Holonomy," hep-th/0109152.

[190] T. Friedmann and E. Witten, "Unification Scale, Proton Decay, And Manifolds Of G_2 Holonomy," hep-th/0211269.

[191] I. R. Klebanov and E. Witten, "Proton Decay in Intersecting D-brane Models," hep-th/0304079.

[192] B. S. Acharya, "A Moduli Fixing Mechanism in M theory," hep-th/0212294.

[193] M. Cvetic, G. Shiu and A. M. Uranga, "Chiral Four-Dimensional N = 1 Supersymmetric Type IIA Orientifolds from Intersecting D6-branes," Nucl. Phys. B **615**, (2001) 3, hep-th/0107166.

[194] M. Cvetic, P. Langacker and J. Wang, "Dynamical Supersymmetry Breaking in Standard-like Models with Intersecting D6-branes," Phys. Rev. D **68**, (2003) 046002, hep-th/0303208.

[195] M. Cvetic and I. Papadimitriou, "Conformal Field Theory Couplings for Intersecting D-branes on Orientifolds," Phys. Rev. D **68**, (2003) 046001, hep-th/0303083

[196] S. Gukov, S. T. Yau, and E. Zaslow, "Duality and Fibrations on G(2) manifolds," hep-th/0203217.

[197] H. Ita, Y. Oz, and T. Sakai, "Comments on M Theory Dynamics on G2 Holonomy Manifolds," JHEP **0204** (2002) 001, hep-th/0203052.

[198] T. Friedmann, "On the Quantum Moduli Space of M-theory Compactifications," Nucl. Phys. B **635** (2002) 384, hep-th/0203256.

[199] E. Witten, "Anomaly Cancellation On Manifolds Of G_2 Holonomy," hep-th/0108165.

[200] J. de Boer, R. Dijkgraaf, K. Hori, A. Keurentjes, J. Morgan, D. R. Morrison, and S. Sethi, "Triples, fluxes, and strings," Adv. Theor. Math. Phys. **4**, (2002) 995, hep-th/0103170.

[201] A. Keurentjes and S. Sethi, "Twisting E_8 Five-Branes," Phys. Rev. D **66**, (2002) 046001, hep-th/0205162.

[202] D.M. Austin, "$SO(3)$-Instantons on $L(p,q) \times R$," J. Diff. Geom. **32** (1990) 383.

[203] R. Fintushel and R. Stern, "Invariants for Homology 3-Spheres," London Math. Soc. Lect. Notes **150** (1989) 125.

[204] R. Fintushel and T. Lawson, "Compactness of the Moduli Spaces for Orbifold Instantons," Topology Appl. **23** (1986) 305.

[205] E. Witten, "New Issues in Manifolds of $SU(3)$ Holonomy," Nucl. Phys. B **268** (1986) 79.

[206] E. Silverstein and E. Witten, "Criteria for Conformal Invariance of (0,2) Models," Nucl. Phys. B **444**, (1995) 161, hep-th/9503212;
A. Basu and S. Sethi, "World-sheet Stability of (0,2) Linear Sigma Models," hep-th/0303066;

C. Beasley and E. Witten, "Residues and World-Sheet Instantons," hep-th/0304115.

[207] G. Moore, G. Peradze, and N. Saulina, "Instabilities in Heterotic M-theory Induced by Open Membrane Instantons," Nucl. Phys. B **607**, (2001) 117, hep-th/0012104;
E. Buchbinder, R. Donagi, and B. Ovrut, "Superpotentials for Vector Bundle Moduli," Nucl. Phys. B **653**, (2003) 400, hep-th/0205190;
G. Curio and A. Krause, "G-Fluxes and Nonperturbative Stabilisation of Heterotic M-theory," Nucl. Phys. B **643**, (2002) 131, hep-th/0108220.

[208] C.P. Burgess, R. Kallosh, and F. Quevedo, "de Sitter String Vacua from Supersymmetric D-terms," hep-th/0309187.

[209] E. Silverstein, "(A)dS Backgrounds from Asymmetric Orientifolds," hep-th/0106209.

[210] C. Escoda, M. Gomez-Reino and F. Quevedo, "Saltatory de Sitter String Vacua," hep-th/0307160;
A. Frey, M. Lippert and B. Williams, "The Fall of Stringy de Sitter," Phys. Rev. D **68** (2003) 046008, hep-th/0305018.

[211] L. Susskind, "The Anthropic Landscape of String Theory," hep-th/0302219.

[212] B. Acharya, "Compactification with Flux and Yukawa Hierarchies," hep-th/0303234.

[213] J. Feng, J. March-Russell, S. Sethi, and F. Wilczek, "Saltatory Relaxation of the Cosmological Constant," Nucl. Phys. B **602** (2001) 307, hep-th/0005276.

[214] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood, Chur, Switzerland (1990).

[215] A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, Cambridge, England (2000).

[216] S. Hellerman, N. Kaloper and L. Susskind, "String Theory and Quintessence," JHEP **0106**, (2001) 003, hep-th/0104180;
W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, "The Acceleration of the Universe: A Challenge for String Theory," JHEP **0107**, (2001) 003, hep-th/0104181.

[217] F. Quevedo, "Lectures on String/Brane Cosmology," hep-th/0210292.

[218] O. DeWolfe and S. Giddings, "Scales and Hierarchies in Warped Compactifications and Brane Worlds," Phys. Rev. D **67**, (2003) 066008, hep-th/0208123.

[219] A. Frey, M. Lippert and B. Williams, "The Fall of Stringy de Sitter," hep-th/0305018.

[220] C. Escoda, M. Gomez-Reino and F. Quevedo, "Saltatory de Sitter String Vacua," hep-th/0307160.

[221] G. Dvali and S.H. Tye, "Brane Inflation," Phys. Lett. B **450**, (1999) 72, hep-th/9812483.

[222] S. Alexander, "Inflation from D - Anti-D-Brane Annihilation," Phys. Rev. D **65**, (2002) 023507, hep-th/0105032;
G. Dvali, Q. Shafi and S. Solganik, "D-brane Inflation," hep-th/0105203;
C.P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R.J. Zhang, "The Inflationary Brane-Antibrane Universe," JHEP **07**, (2001) 047, hep-th/0105204;
G. Shiu and S.H. Tye, "Some Aspects of Brane Inflation," Phys. Lett. B **516**, (2001) 421, hep-th/0106274 ;
D. Choudhury, D. Ghoshal, D.P. Jatkar, S. Panda, "Hybrid Inflation and Brane-Antibrane System," hep-th/0305104.

[223] R. Brustein, S. De Alwis and E. Novak, "Inflationary Cosmology in the Central Region of String/M-theory Moduli space," Phys. Rev. D **68**, (2003) 023517, hep-th/0205042;
R. Brustein, S. De Alwis and E. Novak, "M-theory Moduli Space and Cosmology," Phys. Rev. D **68**, (2003) 043507, hep-th/0212344.

[224] C. Herdeiro, S. Hirano and R. Kallosh, "String Theory and Hybrid Inflation/Acceleration," JHEP **0112**, (2001) 027, hep-th/0110271.

[225] J. Garcia-Bellido, R. Rabadan and F. Zamora, "Inflationary Scenarios from Branes at Angles," JHEP **0201**, (2002) 036, hep-th/0112147.

[226] B. Kyae and Q. Shafi, "Branes and Inflationary Cosmology," Phys. Lett. B **526**, (2002) 379, hep-ph/0111101.

[227] N. Jones, H. Stoica and S.H. Tye, "Brane Interaction as the Origin of Inflation," JHEP **0207**, (2002) 051, hep-th/0203163.

[228] N. Seiberg and E. Witten, "The D1/D5 System and Singular CFT," JHEP **9904**, (1999) 017, hep-th/9903224.

[229] S. Kachru, J. Pearson and H. Verlinde, "Brane/Flux Annihilation and the String Dual of a Nonsupersymmetric Field Theory," JHEP **0206**, (2002) 021, hep-th/0112197.

[230] E. Witten, "Dimensional Reduction Of Superstring Models," Phys. Lett. B **155** (1985) 151.

[231] O. Ganor, "A Note on Zeroes of Superpotentials in F-theory," Nucl. Phys. B **499**, (1997) 55, hep-th/9612077.

[232] J. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," Adv. Th. Math. Phys. **2**, (1998) 231, hep-th/9711200.

[233] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, "False Vacuum Inflation with Einstein Gravity," Phys. Rev. D **49**, (1994) 6410, astro-ph/9401011; G. R. Dvali, Q. Shafi and R. Schaefer, "Large Scale

Structure and Supersymmetric Inflation without Fine Tuning," Phys. Rev. Lett. **73**, (1994) 1886, hep-ph/9406319; A. D. Linde and A. Riotto, "Hybrid Inflation in Supergravity," Phys. Rev. D **56**, (1997) 1841, hep-ph/9703209.

[234] P. Binetruy and G. R. Dvali, "D-term inflation," Phys. Lett. B **388**, (1996) 241, hep-ph/9606342.

[235] T. Banks and M. Dine, "Coping with Strongly Coupled String Theory," hep-th/9406132.

[236] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry Breaking and Alpha-Prime Corrections to Flux Induced Potentials," JHEP **0206**, (2002) 060, hep-th/0204254.

[237] S. Giddings, "The Fate of Four Dimensions," Phys. Rev. D **68**, (2003) 026006, hep-th/0303031.

[238] A. Linde, "Inflation, quantum cosmology and the anthropic principle," hep-th/0211048.

[239] S. Ashok and M. Douglas, "Counting Flux Vacua," hep-th/0307049;
M. Douglas, "The statistics of string/M theory vacua," hep-th/0303194.

[240] R. Kallosh, work in progress.

[241] A. Vilenkin, "The Birth of Inflationary Universes," Phys. Rev. D **27** (1983) 2848.

[242] A. D. Linde, "Eternally Existing Selfreproducing Chaotic Inflationary Universe," Phys. Lett. B **175** (1986) 395.

[243] A. D. Linde, D. A. Linde and A. Mezhlumian, "From the Big Bang Theory to the Theory of a Stationary Universe," Phys. Rev. D **49**, (1994) 1783, gr-qc/9306035.

[244] A. D. Linde, "Axions in Inflationary Cosmology," Phys. Lett. B **259**, (1991) 38; A. D. Linde, "Hybrid Inflation," Phys. Rev. D **49**, (1994) 748, astro-ph/9307002.

[245] M.R. Garousi, "Tachyon Couplings on Non-BPS D-branes and Dirac-Born-Infeld Action," Nucl. Phys. B **584**, (2000) 584, hep-th/0003122;
E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras and S. Panda, "T-Duality and Actions for Non-BPS D-branes," JHEP **0005**, (2000) 009, hep-th/0003221;
J. Kluson, "Proposal for Non-BPS D-brane Action," Phys. Rev. D **62**, (2000) 126003, hep-th/0004106.

[246] L. Kofman and A. Linde, "Problems with Tachyon Inflation," JHEP **0207**, (2002) 004, hep-th/0205121.

[247] C. L. Bennett *et al.*, "First Year Wilkinson Microwave Anisotropy Probe Observations: Preliminary Maps and Basic Results," astro-ph/0302207.

[248] P. P. Avelino and A. R. Liddle, "Cosmological Perturbations and the Reionization Epoch," astro-ph/0305357.

[249] A. Sen, "Dirac-Born-Infeld Action on the Tachyon Kink and Vortex," hep-th/0303057.

[250] C. R. Contaldi, H. Hoekstra and A. Lewis, "Joint CMB and Weak Lensing Analysis: Physically Motivated Constraints on Cosmological Parameters," Phys. Rev. Lett. **90**, (2003) 221303, astro-ph/0302435.

[251] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, "False vacuum inflation with Einstein gravity," Phys. Rev. D **49**, 6410 (1994), astro-ph/9401011.

[252] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, "Fayet-Iliopoulos terms in supergravity and cosmology," Class. Quant. Grav. **21**, 3137 (2004), hep-th/0402046.

[253] C. P. Burgess, J. M. Cline, H. Stoica and F. Quevedo, "Inflation in realistic D-brane models," JHEP **0409**, 033 (2004), hep-th/0403119;
N. Iizuka and S. P. Trivedi, "An inflationary model in string theory," Phys. Rev. D **70**, 043519 (2004), hep-th/0403203;
X. G. Chen, "Multi-throat brane inflation," hep-th/0408084;
X. G. Chen, "Inflation from warped space," hep-th/0501184;
H. Firouzjahi and S. H. Tye, "Brane inflation and cosmic string tension in superstring theory," hep-th/0501099.

[254] J. P. Hsu and R. Kallosh, "Volume stabilization and the origin of the inflaton shift symmetry in string theory," JHEP **0404**, 042 (2004), hep-th/0402047;
H. Firouzjahi and S. H. H. Tye, "Closer towards inflation in string theory," Phys. Lett. B **584**, 147 (2004), hep-th/0312020;
J. P. Hsu, R. Kallosh and S. Prokushkin, "On brane inflation with volume stabilization," JCAP **0312**, 009 (2003), hep-th/0311077.

[255] S. Buchan, B. Shlaer, H. Stoica and S. H. H. Tye, "Inter-brane interactions in compact spaces and brane inflation," JCAP **0402**, 013 (2004), hep-th/0311207.

[256] A. Buchel and R. Roiban, "Inflation in warped geometries," Phys. Lett. B **590**, 284 (2004), hep-th/0311154.

[257] F. Koyama, Y. Tachikawa and T. Watari, "Supergravity analysis of hybrid inflation model from D3-D7 system," Phys. Rev. D **69**, 106001 (2004), hep-th/0311191.

[258] C. Angelantonj, R. D'Auria, S. Ferrara and M. Trigiante, "K3 x T**2/Z(2) orientifolds with fluxes, open string moduli and critical points," Phys. Lett. B **583**, 331 (2004), hep-th/0312019.

[259] A. Saltman and E. Silverstein, "A new handle on de Sitter compactifications," hep-th/0411271.

[260] K. Bobkov, "Volume stabilization via alpha' corrections in type IIB theory with fluxes," hep-th/0412239.

[261] E. Halyo, "Hybrid inflation from supergravity D-terms," Phys. Lett. B **387**, 43 (1996), hep-ph/9606423.

[262] G. Dvali, personal communication.

[263] M. Dine, L. Randall and S. Thomas, "Supersymmetry breaking in the early universe," Phys. Rev. Lett. **75**, 398 (1995), hep-ph/9503303.

[264] S. Thomas, personal communication.

[265] E. Silverstein, "TASI / PiTP / ISS lectures on moduli and microphysics," hep-th/0405068.

[266] L. Görlich, S. Kachru, P. K. Tripathy and S. P. Trivedi, "Gaugino condensation and nonperturbative superpotentials in flux compactifications," hep-th/0407130.

[267] L. Görlich, P. Tripathy and S. Trivedi, work in progress.

[268] R. Kallosh and D. Sorokin, "Dirac action on M5 and M2 branes with bulk fluxes," hep-th/0501081.

[269] M. A. Shifman and A. I. Vainshtein, "Solution Of The Anomaly Puzzle In Susy Gauge Theories And The Wilson Operator Expansion," Nucl. Phys. B **277**, 456 (1986).

[270] H. P. Nilles, "The Role Of Classical Symmetries In The Low-Energy Limit Of Superstring Theories," Phys. Lett. B **180**, 240 (1986).

[271] O. DeWolfe and S. B. Giddings, "Scales and hierarchies in warped compactifications and brane worlds," Phys. Rev. D **67**, 066008 (2003), hep-th/0208123.

[272] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha'-corrections to flux induced potentials," JHEP **0206**, 060 (2002), hep-th/0204254.

[273] M. Berg, M. Haack, and B. Körs, work in progress.

[274] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory, Volume 2*, Cambridge, 1986.

[275] S. E. Shandera, "Slow Roll in Brane Inflation," hep-th/0412077.

[276] S. Kachru, M. B. Schulz and S. Trivedi, "Moduli stabilization from fluxes in a simple IIB orientifold," JHEP **0310**, 007 (2003), hep-th/0201028.

[277] P. K. Tripathy and S. P. Trivedi, "Compactification with flux on K3 and tori," JHEP **0303**, 028 (2003), hep-th/0301139.

[278] F. Denef, M. R. Douglas and B. Florea, "Building a better racetrack," JHEP **0406**, 034 (2004), hep-th/0404257.

[279] C. P. Burgess, R. Kallosh and F. Quevedo, "de Sitter string vacua from supersymmetric D-terms," JHEP **0310**, 056 (2003), hep-th/0309187.

[280] A. Saltman and E. Silverstein, "The scaling of the no-scale potential and de Sitter model building," JHEP **0411**, 066 (2004), hep-th/0402135.

[281] K. Dasgupta, J. P. Hsu, R. Kallosh, A. Linde and M. Zagermann, "D3/D7 brane inflation and semilocal strings," JHEP **0408**, 030 (2004), hep-th/0405247.

[282] P. Chen, K. Dasgupta, K. Narayan, M. Shmakova and M. Zagermann, "Brane inflation, solitons and cosmological solutions: I," hep-th/0501185.

[283] R. Kallosh and A. Linde, "Landscape, the scale of SUSY breaking, and inflation," hep-th/0411011.

[284] E. I. Buchbinder and B. A. Ovrut, "Vacuum stability in heterotic M-theory," Phys. Rev. D **69**, 086010 (2004), hep-th/0310112;
S. Gukov, S. Kachru, X. Liu and L. McAllister, "Heterotic moduli stabilization with fractional Chern-Simons invariants," Phys. Rev. D **69**, 086008 (2004), hep-th/0310159;
M. Becker, G. Curio and A. Krause, "De Sitter vacua from heterotic M-theory," Nucl. Phys. B **693**, 223 (2004), hep-th/0403027.

[285] B. de Carlos, A. Lukas and S. Morris, "Non-perturbative vacua for M-theory on G(2) manifolds," hep-th/0409255.

[286] E. I. Buchbinder, "Five-brane dynamics and inflation in heterotic M-theory," hep-th/0411062;
K. Becker, M. Becker and A. Krause, "M-Theory Inflation from Multi M5-Brane Dynamics," hep-th/0501130.

[287] J. J. Blanco-Pillado *et al.*, "Racetrack inflation," JHEP **0411**, 063 (2004), hep-th/0406230.

[288] K. A. Intriligator and N. Seiberg, "Lectures on supersymmetric gauge theories and electric-magnetic duality," Nucl. Phys. Proc. Suppl. **45** BC, 1 (1996), hep-th/9509066.