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Correlation functions in $\mathcal{N} = 2$ Supersymmetric vector matter Chern-Simons theory

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Abstract: Correlation functions of the higher-spin current operators in large N Chern-Simons theories are important to understand approximate higher-spin symmetries in these theories. Moreover, they also provide stronger checks for conjectured dualities in these theories. In this paper, we compute the two, three and four-point functions of the operators in the spin zero multiplet of $\mathcal{N} = 2$ Supersymmetric vector matter Chern-Simons theory at large N to all orders of 't Hooft coupling. While the two- and three-point functions are computed by solving the Schwinger-Dyson equation, this method becomes intractable for the computation of the four-point functions. Thereby, we use bootstrap method to evaluate four-point function of scalar operator $J_0^f = \bar{\psi}\psi$ and $J_0^b = \bar{\phi}\phi$. Interestingly, because $\langle J_0^f J_0^f J_0^b \rangle$ is a contact term, the four point function of J_0^f operator resembles the corresponding correlation function in the free theory, up to overall coupling constant

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dependent factors and up to some 'bulk AdS' contact terms. On the other hand the J_0^b four-point function receives an additional contribution compared to the free theory expression due to the J_0^f exchange. We find that the double discontinuity of this single trace operator J_0^f vanishes and hence it only contributes to AdS-contact term.

Keywords: 1/N Expansion, Chern-Simons Theories, Higher Spin Symmetry, Field Theories in Lower Dimensions

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Contents

1 Introduction

The perturbative technique to compute observables in quantum field theories involving Feynman diagrams is effective only when the coupling is weak and breaks down in the strong coupling regime. In the past few decades various strong-weak dualities have been discovered which have proven to be extremely useful in understanding some of the most interesting non-perturbative properties of strongly coupled quantum field theories. One such class of dualities which have been studied extensively in recent times are the *Bosonization* *dualities* in Chern Simons gauge theories coupled to fundamental matter at large N [1–7]. Though, one of the main indications for these dualities initially came from their holographic duality with Vasiliev theories in AdS_4 [8–13], by now there exists a plethora of evidence for these dualities coming from exact computations of correlation functions, thermal partition functions, anomalous dimensions, scattering amplitudes and RG flow analysis relating these theories to known supersymmetric dualities [3–5, 14–36].

Primary example of these bosonization dualities are those among the quasi-fermionic (critical bosonic and regular fermionic theory) and the quasi-bosonic theories (regular bosonic and critical fermionic theory).[1] A particularly interesting case of these dualities is present in the $\mathcal{N} = 2$ supersymmetric U(N) Chern Simons theory coupled to a single fundamental chiral multiplet. This theory exhibits a strong-weak self duality [37–39] generalizing the well known Giveon-Kutosov duality [37, 40]. The self duality of this supersymmetric theory serves as a parent duality for the non-supersymmetric bosonization dualities mentioned above since they can obtained from the supersymmetric theory via RG flows seeded by mass deformations [41, 42]. Taking hints from the supersymmetric dualities and the Level-Rank duality of pure Chern Simons theory, finite N extensions for the non supersymmetric dualities have also been proposed [43–52]. These theories were also investigated recently in presence of background magnetic field [53].

In this article, we will focus our attention on the $\mathcal{N} = 2$ theory. Various large N computations in this theory show remarkable features which are absent in the non supersymmetric couterparts. For example, the all loop $2 \to 2$ scattering amplitude is tree-level exact except in anyonic channel [27] and it was shown that these amplitudes are also invariant under *Dual superconformal symmetry* [27, 30]. [29] further showed that the tree level $m \to n$ scattering amplitudes in this theory can be constructed using the BCFW recursions relations.

Although many interesting non supersymmetric physical observables, as mentioned above, are amenable to direct exact computations by solving corresponding Dyson-Schwinger equations, the computation of 4-point correlation function of even the simplest of single trace operators, namely the scalar operators $\bar{\phi}\phi$ and $\bar{\psi}\psi$, appears prohibitively difficult[2] to compute via this direct approach. Given the remarkable simplicity of the results for other known observables one expects the 4-point functions in this theory to also have a simple structure. In the present article, our main goal will be to determine the exact 2, 3 and leading connected 4-point correlation functions of scalar operators in this supersymmetric theory.

For the quasi-bosonic and quasi-fermionic theories mentioned above, in [54–57], the 4-point correlation functions of various scalars as well as some higher spin operators were determined using recently developed ideas from conformal Bootstrap. In particular, one of the central objects used in [55] is the double discontinuity of the 4-point function which determines the coefficients in the OPE expansion of external operators via the Lorenzian inversion formula (LIF) discovered by Caron-Huot in [58]. The authors of [55] first demon-

[1] In the terminology of [3, 4].

[2] See appendix D for a discussion of our attempt.

strated that for large-N CFTs the double discontinuity of the 4-point function of identical scalars determines the full 4-point function up to three AdS_4 contact Witten diagrams. The authors further showed that for the quasi-bosonic and quasi-fermionic theories the coefficients of these contact terms vanish. This is consistent with the results of [56] which used the large spin perturbation theory developed in [54] instead of the LIF. In the present work, we apply some of these ideas in conjunction with the self duality, to the case of scalar 4-point functions in our $N = 2$ theory.

Our article is structured as follows. In section 2, we review the $\mathcal{N} = 2$ theory of interest in this paper and its operators spectrum in some detail. In section 3, we determine the scalar multiplet 2 and 3-point functions via a directl computation. In section 4, we determine the 4-point function of the bosonic and the fermionic scalar operators in this theory using the double discontinuity technique developed in [55]. Finally, in the section 5, we summarize our results and outline related open questions and future directions. In various appendix, we collect our notation and conventions, some technical details of the results in main text of the paper and briefly summarize our attempt at the direct computation of 4-point function.

Note added. While we were in the process of finishing up our article, we were informed about the related work [59] by the authors which has overlap with the results of our section 3.

2 $\mathcal{N} = 2$ theory and its operator spectrum

In this paper, we are interested in $\mathcal{N} = 2$ U(N) Chern-Simons theory coupled to single chiral multiplet, $\Phi \equiv (\phi, \psi)$, in the fundamental representation of the gauge group. The position space Lagrangian for the theory is

$$\mathcal{S}^L_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) - \bar{\psi}i\not{D}\psi + \mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 - \frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi) - \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]. \tag{2.1}$$

The theory above has two parameters: the rank of the gauge group, N, and the Chern-Simons level, κ, which is quantized to take only integer values [60]. κ^{-1} controls the strength of gauge interactions and the theory is perturbative for large values of κ at any finite N.

This theory is conjectured to be self-dual under a strong-weak type duality, [37]. In the 't Hooft like large N limit

$$\kappa \to \infty, N \to \infty \quad \text{with} \quad \lambda = \frac{N}{\kappa} \quad \text{fixed} \tag{2.2}$$

of interest in this paper, the duality transformation is

$$\kappa \to -\kappa, \quad \lambda \to \lambda - \mathrm{sgn}(\lambda). \tag{2.3}$$

Apart from the matching of many of the supersymmetric observables which can be computed at finite N and κ using supersymmetric localization techniques, recent exact computation of many non-supersymmetric observables, e.g. the thermal partition function, in the large N limit [1, 2, 5, 26, 31, 41] has provided ample evidence for this conjectured duality.

The theory is quantum mechanically (super) conformal for all values of κ and N. In the 't Hooft limit, one can focus on the single trace superconformal primary operator spectrum of the theory. Though our theory has $\mathcal{N}=2$ superconformal symmetry, in this paper we will work in the $\mathcal{N}=1$ superspace formulation to allow us to use the relevant results of [27] for our computations. In the $\mathcal{N}=1$ language, the operators spectrum of the theory consists of a set of supercurrent operators [61]

$$J^{(s)} = \sum_{r=0}^{2s} (-1)^{\frac{r(r+1)}{2}} \binom{2s}{r} \nabla^r \bar{\Phi} \nabla^{2s-r} \Phi\,, \tag{2.4}$$

which are written in terms of the superfields,

$$\Phi = \phi + \theta\psi - \theta^2 F, \quad \bar{\Phi} = \bar{\phi} + \theta\bar{\psi} - \theta^2 \bar{F}\,.$$

and the superscript s in (2.4) takes values in $\{0, \frac{1}{2}, 1, \frac{3}{2}, \ldots\}$. Here, we have also defined

$$J^{(s)} = \lambda^{\alpha_1}\lambda^{\alpha_2}\ldots\lambda^{\alpha_{2s}} J_{\alpha_1\alpha_2\ldots\alpha_{2s}}, \quad \nabla = \lambda^\alpha \nabla_\alpha \tag{2.5}$$

using the auxiliary commuting polarisation spinors, λ^{α_i}, which keep track of the spin; and ∇_{α_i} are the standard supersymmetry invariant gauge-covariant derivatives. Their action on the matter superfields of our theory is given by,

$$\begin{aligned} \nabla_\alpha \Phi &= D_\alpha \Phi - i\Gamma_\alpha \Phi \\ \nabla_\alpha \bar{\Phi} &= D_\alpha \bar{\Phi} + i\Gamma_\alpha \bar{\Phi} \end{aligned} \tag{2.6}$$

The explicit expressions for spin 0 operator and the first few spin-s currents are,

$$\begin{aligned} J_0 &= \bar{\Phi}\Phi \\ J_\alpha &= \bar{\Phi}\nabla_\alpha\Phi - \nabla_\alpha\bar{\Phi}\Phi = \bar{\Phi}D_\alpha\Phi - D_\alpha\bar{\Phi}\Phi - 2i\bar{\Phi}\Gamma_\alpha\Phi \\ J_{\alpha\beta} &= \bar{\Phi}\nabla_\alpha\nabla_\beta\Phi - 2\nabla_\alpha\bar{\Phi}\nabla_\beta\Phi + \nabla_\alpha\nabla_\beta\bar{\Phi}\Phi \\ J_{\alpha\beta\gamma} &= \bar{\Phi}\nabla_\alpha\nabla_\beta\nabla_\gamma\Phi - 3\nabla_\alpha\bar{\Phi}\nabla_\beta\nabla_\gamma\Phi - 3\nabla_\alpha\nabla_\beta\bar{\Phi}\nabla_\gamma\Phi + \nabla_\alpha\nabla_\beta\nabla_\gamma\bar{\Phi}\Phi \end{aligned} \tag{2.7}$$

In the free limit of the theory i.e. $\lambda \to 0$, each of these supercurrents, $J^{(s)}$ with $s \neq 0$, satisfies the conservation equation

$$\mathcal{D}^\alpha \left(\frac{\partial}{\partial\lambda^\alpha} J^{(s)} \right) = 0 \tag{2.8}$$

and constitutes two component conserved current operators $\{J^{(s)}, J^{(s+\frac{1}{2})}\}$ in its θ expansion [61]. At finite λ, the conservation equation (2.8) is violated at order $\frac{1}{N}$ by double trace operators for $s \geq 2$ [1, 61].

In this article, we are interested in the scalar operator $J_0(\theta, x)$. There is no conservation equation associated with this operator and it constitutes 2 scalar and 1 spin half operator as follows

$$J^{(0)}(\theta, x) = J_0^b(x) + \theta^\alpha \Psi_\alpha(x) - \theta^2 J_0^f(x) \tag{2.9}$$

where

$$J_0^b(x) = \bar{\phi}\phi(x), \quad \Psi_\alpha(x) = (\bar{\phi}\psi_\alpha + \bar{\psi}_\alpha\phi)(x), \quad J_0^f(x) = \bar{\psi}\psi(x). \tag{2.10}$$

In the subsequent sections, we compute the 2 and 3-point functions of the $J^{(0)}$ operator and two component of the 4-point function.

3 Correlation functions

In this section, we compute the two and three point correlation function of the $J_0(\theta, p)$ operator in momemtum space. Two of the main ingredients for these computations are the exact propagator (3.2) and the renormalized four point vertex for the fundamental superfield $\Phi(\theta, p)$ (ν_4 in (3.3)). These were computed in [27] for a more general class of theories with $\mathcal{N} = 1$ supersymmetry which can be thought of as one parameter[3] deformation of the $\mathcal{N} = 2$ theory of interest in this paper. Below, we list these results for our $\mathcal{N} = 2$ theory, conveniently stated in term of the exact quantum effective action

$$\begin{aligned}
S &= S_2 + S_4\,, \\
S_2 &= \int \frac{d^3p}{(2\pi)^3} d^2\theta_1 d^2\theta_2 \left[\bar{\Phi}(\theta_1, -p) e^{-\theta_1^\alpha p_{\alpha\beta}\theta_2^\beta} \Phi(\theta_2, p)\right], \\
S_4 &= \frac{1}{2}\int \frac{d^2p}{(2\pi)^3}\frac{d^2q}{(2\pi)^3}\frac{d^2k}{(2\pi)^3} d^2\theta_1 d^2\theta_2 d^2\theta_3 d^2\theta_4 \\
&\qquad \left[\nu_4(\theta_1,\theta_2,\theta_3,\theta_4; p,q,k)\Phi_i(\theta_1, -(p+q))\bar{\Phi}^i(\theta_2, p)\bar{\Phi}^j(\theta_3, k+q)\Phi_j(\theta_4, -k)\right]
\end{aligned} \tag{3.1}$$

The quadratic part of the effective action receives no quantum corrections at large N in the $\mathcal{N} = 2$ theory. The propagator is thus tree level exact and given by

$$\begin{aligned}
\langle \bar{\Phi}(\theta_1, p_1)\Phi(\theta_2, p_2)\rangle &= (2\pi)^3\delta^3(p_1 + p_2)\mathcal{P}(\theta_1, \theta_2; p_1) \\
&= (2\pi)^3\delta^3(p_1 + p_2)\frac{e^{-\theta_1^\alpha\theta_2^\beta (p_1)_{\alpha\beta}}}{p_1^2}.
\end{aligned} \tag{3.2}$$

The quartic superspace vertex, ν_4, does receive quantum corrections and takes the following form

$$\begin{aligned}
\nu_4(\theta_1,\theta_2,\theta_3,\theta_4; p,q,k) &= e^{\frac{1}{4}X.(p.\theta_{12}+q.\theta_{13}+k.\theta_{43})} F_4(\theta_{12},\theta_{13},\theta_{43}; p,q,k), \\
\text{with} \quad F_4 &= \theta_{12}^+\theta_{43}^+\Big[A(p,q,k)\theta_{12}^-\theta_{43}^-\theta_{13}^+\theta_{13}^- + C(p,q,k)\theta_{12}^-\theta_{13}^+ \\
&\qquad + D(p,q,k)\theta_{13}^+\theta_{43}^-\Big]
\end{aligned} \tag{3.3}$$

[3] Quartic superpotential term: $-\frac{\pi(\omega-1)}{\kappa}\int d^3x\, d^2\theta\, (\bar{\Phi}\Phi)^2$. $\omega = 1$ is the $\mathcal{N} = 2$ point.

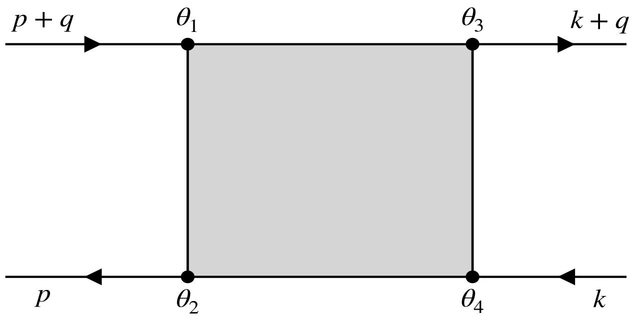

Figure 1. Diagrammatic representation of the exact four point vertex, ν_4 in (3.3).

Here, we have used following notation for the sum and the difference of Grassmann variables to avoid clutter,

$$X^\alpha = \sum_{i=1}^{n} \theta_i^\alpha\,, \quad \theta_{\text{in}}^\alpha = \theta_i^\alpha - \theta_n^\alpha. \tag{3.4}$$

The overall exponential factor is determined by supersymmetric Ward identity (3.8), while the coefficient functions A, C and D require explicit computation and given by [27]

$$\begin{aligned} A(p,q,k) &= -\frac{2\pi i}{\kappa} e^{2i\lambda\left[\tan^{-1}\left(\frac{2k_s}{q_3}\right) - \tan^{-1}\left(\frac{2p_s}{q_3}\right)\right]}, \\ C(p,q,k) = D(p,q,k) &= \frac{2A(p,q,k)}{(k-p)_-}. \end{aligned} \tag{3.5}$$

Note that the vertex ν_4 was computed in a special momentum configuration, namely

$$q_+ = q_- = 0, \tag{3.6}$$

while the momenta p and k are arbitrary.[4] For this reason, our computation of correlation functions will also be restricted configuration in which the momentum of J_0 operators are restricted to lie only in the 3-direction. Diagrammatically, the exact four point vertex will be represented as in figure 1.

3.1 Constraints on correlation functions from supersymmetry

To begin with, let us study the constraints on an arbitrary correlation function due to supersymmetry. As stated earlier, although our theory has $\mathcal{N}=2$ supersymmetry, we will be working in $\mathcal{N}=1$ superspace following [27]. A general n-point correlation function of $\mathcal{N}=1$ scalar superfield is constrained by supersymmetry and translation invariance to take the following form [27]

$$\begin{aligned} &\langle \mathcal{O}_1(\theta_1,p_1)\ldots\mathcal{O}_n(\theta_n,p_n)\rangle \\ &\quad = (2\pi)^3\delta^3\left(\sum_{i=1}^n p_i\right)\exp\left[\left(\frac{1}{n}\sum_{i=1}^n\theta_i\right).\left(\sum_{i=1}^n p_i.\theta_i\right)\right]F_n(\{\theta_{\text{in}}\};\{p_i\}). \end{aligned} \tag{3.7}$$

[4]We refer the reader to appendix A for conventions for labelling momenta.

The $\delta^3(\sum_i p_i)$ follows from translation invariance while the overall Grassmann exponential factor follows from invariance under $\mathcal{N} = 1$ supersymmetry. Note that the function F_n above only depend on the differences of the Grassmann coordinates. Following [27], the form is easily derived as follows

$$\begin{aligned} 0 &= \left[\sum_{i=1}^{n} \mathcal{Q}_\alpha^{(i)}\right] \langle \mathcal{O}_1(\theta_1, p_1) \dots \mathcal{O}_n(\theta_n, p_n)\rangle \\ &= \left[\sum_{i=1}^{n} \left(\frac{\partial}{\partial \theta_i^\alpha} - (p_i)_{\alpha\beta}\theta_i^\beta\right)\right] \langle \mathcal{O}_1(\theta_1, p_1) \dots \mathcal{O}_n(\theta_n, p_n)\rangle \\ &= \left(n\frac{\partial}{\partial X^\alpha} - \sum_{i=1}^{n-1}(p_i)_{\alpha\beta}\theta_{\text{in}}^\beta\right) \langle \mathcal{O}_1(\theta_1, p_1) \dots \mathcal{O}_n(\theta_n, p_n)\rangle. \end{aligned} \tag{3.8}$$

In the last line above, we used the momentum conservation to replace p_n with $\sum_{i=1}^{n-1}(-p_i)$. The factorized form in (3.7) follows as the solution to last equation in (3.8).

3.2 J_0-vertex

Before proceeding to the computation of correlation functions, it would be useful to compute an intermediate quantity, the J_0-vertex. It is defined by stripping of the propagators from $\langle J_0\Phi\bar{\Phi}\rangle$ as follows

$$\begin{aligned} &\langle J_0(\theta_1, p_1)\Phi(\theta_2, p_2)\bar{\Phi}(\theta_3, p_3)\rangle = \\ &\int \frac{d^3p_2'}{(2\pi)^3}\frac{d^3p_3'}{(2\pi)^3} d^2\theta_2' d^2\theta_3' \ \left[\langle J_0(\theta_1, p_1)\Phi(\theta_2', p_2')\bar{\Phi}(\theta_3', p_3')\rangle_{\text{ver}} \mathcal{P}(\theta_2', \theta_2; -p_2)\mathcal{P}(\theta_3', \theta_3; p_3)\right] \end{aligned} \tag{3.9}$$

and satisfies the same Ward identity as a three point function (3.7).

The vertex receives contribution both from the free propagation of the fundamental field as well as from the interaction vertices in the theory. The free part vertex is simply proportional to the momentum and the Grassmannian δ-functions while the interacting part of the vertex can be computed from the exact ν_4 vertex. Figure 2 shows the relevant diagrams.

$$\begin{aligned} \langle J_0(\theta_1, p)\Phi(\theta_2, r)\bar{\Phi}(\theta_3, s)\rangle_{\text{ver}} = &\langle J_0(\theta_1, p)\Phi(\theta_2, r)\bar{\Phi}(\theta_3, s)\rangle_{\text{ver,free}} \\ &+ \langle J_0(\theta_1, p)\Phi(\theta_2, r)\bar{\Phi}(\theta_3, s)\rangle_{\text{ver,int}} \end{aligned}$$

where

$$\begin{aligned} \langle J_0(\theta_1, p)\Phi(\theta_2, r)\bar{\Phi}(\theta_3, s)\rangle_{\text{ver,free}} &= (2\pi)^3\delta^3(p+r+s) \ \ \nu_{3,\text{free}}(\theta_{12}, \theta_{32}; p, s) \\ &= (2\pi)^3\delta^3(p+r+s)\theta_{32}^+\theta_{32}^-\theta_{12}^+\theta_{12}^- \end{aligned}$$

and

$$\begin{aligned} &\langle J_0(\theta_1, p)\Phi(\theta_2, r)\bar{\Phi}(\theta_3, s)\rangle_{\text{ver,int}} \\ &\quad = (2\pi)^3\delta^3(p+r+s)\left[\int \frac{d^3q}{(2\pi^3)} d^2\theta_a d^2\theta_b \ \mathcal{P}(\theta_1, \theta_a; q+p)\mathcal{P}(\theta_b, \theta_1; q)\nu_4(\theta_a, \theta_b, \theta_2, \theta_3; q, p, s)\right] \\ &\quad = (2\pi)^3\delta^3(p+r+s)e^{\frac{1}{3}\theta_{123}.(p.\theta_{12}+s.\theta_{32})}\nu_{3,\text{int}}(\theta_{12}, \theta_{32}, p, s) \end{aligned} \tag{3.10}$$

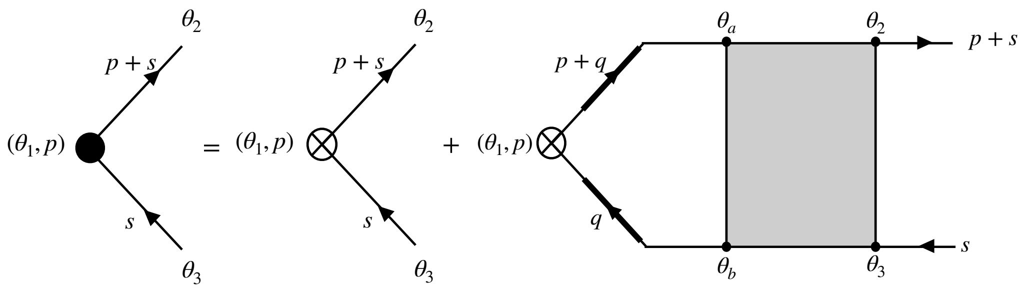

Figure 2. Solid circle on the l.h.s. represents the full exact J_0 vertex and the first diagram on r.h.s. is the free vertex. The second diagram on r.h.s. includes all the interactions which are accounted by insertion of exact 4 point vertex (3.3) connected to the external $\mathcal{J}^{(0)}$ operator using the exact propagator.

Explicit computation of the above integral, with constraint $p_+ = p_- = 0$ following from the (3.6), leads to the following result for the full J_0-vertex factor

$$\begin{aligned}\nu_3 &= (\nu_{3,\text{free}} + \nu_{3,\text{int}})(\theta_{12}, \theta_{32}, p, s)\\ &= \frac{1}{2s^+}\left[1 - e^{2i\lambda \tan^{-1}\left(\frac{2s_s}{p_3}\right)}\right]\theta^+_{32}\theta^+_{12} + \frac{1}{2p_3}\left(e^{2i\lambda\tan^{-1}\left(\frac{2s_s}{p_3}\right) - i\pi\lambda sgn(p_3)} - 1\right)\theta^+_{32}\theta^-_{32}\\ &\quad + \left(1 + \frac{1}{6}\left(-4 + e^{2i\lambda\tan^{-1}\left(\frac{2s_s}{p_3}\right)} + 3e^{2i\lambda\tan^{-1}\left(\frac{2s_s}{p_3}\right) - i\pi\lambda sgn(p_3)}\right)\right)\theta^+_{32}\theta^-_{32}\theta^+_{12}\theta^-_{12}\end{aligned} \tag{3.11}$$

The J_0-vertex computed above will be useful in further computations of 2 and 3 point functions of the J_0 operator.

3.3 $\langle J_0 J_0\rangle$ correlation function

The 2 point function can be straightforwardly computed from the J_0-vertex determined in the previous section by combining the exact vertex on one side with the free vertex on the other side. Figure 3 shows the relevant diagram which leads to the following integral for the two point function

$$\begin{aligned}&\langle J_0(\theta_1, p) J_0(\theta_2, r)\rangle\\ &\quad = (2\pi)^3\delta^3(p+r)\left[N\int \frac{d^3q}{(2\pi)^3} d^2\theta_a d^2\theta_b\ \nu_3(\theta_1, \theta_a, \theta_b; p, -p-q, q)\mathcal{P}(\theta_a, \theta_2; q+p)\mathcal{P}(\theta_2, \theta_b; q)\right]\end{aligned} \tag{3.12}$$

Again, the collinear constraint (3.6) restricts the momenta p and s to lie in 3-direction. Computing the integrals with this constraint leads to the following result

$$\langle J_0(\theta_1, p) J_0(\theta_2, r)\rangle = (2\pi)^3\delta^3(p+r) N \frac{e^{-\theta_1 . p . \theta_2}}{8|p_3|}\left(\frac{\sin(\pi\lambda)}{\pi\lambda} + |p_3|\delta^2(\theta_{12})\frac{1 - \cos(\pi\lambda)}{\pi\lambda}\right) \tag{3.13}$$

The result can be straightforwardly generalized for arbitrary external momenta to give

$$\langle J_0(\theta_1, p) J_0(\theta_2, r)\rangle = (2\pi)^3\delta^3(p+r) N \frac{e^{-\theta_1 . p . \theta_2}}{8|p|}\left(\frac{\sin(\pi\lambda)}{\pi\lambda} + |p|\delta^2(\theta_{12})\frac{1 - \cos(\pi\lambda)}{\pi\lambda}\right) \tag{3.14}$$

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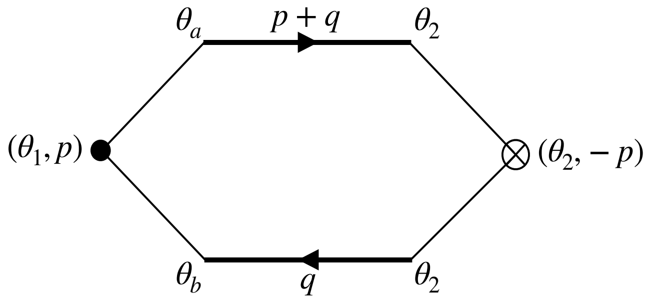

Figure 3. The full J_0 2 point function is obtained by connecting the exact J_0-vertex (solid circle) to the free vertex (cross) with exact propagators (thick line).

The non vanishing component correlators can easily be read off to give

$$\begin{aligned}
\langle J_0^b(p) J_0^b(-p)\rangle &= \frac{N}{8|p|}\frac{\sin(\pi\lambda)}{\pi\lambda} \\
\langle J_0^f(p) J_0^f(-p)\rangle &= -\frac{N|p|}{8}\frac{\sin(\pi\lambda)}{\pi\lambda} \\
\langle \Psi_\alpha(p)\Psi_\beta(-p)\rangle &= \frac{N}{8}\left(\frac{p_{\alpha\beta}}{|p|}\frac{\sin(\pi\lambda)}{\pi\lambda} + C_{\alpha\beta}\frac{1-\cos(\pi\lambda)}{\pi\lambda}\right) \\
\langle J_0^b(p) J_0^f(-p)\rangle &= -\frac{N}{8}\frac{(1-\cos(\pi\lambda))}{\pi\lambda}
\end{aligned} \tag{3.15}$$

Let us compare the above two-point functions with the corresponding two-point functions in the regular fermionic and regular bosonic theories studied in [5] and [31] respectively.

Note that as opposed to the regular bosonic and regular fermionic theories studied in [5] and [31], the λ dependence of the two-point function of J_0^b and J_0^f operators is the same as that of the higher spin currents in the non-supersymmetric cases. Further, using the double trace factorization argument of [21] relating the two-point function of current operators in the supersymmetric and the above mentioned non-supersymmetric theories, we know that the two-point function of all the current operators in our supersymmetric theory is exactly the same as those of the corresponding regular boson/fermion theory. Thus, we see that in our theory the two-point function of scalar operators is the same as that for the higher spin current operators. The reason for this is supersymmetry. Though we are working in $\mathcal{N}=1$ superspace language, our theory has underlying $\mathcal{N}=2$ supersymmetry under which the scalar operators J_0^b, J_0^f belong to the same supersymmetry multiplet as the spin 1 conserved current and thus the two-point function of the two are thus related by supersymmetry.

3.4 $\langle J_0 J_0 J_0\rangle$ correlation function

The full 3-point function can be constructed by combining three J_0 vertices with exact propagators. There are two such diagrams shown in figure 4. Each of these two diagrams can easily be shown to be cyclically symmetric and related to each other by pair-exchange of any two J_0 insertions. An explicit computation of the diagram shows that each of the

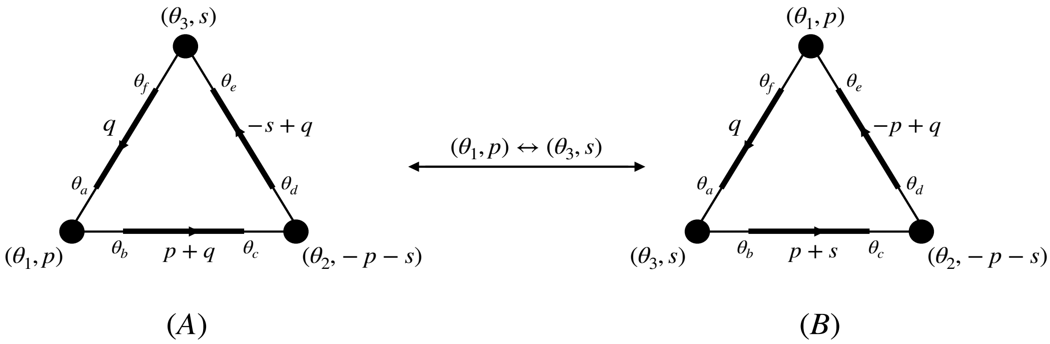

Figure 4. The full J_0 3 point function is obtained by connecting three exact J_0-vertices with exact propagators. There are two such diagrams, as shown above, which turn out to be equal.

diagrams is completely symmetric (cyclic as well as under pair-exchange) by itself and the two diagrams are equal. The full 3 point function is then just twice the contribution of the first diagram which we write down below.

$$\langle J_0(\theta_1, p) J_0(\theta_2, r) J_0(\theta_3, s)\rangle = (2\pi)^3 \delta^3(p+r+s) G_3(\theta_1, \theta_2, \theta_3; p, s), \tag{3.16}$$

where

$$\begin{aligned} G_3(\theta_1, \theta_2, \theta_3; p, s) = 2N \int \frac{d^3q}{(2\pi)^3} \left(\prod_{i=a}^{f} d^2\theta_i\right) \Big[& \nu_3(\theta_1, \theta_a, \theta_b; p, -p-q) \\ & \times \nu_3(\theta_2, \theta_c, \theta_d; -p-s, s-q)\ \nu_3(\theta_3, \theta_e, \theta_f; s, -q) \\ & \times \mathcal{P}(\theta_a, \theta_f; -q) \mathcal{P}(\theta_a, \theta_f; s-q) \mathcal{P}(\theta_c, \theta_b; -p-q) \Big] \end{aligned}$$

The overall factor of 2 in the above equation is from the sum over two triangle diagrams in figure 4 which turn out to be equal while the factor of N results from index contractions. Explicit computation of the above integrals in the collinear limit of the external momenta gives the following result

$$\begin{aligned} G_3(\theta_1, \theta_2, \theta_3; p, s) &= e^{\frac{1}{3}\theta_{123}.(p.\theta_{12}+s.\theta_{32})} F_3(\theta_{12}, \theta_{32}, p, s), \\ F_3(\theta_{12}, \theta_{32}, p, s) &= 2N \left(A_1 + A_2 \theta_{12}^+ \theta_{12}^- + A_3 \theta_{32}^+ \theta_{32}^- + A_4 \theta_{12}^+ \theta_{32}^- + A_5 \theta_{32}^+ \theta_{12}^- + A_6 \theta_{12}^+ \theta_{12}^- \theta_{32}^+ \theta_{32}^-\right) \end{aligned} \tag{3.17}$$

The overall factor of 2 in the expression of F_3 above is from the sum over two triangle diagrams which turn out to be equal. The coefficients $\{A_i\}$ are given by

$$\begin{aligned} A_1 &= \frac{\sin(2\pi\lambda)}{2\pi\lambda} \frac{1}{8|p_3||s_3||p_3+s_3|}, \\ A_2 &= -i\frac{(\sin(\pi\lambda))^2}{\pi\lambda} \frac{1}{8|s_3||p_3+s_3|}, \\ A_3 &= -i\frac{(\sin(\pi\lambda))^2}{\pi\lambda} \frac{1}{8|p_3||p_3+s_3|}, \end{aligned}$$

$$A_4 = -\frac{1}{48p_3s_3(p_3+s_3)}\Bigg[\frac{\sin(2\pi\lambda)}{2\pi\lambda}\Big(-(p_3+2s_3)\text{sgn}(p_3)+(2p_3+s_3)\text{sgn}(s_3)+(p_3-s_3)\text{sgn}(p_3+s_3)\Big)$$
$$-3i\frac{\sin^2(\pi\lambda)}{\pi\lambda}\text{sgn}(p_3+s_3)\Big(|p_3+s_3|-(|p_3|+|s_3|)\Big)\Bigg]$$
$$A_5 = \frac{1}{48p_3s_3(p_3+s_3)}\Bigg[\frac{\sin(2\pi\lambda)}{2\pi\lambda}\Big(-(p_3+2s_3)\text{sgn}(p_3)+(2p_3+s_3)\text{sgn}(s_3)+(p_3-s_3)\text{sgn}(p_3+s_3)\Big)$$
$$+3i\frac{\sin^2(\pi\lambda)}{\pi\lambda}\text{sgn}(p_3+s_3)\Big(|p_3+s_3|-(|p_3|+|s_3|)\Big)\Bigg]$$
$$A_6 = -\frac{\sin(2\pi\lambda)}{2\pi\lambda}\Bigg[\frac{1}{72p_3s_3(p_3+s_3)}\Big((p_3-s_3)(2p_3+s_3)\text{sgn}(p_3)-(p_3+2s_3)(p_3-s_3)\text{sgn}(s_3)$$
$$-(p_3+2s_3)(2p_3+s_3)\text{sgn}(p_3+s_3)\Big)\Bigg] \tag{3.18}$$

The non vanishing components of the three point functions can easily be extracted from (3.17) and (3.18) to be

$$\langle J_0^b(p_3)J_0^b(s_3)J_0^b(-p_3-s_3)\rangle = \frac{\sin(2\pi\lambda)}{2\pi\lambda}\frac{N}{8|p_3s_3(p_3+s_3)|}$$
$$\langle J_0^f(p_3)J_0^f(s_3)J_0^f(-p_3-s_3)\rangle = \frac{-iN}{8}\frac{(\sin(\pi\lambda))^2}{\pi\lambda}$$
$$\langle J_0^b(p_3)J_0^b(s_3)J_0^f(-p_3-s_3)\rangle = \frac{(\sin(\pi\lambda))^2}{\pi\lambda}\frac{(-iN)}{8|p_3s_3|}$$
$$\langle J_0^f(p_3)J_0^f(s_3)J_0^b(-p_3-s_3)\rangle = \frac{\sin(2\pi\lambda)}{2\pi\lambda}\frac{N}{16|p_3+s_3|}$$
$$\langle \Psi_+(p_3)\Psi_-(s_3)J_0^b(-p_3-s_3)\rangle = \frac{N}{16p_ss_3(p_3+s_3)}\Bigg(\frac{\sin(2\pi\lambda)}{2\pi\lambda}\big(|p_3|-|s_3|-(p_3-s_3)\text{sgn}(p_3+s_3)\big)$$
$$-i\frac{(\sin(\pi\lambda))^2}{\pi\lambda}\text{sgn}(p_3+s_3)\big(|p_3+s_3|-|p_3|+|s_3|\big)\Bigg)$$
$$\langle \Psi_+(p_3)\Psi_-(s_3)J_0^f(-p_3-s_3)\rangle = \frac{N}{16p_ss_3}\Bigg(\frac{\sin(2\pi\lambda)}{2\pi\lambda}\big(|p_3+s_3|-|p_3|-|s_3|\big)$$
$$+i\frac{(\sin(\pi\lambda))^2}{\pi\lambda}\text{sgn}(p_3+s_3)\big((p_3-s_3)|p_3+s_3|-|p_3|+|s_3|\big)\Bigg) \tag{3.19}$$

Notice that in the above result for 3 point functions, two different functional forms of λ dependences appear, namely $\frac{\sin(2\pi\lambda)}{2\pi\lambda}$ and $\frac{\sin^2\pi\lambda}{\pi\lambda}$. The two of them differ in a crucial way. The first one has a finite $\lambda \to 0$ limit and is invariant under parity under which λ is odd. The second is odd under parity and vanishes in $\lambda \to 0$ limit. This result thus provides some support for the conjecture made in [61] that the three-point functions in $\mathcal{N}=1$ superconformal theories with higher spin symmetry have exactly one parity even and one parity odd structure. The results (3.14) and (3.18) for the 2 and 3-point are clearly invariant under the duality transformation (2.3).

4 Four point functions

In the previous section, we evaluated the 3-point functions involving the $\mathcal{J}_0$ operator in the $\mathcal{N}=2$ supersymmetric theory by computing the required vertex. However, the direct

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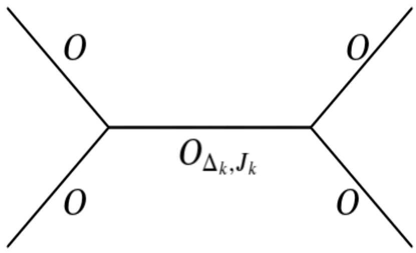

Figure 5. Schematic for the conformal block expansion.

computation of the four-point function of J_0 operator following the same technique has proven to be intractable in our attempt till now. We describe our attempt to evaluate this four-point function in momentum space through the required vertices in the appendix D.

In this section, we determine the four-point correlators of the J_0^b and J_0^f operators using a novel method developed in [55], which we briefly review below. Note that we will be evaluating the 4-point correlation function in the position space as in [55].

Consider the position space four-point correlator of the identical external operators with conformal dimensions Δ. The function $\mathcal{A}$ which is known as the reduced correlator is defined as follows

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}}\frac{1}{x_{34}^{2\Delta}}\mathcal{A}(u,v) = \frac{1}{x_{13}^{2\Delta}}\frac{1}{x_{24}^{2\Delta}}\frac{\mathcal{A}(u,v)}{u^{\Delta}}\,. \tag{4.1}$$

Here, u, v are the standard cross-ratios:

$$u = \left(\frac{|x_{12}||x_{34}|}{|x_{13}||x_{24}|}\right)^2, \quad v = \left(\frac{|x_{14}||x_{23}|}{|x_{13}||x_{24}|}\right)^2.$$

The conformal block expansion expressed in terms of the reduced correlator $\mathcal{A}(u,v)$ is given as

$$\frac{\mathcal{A}(u,v)}{u^{\Delta}} = \frac{1}{u^{\Delta}}\sum_k C^2_{\mathcal{O}\mathcal{O}O_k} G_{\Delta_k,J_k}(u,v) \tag{4.2}$$

where $G_{\Delta_k,J_k}(u,v)$ is known as the conformal block corresponding to the operator $\mathcal{O}_k$ with scaling dimension Δ_k and spin J_k (see figure 5).

In the supersymmetric four point functions of J_0 operators, the relevant exchanges are schematically shown in figure 6.

4.1 Review of the double discontinuity technique

In [55], the authors determine the four-point correlation functions of the scalar operator in the non-supersymmetric scalar/fermion coupled to Chern Simons gauge field i.e. quasi-bosonic and quasi-fermionic theory respectively. In order to obtain the required four-point functions, the authors utilize the inversion formula which relates the double discontinuity to the OPE coefficients [58]. The authors first prove an interesting theorem that in the large-N limit of a CFT_d, the double discontinuity constrains the four-point correlator up

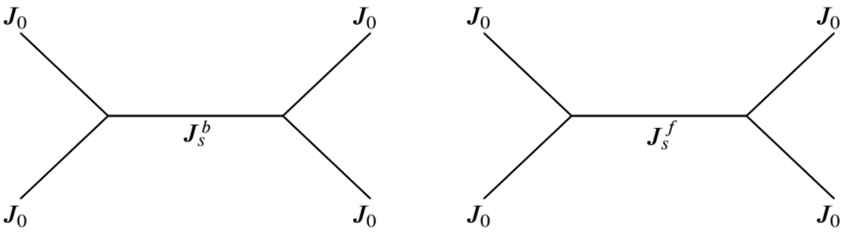

Figure 6. Schematic for the exchanges relevant in the supersymmetric scalar correlators.

to three contact terms in AdS_{d+1}. Suppose there are two solutions G_1 and G_2 to the crossing equation with the same double discontinuity then they are related by the contact interactions in the AdS as follows

$$G_1 = G_2 + c_1 G^{\rm AdS}_{\phi^4} + c_2 G^{\rm AdS}_{(\partial\phi)^4} + c_2 G^{\rm AdS}_{\phi^2(\partial^3\phi)^2} \tag{4.3}$$

Furthermore, the authors showed[5] that for the four-point function of single trace scalar operator in Chern-Simons coupled fundamental scalar/fermion theories these AdS_4 contact terms do not contribute and hence the double discontinuity completely determines the four-point functions.

Consider the normalized three point functions of the operators $\mathcal{O}_i (i = 1, 2, 3)$.[6] In [3, 4, 55], it was noticed that the square of this normalized coefficients in the quasi-fermionic theories ($C^2_{s,qf}$) are related to that of a single free Majorana fermion ($C^2_{s,ff}$) as follows

$$C^2_{s,qf} = \frac{1}{\tilde{N}} C^2_{s,ff} \tag{4.4}$$

where $\tilde{N}$ is related to the rank of the gauge group N and coupling λ_{qf} by,

$$\tilde{N} = 2N \frac{\sin(\pi\lambda_{qf})}{\pi\lambda_{qf}}. \tag{4.5}$$

Note that the normalized coefficients of quasi-fermionic theory and free fermionic theory are proportional to each other as given in (4.4). Hence, the double discontinuity of the scalar four point function in the free fermionic theory is same as that of the quasi-fermionic theories up to an overall factor which depends only on N and λ_{qf}.

On the other hand, the square of the normalized coefficients of the quasi-bosonic theories ($C^2_{s,qb}$) are related to the theory of a free real boson ($C^2_{s,fb}$) as follows

$$C^2_{s,qb} = \frac{1}{\tilde{N}} C^2_{s,fb} \quad s > 0, \tag{4.6}$$

$$C^2_{0,qb} = \frac{1}{\tilde{N}} \frac{1}{(1+\tilde{\lambda}^2_{qb})} C^2_{0,fb} = \frac{1}{\tilde{N}} C^2_{0,fb} - \frac{1}{\tilde{N}} \frac{\tilde{\lambda}^2_{qb}}{(1+\tilde{\lambda}^2_{qb})} C^2_{0,fb}, \tag{4.7}$$

[5]via explicit numerical computation.

[6]For the conventions of normalization correlation functions please refer to appendix C.

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where $\tilde{N}$ and $\tilde{\lambda}$ are related to N and coupling λ_{qb} as

$$\tilde{N} = 2N\frac{\sin(\pi\lambda_{qb})}{\pi\lambda_{qb}}, \tag{4.8}$$

$$\tilde{\lambda}_{qb} = \tan\left(\frac{\pi\lambda_{qb}}{2}\right). \tag{4.9}$$

Note that unlike the normalized coefficients of the quasi-fermionic theories, in the quasi-bosonic theories, the spin $s = 0$ and $s \neq 0$ coefficients given above have different factors in front of their free bosonic counterparts. In order to account for the second term on the r.h.s. of (4.7) one needs to add a conformal partial wave with spin-0 exchange which is given by the well known $\bar{D}$-function with the correct pre-factor [55]. We now proceed to employ this technique for the supersymmetric case.

4.2 Double discontinuity and the supersymmetric correlators

Here, we utilize the technique described above to compute the four-point correlators for spin-0 operators J_0^b and J_0^f in our supersymmetric theory. Since we are considering correlators of identical external operators,[7] only even spin operators will contribute to the block expansion.

4.2.1 $\langle J_0^b(x_1)J_0^b(x_2)J_0^b(x_3)J_0^b(x_4)\rangle$

The four point function of the J_0^b operators is expressed as follows[8]

$$\langle J_0^b(x_1)J_0^b(x_2)J_0^b(x_3)J_0^b(x_4)\rangle = \text{disc} + \frac{1}{x_{13}^2 x_{24}^2}F(u,v). \tag{4.10}$$

Here, disc corresponds to the disconnected part given by

$$\text{disc} = \frac{1}{x_{12}^2 x_{34}^2} + \frac{1}{x_{13}^2 x_{24}^2} + \frac{1}{x_{14}^2 x_{23}^2} \tag{4.11}$$

while $F(u,v)$ is given by

$$F(u,v) = \frac{1}{u}\sum_k C^2_{\mathcal{O}\mathcal{O}O_k} G_{\Delta_k,J_k}(u,v) \tag{4.12}$$

In order to determine the double discontinuity and hence the 4-point functions in the supersymmetric case using the method described above, we need the normalized 3-point function coefficients for the operators running in OPE of two J_0^b operators. For the case of spin 0 operators, i.e. J_0^b, J_0^f, these normalized coefficients can directly be obtained from our explicit computations for the 2 and 3-point functions in (3.15), (3.19). For the contribution of higher spin operators (J_s^b, J_s^f), these coefficients can be computed by relating to them

[7] Although we have all the three-point correlators required, we do not compute mixed correlators such as $\langle J_0^b J_0^b J_0^f J_0^f\rangle$ here, currently a free theory analogue for such correlators is not clear. We reserve this issue for future investigations.

[8] Note that, it is useful to redefine operators such that the normalization is fixed to be $\langle J_0 J_0\rangle = x^{-2\Delta}$ [55]. We work with this normalization in this section.

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to the regular boson (fermion) theories using the large N *double trace factorization* (see e.g. [21]) of correlation functions. We relegate the computation of these to appendix C and only collect the final result here.

For scalar operators $J_0^{b,f}$, we have

$$\begin{aligned} C^{2(BBB)}_{0,\text{susy}} &= \frac{1}{\tilde{N}}\frac{(1-\tilde{\lambda}^2)^2}{(1+\tilde{\lambda}^2)^2}\, C^2_{0,fb}, \\ C^{2(BBF)}_{0,\text{susy}} &= \frac{8}{\pi^2}\frac{\tilde{\lambda}^2}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{0,fb}. \end{aligned}$$

For the higher spin operators, $J_s^{b,f}$ $(s \in (2,4,6,\ldots))$, we get

$$\begin{aligned} C^{2(BBB)}_{s,\text{susy}} &= \frac{1}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,fb} \qquad s>0, \\ C^{2(BBF)}_{s,\text{susy}} &= \frac{\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,fb} \qquad s>0. \end{aligned} \tag{4.13}$$

Note that we may re-express the spin 0 coefficient $C^{2(BBB)}_{0,\text{susy}}$ above as follows

$$C^{2(BBB)}_{0,\text{susy}} = \frac{1}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{0,fb} + \frac{\tilde{\lambda}^4 - 2\tilde{\lambda}^2}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{0,fb}. \tag{4.14}$$

Observe that $C^{2(BBB)}_{s,\text{susy}}$ in (4.13) and the first term of $C^{2(BBB)}_{0,\text{susy}}$ in (4.14) have the same pre-factor. This is similar to the case of the quasibosnic case given in (4.6) and (4.7) reviewed earlier. Consider, now, the double discontinuity of the conformal blocks

$$\text{dDisc}\left[G_{\Delta,J}\left(1-z,1-\bar{z}\right)\right] = \sin^2\left(\frac{\pi}{2}(\Delta - J - 2\Delta_\phi)\right) G_{\Delta,J}(1-z,1-\bar{z}) \tag{4.15}$$

where Δ_ϕ being the conformal dimension of the external operator. Notice that for $\Delta = 2\Delta_\phi + J + 2m$, the double-discontinuity vanishes. Therefore, for the double-trace exchange, the double-discontinuity vanishes. That is why the OPE of single-trace operators are sufficient to construct a function that has a double-discontinutiy equal to the four-point correlator. However, notice that the single-trace exchange J_0^{FF} with quantum numbers $(\Delta, J) = (2,0)$ also vanish. Coincidently, the double-trace operator $[J_0^b, J_0^b]_{0,0}$ also has the same quantum numbers.[9] By inspection, we can see that the function below has the right double-discontinuity

$$\begin{aligned} F\left(u,v\right) = &\frac{1+\tilde{\lambda}^4}{\tilde{N}\left(1+\tilde{\lambda}^2\right)^2} f_{fb}\left(u,v\right) \\ &- \frac{8}{\tilde{N}}\frac{2\tilde{\lambda}^2}{\pi^{5/2}\left(1+\tilde{\lambda}^2\right)^2}\left[\bar{D}_{11\frac{1}{2}\frac{1}{2}}\left(u,v\right) + \bar{D}_{11\frac{1}{2}\frac{1}{2}}\left(v,u\right) + \frac{1}{u}\bar{D}_{11\frac{1}{2}\frac{1}{2}}\left(\frac{1}{u},\frac{v}{u}\right)\right] \\ &+ c_1 G^{\text{AdS}}_{\phi^4} + c_2 G^{\text{AdS}}_{(\partial\phi)^4} + c_3 G^{\text{AdS}}_{\phi^2(\partial^3\phi)^2} \end{aligned} \tag{4.16}$$

[9] $[\mathcal{O},\mathcal{O}]_{n,l} = \mathcal{O}\Box^n \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_l}\mathcal{O}$ − traces where $\mathcal{O}$ is a single-trace operator.

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where, the function $f_{fb}(u,v)$ is the free bosonic part given by.[10]

$$f_{fb}(u,v) = 4\frac{1+u^{1/2}+v^{1/2}}{u^{1/2}v^{1/2}} \tag{4.17}$$

The contact terms are explicitly provided in (E.18). Note that c_1 contains contribution from both single-trace and double-trace operators which we have separated in the following equation as a_1 and $\tilde{c}_1$

$$\begin{aligned} F(u,v) &= \frac{1+\tilde{\lambda}^4}{\tilde{N}\left(1+\tilde{\lambda}^2\right)^2} f_{fb}(u,v) \\ &\quad - \frac{8}{\tilde{N}} \frac{2\tilde{\lambda}^2}{\pi^{5/2}\left(1+\tilde{\lambda}^2\right)^2}\left[\bar{D}_{11\frac{1}{2}\frac{1}{2}}(u,v) + \bar{D}_{11\frac{1}{2}\frac{1}{2}}(v,u) + \frac{1}{u}\bar{D}_{11\frac{1}{2}\frac{1}{2}}\left(\frac{1}{u},\frac{v}{u}\right)\right] \\ &\quad + a_1\bar{D}_{1111}(u,v) + \tilde{c}_1 G^{\text{AdS}}_{\phi^4} + c_2 G^{\text{AdS}}_{(\partial\phi)^4} + c_3 G^{\text{AdS}}_{\phi^2(\partial^3\phi)^2} \end{aligned} \tag{4.18}$$

To determine a_1 we take the OPE limit. In the OPE limit[11] the conformal blocks behaves as follow [63]

$$G_{\Delta,J}(u,v) \approx \frac{J!}{2^J(h-1)_J} u^{\Delta/2} C_J^{h-1}\left(\frac{v-1}{2\sqrt{u}}\right) \qquad \left(\text{here } h = \frac{d}{2} = \frac{3}{2}\right) \tag{4.19}$$

For $(\Delta, J) = (2,0)$ i.e. for J_0^f exchange, we have $G_{2,0}(u,v) \approx u$ in the OPE limit. Since, we are interested in the single-trace operator J_0^f, hence, we have

$$F(u,v) \approx C^{2(BBF)}_{0,\text{susy}} \tag{4.20}$$

In the OPE limit, we have for ϕ^4 contact term

$$\bar{D}_{1111}(u,v) \approx 2 \tag{4.21}$$

By only looking at the single-trace contributions we obtain

$$a_1 = \frac{C^{2(BBF)}_{0,\text{susy}}}{2} \tag{4.22}$$

Now, we focus our attention to double-trace operators. Coefficient $\tilde{c}_1$ can now be determined by looking at the double-trace trace operator $[J_0^b J_0^b]_{0,0}$. Since, $(\Delta, J) = (2,0)$ for the double-trace is same as that of the single-trace operator J_0^f, we use the same method to obtain $\tilde{c}_1$.[12]

[10] Note that we may have used two separate tree-level ϕ^3 exchange Witten diagrams corresponding to $\Delta = 1$ and $\Delta = 2$ bulk exchange with arbitrary coefficients instead [62]. But Witten diagrams themselves admitting an expansion in contact terms would compound the problem. The $\bar{D}$-functions, therefore, represents the choice with the least number of contact terms and the right double-discontinuity.

[11] OPE limit: $u \to 0,\ v \to 1,$ with $(v-1)/u^{1/2}$ fixed.

[12] The procedure above thus determines the coefficient $\tilde{c}_1$ of the first AdS_4 contact Witten diagram in term of the contribution of operators J_0^f and $[J_0^b J_0^b]_{0,0}$. We collect the formal relation below and leave the

4.2.2 $\langle J_0^f(x_1)J_0^f(x_2)J_0^f(x_3)J_0^f(x_4)\rangle$

The four point function of J_0^f is given by the following expression

$$\langle J_0^f(x_1)J_0^f(x_2)J_0^f(x_3)J_0^f(x_4)\rangle = \text{disc} + \frac{1}{x_{13}^4 x_{24}^4}\mathcal{G}(u,v) \tag{4.23}$$

where, "disc" denotes the disconnected piece given by

$$\text{disc} = \frac{1}{x_{12}^4 x_{34}^4} + \frac{1}{x_{13}^4 x_{24}^4} + \frac{1}{x_{14}^4 x_{23}^4} \tag{4.24}$$

while $F(u,v)$ is given by

$$\mathcal{G}(u,v) = \frac{1}{u}\sum_k C^2_{\mathcal{O}\mathcal{O}\mathcal{O}_k} G_{\Delta_k,J_k}(u,v) \tag{4.25}$$

We now proceed to determine the four-point function J_0^f using the same technique as above. The relevant normalized 3-point function coefficient squared are collected below[13] (see appendix C for details)

$$C^{2(FFF)}_{s,\text{susy}} = \frac{1}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,ff}, \tag{4.26}$$

$$C^{2(FFB)}_{s,\text{susy}} = \frac{\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,ff}, \tag{4.27}$$

where $C^2_{s,ff}$ is the normalized three point functions for free fermionic theory. Note that the 3-point functions of the spin-0 exchanges given by $C^{2(FFF)}_{0,\text{susy}}$ and $C^{2(FFB)}_{0,\text{susy}}$ are contact terms in this case which, therefore, may be set to zero. This implies that the above relation is trivially satisfied for the spin $s = 0$ case as the free fermionic coeffcient $C^2_{0,ff} = 0$. Hence, both the $s = 0$ and $s \neq 0$ coefficients in this case come with the same pre-factor. This implies that the function which has the correct double discontinuity is given by

$$\mathcal{G}(u,v) = \frac{1+\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2} f_{ff}(u,v) + \bar{c}_1 G^{\text{AdS}}_{\phi^4} + \bar{c}_2 G^{\text{AdS}}_{(\partial\phi)^4} + \bar{c}_3 G^{\text{AdS}}_{\phi^2(\partial^3\phi)^2}, \tag{4.28}$$

where $f_{ff}(u,v)$ is the free fermionic part given by

$$f_{ff}(u,v) = \frac{1 + u^{5/2} + v^{5/2} - u^{3/2}(1+v) - v^{3/2}(1+u) - u - v}{u^{3/2}v^{3/2}} \tag{4.29}$$

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explicit computation of these OPE coefficients for future work.

$$\tilde{c}_1 = \frac{1}{2}\left([C^{2(BBB)}_{0,\text{susy}}]_{(J_0^b)^2} - \frac{1}{\tilde{N}} \frac{4\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2\pi^2} C^2_{0,fb} - \frac{1+\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2} [C^2_{0,fb}]_{(J_0^b)^2} \right)$$

Note that computationally $[\tilde{C}^{2(BBB)}_{0,\text{susy}}]_{(J_0^b)^2} = \frac{\langle J_0^b J_0^b (J_0^b)^2\rangle}{\langle J_0^b J_0^b\rangle \sqrt{\langle (J_0^b)^2 (J_0^b)^2\rangle}}$, the OPE coefficient involving double trace operator $(J_0^b)^2$ is as difficult as computing 4-point function. However, it may be of use to write contact term coefficients in terms of these ope coefficient.

[13] Note that $C^{FFF}_{s,\text{susy}} = \frac{\langle J_0^f J_0^f J_s^f\rangle}{\langle J_0^f J_0^f\rangle\sqrt{\langle J_s^f J_s^f\rangle}}$ and $C^{FFB}_{s,\text{susy}} = \frac{\langle J_0^f J_0^f J_s^b\rangle}{\langle J_0^f J_0^f\rangle\sqrt{\langle J_s^b J_s^b\rangle}}$.

The coefficients $\bar{c}_i$ can be related to OPE coefficients involving double trace operator as discussed in the previous section. We leave this for future work.[14]

5 Summary and discussion

In this article, we have focused our attention on the $\mathcal{N}=2$ U(N) Chern Simons theory coupled with a single fundamental chiral multiplet in the 't Hooft large N limit and presented the computations for the exact 2 and 3-point functions for the scalar supermultiplet. The result are invariant under duality transformation (2.3) and can be seen as an independent confirmation of the duality. For the case of 4-point function, though we are not able to perform the direct computation for the full scalar supermultiplet, we are able to use a combination of techniques from conformal bootstrap, factorization of 3-point functions via double trace interactions along with the self duality of our theory to determine two of the component 4 point function, namely $\langle J_0^b J_0^b J_0^b J_0^b\rangle$ and $\langle J_0^f J_0^f J_0^f J_0^f\rangle$, up to 3 undetemined coefficients. These undertermined coefficients can be fixed in terms of the OPE coefficients involving specific double trace operators. We plan to report on this in near future.

Though we have focused on the $\mathcal{N}=2$ theory in this paper, the approach used to compute the four point function can be straightforwardly applied to the one parameter deformed $\mathcal{N}=1$ theory. These differ from our $\mathcal{N}=2$ theory only via a double trace term in $\mathcal{N}=1$ superspace.[15] The 2 and 3-point functions of the two theories can thus be related via the double trace type factorization also used in this paper.

The approach used in this paper, following [55], to compute the J_0^b and J_0^f 4-point functions relies crucially on the fact that the double discontinuity of the 4-point function in the interacting theory is almost the same as that of the free theory. We could thus write down the full interacting 4-point function in term of the free 4-point function. For the case of mixed 4-point functions, e.g. $\langle J_0^b J_0^b J_0^f J_0^f\rangle$, this approach is not directly useful as a free theory analogue of such mixed correlator is not available since bosons and fermions decouple from each other the mixed 4 point correlators vanish in $\lambda \to 0$ limit. One approach that might be useful in this regard is to first study the single trace OPE coefficients in the $\mathcal{N}=1$ deformed theory (for general w) in $\lambda=0$ limit. We expect this limit to be significantly simpler then $\mathcal{N}=2$ theory and one can compute not only the exact 2, 3 point functions (see e.g. [59]) but perhaps even the exact 4-point function (we expect it to be non vanishing for $w \neq 0$) of J_0 operators in this limit since the only interaction term present is a double trace term. If this indeed turn out to be the case, one can compare the double discontinuity of mixed J_0^b, J_0^f correlators in $\mathcal{N}=2$ theory with this limit and see these are closely related in a similar way as in [55] and in this paper for the identical scalar 4-point function.

[14] The coefficient $\bar{c}_1$ can be evaluated easily and is given by

$$\bar{c}_1 = \frac{3\pi^{1/2}}{8P_1^{(2)}(0,0)}\left([\tilde{C}^{2(FFF)}_{0,\text{susy}}]_{(J_0^f)^2} - \frac{1+\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2}[\tilde{C}^2_{0,ff}]_{(J_0^f)^2}\right)$$

where $P_1^{(2)}(0,0)$ is defined in appendix E.2.

[15] $\delta S = \frac{\pi w}{\kappa}\int d^3x d^2\theta (\bar{\Phi}\Phi)^2$.

As we have noticed in this paper, the coefficients $\{c_i\}$ can be determined in term of the normalized 3-point function coefficients of specific double trace operator. An interesting property of the AdS_4 contact Witten diagrams is that their series expansions contain *Log* terms. This implies that the coefficients $\{c_i\}$ not only contribute to the OPE coefficients of double trace terms but also to their leading anomalous dimensions as well, but in a coordinated way. The absence of *Log* term in the free 4-point function along with the vanishing of these coefficients for quasi-bosonic and quasi-fermionic theories [55] thus means that these double trace operators in the leading large-N order do not receive corrections to there anomalous dimensions in these theories. Whether this is also the case in the supersymmetric theory studied in this paper, requires the computation of anomalous dimensions of these double trace operators which we leave for future investigation.

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A Notations and conventions

$$\begin{aligned}
\text{Metric}: &\quad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1)\\
\text{Gamma Matrices}: &\quad (\gamma^\mu)_\alpha^{\ \beta} = (\sigma_2, -i\sigma_1, i\sigma_3)_\alpha^{\ \beta} \quad \Rightarrow \{\gamma^\mu,\gamma^\nu\} = -2\eta^{\mu\nu} I_2\\
\text{Charge Conjugation}: &\quad C_{\alpha\beta} = -C_{\beta\alpha} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = -C^{\alpha\beta} = C^{\beta\alpha}\\
\text{Raising-Lowering}: &\quad \psi^\alpha = C^{\alpha\beta}\psi_\beta\,; \quad \psi_\alpha = -C_{\alpha\beta}\psi^\beta = \psi^\beta C_{\beta\alpha}\\
&\quad \Rightarrow \psi^+ = i\psi_-\,; \qquad \psi^- = -i\psi_+\\
\text{Vector} \leftrightarrow \text{Bi-spinor}: &\quad p_{\alpha\beta} = p_\mu(\gamma^\mu)_{\alpha\beta} = \begin{pmatrix} p_0+p_1 & p_3 \\ p_3 & p_0-p_1 \end{pmatrix} = \begin{pmatrix} p_+ & p_3 \\ p_3 & -p_- \end{pmatrix}\\
\text{Squared Grassmann variables}: &\quad \theta^2 = \frac{1}{2}\theta^\alpha\theta_\alpha,\ d^2\theta = \frac{1}{2}d\theta^\alpha d\theta_\alpha
\end{aligned}$$

$$
\begin{aligned}
\text{Superspace integrals}: \quad & \int d\theta = 0, \qquad \int d\theta \ \theta = 1 \\
& \int d^2\theta \ \theta^2 = -1, \ \int d^2\theta \ \theta^\alpha\theta^\beta = C^{\alpha\beta} \\
\text{Grassmann } \delta\text{-function}: \quad & \delta^2(\theta) = -\theta^2 \\
\text{Superfields}: \quad & \Phi = \phi + \theta\psi - \theta^2 F, \ \bar\Phi = \bar\phi + \theta\bar\psi - \theta^2\bar F \\
& \bar\Phi\Phi = \bar\phi\phi + \theta^\alpha\left(\bar\phi\psi_\alpha + \bar\psi_\alpha\phi\right) - \theta^2\left(\bar F\phi + \bar\phi F + \bar\psi\psi\right)
\end{aligned}
$$

B Component 3 point functions

In this appendix, we write down the component 3 functions abstractly in term of the functions $\{A_i\}$ appearing in form of full superspace 3 point function (3.17) determined by supersymmetric Ward identity.

$$
\begin{aligned}
\langle J_0^b(p)J_0^b(-p-s)J_0^b(s)\rangle &= 2A_1 \\
\langle J_0^f(p)J_0^f(-p-s)J_0^f(s)\rangle &= 2(A_3p_3^2 + s_3(-A_4p_3 - A_5p_3 + A_2s_3)) \\
\langle J_0^b(p)J_0^f(-p-s)J_0^b(s)\rangle &= 2(A_2 + A_3 + A_4 + A_5) \\
\langle J_0^f(p)J_0^b(-p-s)J_0^f(s)\rangle &= \frac{2}{9}(9A_6 + (p_3 - s_3)(3A_4 - 3A_5 + A_1p_3 - A_1s_3)) \\
\langle \Psi_+(p)J_0^b(-p-s)\Psi_-(s)\rangle &= -\frac{2}{3}(3A_5 + A_1(-p_3 + s_3)) \\
\langle \Psi_+(p)J_0^b(-p-s)\Psi_-(s)\rangle &= -\frac{2}{9}(-9A_6 + p_3(-3(3A_3 + A_4 + 2A_5) + 2A_1p_3) \\
&\quad + (9A_2 + 3A_4 + 6A_5 + 5A_1p_3)s_3 + 2A_1s_3^2)
\end{aligned}
\tag{B.1}
$$

C $\langle J_0J_0J_s\rangle_{\mathcal{T}_{\kappa,N}}$ via double trace factorization

In this section, we will derive the expression for normalized 3-point coefficient used in subsection 4.2 in the main text of the paper. The main idea is to use the fact the supersymmetric theory differs from the regular boson (fermion) theory only via double trace interaction term involving the scalar and spin half operators. This allows one to use large N factorisation to relate the 2 and 3-point function between the supersymmetric and regular boson (fermion) theory.[16]

[16]From the diagrammatic point of view one might wonder as to how is possible to derive any such relation since the supersymmetric theory contain more fields which can run in the internal loops of Feynman diagrams in supersymmetric theory. It is easy to see that in these Chern-Simons vector models any diagrams which has gauge boson converting into matter in the loops is suppressed in the large N 't Hooft limit of interest in this paper.

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Let us start by writing the action for our $\mathcal{N}=2$ theory in a way which makes it easier to compare it with the regular boson (fermion) theory.

$$\begin{aligned}
S_{\mathcal{T}_{\kappa,N}} &= \frac{i\kappa}{4\pi} S_{\rm CS}(A) + S_b(\phi, A) + S_f(\psi, A) + S_{bf}(\phi, \psi) \\
\text{where} \quad S_{\rm CS}(A) &= \int d^3x\ \epsilon_{\mu\nu\rho} {\rm Tr}(A^\mu \partial^\nu A^\rho - \frac{2i}{3} A^\mu A^\nu A^\rho) \\
S_b(\phi, A) &= \mathcal{D}_\mu \bar{\phi} \mathcal{D}^\mu \phi\,, \quad S_f(\psi, A) = -i\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi, \\
S_{bf}(\phi, \psi) &= \int d^3x \left(\frac{4\pi^2}{\kappa^2} (\bar{\phi}\phi)^3 - \frac{4\pi}{\kappa} (\bar{\phi}\phi)(\bar{\psi}\psi) - \frac{2\pi}{\kappa} (\bar{\psi}\phi)(\bar{\phi}\psi) \right).
\end{aligned} \tag{C.1}$$

Similarly, the action for regular boson (fermion) theory in term of these building blocks can be written as follows

$$\begin{aligned}
S_{\mathcal{B}_{\kappa,N}} &= \frac{i\kappa}{4\pi} S_{\rm CS}(A) + S_b(\phi, A) + \frac{\lambda_6}{3!N^2} (\bar{\phi}\phi)^3 \\
S_{\mathcal{F}_{\kappa,N}} &= \frac{i\kappa}{4\pi} S_{\rm CS}(A) + S_f(\phi, A).
\end{aligned} \tag{C.2}$$

Note that the regular boson theory above has an extra parameter, λ_6. To leading order in the 't Hooft large N limit, of interest in this paper, λ_6 is exactly marginal while it develops a non-trivial beta function at subleading orders. The question of beta function and fixed points structure for this deformations have been studied in details in [24, 42, 59]. The particular value of λ_6 for the regular bosonic theory that will be relevant for us in this paper is the one in supersymmetric theory, namely

$$\lambda_6 = 24\pi^2 \lambda^2. \tag{C.3}$$

Henceforth, in this paper 'regular boson theory' should be understood as with this values of λ_6 coupling.

For notational convenience, we will use the subscripts $\mathcal{T}_{\kappa,N}$, $\mathcal{B}_{\kappa,N}$ and $\mathcal{F}_{\kappa,N}$ to refer to quantities computed in the supersymmetric, regular boson (with (C.3)) and regular fermion theory respectively. For later use, let us further define

$$S_{(\mathcal{BF})_{\kappa,N}} = \frac{i\kappa}{4\pi} S_{\rm CS}(A) + S_b(\phi, A) + S_f(\psi, A) + \frac{4\pi^2}{\kappa^2} \int (\bar{\phi}\phi)^3\,. \tag{C.4}$$

As discussed in section 2, our supersymmetric theory consists of a pair of approximately conserved single trace higher spin operators at each value of half integer spin. At any integers values 's' of the spin, the two currents can be taken to be the ones existing in theories $\mathcal{B}_{\kappa,N}$ and $\mathcal{F}_{\kappa,N}$. We will refer to these current operators as J_s^b and J_s^f respectively. The explicit expressions for these currents for low value of spins can be found in [1, 5, 31].

Let us first consider $\langle J_s^b J_0^b J_0^b\rangle_{\mathcal{T}_{\kappa,N}}$. Taylor expanding the double trace interaction terms in the action, the path integral expression for the correlator can be written as follows

$$\begin{aligned}
&\langle J_s^b J_0^b J_0^b\rangle_{\mathcal{T}_{\kappa,N}} \\
&= \int [D\Phi]\ e^{-S_{\mathcal{BF}}} \left(J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\ e^{\int d^3q\left(\frac{4\pi}{\kappa}J_0^b(q)J_0^f(-q)+\frac{2\pi}{\kappa}(\bar\psi\phi)(q)(\bar\phi\psi)(-q)\right)}\right) \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}\left\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\left[\int d^3q\left(\frac{4\pi}{\kappa}J_0^b(q)J_0^f(-q)+\frac{2\pi}{\kappa}(\bar\psi\phi)(q)(\bar\phi\psi)(-q)\right)\right]^n\right\rangle_{(\mathcal{BF})_{\kappa,N}} \\
&\equiv \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{4\pi}{\kappa}\right)^n\left\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\prod_{i=1}^{n}\left(\int d^3q_i\ J_0^b(q_i)J_0^f(-q_i)\right)\right\rangle_{(\mathcal{BF})_{\kappa,N}}
\end{aligned} \tag{C.5}$$

In the third line above we dropped the fermion double trace terms $((\bar\psi\phi)(\phi\bar\psi))$ since they do not contribute to the leading order result. The leading $\mathcal{O}(N)$ contribution from the last line of (C.5) can be computed using large N factorization as we outline now. Let's look at the general n-th term in the sum

$$\left\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\prod_{i=1}^{n}\left(\int d^3q_iJ_0^b(q_i)J_0^f(-q_i)\right)\right\rangle_{(\mathcal{BF})_{\kappa,N}} \tag{C.6}$$

The leading $\mathcal{O}(N)$ contribution from this term comes from its factorization into a product of (n+1) correlators, namely n 2-point functions and one 3-point function. Since $S_{\mathcal{BF}}$ doesn't have any explicit interaction term between fermions and bosons, this can only happen for even values of n (say $n=2m$) in the 't Hooft limit, in which case the factorized contribution (schematically, suppressing the argument momenta) looks like

$$\langle J_s^bJ_0^bJ_0^b\rangle_{(\mathcal{BF})_{\kappa,N}}\langle J_0^bJ_0^b\rangle^n_{(\mathcal{BF})_{\kappa,N}}\langle J_0^fJ_0^f\rangle^n_{(\mathcal{BF})_{\kappa,N}}$$

More precisely, there are three different type of such factorized contribution which are represented in figure 7. The contribution from each of these type of factorization channels is exactly the same.[17] Carefully counting the numerical factor for each and summing up gives the total contribution to be

$$\begin{aligned}
&\left\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\prod_{i=1}^{n}\left(\int d^3q_iJ_0^b(q_i)J_0^f(-q_i)\right)\right\rangle_{(\mathcal{BF})_{\kappa,N}} \\
&\qquad = (n+1)\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\rangle_{(\mathcal{BF})_{\kappa,N}}\left(\langle J_0^bJ_0^b\rangle_{(\mathcal{BF})_{\kappa,N}}\langle J_0^fJ_0^f\rangle_{(\mathcal{BF})_{\kappa,N}}\right)^n
\end{aligned} \tag{C.7}$$

Now we further notice that the absence of explicit interaction terms between bosons and fermions[18] in the action $S_{\mathcal{BF}}$ implies the following relations in the large N limit

$$\begin{aligned}
\langle J_0^b(q)J_0^b(-q)\rangle_{(\mathcal{BF})_{\kappa,N}} &= \langle J_0^b(q)J_0^b(-q)\rangle_{\mathcal{B}_{\kappa,N}} \\
\langle J_0^f(q)J_0^f(-q)\rangle_{(\mathcal{BF})_{\kappa,N}} &= \langle J_0^f(q)J_0^f(-q)\rangle_{\mathcal{F}_{\kappa,N}} \\
\langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\rangle_{(\mathcal{BF})_{\kappa,N}} &= \langle J_s^b(p_1)J_0^b(p_2)J_0^b(p_3)\rangle_{\mathcal{B}_{\kappa,N}}
\end{aligned} \tag{C.8}$$

[17]This is because of the fact that the product $\langle J_0^b(q)J_0^b(-q)\rangle\langle J_0^f(q)J_0^f(-q)\rangle$ is independent of the momenta q.

[18]I.e. $\bar\phi\phi\bar\psi\psi$ and $\bar\phi\psi\phi\bar\psi$ terms.

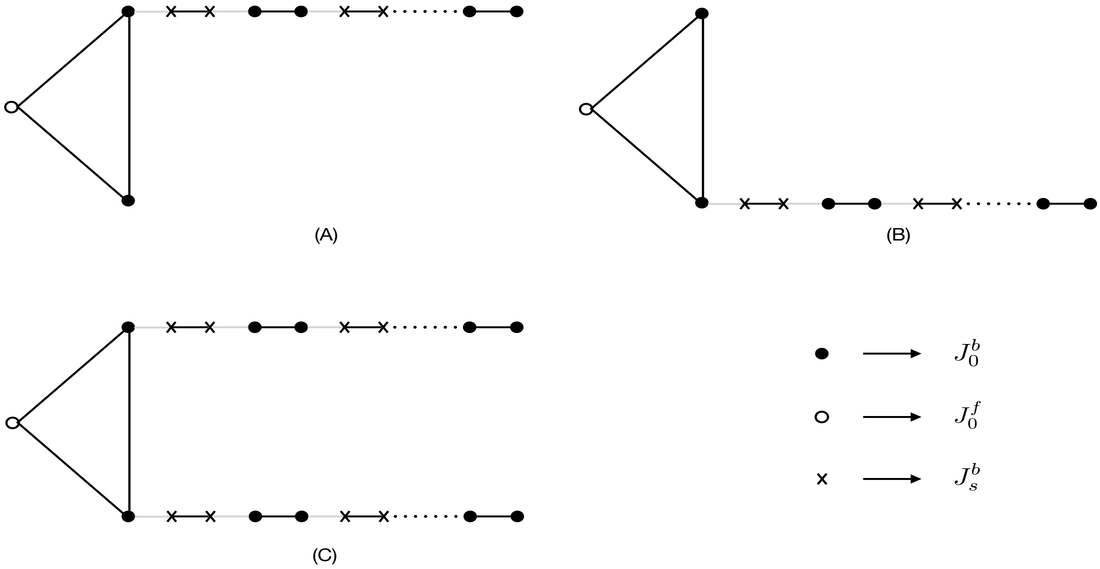

Figure 7. Schematic representation of 3 type of diagrams contributing to the factorization via the double trace term $J_0^b J_0^f$ in the action. The dots (crosses) connected with solid lines are factorized correlation functions while the grey line connecting a dot with a cross means the corresponding operators have same momenta.

Combining (C.5), (C.7) and (C.8) and summing the series over n, we arrive at the following expression for the supersymmetric correlator

$$\langle J_s^b J_0^b J_0^b \rangle_{\mathcal{T}_{\kappa,N}} = \langle J_s^b J_0^b J_0^b \rangle_{\mathcal{B}_{\kappa,N}} \left[\sum_{n=0}^{\infty} \left(\left(\frac{4\pi}{\kappa} \right)^2 \langle J_0^b J_0^b \rangle_{\mathcal{B}} \langle J_0^f J_0^f \rangle_{\mathcal{F}_{\kappa,N}} \right)^n \right]^2 \tag{C.9}$$

Further using the relation [21][19]

$$\langle J_0^b J_0^f \rangle_{\mathcal{T}_{\kappa,N}} = \frac{\kappa}{4\pi} \sum_{n=1}^{\infty} \left(\left(\frac{4\pi}{\kappa} \right)^2 \langle J_0^b J_0^b \rangle_{\mathcal{B}_{\kappa,N}} \langle J_0^f J_0^f \rangle_{\mathcal{F}_{\kappa,N}} \right)^n , \tag{C.10}$$

we can write (C.9) as

$$\langle J_s^b(p_1) J_0^b(p_2) J_0^b(p_3) \rangle_{\mathcal{T}_{\kappa,N}} = \langle J_s^b(p_1) J_0^b(p_2) J_0^b(p_3) \rangle_{\mathcal{B}_{\kappa,N}} \left[1 + \frac{4\pi}{\kappa} \langle J_0^b J_0^f \rangle_{\mathcal{T}_{\kappa,N}} \right]^2 . \tag{C.11}$$

Following exactly the same procedure, one can also derive the following relation[20]

$$\langle J_s^f(p_1) J_0^f(p_2) J_0^f(p_3) \rangle_{\mathcal{T}_{\kappa,N}} = \langle J_s^f(p_1) J_0^f(p_2) J_0^f(p_3) \rangle_{\mathcal{F}_{\kappa,N}} \left[1 + \frac{4\pi}{\kappa} \langle J_0^b J_0^f \rangle_{\mathcal{T}_{\kappa,N}} \right]^2 . \tag{C.12}$$

The correlators $\langle J_s^f(p_1) J_0^f(p_2) J_0^f(p_3) \rangle_{\mathcal{F}_{\kappa,N}}$ and $\langle J_s^b(p_1) J_0^b(p_2) J_0^b(p_3) \rangle_{\mathcal{B}_{\kappa,N}}$ are known from [4] where the authors determined the all 3-point correlators of single trace operators in *quasi bosonic* and *quasi fermionic* theories in term of two abstract parameters $\tilde{\lambda}$

[19]This can also be derived in a very similar fashion using the large N factorization via double trace $(J_0^b J_0^f)$ interaction term in the SUSY lagrangian.

[20]We have difference in signs compared to [21] due to spinor convention difference.

and $\tilde{N}$ using the constraints of weakly broken higher spin symmetry in these theories. The result for the 2-point and 3-point functions relevant to our analysis are as follows

$$\begin{aligned}
\langle \tilde{J}_0(x_1)\tilde{J}_0(x_2)\rangle_{\mathcal{B}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_0(x_1)\tilde{J}_0(x_2)\rangle_{\text{bos}} \\
\langle \tilde{J}_0(x_1)\tilde{J}_0(x_2)\rangle_{\mathcal{F}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_0(x_1)\tilde{J}_0(x_2)\rangle_{\text{fer}} \\
\langle \tilde{J}_s(x_1)\tilde{J}_s(x_2)\rangle_{\mathcal{B}_{\kappa,N}} &= \langle \tilde{J}_s(x_1)\tilde{J}_s(x_2)\rangle_{\mathcal{F}_{\kappa,N}} = \tilde{N}\langle \tilde{J}_s(x_1)\tilde{J}_s(x_2)\rangle_{\text{bos}} \\
\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\mathcal{B}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\text{bos}} \\
\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\mathcal{F}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\text{fer}}
\end{aligned} \tag{C.13}$$

Here the subscript *bos* (fer) refers to the quantity computed in theory of a free single real boson (Majorana fermion) respectively. Further, in above relation we denote the operators with a *tilde* on top to emphasize that the normalization used in [4] is in general different from the usual normalization used for these operators in Chern Simons vector models.

The exact relation between these operators normalizations and the abstract parameters $(\tilde{\lambda}, \tilde{N})$ to the parameter (λ, N) of the regular boson theory $(\mathcal{B}_{\kappa,N})$ were obtained in [5] while the equivalent relations for the regular fermion theory $(\mathcal{F}_{\kappa,N})$ were obtained in [31] via explicit computation of 3 point function for some of the low spin operators. These relations are as follows

$$\begin{aligned}
\mathcal{B}_{\kappa,N}: &\qquad (\tilde{J}_0,\tilde{J}_s) = \left(\frac{J_0^b}{1+\tilde{\lambda}^2}, J_s^b\right) \\
\mathcal{F}_{\kappa,N}: &\qquad (\tilde{J}_0,\tilde{J}_s) = \left(\frac{J_0^f}{1+\tilde{\lambda}^2}, J_s^f\right) \\
&\text{where} \quad (\tilde{\lambda},\tilde{N}) = \left(\tan\left(\frac{\pi\lambda}{2}\right), 2N\frac{\sin(\pi\lambda)}{\pi\lambda}\right)
\end{aligned} \tag{C.14}$$

Combining (C.11), (C.12), (C.13) and (C.14), we get the following expression for our desired 3-point function in supersymmetric theory $\mathcal{T}_{\kappa,N}$

$$\begin{aligned}
\langle J_s^b(x_1)J_0^b(x_2)J_0^b(x_3)\rangle_{\mathcal{T}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\text{bos}} \\
\langle J_s^f(x_1)J_0^f(x_2)J_0^f(x_3)\rangle_{\mathcal{T}_{\kappa,N}} &= \frac{\tilde{N}}{1+\tilde{\lambda}^2}\langle \tilde{J}_s(x_1)\tilde{J}_0(x_2)\tilde{J}_0(x_3)\rangle_{\text{fer}}
\end{aligned} \tag{C.15}$$

with $\tilde{\lambda}$ and $\tilde{N}$ as in (C.14).

Now that we have all the requisite 2 and 3 point functions, we can compute the normalization independent squared 3-point function coefficients to be

$$\begin{aligned}
C_{0,\text{susy}}^{2(BBB)} &= \frac{(1-\tilde{\lambda}^2)^2}{\tilde{N}(1+\tilde{\lambda}^2)^2}C_{0,fb}^2 \\
C_{s,\text{susy}}^{2(BBB)} &= \frac{1}{\tilde{N}(1+\tilde{\lambda}^2)^2}C_{s,fb}^2 \qquad s = 2,4,6\ldots
\end{aligned} \tag{C.16}$$

where $C^2_{s,fb}$ ($C^2_{s,ff}$) denote the corresponding coefficients in a free real scalar (majorana fermion) theory. The normalized coefficients above and in the rest of the paper can formally be defined as follows. Conformal invariance uniquely fixes the position dependence of all the 2 point functions and the relevant 3 point functions we are interested in, namely of the type $\langle J_0(x_1)J_0(x_2)J^{(s)}(x_3)\rangle$. Lets define the normalization N_* and 3 point function coefficient C_{***} as our operators to be

$$\begin{aligned}\langle J^{(s)}(x_1,\lambda_1)J^{(s)}(x_2,\lambda_2)\rangle &= N_s^2\frac{P_3^{2s}}{|x_{12}|^2},\\ \langle J^{(s)}(x_1,\lambda_1)J_0(x_2)J_0(x_3)\rangle &= \tilde{C}_{s00}\frac{Q_1^s}{|x_{12}||x_23|^{2\Delta_0-1}|x_{31}|},\\ \text{where}\quad P_3 = \frac{\lambda_1 X_{12}\lambda_2^{2s}}{|x_{12}|^2}\,,\quad Q_1 &= \frac{\lambda_1 X_{12}X_{23}X_{31}\lambda_1}{x_{12}^2x_{31}^2}\ \text{with}\ X = x_i\sigma^i.\end{aligned} \tag{C.17}$$

We refer the reader to [64] for further details of the conformally invariant structures involved in 2 and 3 point functions. The relevant normalized 3-point function coefficient squares we are interested in are then defined as

$$C^2_{ijk} = \frac{\tilde{C}^2_{ijk}}{N_i^2N_j^2N_k^2} \tag{C.18}$$

where the i, j, k are just labels for the operators involved.

The mixed correlators $\langle J_s^f J_0^b J_0^b\rangle$ and $\langle J_s^b J_0^f J_0^f\rangle$ of our theory cannot directly be related to correlators of $\mathcal{B}_{\kappa,N}$ or $\mathcal{F}_{\kappa,N}$ theories via double trace type factorization used above. We will instead use the self duality of our theory to determine these correlators. Under the self duality transformation (2.3) the operators in our theory map in the following way [21]

$$J_0^b \leftrightarrow J_0^b\,,\quad J_0^f \leftrightarrow J_0^f\,,\quad J_s^b \leftrightarrow (-1)^s J_s^f. \tag{C.19}$$

Thus, we have following relations for the mixed 3-point functions

$$\begin{aligned}\langle J_0^bJ_0^bJ_s^f\rangle_{\mathcal{T}_{\kappa,N}} &= (-1)^s\langle J_s^bJ_0^bJ_0^b\rangle_{\mathcal{T}_{-\kappa,|\kappa|-N}}\\ \langle J_0^fJ_0^fJ_s^b\rangle_{\mathcal{T}_{\kappa,N}} &= (-1)^s\langle J_s^fJ_0^fJ_0^f\rangle_{\mathcal{T}_{-\kappa,|\kappa|-N}}\end{aligned} \tag{C.20}$$

The 2-point functions are, of course, invariant under the duality while the parameters $\tilde{\lambda}$ and $\tilde{N}$ transform as follows

$$\tilde{N} \to \tilde{N}\,,\quad \tilde{\lambda} \to \tilde{\lambda}^{-1}. \tag{C.21}$$

Using (C.20), the result of our explicit computation (3.19) for the mixed 3-point function $\langle J_0^bJ_0^bJ_0^f\rangle$ and the duality transformation (C.21), we can determine the other 3 point function coefficients, $C^{2(BBF)}_{0,\text{susy}}$ and $C^{2(BBF)}_{s,\text{susy}}$ to be

$$\begin{aligned}C^{2(BBF)}_{0,\text{susy}} &= \frac{2}{\pi^2}\frac{(2\tilde{\lambda})^2}{\tilde{N}(1+\tilde{\lambda}^2)^2}C^2_{fb}\\ C^{2(BBF)}_{s,\text{susy}} &= \frac{\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2}C^2_{s,fb}\end{aligned} \tag{C.22}$$

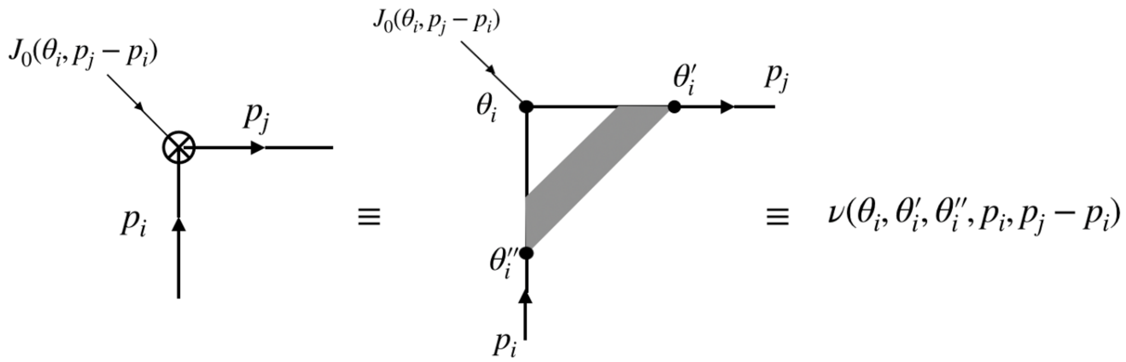

Figure 8. Convention for the definition of each vertex in the 4-point function. The 'internal' Grassmann variables, θ_i', θ_i'' that are explicitly shown here are suppressed in figure 9 to avoid clutter. These internal variables are integrated over in the computation of the correlation functions. The convention of various momenta entering or leaving the vertex is also demonstrated here.

Note that since our result for the 2 and 3-point function (3.15) and (3.19) are obtained in the momentum space, in order to compare $C^{2(BBF)}_{0,\text{susy}}$ with C^2_{fb} (as we have done in the first line of (C.22)) we need to read out the 3-point function coefficient in position space by taking the appropriate Fourier transform of our result to go to the position space expression. This can be implemented in a straightforward manner, e.g. using the Fourier transform result in [65]. This leads to the extra factor of $(2/\pi^2)$ in the first line of (C.22).

Using the method described above the relevant normalized 3-point function coefficients required for the J_0^f 4-point function can also be computed. We simply quote the results below

$$\begin{aligned} C^{2(FFF)}_{s,\text{susy}} &= \frac{1}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,ff} \\ C^{2(FFB)}_{s,\text{susy}} &= \frac{\tilde{\lambda}^4}{\tilde{N}(1+\tilde{\lambda}^2)^2} C^2_{s,ff} \end{aligned} \tag{C.23}$$

We do not write down the coefficients $C^{2(FFF)}_{0,\text{susy}}$ and $C^{2(FFB)}_{0,\text{susy}}$ since the corresponding 3-point functions are contact terms.

D Comments on direct computation of $J^{(0)}$ 4 point function

In this appendix, we describe the relevant diagrams, and corresponding integrals, constructed using the exact 4 point vertex which contribute to the full $J^{(0)}$ four point function. Figure 8 shows the exact 4-point vertex used to construct all the relevant diagrams in figure 9.

For diagrams in figure 9, note that the exact vertex (3.10) is a function of two internal grassmann variables (θ_i', θ_i'' as depicted in figure 8). The internal propagators in figure 9 that emanate from/to the exact vertices connect these internal Grassmann variables, which are integrated over in the computation of the relevant diagrams. In figure 9 the value of

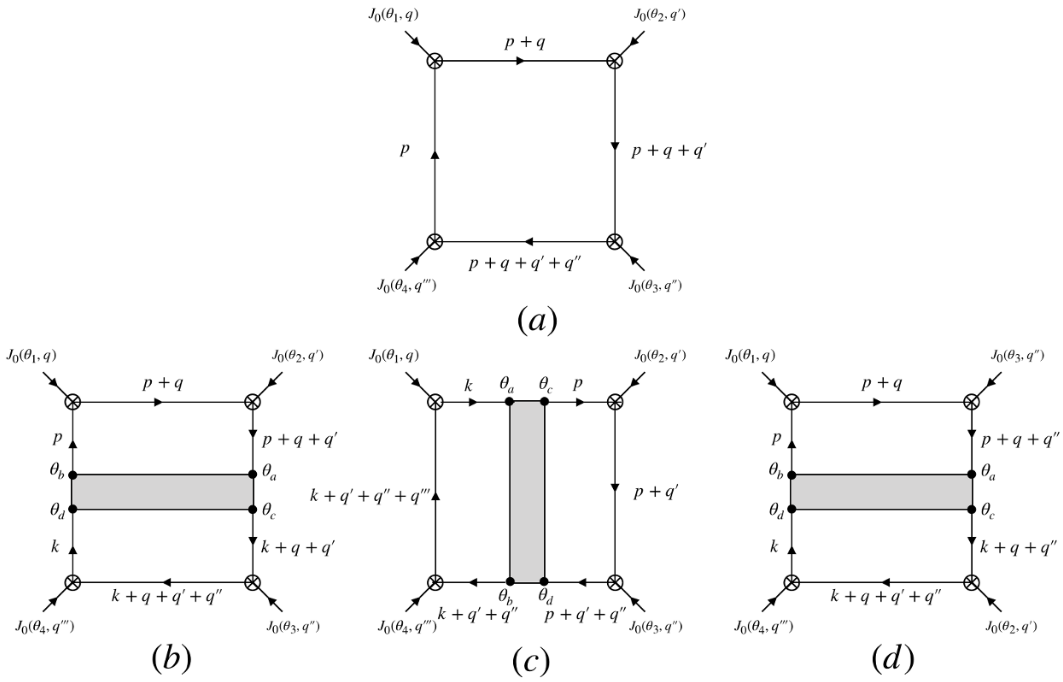

Figure 9. The contributing diagrams for the four point function of currents. The first diagram is diagram type (a). The grey blob in (b), (c), (d) represents the all loop four point correlator. The remaining diagrams are obtained by permutations of the external operators.

diagram (a) is given by

$$
\begin{aligned}
&V^{(A)}(q,q',q'',\theta_1,\theta_2,\theta_3,\theta_4) \\
&= N\int \frac{d^3p}{(2\pi)^3} d^2\theta'_1 d^2\theta''_1 d^2\theta'_2 d^2\theta''_1 d^2\theta'_3 d^2\theta''_3 d^2\theta'_4 d^2\theta''_4 \\
&\Big(P(\theta'_1,\theta''_4,p+q)P(\theta'_4,\theta''_3,p-q'-q'')P(\theta'_3,\theta''_2,p-q')P(\theta'_2,\theta''_1,p) \\
&\quad \mathcal{V}_3(\theta_1,\theta'_1,\theta''_1,q,p)\mathcal{V}_3(\theta_2,\theta'_2,\theta''_2,q',p-q')\mathcal{V}_3(\theta_3,\theta'_3,\theta''_3,q'',p-q'-q'') \\
&\quad \mathcal{V}_3(\theta_4,\theta'_4,\theta''_4,-q-q'-q'',p+q)\Big)
\end{aligned} \tag{D.1}
$$

There are a total of 6 additional diagrams due to permutations of the operators. and the interaction part is given by

$$
\begin{aligned}
&V_4^{(B)}(q,q',q'',\theta_1,\theta_2,\theta_3,\theta_4) \\
&= N^2\int \frac{d^3p}{(2\pi)^3}\frac{d^3k}{(2\pi)^3} d^2\theta_a d^2\theta_b d^2\theta_c d^2\theta_d d^2\theta'_1 d^2\theta''_1 d^2\theta'_2 d^2\theta''_1 d^2\theta'_3 d^2\theta''_3 d^2\theta'_4 d^2\theta''_4 \\
&\Big(P(\theta'_1,\theta''_4,p+q)P(\theta'_4,\theta_a,p-q'-q'')P(\theta_c,\theta''_3,k-q'-q'')P(\theta'_3,\theta''_2,k-q')P(\theta'_2,\theta_d,k)P(\theta_b,\theta''_1,p) \\
&\quad \mathcal{V}_3(\theta_1,\theta'_1,\theta''_1,q,p)\mathcal{V}_3(\theta_2,\theta'_2,\theta''_2,q',k-q')\mathcal{V}_3(\theta_3,\theta'_3,\theta''_3,q'',k-q'-q'') \\
&\quad \mathcal{V}_3(\theta_4,\theta'_4,\theta''_4,-q-q'-q'',p+q)\mathcal{V}_4(\theta_a,\theta_b,\theta_c,\theta_d,p,-q'-q'',k)\Big)
\end{aligned} \tag{D.2}
$$

The bosonic and fermionic correlators for the diagram figure 9 are given by

$$\langle J_0^b(q)J_0^b(q')J_0^b(q'')J_0^b(-q-q'-q'')\rangle = V_4^{(1)}(q,q',q'',\theta_1,\theta_2,\theta_3,\theta_4)\Big|_{\theta_1\to 0,\theta_2\to 0,\theta_3\to 0,\theta_4\to 0}$$
$$\langle J_0^f(q)J_0^f(q')J_0^f(q'')J_0^f(-q-q'-q'')\rangle = \prod_{i=1}^{4}\frac{\partial}{\partial\theta_{\alpha i}}\frac{\partial}{\partial\theta_i^{\alpha}}V_4^{(1)}(q,q',q'',\theta_1,\theta_2,\theta_3,\theta_4) \tag{D.3}$$

Although we were able to successfully perform the integrals for the components p_3,θ_p and k_3,θ_k in the expression for $V_4^{(B)}$ given by (D.2) k_s and p_s integrals out be intractable analytically. Due to this difficulty we were not able to obtain a closed form expression for the four point function of the scalar operatorsJ_0^b and J_0^f in (D.3).

E AdS contact diagrams

E.1 Closed-form

$$\begin{aligned}
\bar{D}_{1111}(z,\bar{z}) &= \frac{1}{z-\bar{z}}[\ln(z\bar{z})\ln\left(\frac{1-z}{1-\bar{z}}\right)+2\mathrm{Li}_2(z)-2\mathrm{Li}_2(\bar{z})]\\
\bar{D}_{2222}(z,\bar{z}) &= \frac{12uv}{(z-\bar{z})^5}+\frac{1+u+v}{(z-\bar{z})^3}[\ln(z\bar{z})\ln\left(\frac{1-z}{1-\bar{z}}\right)+2\mathrm{Li}_2(z)-2\mathrm{Li}_2(\bar{z})]\\
&\quad+\frac{6}{(z-\bar{z})^4}\Big((1+u-v)v\ln v+(1+v-u)u\ln u\Big)+\frac{2}{(z-\bar{z})^2}(\ln uv+1)\\
\bar{D}_{3333}(u,v) &= \left(\frac{1680u^2v^2}{(z-\bar{z})^9}+\left(\frac{240uv}{(z-\bar{z})^7}+\frac{24}{(z-\bar{z})^5}\right)(1+u+v)+\frac{4}{(z-\bar{z})^3}\right)\\
&\quad\times\left[\ln(z\bar{z})\ln\left(\frac{1-z}{1-\bar{z}}\right)+2\mathrm{Li}_2(z)-2\mathrm{Li}_2(\bar{z})\right]\\
&\quad+\left(\left(\frac{840u}{(z-\bar{z})^8}+\frac{100}{(z-\bar{z})^6}\right)v^2(1+u-v)+\frac{480uv}{(z-\bar{z})^6}+\frac{12(1+u)+76v}{(z-\bar{z})^4})\right)\ln v+u\leftrightarrow v\\
&\quad+\frac{260uv}{(z-\bar{z})^6}+\frac{26}{(z-\bar{z})^4}(1+u+v)
\end{aligned} \tag{E.1}$$

$$\begin{aligned}
\bar{D}(u,v)_{3322} &= -\partial_u\bar{D}_{2222}(u,v)\\
\bar{D}(u,v)_{4433} &= -\partial_u\bar{D}_{3333}(u,v)
\end{aligned} \tag{E.2}$$

E.2 Decomposition in terms of conformal blocks

The contact diagrams may be written as an expansion in conformal blocks [66]

$$D_{\Delta\Delta\Delta'\Delta'}(x_i) = \sum_m a_m^{\Delta\Delta}\alpha_m^{\Delta'\Delta'}\mathcal{W}_{\Delta_m,0}(x_i) + \sum_n a_n^{\Delta\Delta}\alpha_n^{\Delta'\Delta'}\mathcal{W}_{\Delta_n,0}(x_i) \tag{E.3}$$

$$D_{\Delta\Delta\Delta\Delta}(x_i) = \sum_n 2a_n^{\Delta\Delta}\left(\sum_{m\neq n}\frac{a_m^{\Delta\Delta}}{m_n^2-m_m^2}\right)\mathcal{W}_{\Delta_n,0}(x_i) + \sum_n (a_n^{\Delta\Delta})^2\frac{\partial}{\partial m_n^2}\mathcal{W}_{\Delta_n,0}(x_i) \tag{E.4}$$

where $\mathcal{W}_{\Delta,0} = \beta_{\Delta 34}\beta_{\Delta 12} W_{\Delta,0}$. For $\Delta_i = \Delta$

$$D_{\Delta\Delta\Delta\Delta}(x_i) = \sum_n 2a_n^{\Delta\Delta}\eta_n^{\Delta\Delta}\mathcal{W}_{\Delta_n,0}(x_i) + \sum_n (a_n^{\Delta\Delta})^2 \frac{\partial}{\partial m_n^2}\mathcal{W}_{\Delta_n,0}(x_i) \tag{E.5}$$

$$= \sum_n \left[(2a_n^{\Delta\Delta}\eta_n^{\Delta\Delta} + (a_n^{\Delta\Delta})^2)\beta^2_{\Delta_n\Delta\Delta} + \frac{\partial}{\partial m_n^2}\beta^2_{\Delta_n\Delta\Delta}\right] W_{\Delta_n,0}(x_i)$$
$$+ \sum_n (a_n^{\Delta\Delta})^2 \beta^2_{\Delta_n\Delta\Delta}\frac{\partial}{\partial m_n^2} W_{\Delta_n,0}(x_i) \tag{E.6}$$

with

$$\eta_n^{\Delta\Delta} = \sum_{m\neq n} \frac{a_m^{\Delta\Delta}}{m_n^2 - m_m^2} \tag{E.7}$$

$$\beta_{\Delta 34} \equiv \frac{\Gamma\left(\frac{\Delta+\Delta_{34}}{2}\right)\Gamma\left(\frac{\Delta-\Delta_{34}}{2}\right)}{2\Gamma(\Delta)} \tag{E.8}$$

$$m^2_{\Delta_k} = \Delta_k(\Delta_k - d)$$

$$a_m^{12} = \frac{(-1)^m}{\beta_{\Delta_m 12} m!}\frac{(\Delta_1)_m(\Delta_2)_m}{(\Delta_1+\Delta_2+m-d/2)_m} \tag{E.9}$$

with the anomalous dimension being proportional to the coefficient of the third term which involves derivative of the conformal block. Writing the above in terms of the $\bar{D}$ functions

$$\bar{D}_{\Delta\Delta\Delta\Delta}(u,v) = \frac{1}{u^\Delta}\left[\sum_n \left[(2a_n^{\Delta\Delta}\eta_n^{\Delta\Delta})\beta^2_{\Delta_n\Delta\Delta} + (a_n^{\Delta\Delta})^2\frac{\partial}{\partial m_n^2}\beta^2_{\Delta_n\Delta\Delta}\right] G_{\Delta_n,0}(u,v)\right.$$
$$\left. + \sum_n (a_n^{\Delta\Delta})^2\beta^2_{\Delta_n\Delta\Delta}\frac{\partial}{\partial m_n^2} G_{\Delta_n,0}(u,v)\right] \tag{E.10}$$

We will re-label

$$P_1^{(\Delta)}(n,0) = (2a_n^{\Delta\Delta}\eta_n^{\Delta\Delta} + (a_n^{\Delta\Delta})^2)\beta^2_{\Delta_n\Delta\Delta} + \frac{\partial}{\partial m_n^2}\beta^2_{\Delta_n\Delta\Delta}$$
$$P_0^{(\Delta)}(n,0)\gamma_1^{(\Delta)}(n,0) = 2(a_n^{\Delta\Delta})^2\beta^2_{\Delta_n\Delta\Delta} \tag{E.11}$$

so that

$$\bar{D}_{\Delta\Delta\Delta\Delta}(u,v) = \frac{2\Gamma(\Delta)^4}{\Gamma(2\Delta-d/2)}\frac{1}{u^\Delta}\sum_n \left[P_1^{(\Delta)}(n,0)G_{\Delta_n,0}(u,v)\right.$$
$$\left. + \frac{1}{2}P_0^{(\Delta)}(n,0)\gamma_1^{(\Delta)}(n,0)\frac{\partial}{\partial m_n^2}G_{\Delta_n,0}(u,v)\right] \tag{E.12}$$

satisfying [67]

$$P_1^{(\Delta)}(n,0) = \frac{1}{2}\partial_n\left(P_0^{(\Delta)}(n,0)\,\gamma_1^{(\Delta)}(n,0)\right) \tag{E.13}$$

Similarly, for (141)

$$\begin{aligned}\bar{D}_{\Delta+1\Delta+1\Delta\Delta}(u,v)=&\frac{2\Gamma(\Delta)^2\Gamma(\Delta+1)^2}{\Gamma(2\Delta+1-d/2)}\frac{1}{u^\Delta}\\&\left[\sum_m \bar{P}_1^{(\Delta)}(n,0)G_{\Delta_m,0}(u,v)+\frac{1}{2}\bar{P}_0^{(\Delta)}(n,0)\bar{\gamma}_1^{(\Delta)}(n,0)\frac{\partial}{\partial m_n^2}G_{\Delta_n,0}(u,v)\right]\\&+\beta_{2\Delta}^2 a_0^{\Delta\Delta}\eta_0^{\Delta+1\Delta+1}G_{2\Delta,0}(u,v)\end{aligned}\tag{E.14}$$

$$\begin{aligned}\bar{P}_1^{(\Delta)}(n,0)&=(a_n^{\Delta+1\Delta+1}\eta_n^{\Delta\Delta}+a_n^{\Delta\Delta}\eta_n^{\Delta+1\Delta+1})\beta^2_{\Delta_n\Delta\Delta}+a_n^{\Delta\Delta}a_n^{\Delta+1\Delta+1}\frac{\partial}{\partial m_n^2}\beta^2_{\Delta_n\Delta\Delta}\\\bar{P}_0^{(\Delta)}(n,0)\bar{\gamma}_1^{(\Delta)}(n,0)&=2a_n^{\Delta\Delta}a_n^{\Delta+1\Delta+1}\beta^2_{\Delta_n\Delta\Delta}\end{aligned}\tag{E.15}$$

E.2.1 Examples

$$\begin{aligned}\bar{D}_{1111}(u,v)&=\frac{2}{\pi^{1/2}u}\sum_n\left[P_1^{(1)}(n,0)G_{2+2n,0}(u,v)+\frac{1}{2}P_0^{(1)}(n,0)\gamma_1^{(1)}(n,0)\frac{\partial_n G_{2+2n,0}(u,v)}{8n+2}\right]\\\bar{D}_{2222}(u,v)&=\frac{8}{3\pi^{1/2}u^2}\sum_n\left[P_1^{(2)}(n,0)G_{4+2n,0}(u,v)+\frac{1}{2}P_0^{(2)}(n,0)\gamma_1^{(2)}(n,0)\frac{\partial_n G_{4+2n,0}(u,v)}{8n+10}\right]\\\bar{D}_{3333}(u,v)&=\frac{256}{105\pi^{1/2}u^3}\sum_n\left[P_1^{(3)}(n,0)G_{6+2n,0}(u,v)+\frac{1}{2}P_0^{(3)}(n,0)\gamma_1^{(3)}(n,0)\frac{\partial_n G_{6+2n,0}(u,v)}{8n+18}\right]\end{aligned}\tag{E.16}$$

$$\begin{aligned}\bar{D}_{3322}(u,v)=&\frac{64}{15\pi^{1/2}u^3}\left[\sum_m\bar{P}_1^{(3)}(m,0)G_{6+2m,0}(u,v)+\frac{1}{2}\bar{P}_0^{(3)}(m,0)\bar{\gamma}_1^{(3)}(m,0)\frac{\partial}{\partial m_n^2}G_{6+2m,0}(u,v)\right.\\&\left.+\beta_4^2a_0^{22}\eta_0^{33}G_{4,0}(u,v)\right]\\\bar{D}_{4433}(u,v)=&\frac{1024}{105\sqrt{\pi}u^4}\left[\sum_m\bar{P}_1^{(4)}(m,0)G_{8+2m,0}(u,v)+\frac{1}{2}\bar{P}_0^{(4)}(m,0)\bar{\gamma}_1^{(4)}(m,0)\frac{\partial}{\partial m_n^2}G_{8+2m,0}(u,v)\right.\\&\left.+\beta_6^2a_0^{33}\eta_0^{44}G_{6,0}(u,v)\right]\end{aligned}\tag{E.17}$$

Contact terms for bosonic correlator

$$\begin{aligned}G^{\text{AdS}}_{\phi^4}&=\bar{D}_{1111}(u,v)\\G^{\text{AdS}}_{(\partial\phi)^4}&=(1+u+v)\bar{D}_{2222}(u,v)\\G^{\text{AdS}}_{\phi^2(\partial^3\phi)^2}&=2(u^2\bar{D}_{3322}(u,v)+v^2\bar{D}_{3322}(v,u)+\frac{1}{v^3}\bar{D}_{3322}(1/v,u/v))\end{aligned}\tag{E.18}$$

Contact terms for fermionic correlator

$$\begin{aligned}G^{\text{AdS}}_{\phi^4}&=\bar{D}_{2222}(u,v)\\G^{\text{AdS}}_{(\partial\phi)^4}&=(1+u+v)\bar{D}_{3333}(u,v)\\G^{\text{AdS}}_{\phi^2(\partial^3\phi)^2}&=2(u^2\bar{D}_{4433}(u,v)+v^2\bar{D}_{4433}(v,u)+\frac{1}{v^3}\bar{D}_{4433}(1/v,u/v))\end{aligned}\tag{E.19}$$

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