POTENTIAL CONCEPTS, LONG-RANGE FORCES AND BOUND-STATE QED

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Abstract

The effects of one- and two-photon exchange on the energy levels of composite systems have been studied for many years, especially in the case of two-body bound states. However, a systematic analysis of the extent to which such effects can be described as resulting from the action of configuration-space potentials, deter-mined by field theory but acting directly between the constituents, appears not to have been carried out. A method for studying this problem will be presented and the results obtained to date will be summarized, with emphasis on the long-range character and spin-dependence of such potentials, as well as on conceptual aspects; there are some surprises here. Some comments will also be made on the long-range forces arising from two-neutrino exchange, which have recently been the object of renewed interest, and on aspects of the long-range force between a hydrogen and helium atom, which involves a competition between a repulsive two-photon exchange force and the attractive force of gravity.

I. Introduction

The main purpose of this talk is to discuss some aspects of the use of effective potentials in few-body problems and to report results of a recent paper on this topic, in the context of the long-range forces arising from photon exchange.[1] I will also make some brief remarks on gravity and (virtual!) neutrinos, major themes at this meeting. As befits the pre-dinner time slot, my sermon will be very informal and start with an ever popular type of tale: a referee story. The first referee on the above-mentioned paper indicated that he might be old-fashioned but he thought it should not be published because it does not contain a calculation whose results could be immediately compared with experiment. This was a criterion I had not encountered before; it would certainly lead to a much thinner journal! A second referee was very positive, agreeing that the issues raised were important. I conclude that the latter is not only modern in outlook but, if male, both a gentleman and a scholar.

By way of an aperitif, here's a quiz. Recall that the usual starting point for a relativistic theory of hydrogen or H-like ions is the Dirac equation:

$$h(1)\psi(1) \equiv [\boldsymbol{\alpha}_1\cdot\mathbf{p}_1+\beta_1 m+V_{ext}(1)]\psi(1) = E\psi(1) \qquad (1)$$

Now consider the statement: "With $U_C = e_1e_2/4\pi r$, a reasonable starting point for a relativistic theory of helium or He-like ions is the equation:

$$h(1,2)\psi(1,2) \equiv [h(1)+h(2)+U_C]\psi(1,2) = E\psi(1,2)." \qquad (2)$$

Question 1: The last statement is a) true, b) false, c) not well posed.

Having given you five seconds to think about it, I will tell you that (c), which I would normally chose in such a pressure situation (one can always argue about what is meant by "reasonable"), is incorrect. By any standard, the answer is (b), because (1) has no normalizable solutions associated with bound states.

Another example: In the late 1920's Breit and Gaunt independently considered the question of the leading correction to the Coulomb interaction, in the context of a Dirac description of electrons, arriving at different results:

$$U_B = -\tfrac{1}{2}(\boldsymbol{\alpha}_A\cdot\boldsymbol{\alpha}_B+\boldsymbol{\alpha}_A\cdot\hat{\mathbf{r}}\,\boldsymbol{\alpha}_B\cdot\hat{\mathbf{r}})U_C, \qquad U_G = -\boldsymbol{\alpha}_A\cdot\boldsymbol{\alpha}_B U_C. \qquad (3)$$

Question 2: Which one, if any, is right?

I will return to this later. The point is that in the context of relativistic quantum field theory (RQFT) the concept of an effective two-body interaction operator or potential V is subtle and beset with ambiguity.[2] In RQFT the interaction Lagrangian is primary; V is secondary and requires sharp definition. Historically, effective potentials have often "emerged" in the context of level-shift calculations for a specific physical system, initially in the context of time-independent perturbation theory, exemplified by Tamm-Dancoff (TD) type of calculations, and later from four-dimensional Bethe-Salpeter (BS) type of equations, via a relatively messy reduction to equal times.[3] However such potentials really merit *a priori* definitions and delineation of their use.

II. A different approach to effective potentials and bound states

I want to briefly sketch another approach, whose genesis is in work done long ago with the late Gary Feinberg on the quantum theory of long range forces (LRF). Using the techniques of particle theory (Lorentz and gauge invariance, analyticity and unitarity) we studied, in particular, the LRF arising from photon exchange between two composite neutral spinless systems. I review some of the results very briefly here, in order to write down some formulas which are needed later. As one would expect, the potential $V_{1\gamma}$ associated with one-photon exchange turns out to be short-range. However, the potential $V_{2\gamma}$ from two-photon exchange between two such systems, A and B, is long-range. For large r and low energies one finds that

$$V_{2\gamma} \approx -D/r^7, \quad D \equiv (23/4\pi)(\alpha_E^A\alpha_E^B+\alpha_M^A\alpha_M^B)-(7/4\pi)(\alpha_E^A\alpha_M^B+\alpha_M^A\alpha_E^B), \tag{4}$$

where the α's denote electric and magnetic polarizabilities. The purely electric terms were first obtained in the classic work of Casimir and Polder.[4] Application of the same techniques to the case of a neutral composite A and a charged elementary particle B ($V_{1\gamma}$ is then still short-range) yields

$$V_{2\gamma}(r) = (e_B^2/4\pi)[(-1/2)\alpha_E r^{-4}+(11/4\pi)\alpha_E r^{-4}(\lambda_B/r) +(5/4\pi)\alpha_M r^{-4}(\lambda_B/r)+\ldots] \tag{5}$$

where $\lambda_B = m_B^{-1}$ and the dots denote terms which fall off as $1/r^7$ or faster; these terms are however important in applications of the results to the fine structure of He Rydberg levels.[5]

Returning to our problem, both for practical reasons and to gain insight, it is obviously desirable, on the one hand, to try (i) to describe the interaction of particles in terms of potentials which can be used in 3-dimensional equations and to define such potentials directly. On the other hand, we wish (ii) to retain the enormous simplification achieved by the use of Feynman graphs and techniques in the computation of higher-order effects. Of course we want (iii) to avoid any *a priori* nonrelativistic approximations and if possible, (iv) to avoid any approximations which destroy gauge-invariance (GI). In contrast, the BS equation involves a kernel K which must be truncated in practice; this destroys GI in gauge theories such as QED. Further, in bound-state problems the use of Coulomb gauge is a practical necessity, which destroys manifest Lorentz invariance. While these are not fatal flaws, the approach I will sketch has, at a minimum, some conceptual advantages; unlike TD or BS, it retains both Lorentz and gauge invariance at any stage of approximation and has some other practical advantages. In any case, I believe it has a higher ISQ (intellectual satisfaction quotient) than the traditional approaches, which to some extent have the character of a black box.

I will focus on the two-body problem. The basic idea is quite simple,[6] a sort of a geometric mean between TD and BS. Somewhat paradoxically, we first consider the scattering problem and the associated two-body transition amplitude T. We then ask to what extent T can be regarded as arising from an effective two-body potential, to be used in a Schroedinger type of equation. To be more explicit, we define an interaction operator V, acting directly in configuration space, as a Fourier transform of an on-shell amplitude, obtained from gauge-invariant subsets of Feynman diagrams, modified by appropriate subtractions to avoid double counting; V is constrained by the requirement that when used in a specified type of center-or momentum system (c.m.s.) relativistic Schroedinger equation it reproduce $T_{c.m.}$, the value of T in the c.m.s. The equation has the natural form

$$h_{op}\,\phi = W\phi, \quad h_{op} = h_{op}^{(0)} + V \tag{6a}$$

with $h_{op}^{(0)}$ defined by

$$h_{op}^{(0)} = E_A^{op} + E_B^{op}\ , \quad [E_i^{op} \equiv (m_i^2+\mathbf{p}_{op}^2)^{1/2},\ \mathbf{p}_{op} = -i\partial/\partial\mathbf{r}]. \tag{6b}$$

The associated potential theory transition amplitude T_{pot} is given by

$$T_{pot} = \langle\mathbf{p}'|V + V(W-h_0^{op}-V+i\epsilon)^{-1}V|\mathbf{p}\rangle. \tag{7}$$

The field-theory transition amplitude T is given, in the c.m.s. by

$$T_{c.m.} = M(s,t)/4E_AE_B \tag{8}$$

where $M(s,t)$ denotes the invariant Feynman amplitude and $s\equiv(p_A+p_B)^2$, $t\equiv(p_A-p_A')^2$. The constraint on V, which in general will depend parametrically on s, is then simply that $T_{pot} = T_{c.m.}$, a condition which is to be satisfied order-by-order in perturbation theory. To apply this to bound states, we look for normalizable solutions of (6); these correspond to poles of M at values of s below $s_0 = (m_A+m_B)^2$.

III. LRF between charged particles: Beyond the Coulomb potential

For concreteness, consider two point-like spin-0 particles, with charges e_A and e_B, and confine attention to the so-called generalized ladder approximation to M(s,t), i.e. to graphs which involve photon exchange only between the particles. Consider first the one-photon exchange potential $V_{1\gamma}$. If one uses Feynman gauge in writing down the (gauge invariant) one-photon exchange amplitude $M_{1\gamma}$, one is led[6] to a Feynman-gauge inspired (FGI) potential $V_{1\gamma}^{FGI}$,

$$V_{1\gamma}^{FGI} = z'_{op}U_C z'_{op} + y_{op}(\mathbf{p}_{op}\cdot U_C\mathbf{p}_{op}/2m_Am_B)y_{op} \tag{9a}$$

where

$$z'_{op} \equiv (1+\mathbf{p}_{op}^2/2E_A^{op}E_B^{op})^{1/2}, \quad y_{op} = (m_Am_B/E_A^{op}E_B^{op})^{1/2}. \tag{9b}$$

The corresponding Coulomb-gauge inspired (CGI) one-photon exchange potential $V_{1\gamma}^{CGI}$ is given by[1]

$$V_{1\gamma}^{CGI} \equiv y_{op}[\{E_A^{op},\{E_B^{op},U_C\}\}+(1/2)\{\mathbf{p}_i^{op},\{\mathbf{p}_j^{op},(\delta_{ij}+\hat{r}_i\hat{r}_j)U_C\}\}]y_{op}/4m_Am_B \tag{10}$$

In the n.r. limit (9) yields as the leading correction to U_C an orbit-orbit interaction U_{o-o} of the form

$$U_{o-o}^{FGI} = \{\mathbf{p}_i^{op},\{\mathbf{p}_j^{op},\delta_{ij}U_C\}\}/4m_Am_B. \tag{11}$$

whereas (10) yields

$$U_{o-o}^{CGI} = (1/2)\{\mathbf{p}_i^{op},\{\mathbf{p}_j^{op},(\delta_{ij}+\hat{r}_i\hat{r}_j)U_C\}\}/4m_Am_B. \tag{12}$$

This is a manifestly hermitian form of the orbit-orbit interaction U_{o-o} familiar from atomic physics, usually described as arising from reduction of the Breit operator (3) to n.r. form. But, of course, spin has nothing to do with it!

Some insight into the difference between the two choices comes from examining the potential $V_{2\gamma}$ from 2γ exchange. Surprisingly, the computation of this is more difficult than in the case where at least one of A or B is neutral, because of the presence of infrared divergences (IR); the cure for these turns out to be precisely the subtractions necessary anyhow to avoid double counting.[6] *These subtractions depend on the choice of* $V_{1\gamma}$ *and turn out to affect even the asymptotic form of* $V_{2\gamma}$. At low energies and large r one finds, with $k \equiv e_Ae_B/4\pi$, that[1]

$$V_{2\gamma} = c_2r^{-2} + c_3r^{-3} + \ \ldots \tag{13}$$

where $c_3 = -\ 7k^2/6\pi m_Am_B$ in both cases, but

$$c_2^{FGI} = k^2/2(m_A+m_B), \quad c_2^{CGI} = 0. \tag{14}$$

This observation resolves a long-standing puzzle in the literature and shows that in the case of two charged particles the concept of the asymptotic form of the effective potential has an unexpected ambiguity. Further, as was noted some time ago by L. Spruch, c_2^{FGI} is classical in character. It turns out that this can also be understood, by an extension of the classic work of Darwin.[7]

IV. Inclusion of spin

Similar results hold when *A* and *B* are point particles but *B*, say, has spin-1/2. The FGI $V_{1\gamma}$ is then given by [8]

$$V_{1\gamma}^{FGI} = (y_A^{op}\Lambda_+^{op})U^{(2)}(y_A^{op}\Lambda_+^{op}), \quad U^{(2)} \equiv \{E_A^{op}-\boldsymbol{\alpha}\cdot\mathbf{p}_{op}, U_C\}/2m_A, \tag{15}$$

where $\Lambda_+^{op} = (E_B^{op}+h_B^{op})/2E_B^{op}$ is a Casimir positive-energy projection operator and $y_A^{op} = (m_A/E_B^{op})^{1/2}$. The corresponding CGI $V_{1\gamma}$ is

$$V_{1\gamma}^{CGI} = (y_A^{op}\Lambda_+^{op})(U_C'+U_T')(y_A^{op}\Lambda_+^{op}) \tag{16a}$$

where

$$U_C' \equiv \{E_A^{op},U_C\}/2m_A, \quad U_T' \equiv -(1/2)\{\mathbf{p}_i^{op}\alpha_j,(\delta_{ij}+\hat{r}_i\hat{r}_j)U_C\}/2m_A. \tag{16b}$$

Reduction of (15) and (16) to the n.r. limit yields the same spin-dependent (s.d) interaction operator $-[1+2m_B/m_A]U_C\boldsymbol{\sigma}\cdot\boldsymbol{\ell}/4m_B^2r^2$, while the s.i. potentials differ in the same way as when *B* has spin-0. If $V_{1\gamma}^{FGI}$ is used, the two-photon exchange yields a s.i. term with c_2 as in (14) and a s.d. correction given at large r by [8]

$$V_{2\gamma;pt}^{s.o.} \approx -k^2[(3m_A+5m_B)/m_A(m_A+m_B)](\boldsymbol{\sigma}\cdot\boldsymbol{\ell}/4m_B^2r^4). \tag{17}$$

If A has structure, $V_{2\gamma}$ also contains a spin-orbit polarizability potential

$$V_{2\gamma;pol}^{s.o.} = (e_B^2/4\pi)[(\alpha_E^A m_B+\alpha_M^A(m_A+m_B))/2m_Am_B^2](\boldsymbol{\sigma}\cdot\boldsymbol{\ell}/r^6)+O(r^{-7}). \tag{18}$$

There are a number of physical situations in which it may be possible to detect the effects of $V_{2\gamma}^{s.o}$. Typically these involve measurements of bound state energies in exotic atoms, where one particle has spin 1/2 and another has spin 0. Examples include antiprotonic atoms with a spin-0 nucleus, such as p-He^4, pionic atoms with a spin-1/2 nucleus, such as pionic hydrogen, and the pi-muon bound state known as pi-muonium. Certain aspects of $V_{2\gamma}^{s.o.}$ may be observable in Rydberg states of helium-like ions whose nuclei have spin 1/2. For details see Ref.8.

For two-spin 1/2 particles one finds, with Λ_{++} a projection operator product,

$$V_{1\gamma}^{FGI} = \Lambda_{++}(U_C+U_G)\Lambda_{++}, \quad V_{1\gamma}^{CGI} = \Lambda_{++}(U_C+U_B)\Lambda_{++}. \tag{19}$$

Thus the answer to the question posed earlier is "neither". The computation of $V_{2\gamma}$ for this case is a major undertaking, which is currently in progress.[9] When this is completed one will be able to reanalyze the spin-dependent level structure of a number of physical systems and gain new insight into some aspects of QED.

IV. Some exotic long-range forces

Since both gravity and neutrinos are topics of this meeting I thought it would be of interest to discuss briefly some unusual aspects of long-range forces, which touch on these areas.

A. *The long-range H-He force and gravity.* From (4) we can see that, contrary to folklore, there are cases in which $V_{2\gamma}$ is repulsive at large distances. Indeed, although the coefficient D in (4) is necessarily positive if A = B, already the very simplest departure from this, viz. A = H and B = He^4 gives a negative D. This is because $\alpha_M \ll \alpha_E$ for He while for H, $\alpha_M \approx 129a^3$, which is about 30 times larger than $\alpha_E \approx 9a^3/2$ (a = Bohr radius) and more than compensates for the ratio $23/7 \approx 3$. Thus $D \approx -(7/4\pi)\alpha_M{}^H\alpha_E{}^{He} < 0$ and $V_{2\gamma}{}^{as} \equiv -D/r^7$ is repulsive. Since $V_{2\gamma}$ is attractive at distances of a few Bohr radii, when graphed as a function of r it must cross the V = 0 axis for large enough r and then approach this axis from above. If we now add the one-graviton exchange potential $V_{1g} = -Gm_Hm_{He}/r$ to $V_{2\gamma}$ the total potential $V_{tot} = V_{2\gamma} + V_{1g}$ must eventually approach the axis from below. It is possible that there is a little potential well at some sufficiently large r ? If so, perhaps something could be made of it. Indeed the sum $V_{tot}{}^{as} \equiv V_{2\gamma}{}^{as}+V_{1g}$ vanishes at $r_0 \approx [(7/4\pi)\hbar c\alpha_M{}^H\alpha_E{}^{He}/Gm_Hm_{He}]^{1/6}$ and its derivative vanishes at $r_1 = 7^{1/6}r_0 \approx 1.4r_0$. With $\alpha_E{}^{He} \approx 1.4a^3$ one gets $r_0 \approx 4\text{x}10^6a \approx 2\text{x}10^{-2}$ cm. However, one must now ask whether $V_{2\gamma}$ is well approximated by $V_{2\gamma}{}^{as}$ for such values of r. This requires a detailed knowledge of $V_{2\gamma}$ for the case at hand. Recently Chi Kwan Au and I decided to have a closer look at this.[10]

We may write, in an obvious notation, $V_{2\gamma} = V_{EE} + V_{ME} + V_{EM} + V_{MM}$. Although V_{EM} and V_{MM} can be neglected, analysis shows that the actual V_{ME} is canceled by V_{1g} at a much smaller value of r. To be quantitative, for values of r such that $2\omega_H r/c \ll 1$, where $\hbar\omega_H$ is the hyperfine splitting of the hydrogen ground state, one finds that

$$V_{ME} \approx (5/8\pi r^5)\alpha^3a^2\alpha_E{}^{He}(\hbar\omega_H). \tag{20}$$

Then $V_{ME} + V_{1g} = 0$ for $r_0' = [(5/8\pi)\alpha^3a^2\alpha_E{}^{He}(\hbar\omega_H)/Gm_Hm_{He}]^{1/4} \approx 2\text{x}10^{-3}$ cm and the derivative vanishes for $r_1' = 5^{1/4}r_0' = 1.5r_0'$. Since V_{EE} is well approximated by its asymptotic form for $r \gg \alpha^{-1}a \approx 10^{-6}$cm, we have

$$-V_{ME}/V_{EE} \approx (5/46)\alpha^3(2\pi a^2/\Lambda_H\alpha_E{}^H)r^2 \approx .015\alpha^3(r^2/\Lambda_H a) \tag{22}$$

in the region of interest, with λ_H = 21 cm. For $r = r_0'$ the ratio (22) if of order 10^{-6} and so is much less than unity. The V_{EE} potential therefore dominates in this region and there is no minimum. Thus, this intriguing possibility for detecting an interplay between electromagnetism and gravity in an atomic system appears to be out of reach for the foreseeable future.

B. *The two-neutrino exchange force.* The exchange of neutrino-antineutrino pair between two spin-1/2 particles also gives rise to a long-range potential, $V_{2\nu}$, which falls of as r^{-5}, as shown long-ago.[11] My interest in this subject was reawakened by a preprint in of Hsu and Sikivie.[12] These authors had come across some lectures of Feynman in which he studied the question of whether neutrino-pair exchange forces might be responsible for gravity.[13] In this connection Feynman considered the possibility that the two-neutrino exchange potential might fall off as r^{-3}. On dimensional grounds the simplest form of $V_{2\nu}$ compatible with this is $V_{2\nu} \propto G_F{}^2 m_A m_B/r^3$, in which case some experimental tests might be feasible. Because the results of Ref. 12 disagreed with those found in Ref. 11 (in the coefficients of both the s.i. and s.d. parts of $V_{\nu\nu}$), I was led to check the old work in a more general way, without making any n.r. approximations. The result turns out to be of some interest, especially in view of Feynman's considerations.

Consider a current-current interaction of Dirac fields ψ_A and ψ_B with a massless neutrino field ψ_ν of the form

$$L_{eff} = - G_A(\bar{\psi}_A\Gamma_A{}^\sigma\psi_A)(\bar{\psi}_\nu\Gamma_\sigma\psi_\nu) - (A\rightarrow B) \tag{22}$$

where the Γ's have the generic form

$$\Gamma_A{}^\sigma = \gamma^\sigma(1+\xi_A\gamma_5),\quad \Gamma_B{}^\sigma = \gamma^\sigma(1+\xi_B\gamma_5),\quad \Gamma^\sigma = \gamma^\sigma(1+c\gamma_5) \tag{23}$$

and G's denote effective Fermi coupling constants. One then finds, on use of the methods of Ref. 11, that the long-range part of $V_{\nu\nu}$ is given by $V_1 + V_2$, where, with $G^2 \equiv (1+c^2)G_AG_B$,

$$V_1 \equiv (G^2/4\pi^3)(\gamma_A{}^0\gamma_B{}^0)(\Gamma_A\cdot\Gamma_B r^{-5}),\quad V_2 \equiv (3/2)(G^2/4\pi^3)(\gamma_A{}^0\gamma_B{}^0)(m_Am_B\gamma_A{}^5\gamma_B{}^5/r^3). \tag{24}$$

Thus, within a Dirac description of spin-1/2 particles, there is indeed a term proportional to $m_Am_Br^{-3}$, but it comes accompanied with a γ^5 factor for each particle. Reduction to Schroedinger-Pauli form of V_2 then yields, in the low-energy limit, only spin-dependent terms of order r^{-5}. To be precise,

$$(V_1)_{red} = (G^2/4\pi^3r^5)(1-\xi_A\xi_B\sigma_A\cdot\sigma_B),\ (V_2)_{red} = (G^2/4\pi^3r^5)(\xi_A\xi_B/2)(5\sigma_A\cdot\mathbf{r}\sigma_B\cdot\mathbf{r}-\sigma_A\cdot\sigma_B)$$

so that

$$(V_{\nu\nu})_{red} = (G^2/4\pi^3r^5)[1+(\xi_A\xi_B/2)(5\sigma_A\cdot\mathbf{r}\sigma_B\cdot\mathbf{r}-3\sigma_A\cdot\sigma_B)]. \tag{25}$$

For $G_i = G_F/2^{1/2}$ and $c = -1$, $\xi_i = -1$, as in the old four-fermion interaction for charged leptons, this reduces to the results of Ref. 11. In the standard model, we have $G_i = c_V G_F/2^{1/2}$, $\xi_i = -c_A/c_V$, with $c_V - 1 = 2\sin^2\theta_W - 1/2$, $c_A - 1 = -1/2$, so that the s.i. part of $(V_w)_{red}$ becomes $(2\sin^2\theta_W + 1/2)^2 G_F^2/4\pi^3 r^5$, in agreement with Ref. 2.

It is amusing to note that if one had two macroscopic bodies which contained an appreciable fraction of relativistic polarized electrons, so that $\langle\gamma_5\rangle \approx 1$, one could indeed generate an interaction of the Feynman type! Unfortunately such systems are hard to come by.

Acknowledgement

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