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A possible origin of superconducting currents in cosmic strings

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Abstract. The scattering and capture of right-handed neutrinos by an Abelian cosmic string in the $SO(10)$ grand unification model are considered. The scattering cross section of neutrinos per unit length due to the interaction with the gauge and Higgs fields of the string is much larger in its scaling regime than in the friction regime because of the larger infrared cutoff of the former. The probability of capture in a zero mode of the string accompanied by the emission of a gauge or Higgs boson shows a resonant peak for the neutrino momentum of the order of its mass. Due to the decrease in the number of strings per unit of comoving volume in the scaling epoch, the cosmological consequences of the superconducting strings formed in this regime will be much smaller than those which may already be produced in the friction regime; in particular as possible sources of ultraenergetic cosmic rays.

1. Introduction

It is possible that the early universe suffered a sequence of phase transitions breaking symmetries of the grand unification theories (GUTs) and generating topological defects [1] such as the cosmic strings.

Whereas other topological defects such as monopoles are unacceptable because they would quickly give an excessive energy density to the universe, cosmic strings undergo certain dynamics that may allow them to play a relevant cosmological rôle.

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Ordinary cosmic strings are formed by the gauge field corresponding to the symmetry which is broken and the Higgs field responsible for spontaneous breaking. During the universe expansion the decrease of the number of long strings compensates the increasing contribution of each one leading to the so called scaling regime in which they represent a constant fraction of the energy of universe.

These dynamics, however, are altered for the small part of strings which may become superconducting by the incorporation of fermions and could give rise to astrophysical phenomena [2]. In particular, closed superconducting loops called vortons are classically stable [3] and might constitute a fraction of cold dark matter in the galactic halo that by a slow quantum decay is a possible origin [4] of the ultra-high energy cosmic rays (UHECRs) which are difficult to explain [5].

After their formation, the ordinary strings are in a bath of high-energy particles some of which may be the heavy fermions that if captured would become massless and give rise to the superconducting current. It is interesting to see whether the rate of this process is large enough to form vortons before the density of ordinary strings has decreased too much to allow sizeable astrophysical effects regarding cosmic rays.

In addition to the possibility of capture, the interaction with the string may produce scattering of fermions which are partially Aharonov–Bohm in type.

This effect has been described [6] for the interaction with the string gauge field of fermions of a charge which is one half that of the Higgs field responsible for the breaking of the Abelian symmetry generating the defect.

The corresponding cross section diverges in the forward direction due to the long-range nature of the interaction which gives contributions even for wavepackets passing at an infinite distance from the string. However, the unitarity of the S matrix has been proved [7] and the cross section can be taken as finite introducing a cutoff for the distance from the string.

This cutoff has a physical motivation in the correlation length, ξ, between the strings. At the beginning of the so called friction regime after their formation $\xi \simeq \frac{1}{\lambda\eta}$ where λ is the coupling constant of the Higgs potential which breaks the symmetry at the energy scale η. Afterwards, if the scaling regime is achieved, $\xi \backsim t$.

The purpose of our work is twofold. On one hand it is to include in the scattering the effect of the string Higgs field whose order of magnitude has been estimated [8] without considering the simultaneous interaction with the gauge field. On the other hand it is to evaluate the capture of fermions by the string to form superconducting currents if there are bound zero modes for the Dirac equation in the plane transverse to it [9]. With this last process fermion capture has been considered [8] together with the emission of a Higgs particle, whereas we also include the alternative emission of a gauge boson.

The simplest fermionic candidate suitable for our analysis is the right-handed neutrino ν_R, which is a $SU(5)$ singlet [10] in the representation **16** of $SO(10)$ and that acquires mass through a Majorana coupling with a $SU(5)$ singlet Higgs in the representation **126** of $SO(10)$, responsible for the breaking of the Abelian $\tilde{U}(1)$ contained in the latter symmetry. Therefore ν_R is the only fermion in this model which may form a zero mode when it is captured by the string generated at the breaking of $\tilde{U}(1)$.

In section 2 we describe the Abelian string in $SO(10)$ and the relevant fermionic field. In section 3 the cross section for the scattering of the Majorana neutrinos is calculated perturbatively in the approximation of large momentum both in the friction and scaling string regimes. Section 4 is devoted to the capture of fermions with emission either of a Higgs or a gauge boson; finding in both cases a resonant peak for comparatively low momentum values. This allows us to include in the cosmological implications of section 5 the larger influence of superconducting strings formed in the friction epoch as the origin of UHECRs.

2. $SO(10)$ Abelian cosmic strings and Majorana fermions

The first GUT symmetry which contains an Abelian group $\tilde{U}(1)$ additional to the electromagnetic one is $SO(10)$ which can be broken according to the scheme

$$SO(10) \xrightarrow[\mathbf{45}]{} SU(5)\otimes\tilde{U}(1) \xrightarrow[\mathbf{126}]{} SU(5)\otimes Z_2 \xrightarrow[\mathbf{45}]{} SU(3)_C\otimes SU(2)_L\otimes U(1)_Y\otimes Z_2$$
$$\xrightarrow[\mathbf{10}]{} SU(3)_C\otimes U(1)_{em}\otimes Z_2 \tag{1}$$

where the representations of the relevant Higgs fields are indicated. The expectation value of the Higgs field Φ in $\mathbf{126}$ breaks $\tilde{U}(1)$ producing Abelian strings which are topologically stable because the conserved discrete symmetry Z_2 avoids their fragmentation by monopoles. The expectation value of Φ will be of the GUT order $\eta \backsim 10^{15}$ GeV and since its $\tilde{U}(1)$ charge is 10 whereas that of ν_R denoted by ψ is 5, a Majorana mass term coupling which violates lepton number is possible in the Lagrangian

$$\mathcal{L} = (\mathcal{D}_\mu\,\Phi)^\star(\mathcal{D}^\mu\,\Phi) - \tfrac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \tfrac{1}{4}\lambda(|\Phi|^2-\eta^2)^2 + \psi^\dagger \mathrm{i}\sigma^\mu\mathcal{D}_\mu\psi - \tfrac{1}{2}\{ig\psi^\dagger\Phi\psi^c + \mathrm{h.c.}\}, \tag{2}$$

with $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$, $\mathcal{D}_\mu\,\Phi = (\partial_\mu - \mathrm{i}e\mathcal{A}_\mu)\Phi$, $\mathcal{D}_\mu\,\psi = (\partial_\mu - \frac{1}{2}\mathrm{i}e\mathcal{A}_\mu)\psi$, $\nu_R^c = \mathrm{i}\sigma^2\nu_R^\star$, $\sigma^\mu = (I,\sigma^i)$. In the broken-symmetry vacuum where $\Phi = \eta$ and $\mathcal{A}_\mu = 0$ there is a generation of masses $M_\mathcal{H} = \sqrt{\lambda}\eta$ and $M_\mathcal{A} = \sqrt{2}e\eta$ for the bosons and $M_o = g\eta$ for the fermion.

The string configuration in planar coordinates for unit winding number is

$$\Phi = \eta f(r)\,\mathrm{e}^{\mathrm{i}\varphi}, \qquad \mathcal{A}_\varphi = \frac{1}{e\,r}\,a(r) \tag{3}$$

with the behaviour $f\,(0) = a(0) = 0$, $f\,(\infty) = a(\infty) = 1$.

The free quantum Majorana field of right chirality [11] is

$$\psi(x) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\boldsymbol{p}}\frac{1}{\sqrt{2p_o}}[(c(\boldsymbol{p},+)\mathrm{e}^{-\mathrm{i}p^\mu x_\mu} + c^\dagger(\boldsymbol{p},-)\mathrm{e}^{\mathrm{i}p^\mu x_\mu})\sqrt{p_o+p}\,\chi(\boldsymbol{p},+)$$
$$+ (c(\boldsymbol{p},-)\mathrm{e}^{-\mathrm{i}p^\mu x_\mu} - c^\dagger(\boldsymbol{p},+)\mathrm{e}^{\mathrm{i}p^\mu x_\mu})\sqrt{p_o-p}\,\chi(\boldsymbol{p},-)], \tag{4}$$

where χ are the helicity eigenstates

$$\boldsymbol{\sigma}\cdot\boldsymbol{p}\chi(\boldsymbol{p},\pm) = \pm p\chi(\boldsymbol{p},\pm), \tag{5}$$

with $p = |\boldsymbol{p}|$, being clear that for zero mass only the positive helicity survives in equation (4). Having taken a finite normalization volume $\mathcal{V}$, the anticommutation relation for the corresponding annihilation and creation operators is

$$\{c(\boldsymbol{p},\pm), c^\dagger(\boldsymbol{p}',\pm)\} = \delta_{\boldsymbol{p},\boldsymbol{p}'}. \tag{6}$$

The phase between helicity states has been chosen to satisfy

$$\pm\sigma^2\chi^\star(\boldsymbol{p},\mp) = \chi(\boldsymbol{p},\pm). \tag{7}$$

Convenient bases for a fermion moving in the xy plane are

$$\chi(\boldsymbol{p},+) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \qquad \chi(\boldsymbol{p},-) = \frac{-\mathrm{i}}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{8}$$

$$\chi(\boldsymbol{p}',+) = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{e}^{\mathrm{i}\theta/2} \\ -\mathrm{e}^{-\mathrm{i}\theta/2} \end{pmatrix}, \qquad \chi(\boldsymbol{p}',-) = \frac{-\mathrm{i}}{\sqrt{2}}\begin{pmatrix} \mathrm{e}^{\mathrm{i}\theta/2} \\ \mathrm{e}^{-\mathrm{i}\theta/2} \end{pmatrix}, \tag{9}$$

respectively, for the initial state when it comes from the positive x-axis and the final one where it is scattered with angle θ.

3. Neutrino scattering by strings in friction and scaling regimes

From (2) the interaction of ν_R of mass M_o with the string is given by

$$\mathcal{L}_{int} = \frac{e}{2}\psi^\dagger\sigma^\mu\mathcal{A}_\mu\psi - \frac{M_o}{2}\psi^\dagger\sigma^2\psi^\star(1-\mathrm{e}^{\mathrm{i}\varphi}f(r)) - \frac{M_o}{2}\psi^T\sigma^2\psi(1-\mathrm{e}^{-\mathrm{i}\varphi}f(r)), \tag{10}$$

where, because of the terms of the coupling with the classical Higgs field, the perturbative method will be applicable for $p > M_o$.

In the string rest frame, the cross section due to an interaction time $\mathcal{T}$ will be

$$\sigma = \sum_{\text{final states}} \frac{\mathcal{V}}{\mathcal{T}}\frac{p_o}{p}|S_{fi}|^2, \tag{11}$$

where the sum over final states includes the momenta and helicities.

For elastic scattering, the S-matrix elements will receive three contributions

$$S_{fi} = S_{fi}^{(1)} + S_{fi}^{(2)} + S_{fi}^{(3)}, \tag{12}$$

where the first corresponds to the interaction with the classical gauge field which, due to the half integer ratio of fermion and Higgs charges, is a Aharonov–Bohm type, and the others to do with the Higgs field. For the perturbative evaluation of (12) we will approximate the behaviour of the bosonic classical fields (3) of the string whose core radius is $R \backsim \eta^{-1}$ as

$$f(r) = a(r) = 0, \qquad r < R \tag{13}$$

$$f(r) = a(r) = 1, \qquad r > R. \tag{14}$$

The gauge field contribution to the scattering from positive to positive helicity fermion in a first-order perturbation considering (4), (8), (9) gives

$$S_{+,+}^{(1)} = \mathrm{i}\int \mathrm{d}^4x\,\langle \boldsymbol{p}',+|\frac{e}{2}\psi^\dagger\sigma^\mu\mathcal{A}_\mu\psi|\boldsymbol{p},+\rangle = \frac{\mathrm{i}eL}{2\mathcal{V}}\sqrt{\frac{(p_o'+p')(p_o+p)}{2p_o2p_o}}2\pi\delta(p_o'-p_o)A_{+,+}, \tag{15}$$

for a length L of the string along the z-axis and where

$$A_{+,+} = \int \mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}}\frac{\mathrm{i}a(r)}{2e}(\mathrm{e}^{\mathrm{i}\theta/2+\mathrm{i}\varphi} - \mathrm{e}^{-\mathrm{i}\theta/2-\mathrm{i}\varphi}). \tag{16}$$

Using the expansion in plane waves

$$\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}} = \sum_{l=-\infty}^{\infty}(-\mathrm{i})^l J_l(Q\,r)\mathrm{e}^{-\mathrm{i}l(\varphi-\beta)}, \tag{17}$$

where β is the angle between the momentum transfer $\boldsymbol{Q} = \boldsymbol{p}' - \boldsymbol{p}$ and the x-axis, only the $l = 1$

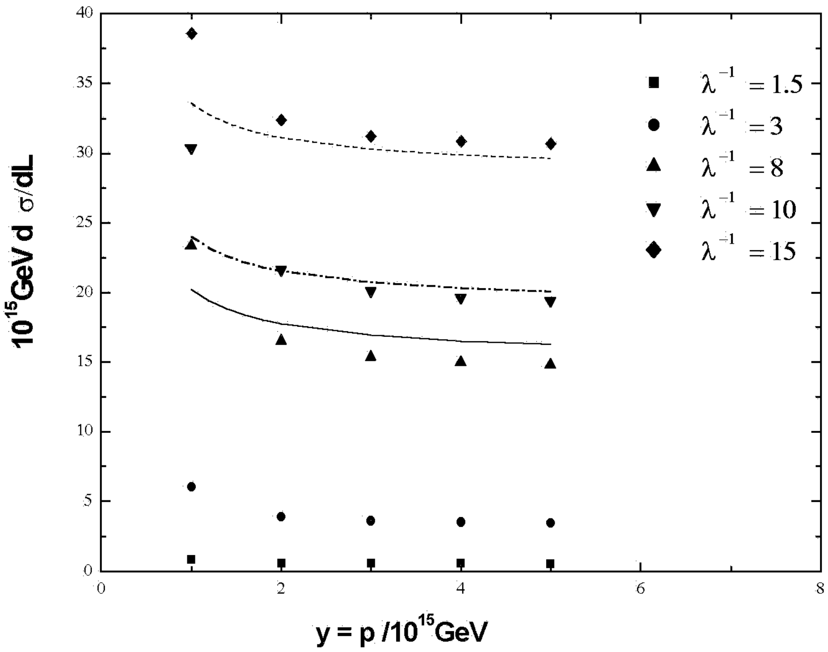

Figure 1. Contribution of the gauge field of the string to the $+ \to +$ helicity scattering cross section in the friction regime for different values of λ. The curves represent the fit using (22).

contribution remains through the integration over φ in (16). The subsequent integration over r considering the approximation of (13), (14) and taking an infrared cutoff ξ gives

$$A_{+,+} = \frac{2\pi \mathrm{i}}{e} \int_R^\xi \mathrm{d}r\, J_1(Qr). \tag{18}$$

We thus obtain the differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}L\,\mathrm{d}\theta}_{+\longrightarrow+} = \frac{\pi}{8p}\left(1 + \frac{M_o^2}{4p^2}\right)^2 \left[\int_{2pR}^{2p\xi} \mathrm{d}z J_1\left(z \sin\frac{\theta}{2}\right)\right]^2, \tag{19}$$

which for $p > M_o$ contains the ordinary relativistic Aharonov–Bohm effect taking $\xi \longrightarrow \infty$, $\theta \neq 0$ since

$$\int_{2pR}^\infty \mathrm{d}z\, J_1\left(z \sin\frac{\theta}{2}\right) = \frac{J_0(2pR \sin\frac{\theta}{2})}{\sin\frac{\theta}{2}}, \tag{20}$$

and for vanishing string radius $R = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}L\,\mathrm{d}\theta}_{+\longrightarrow+} \simeq \frac{\pi}{8p}\frac{1}{\sin^2\frac{\theta}{2}}, \tag{21}$$

which corresponds to the known result [6] in the limit of a small fermion charge.

The first stage following string formation is the friction regime at the beginning of which the correlation length is $\xi = \frac{1}{\lambda\eta}$. The total cross section per unit length can then be computed numerically giving the results of figure 1. For large enough $\lambda^{-1} > 8$, and GUT scale $\eta = 10^{15}$ GeV, the curves can be fitted by

$$\frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}L} = 1.91\xi + \frac{4.93}{p}, \tag{22}$$

showing that the ordinary Aharonov–Bohm behaviour of the second term becomes overrun by the cutoff contribution of the first one. It may be seen that in the friction regime, although the cone in the forward direction which gives rise to the cutoff is very relevant, the non-forward contribution cannot be neglected.

For a final state with negative helicity, it turns out that $S^{(1)}_{-,+} = 0$ which could be expected since the Aharonov–Bohm scattering conserves the helicity [12]. On the other hand the cross section for negative to negative helicity is $\backsim (\frac{M_o^2}{2p^2})^2$ smaller than (19) due to the fact that ν_R is essentially of positive helicity.

Regarding the contribution of the string Higgs field to the scattering of a positive to positive helicity ν_R, using (8) and (9) gives

$$S^{(2)}_{+,+} = -\mathrm{i}\frac{M_o}{2}\int \mathrm{d}^4x\, \langle \boldsymbol{p}', +|\psi^\dagger\sigma^2\psi^\star(1-\mathrm{e}^{\mathrm{i}\varphi}f)|\boldsymbol{p}, +\rangle$$

$$= \frac{\mathrm{i}M_oL}{2\mathcal{V}}\sqrt{\frac{(p_o' + p')(p_o - p)}{2p_o' 2p_o}}\, 2\pi\delta(p_o' - p_o)\cos(\theta/2)\mathcal{F}_{+,+}, \tag{23}$$

where

$$\mathcal{F}_{+,+} = \int \mathrm{d}\varphi\, \mathrm{d}r\, r\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}}(1-\mathrm{e}^{\mathrm{i}\varphi}f). \tag{24}$$

The use of the expansion equation (17) and integration over φ leaves

$$\mathcal{F}_{+,+} = 2\pi\int \mathrm{d}r\, r[J_0(Qr) + \mathrm{i}f\mathrm{e}^{\mathrm{i}\beta}J_1(Qr)] = \frac{\pi}{2p^2}\Xi, \tag{25}$$

where, together with the approximation equations (13), (14) and cutoff ξ,

$$\Xi = \int_o^{2p\xi}\mathrm{d}z\, zJ_0(z\sin(\theta/2)) + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\beta}\int_{2pR}^{2p\xi}\mathrm{d}z\, z\, J_1(z\sin(\theta/2)) = \Xi_o + \mathrm{i}\,\Xi_1\mathrm{e}^{\mathrm{i}\beta}. \tag{26}$$

On changing from positive to negative helicity the calculation is analogous giving

$$S^{(2)}_{-,+} = \mathrm{i}\frac{M_oL}{2\mathcal{V}}\sqrt{\frac{(p_o' - p')(p_o - p)}{2p_o' 2p_o}}\, 2\pi\delta(p_o' - p_o)\sin(\theta/2)\frac{\pi}{2p^2}\Xi. \tag{27}$$

For the matrix element of $S^{(3)}$ without a change of helicity, again using (8) and (9) gives

$$S^{(3)}_{+,+} = -\mathrm{i}\frac{M_o}{2}\int \mathrm{d}^4x\, \langle \boldsymbol{p}', +|\psi^T\sigma^2\psi(1-\mathrm{e}^{-\mathrm{i}\varphi}f)|\boldsymbol{p}, +\rangle$$

$$= \frac{\mathrm{i}M_oL}{2\mathcal{V}}\sqrt{\frac{(p_o' - p')(p_o + p)}{2p_o' 2p_o}}\, 2\pi\delta(p_o' - p_o)\cos(\theta/2)\mathcal{G}_{+,+}, \tag{28}$$

where now

$$\mathcal{G}_{+,+} = \int \mathrm{d}\varphi\, \mathrm{d}r\, r\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}}(1-\mathrm{e}^{-\mathrm{i}\varphi}f). \tag{29}$$

With the same steps as above one gets

$$\mathcal{G}_{+,+} = 2\pi\int \mathrm{d}r\, r[J_0(Qr) + \mathrm{i}f\mathrm{e}^{-\mathrm{i}\beta}J_1(Qr)] = \frac{\pi}{2p^2}(\Xi_o + \mathrm{i}\mathrm{e}^{-\mathrm{i}\beta}\Xi_1). \tag{30}$$

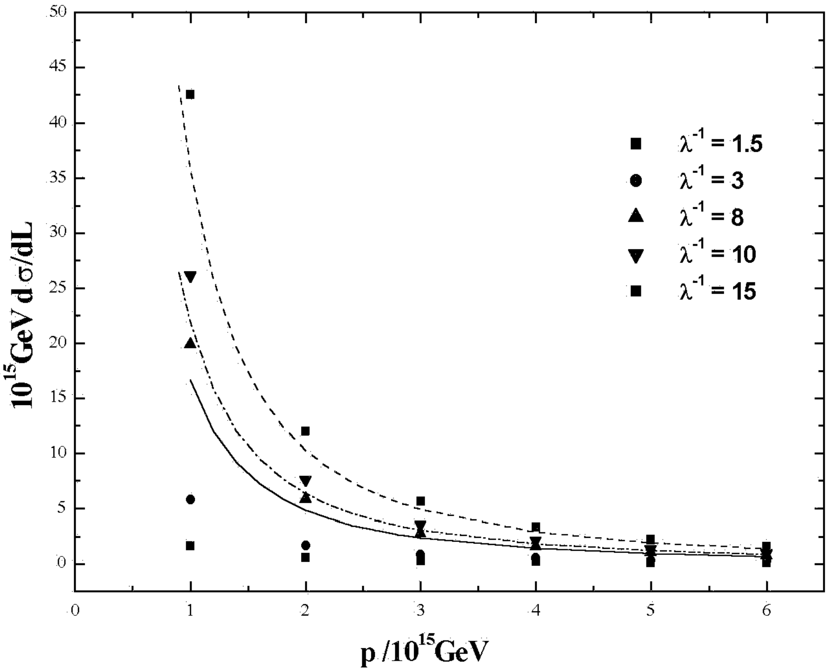

Figure 2. The contribution of the Higgs field of the string to the $+ \rightarrow -$ helicity scattering cross section in the friction regime for different values of λ. The curves represent (34).

Analogously, for the change of helicity

$$S^{(3)}_{-,+} = \frac{\mathrm{i}M_o L}{2\mathcal{V}}\sqrt{\frac{(p'_o + p')(p_o + p)}{2p'_o 2p_o}}\,2\pi\delta(p'_o - p_o)\sin(\theta/2)\frac{\pi}{2p^2}(\Xi_o + \mathrm{i}\mathrm{e}^{-\mathrm{i}\beta}\Xi_1). \tag{31}$$

Using these results

$$|S^{(2)}_{+,+} + S^{(3)}_{+,+}|^2 \backsim \left(\frac{M_o}{p}\right)^2 |S^{(2)}_{-,+} + S^{(3)}_{-,+}|^2 \tag{32}$$

indicating the fact that violation of helicity is favoured by Majorana coupling.

The dominant differential cross section is then

$$\frac{\mathrm{d}\sigma_{\mathcal{H}}}{\mathrm{d}L\,\mathrm{d}\theta}_{+\longrightarrow -} = \frac{\pi}{32p}\left(\frac{M_o}{p}\right)^2 \sin^2\frac{\theta}{2}\left(\Xi_0^2 + 2\cos\frac{\theta}{2}\Xi_0\,\Xi_1 + \Xi_1^2\right). \tag{33}$$

The numerical computation of the total cross section is shown in figure 2 and can be reproduced for $\lambda^{-1} \gtrsim 8$ by the approximate behaviour for $p\xi \gg 1$

$$\frac{\mathrm{d}\sigma_{\mathcal{H}}}{\mathrm{d}L} = 1.04\xi\left(\frac{M_o}{p}\right)^2 [1 + 0.48\ln(p\xi)], \tag{34}$$

where the additional logarithmic dependence on the cutoff is a consequence of the phase of the Higgs field in its interaction with the fermion as seen in (10).

For a later time after their formation, strings may reach [13] the scaling regime, due to a correlation length $\xi \backsim t$, when the universe cooled below $T_{sc} \simeq \frac{T^2_{GUT}}{m_{PL}} \backsim 10^{11}$ GeV where

m_{PL} is the Planck mass. This occurred for the time $t \backsim 10^{-28}$ s, being the expansion of the universe scale due to radiation $a(t) \propto t^{1/2}$. Therefore for momentum $p > M_o$, $\xi p \gg 1$ and $\xi \gg R$ so that now the cutoff contribution dominates clearly over the ordinary Aharonov–Bohm term, it is possible to calculate the approximation for the cross section caused by the gauge field

$$\frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}L} \simeq 2\left(1+\frac{M_o^2}{4p^2}\right)^2 \xi, \tag{35}$$

and obviously the contribution given by the string Higgs field is even better approximated by (34) in the scaling regime. Since the correlation length is much larger in the scaling regime than in the friction one, the above cross sections are correspondingly larger in the former case.

4. Capture of fermions by strings with the emission of bosons

This process is analogous to the capture of an electron by a nucleus with the emission of a photon, where the description is given in terms of the interaction of the electron with the quantized radiation field in addition to the Coulomb attraction.

We thus add the quantum fluctuations to the classical configurations of the string Higgs and gauge fields as

$$\Phi = \Phi^{cl} + \hat{\Phi}, \qquad \mathcal{A}_\mu = \mathcal{A}_\mu^{cl} + \hat{\mathcal{A}}_\mu. \tag{36}$$

Thus we will have as the interaction with the additional quantum boson fields

$$\mathcal{L}^{\mathrm{quan}} = -\frac{\mathrm{i}e}{2}\psi^\dagger\sigma^\mu\hat{\mathcal{A}}_\mu\psi - \frac{\mathrm{i}g}{2}(\psi^\dagger\hat{\Phi}\psi^c - \psi^{c\,\dagger}\hat{\Phi}^\dagger\psi), \tag{37}$$

for which the conditions for the validity of the perturbation treatment are $e \lesssim 1$ and $g \lesssim 1$.

Now for the fermion field we must consider the free solutions and the zero-mode states which can be formed with the background of the classical string configuration, i.e.

$$\psi = \hat{\psi}_{free} + \hat{\psi}_{zm}, \tag{38}$$

where $\hat{\psi}_{free}$ is given by (4) as before, whereas the zero-mode term will be

$$\hat{\psi}_{zm} = \sum_{p_z>0}[c_o(p_z,+)\mathcal{U}_o(p_z,+)\,\mathrm{e}^{-\mathrm{i}\omega t} + c_o^\dagger(p_z,+)\mathcal{U}_o^\star(p_z,+)\,\mathrm{e}^{\mathrm{i}\omega t}], \tag{39}$$

with $\omega = p_z$, describing massless particles which move along the positive z-axis through the anticommuting operators c_o. The zero-mode wavefunction [14] is given by

$$\mathcal{U}_o(p_z,+) = \frac{\tilde{M}}{\sqrt{2\pi L}}\begin{pmatrix}1\\0\end{pmatrix}\exp\left(-\int_0^\rho\left[\frac{M_o}{M_\mathcal{H}}f(\rho') + \frac{a(\rho')}{2\rho'}\right]\mathrm{d}\rho'\right)\exp(\mathrm{i}p_z z), \tag{40}$$

where $\tilde{M}^{-1}$ is its effective radius, $\rho' = M_\mathcal{H} r'$ and the normalization is $\int \mathrm{d}^3x\,|\mathcal{U}_o(p_z,+)|^2 = 1/2$. It is clear that this requires a more detailed description of the classical fields inside the string than that given by (13) and (14), i.e. [15]

$$f(\rho') = f_o\rho' \qquad a(\rho') = a_o\rho'^2, \qquad \rho' < 1, \tag{41}$$

a_o and f_o being constants that, from the normalization condition, give $\tilde{M} = M_{\mathcal{H}}\sqrt{\frac{M_o}{M_{\mathcal{H}}}f_o + \frac{a_o}{2}}$. The boson quantum fields are massive and given in terms of operators with the usual commutators for the complex Higgs and real gauge fields as

$$\hat{\Phi}(x) = \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2k_o\mathcal{V}}}(a_k \mathrm{e}^{-\mathrm{i}k^\mu x_\mu} + b_k^\dagger \mathrm{e}^{\mathrm{i}k^\mu x_\mu}), \tag{42}$$

$$\hat{\mathcal{A}}_\mu(x) = \sum_\lambda \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2k_o\mathcal{V}}}(\varepsilon_\mu(\boldsymbol{k},\lambda)a(\boldsymbol{k},\lambda)\mathrm{e}^{-\mathrm{i}k^\nu x_\nu} + \varepsilon_\mu^\star(\boldsymbol{k},\lambda)\, a^\dagger(\boldsymbol{k},\lambda)\mathrm{e}^{\mathrm{i}k^\nu x_\nu}), \tag{43}$$

with the polarization vectors satisfying

$$\varepsilon_\mu^\star(\boldsymbol{k},\lambda)\varepsilon^\mu(\boldsymbol{k},\lambda') = -\delta_{\lambda\lambda'}, \qquad \sum_\lambda \varepsilon_\mu(\boldsymbol{k},\lambda)\varepsilon_\nu^\star(\boldsymbol{k},\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_{\mathcal{A}}^2}. \tag{44}$$

The capture of a neutrino with the emission of a superheavy Higgs particle $\nu_R \to \nu_{zm} + \Phi$ is produced by the second term of (37) giving with first-order perturbation theory the probability amplitude

$$\mathcal{S}_{zm\Phi,\nu_R} = \frac{g}{4}\frac{\tilde{M}}{\mathcal{V}\sqrt{\pi k_o L}}\sqrt{\frac{p_o+p}{2p_o}}2\pi\delta(p_o - \omega_o' - k_o)2\pi\delta(p_z' + k_z)\Gamma(Q), \tag{45}$$

where

$$\Gamma(Q) = \frac{1}{\sqrt{2}}\int \mathrm{d}^2x\, \mathrm{e}^{\mathrm{i}\boldsymbol{x}_T\cdot\boldsymbol{Q}} \exp\left(-\int_0^\rho \left[\frac{M_o}{M_{\mathcal{H}}}f(\rho') + \frac{a(\rho')}{2\rho'}\right]\mathrm{d}\rho'\right). \tag{46}$$

This integral in the transverse plane of the string, with the momentum transferred to it $\boldsymbol{Q} = \boldsymbol{p} - \boldsymbol{k}_T$, can be calculated approximately [14] expanding the plane wave in Bessel functions to give

$$\int \mathrm{d}\varphi\, \mathrm{e}^{\mathrm{i}\boldsymbol{x}_T\cdot\boldsymbol{Q}} = 2\pi J_0(Qr), \tag{47}$$

which together with the form of the classical fields inside the string (41) (with their outside contribution being negligible) gives

$$\Gamma(Q) \simeq \sqrt{2}\pi\frac{1}{\tilde{M}^2}\exp\left(-\frac{Q^2}{2\tilde{M}^2}\right). \tag{48}$$

From (45) and (48) we get, in the limit of equal masses, the differential cross section for the emission of a Higgs particle with scattering angle θ in the plane transverse to the string

$$\frac{\mathrm{d}\sigma}{\mathrm{d}L\,\mathrm{d}\theta} = \frac{g^2}{64\pi M_o}\left(\frac{1}{y} + \frac{1}{\sqrt{1+y^2}}\right)\int_0^y \mathrm{d}z\, z\, \mathrm{e}^{-(y^2 - 2yz\cos\theta + z^2)}, \tag{49}$$

where $y = p/M_o$.

The numerical evaluation of the corresponding capture total cross section is shown in figure 3.

Regarding the capture with the emission of one gauge vector boson $\nu_R \to \nu_{zm} + \mathcal{A}$, the first term of (37) gives the probability amplitude

$$\mathcal{S}_{zm\mathcal{A},\nu_R} = \frac{\mathrm{i}e}{2\mathcal{V}}\frac{\tilde{M}}{\sqrt{\pi L}}\sqrt{\frac{p_o+p}{2p_o 2k_o}}2\pi\delta(p_o - k_o - \omega')2\pi\delta(p_z' + k_z)\chi_o^\dagger\sigma^\mu\varepsilon_\mu^\star(\boldsymbol{k},\lambda)\,\chi(\boldsymbol{p},+)\Gamma(Q), \tag{50}$$

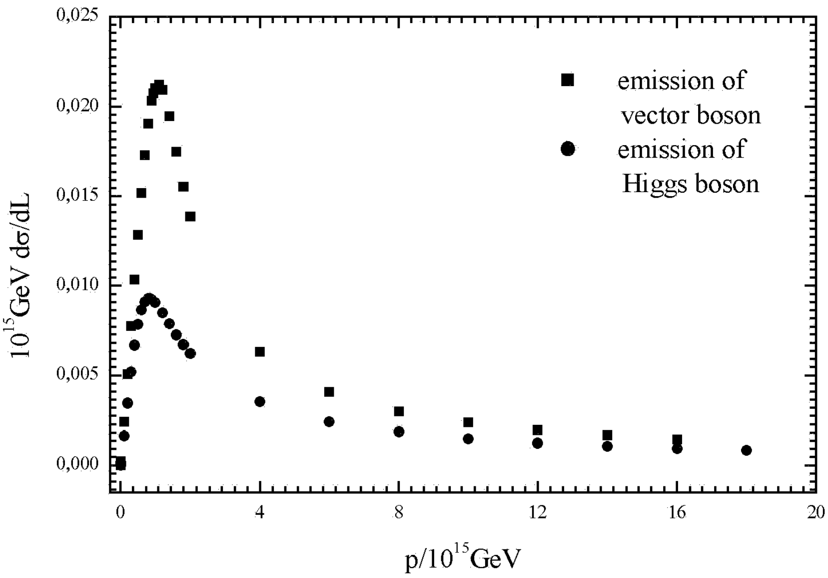

Figure 3. A comparison of the cross sections of the capture of ν_R to form zero modes with the emission either of a Higgs or a vector massive boson.

where

$$\chi_o = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi(\boldsymbol{p}, +) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

and $\Gamma(Q)$ is that of (46).

Considering the sum over the final polarizations, the differential cross section for emission of a gauge boson turns out to be (again taking equal masses)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}L\,\mathrm{d}\theta} = \frac{e^2}{32\pi M_o}\left(\frac{1}{y} + \frac{1}{\sqrt{1+y^2}}\right)\int_0^y \mathrm{d}z\, z\left[\frac{3}{2} + \frac{4 + z^2y^2 - y^2 + 3z^2}{2(1+y^2)} - \frac{z(1+z^2)}{\sqrt{1+y^2}}\cos\theta\right]\mathrm{e}^{-(y^2 - 2yz\cos\theta + z^2)}. \tag{51}$$

The numerical results for the total cross section of the capture taking [16] $\alpha_{GUT}^{-1} = \frac{4\pi}{e^2} = 26$ are also presented in figure 3.

One sees that both cross sections show a resonant behaviour for values of the momentum of ν_R of the order of its mass which is relevant for the following cosmological implications.

5. Cosmological implications and conclusions

We have analysed the possibility that fermions that acquired mass in the GUT epoch of the universe evolution could have been captured by cosmic strings formed by the breaking of an Abelian subgroup at this scale.

In the case that the GUT symmetry that contained this subgroup corresponded to $SO(10)$, the fermion to be considered is ν_R. These neutrinos captured by the string would produce a superconducting current, even though they are neutral, in the sense that inside it they travel at the velocity of light.

This current stabilizes closed strings which otherwise would contract and disappear. The superconducting microscopic loops might, among other models of superheavy relics [17], be the origin of the hard component of UHECRs which has been observed without identification of their astrophysical sources beyond the so called GZK cutoff [18].

To see which is the flux of UHECRs produced by these vortons, it is necessary to estimate their density whose evolution with the temperature of universe T starting from their formation at T_f will be (according to the dilution of stable objects)

$$n_v(T) = n(T_f)\left(\frac{T}{T_f}\right)^3, \tag{52}$$

bearing in mind [19] that $n(T_f) \backsim (\xi(T_f))^{-3}$.

During the friction regime an estimation [14] is

$$\xi^{fr}(T) \backsimeq (m_{PL})^{1/2}\frac{T_{GUT}}{T^{5/2}}, \tag{53}$$

so that for $T_f = T_{GUT}$ from (52) $n_v^{fr}(T) \backsimeq 10^{-6}T^3$, whereas for the formation at the end of this period $T_f \backsimeq 10^{11}$ GeV $n_v^{fr}(T) \backsimeq 10^{-24}\,T^3$.

Regarding the number of fermionic carriers in the loop [20]

$$N \backsimeq \xi(T_f)T_{GUT}, \tag{54}$$

in the friction regime $N^{fr} \backsim 100$ if $T_f = T_{GUT}$, and $N^{fr} \backsim 10^{12}$ at the end of it when $T_f \backsimeq 10^{11}$ GeV.

Looking now at the formation in the scaling regime valid for $T \lesssim 10^{11}$ GeV where

$$\xi^{sc} \backsimeq H^{-1} \backsimeq \frac{m_{PL}}{T^2}, \tag{55}$$

where H is the Hubble parameter, $n_v^{sc}(T) \lesssim 10^{-24}T^3$ and $N^{sc} > 10^{12}$; both in agreement with the limit of the friction epoch.

As a consequence the number of fermions in vortons per unit volume $Nn_v(T)$ if incorporated at the beginning of the friction regime is $10^{-4}\,T^3$, whereas if incorporated at the beginning of the scaling one will be $10^{-12}\,T^3$. Therefore the ratio of these incorporated fermions per unit comoving volume is 10^8, which is equal to the inverse ratio of the universe's times of formation. A similar analysis for the vorton formation during the scaling regime indicates that the density of absorbed fermions per unit of comoving volume is $t_f^{-1/2}$.

It is obvious that the above ratio of fermions equals the one of the lengths of the original closed strings which at formation will be $\xi(T_f)\xi^{-3}(T_f)$. Therefore, with the probability of capture per unit length being independent of time, the formation of vortons should be equally probable in both friction and scaling regimes. However, if one includes the motion of the original strings, which is different in both regimes, the formation of vortons during the scaling period is less likely [21].

Even without this final consideration, from the above estimation of vortons density it is clear that their relevance in UHECRs would be much more important if they were formed at the beginning of the friction epoch.

Our calculation of the capture cross section indicates that it is likely that vortons can be formed in the friction epoch. In fact, according to (54) one needs to incorporate one fermion per Compton length λ_c. Multiplying the peak of $\frac{\mathrm{d}\sigma}{\mathrm{d}L}$ of figure 3 by λ_c and the flux of ν_R proportional to its thermal density $\backsim\eta^3$ at the beginning of the friction regime, the number of incorporated fermions per unit time will be $\backsim 10^{-2}\eta$. Since the duration of the friction period is $\Delta t \backsim 10^{8}\eta^{-1}$, the string may achieve suitable fermion density in the first part of this epoch. A similar conclusion was reached by a simplified diffusion mechanism of the incorporation of fermions [22].

The example considered here of $SO(10)$ is the simplest one since ν_R is the only non-ordinary fermion and acquires mass on the GUT scale. In order to have exotic fermions electrically charged with zero modes giving way to superconducting currents in the common sense of the word, we should take the unification of interactions under a larger group as E_6. In this case, however, the addition of 11 fermions, apart from ν_R, would make the analysis of the problem much harder.

On regarding the scattering of fermions by straight and long strings one may note that to the traditional Aharonov–Bohm effect due to the gauge potential, as in the solenoid case the interaction with the Higgs field which generates the fermion mass must be also added. From our calculation, one sees that this contribution to the total scattering cross section increases with the separation among strings faster than that due to the gauge field by a logarithmic factor which is related to the winding phase also present at large distances.

For these kinds of strings their density length will be $\backsim 1/\xi^2$ and, subtracting the universe expansion, the corresponding one per unit comoving volume in the scaling regime will be $1/t$. Since the cross section per unit string length for neutrino scattering increases at least as t, the effect due to the targets in the unit comoving volume will be roughly constant.

To make the process of the generation of superconducting currents more realistic, one should take into account the propagation of neutrinos in the plasma outside the string, the influence of the motion of the latter and the fluctuations of the field equivalent to the electric one which could produce jumps of the fermions from negative to positive energy states inside the core.

References

[1] Kibble T W B 1976 *J. Phys. A: Math. Gen.* **9** 1387
Vilenkin A and Shellard E P S 1994 *Cosmic Strings and Other Topological Defects* (Cambridge: Cambridge University Press)
Hindmarsh M B and Kibble T W B 1995 *Rep. Prog. Phys.* **58** 477
[2] Witten E 1985 *Nucl. Phys.* B **249** 557
[3] Davis R L 1988 *Phys. Rev.* D **38** 3722
[4] Masperi L and Silva G 1998 *Astropart. Phys.* **8** 173
[5] Nagano N and Watson A A 2000 *Rev. Mod. Phys.* **72** 689
[6] Alford M G and Wilczek F 1989 *Phys. Rev. Lett.* **62** 1071
[7] Perkins W B, Perivolaropoulos L, Davis A C, Brandenberger R H and Matheson A 1991 *Nucl. Phys.* B **353** 237
Sakoda S and Omote M 1997 *J. Math. Phys.* **38** 716
[8] Brandenberger R, Davis A C and Matheson A M 1988 *Nucl. Phys.* **307** 909
[9] Jackiw R and Rossi P 1981 *Nucl. Phys.* B **190** 681
[10] Fritzsch H and Minkowski P 1975 *Ann. Phys., NY* **93** 193
[11] Mannheim P D 1984 *Int. J. Theor. Phys.* **23** 643
[12] Vera F and Schmidt I 1990 *Phys. Rev.* D **42** 3591
[13] Albrecht A and Turok N 1985 *Phys. Rev. Lett.* **54** 1868
Bennett D and Bouchet F 1988 *Phys. Rev. Lett.* **60** 257
Hindmarsh M 1988 *Phys. Rev. Lett.* **80** 2277
[14] Davis S C, Perkins W B and Davis A C 2000 *Phys. Rev.* D **62** 043503
[15] Hindmarsh M and Kibble T W B 1995 *Rep. Prog. Phys.* **58** 477

[16] Amaldi U, de Boer W and Forstenau H 1991 *Phys. Lett.* B **260** 447
[17] Bhattacharjee P and Sigl G 2000 *Phys. Rep.* **327** 109
Olinto A V 2000 *Phys. Rep.* **333–4** 329
[18] Greisen K 1966 *Phys. Rev. Lett.* **16** 748
Zatsepin Z and Kuzmin V 1966 *Zh. Eksp. Teor. Fiz.* **4** 144
[19] Carter B and Davis A C 2000 *Phys. Rev.* D **61** 123501
[20] Brandenberger R, Carter B, Davis A C and Trodden M 1996 *Phys. Rev.* D **54** 6059
[21] Martins C and Shellard E 1998 *Phys. Rev.* D **57** 173
[22] Masperi L and Orsaria M 2002 *Astropart. Phys.* **16** 411