Three Lectures on (Super)String Field Theories*

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Introduction.
The subject of these lectures is a covariant string field theory (SFT). One of the main motivations to construct covariant SFT – an off-shell formulation of a string theory [1] – is the desire to study non-perturbative phenomena in string theories [2, 3]. Recent studies of unstable D-branes and tachyons have led to the realization that string field theory really accommodates significant non-perturbative information ([4]-[8] and refs therein).

SFT is a traditional subject at this school. In the previous set of lectures [9, 10] an introduction to SFT as well as SFT description of unstable D-branes and vacuum string field theory (VSFT) were presented. In the next series rolling tachyon and D-brane decay [11] are considered. Cosmological applications of D-brane dynamics via rolling tachyons description are presented in [12] and in a lecture by one of the present authors [13].

The subject that was never discussed here before concerns loop amplitudes in SFT. Loop amplitudes were studied within the old light-cone SFT [2] and in this formalism the finiteness of superstring diagrams was proved. The finiteness of superstring theories in covariant approaches still remains an open problem and deserves further attention.

In these three lectures we describe specific points that are important to study loop diagrams in the covariant super SFT formalizm. To make these lecture notes self-contained in the first lecture we describe the main building blocks of SFT (for more details see [9, 10]). We will try to present these building blocks in a more transparent way and make presentation related with the diagram technique of the usual quantum field theory [17].

We consider the simplest one-loop diagram, the so-called tadpole diagram, in the cubic SSFT [14, 15, 16]. For the bosonic case this diagram was studied in [18]. There are specific features in the superstring case due to a presence of inevitable picture-changing operators in the cubic SSFT. At formal level (without regularization) one can prove that the tadpole diagram in the cubic SSFT is BRST-exact. However, to deal with picture-changing operators we have to introduce a regularization. An investigation

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of this problem requires a careful study of overlap properties of vertices. Overlap conditions are defining relations to find vertices [19]. There are so-called field and mode overlap conditions. Traditionally there is no difference between the mode overlap and the field overlap and in fact this difference appears only for vertices with insertions. The simplest way to see the difference is to deal with the identity vertex. This point apart from the fact that it is important for loop calculations is an a good point to show main technical features and properties of SFT. We consider this subject in the second lecture.

The simplest way to study overlap conditions gives the CFT language in SFT and it is discussed in the third lecture.

1. Lecture1

1.1. String Fields

In quantum filed theory each elementary particle corresponds to a local field and vice versa [17]. As there is an infinite number of elementary modes (particles) in strings [2, 3] it is natural to associate with a string an infinite tower of usual fields. It turns out that all the fields appear as different coefficients in expansion of a string field $\Psi = \Psi[X(\sigma), c(\sigma), b(\sigma)]$ on specific basic functionals. An origin of the form of these basic functionals comes from vertex operators in the first quantized approach to strings. It is convenient to describe these basic functionals in the Fock representation and the expansion in the Fock representation is

$$|\Psi\rangle = \int d^{26}k \; \left[\phi(k)\,|+;k\rangle + A_\mu(k)\,\alpha^\mu_{-1}|+;k\rangle + \cdots\right], \tag{1}$$

here and below $\alpha^\mu_{-n},\ c_{-l},\ b_{-k}$ are Fourier modes of a string $X^\mu(\sigma)$ and it's ghosts $c(\sigma),\ b(\sigma)$ (here we mean the open bosonic string in the critical dimension D=26, $\mu = 0, ..., 25$) and $|+;k\rangle$ is defined below, see (6). Ghost and antighost fields $c(\sigma)$ and $b(\sigma)$ appear when the bosonic open string action

$$-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu, \tag{2}$$

is quantized in conformal gauge $\gamma_{ab} \sim \delta_{ab},\ a,b = 0,1$ using the BRST approach [2, 3]. The gauge-fixed action is

$$-\frac{1}{4\pi\alpha'}\int d^2\sigma \partial_a X^\mu \partial^a X_\mu + \frac{i}{\pi}\int d^2\sigma\left(b_{++}\partial_- c^+ + b_{--}\partial_+ c^-\right). \tag{3}$$

$\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma),\ \sigma^0 = \sigma,\ \sigma^1 = \tau$. The matter fields X^μ can be expanded in modes as

$$X^\mu(\sigma,\tau) = x^\mu + 2p^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n\cos(n\sigma)e^{-in\tau}, \tag{4}$$

and the canonical commutation relations are $[x^\mu, p^\nu] = i\eta^{\mu\nu}$, $[\alpha^\mu_m, \alpha^\nu_n] = m\eta^{\mu\nu}\delta_{m+n,0}$. Usually α^μ_n's with negative values of n are considered as creation operators, $(\alpha^\mu_n)^\dagger = \alpha^\mu_{-n}$ and $\alpha^\mu_n|k\rangle_{matter} = 0, \quad n \geq 1$, and "$k$" in the ket means that the state $|k\rangle$ is a momentum eigenstate $p^\mu|k\rangle = k^\mu|k\rangle$.

The ghost and antighost fields are decomposed into modes as

$$c^{\pm}(\sigma,\tau) = \sum_{n=-\infty}^{\infty} c_n e^{-in(\tau\pm\sigma)}, \quad b_{\pm\pm}(\sigma,\tau) = \sum_{n=-\infty}^{\infty} b_n e^{-in(\tau\pm\sigma)}. \tag{5}$$

Thereafter we will drop the indexes $\pm$ for $c(\sigma,\tau)$ and $b(\sigma,\tau)$. The ghost and antighost modes satisfy the anticommutation relations $\{c_n, b_m\} = \delta_{n+m,0}$, $\{c_n, c_m\} = \{b_n, b_m\} = 0$. Ghost part, $|+\rangle$, of the physical vacuum is defined as $b_n|+\rangle = 0, \quad n \geq 1, \quad c_n|+\rangle = 0, \quad n \geq 0$, and the physical vacuum is

$$|+, k\rangle = |k\rangle_{matter} \otimes |+\rangle. \tag{6}$$

The ghost part of the SL(2,R) invariant vacuum is defined as $b_n|0\rangle = 0, \quad n \geq -1$, $c_n|0\rangle = 0, \quad n \geq 2$, and $|0;k\rangle = |k\rangle_{matter} \otimes |0\rangle$. The zero-momentum state $|0;0\rangle$ is the SL(2,R) invariant vacuum; we will often write it simply as $|0\rangle$. This vacuum is defined to have ghost number 0, and it is normalized by $\langle 0;k|c_{-1}c_0c_1|0;k'\rangle = (2\pi)^{26}\delta(k-k')$.

1.2. Bosonic String Field Action

There is a question how to write an action for the whole infinite system of local fields or for a string functional, to reproduce standard local actions for all local fields. In the light-cone gauge this problem has been solved in the so-called light-cone string field theory [1].

A covariant open bosonic string field theory action was proposed by Witten [1]

$$S = \frac{1}{2}\int \Psi \star Q\Psi + \frac{g}{3}\int \Psi \star \Psi \star \Psi, \tag{7}$$

here g is the string coupling constant and Ψ is a string field [2]. Here it is assumed that string field Ψ is an element of a string field algebra $\mathcal{A}$. This algebra is equipped with noncommutative star product $\star : \mathcal{A}\otimes\mathcal{A} \to \mathcal{A}$, and BRST operator $Q : \mathcal{A} \to \mathcal{A}$,

The algebra $\mathcal{A}$ is a $\mathbf{Z}$-graded algebra under multiplication and the degree G of Q is equal to one. String fields are integrated using $\int : \mathcal{A} \to \mathbf{C}$. This integral $\int \Psi$ vanishes unless the $G_\Psi = 3$. Thus, the action (7) is only non-vanishing for a string field Ψ of a definite degree 1. Note, that in our conventions $G(|0\rangle) = 0$.

The elements $Q, \star, \int$ that define the string field theory are assumed to satisfy the set of axioms (see [1, 9, 19] for details). When the axioms are satisfied, the action (7) is invariant under the gauge transformations

$$\delta\Psi = Q\Lambda + \Psi \star \Lambda - \Lambda \star \Psi\,, \tag{8}$$

for any gauge parameter $\Lambda \in \mathcal{A}$ with degree 0.

For Witten's cubic open string field theory, the BRST operator Q in (7) is the usual open string BRST operator Q_B, and the degree associated with a Fock space state is the ghost number of that state.

The star product $\star$ is defined by gluing the right half of one string to the left half of the other one using a delta function interaction [1]. The star product is factorized into separate matter and ghost parts. It is given by a formal path integral with δ-function. The integral over a string field also is factorized into matter and ghost parts. In each sector it corresponds to gluing of the left and right halves of the string together with a delta function interaction [1, 19]. The expressions with δ-functions are rather formal. One can give to these expressions precise meaning in Fock space description. Let $\mathcal{H}$ be the Fock space of vectors (1). In the Fock space language, the integral of a string field as well as the integral of the star product of two or three fields is described in terms of one-, two- and three-string vertices $\langle V_1|$, $\langle V_2|$ and $\langle V_3|$. $\langle V_1|$ is an elements of the dual Fock space $(\mathcal{H}^*)$. $\langle V_2|$ and $\langle V_3|$ are two-fold and three-fold product of the dual Fock spaces $(\mathcal{H}^*)^2$ and $(\mathcal{H}^*)^3$, $\langle V_i| \in (\mathcal{H}^*)^i \quad i = 1,2,3$. They are defined so that

$$\begin{aligned}
&\int \Psi \to \langle V_1|\Psi\rangle \equiv \langle I|\Psi\rangle, \\
&\int \Psi_1 \star \Psi_2 \to \langle V_2|\left(|\Psi_1\rangle \otimes |\Psi_2\rangle\right) \equiv \langle V_2|\Psi_1, \Psi_2\rangle, \\
&\int \Psi_1 \star \Psi_2 \star \Psi_3 \to \langle V_3|\left(|\Psi_1\rangle \otimes |\Psi_2\rangle \otimes |\Psi_3\rangle\right) \equiv \langle V_3|\Psi_1, \Psi_2, \Psi_3\rangle.
\end{aligned} \tag{9}$$

Explicit forms for the two- and three-string vertices were found in [19]-[23] and are presented in many lecture notes, and in particular in [9]. In terms of these vertices, the SFT action (7) becomes

$$S = \frac{1}{2}\langle V_2|\Psi, Q\Psi\rangle + \frac{g}{3}\langle V_3|\Psi, \Psi, \Psi\rangle\,. \tag{10}$$

This action is often written using the dual $\langle\Psi|$ of the string field $|\Psi\rangle$ [1], $\langle\Psi| = \langle V_2|\Psi\rangle$, as

$$S = \frac{1}{2}\langle\Psi|Q\Psi\rangle + \frac{g}{3}\langle\Psi|\Psi \star \Psi\rangle\,. \tag{11}$$

1.3. Vertices and their properties

A vertex $\langle V_N|,\ N = 1,2,3$ is a solution of overlap conditions

$$\langle V_N|[X^{(i)}(\sigma) - X^{(i-1)}(\pi - \sigma)] = 0,$$
$$\langle V_N|[c^{(i)}(\sigma) + c^{(i-1)}(\pi - \sigma)] = 0, \quad \langle V_N|[b^{(i)}(\sigma) - b^{(i-1)}(\pi - \sigma)] = 0,$$

[1] In the CFT language the BPZ dual $\langle\Psi|$ is defined by the conformal map $z \to -1/z$; [24]

where $i = 1, ..., N$ means the number of the strings and $X(\sigma) = X(\sigma, 0)$, $c(\sigma) = c(\sigma, 0)$, $b(\sigma) = b(\sigma, 0)$ and $X(\sigma,\tau)$, $c(\sigma,\tau)$ and $b(\sigma,\tau)$ are given by (4) and (5) respectively. According to [19]-[23] $\langle V_2|$ has a form

$$\langle V_2| = \int d^{26}p\,(\langle -;p| \otimes \langle -;-p|)\;\; (c_0^{(1)} + c_0^{(2)})$$
$$\times \exp\left(-\sum_{n=1}^{\infty} (-1)^n [\frac{1}{n}\alpha_n^{(1)}\alpha_n^{(2)} + c_n^{(1)}b_n^{(2)} + c_n^{(2)}b_n^{(1)}] \right),$$

where $\langle -;-p| \equiv \langle 0|c_{-1} \otimes {}_{matter}\langle -p|$. This expression for the two-string vertex can also be derived directly from the conformal field theory approach computing the two-point function of an arbitrary pair of states on the disk [24].

Similar to the two-string vertex the three-string vertex has a form [19]-[23]

$$\langle V_3| = \prod_{i=1}^{3} \left(\int d^{26}p^{(i)} \langle -;p^{(i)}| \right) \quad \delta(p^{(1)} + p^{(2)} + p^{(3)}) c_0^{(1)} c_0^{(2)} c_0^{(3)}$$
$$\times \exp\left(-\frac{1}{2} \sum_{r,s=1}^{3} [\alpha_m^{(r)} \frac{V_{mn}^{rs}}{\sqrt{nm}} \alpha_n^{(s)} + 2\alpha_m^{(r)} \frac{V_{m0}^{rs}}{\sqrt{m}} p^{(s)} + p^{(r)} V_{00}^{rs} p_n^{(s)} + c_m^{(r)} X_{mn}^{rs} b_n^{(s)}] \right).$$

To provide the axioms which have to be obeyed by $\int, \star$ and $BRST$-charge Q one has to assume the BRST-invariance of vertices $\langle V_N| \sum_{i=1}^{N} Q^{(i)} = 0$.

1.4. NSR Fermions String Fields

The action for the matter part of the open superstring in the conformal gauge has the form [2, 3]

$$S = -\frac{1}{2\pi\alpha'} \int d^2\sigma \left[\frac{1}{2} \partial_a X^\mu \partial^a X_\mu - \frac{i}{2} \bar{\Psi}^\mu \gamma^a \partial_a \Psi_\mu \right]. \tag{12}$$

Here Ψ is a Majorana spinor and $\bar{\Psi} = \Psi^T \gamma^0$. We introduce the explicit representation of Euclidean Dirac γ-matrices: $\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The spinor $\Psi^\mu(\sigma,\tau)$ has two components $\Psi^\mu(\sigma,\tau) = \begin{pmatrix} \psi_-^\mu(\sigma,\tau) \\ \psi_+^\mu(\sigma,\tau) \end{pmatrix}$. Performing matrix multiplications in (12) we get the following action for fermions

$$-\frac{1}{2\pi\alpha'} \int d^2\sigma [\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+]. \tag{13}$$

Since $\Psi^\mu(\sigma,\tau)$ is a spinor, we can impose the following boundary conditions (the first one can always be reached by redefinition of the fields), $\psi_+^\mu(\pi,\tau) =$

$\psi^\mu_-(\pi,\tau)$, $\psi^\mu_+(0,\tau) = \pm\psi^\mu_-(0,\tau)$. The "+" sector is called the Ramond (R) sector, and the "$-$" sector is called the Neveu-Schwartz (NS) sector. Therefore, the solution to the equations of motion (merging ψ_- and ψ_+) is of the form

$$\psi^\mu_\pm(\sigma,\tau) = \frac{1}{\sqrt{2}} \sum_{n\in\mathbb{Z}+r} \psi^\mu_n e^{-in(\sigma\pm\tau)}, \tag{14}$$

where $r = 0$ for R-sector, and $r = 1/2$ for NS-sector.

Following the ordinary quantization procedure we get the following commutation relations $\{\psi^\mu_m, \psi^\nu_n\} = \eta^{\mu\nu}\delta_{m+n,0}$. To get the gauge-fixed action one adds to (12) the superghost action

$$S = \frac{1}{2\pi}\int d^2\sigma \left(\beta_+\partial_-\gamma^+ + \beta_-\partial_+\gamma^-\right), \tag{15}$$

where β and γ denote conjugated fields of dimension 3/2 and $-1/2$ respectively; they are Bose fields. These fields have the following mode expansions

$$\gamma^\pm(\sigma) = \sum_n \gamma_n e^{\pm in\sigma}, \quad \beta^\pm(\sigma) = \sum_n \beta_n e^{\pm in\sigma}. \tag{16}$$

The superghosts satisfy the commutation relations $[\gamma_m, \beta_n] = \delta_{m+n,0}$. In the NS sector we have $\beta_n,\ n \in \mathbb{Z} + \frac{1}{2}$, $\gamma_n,\ n \in \mathbb{Z} + \frac{1}{2}$ and in the R sector we have $\beta_n,\ n \in \mathbb{Z}$, $\gamma_n,\ n \in \mathbb{Z}$ A general (ghost number q) state $|q\rangle$ is defined to obey [26, 27]

$$\beta_n|q\rangle = 0,\, n > -q + 3/2, \quad \gamma_n|q\rangle = 0,\, n \geq q - 1/2.$$

Where $q \in \mathbb{Z}$ for the NS sector and $q \in \frac{1}{2} + \mathbb{Z}$ for the R sector.

1.5. NSR Fermions String Field Action

The cubic action for NSR superstring field theory is [14]-[16]:

$$S = \int Y_{-2}\Psi\star Q_{NS}\Psi + \frac{2}{3}\int Y_{-2}\Psi\star\Psi\star\Psi + \int Y_{-1}\Phi\star Q_R\Phi + 2\int Y_{-1}\Phi\star\Psi\star\Phi. \tag{17}$$

Here Q_{NS} and Q_R are the BRST charges in NS and R sectors, $\int$ and $\star$ are Witten's string integral and star product. Y_{-2}, Y_{-1} are picture-changing operators. States in the extended Fock space $\mathcal{H}$ are created by the modes of the matter fields X^μ and ψ^μ, conformal ghosts b, c and superghosts β, γ:

$$\text{NS:} \qquad \Psi = \sum_{m,j,i\in\mathbb{N}+\frac{1}{2}} A_{i\ldots}(x)\, \beta_{-i}\ldots\gamma_{-j}\ldots b_{-k}\ldots c_{-l}\ldots\alpha^\mu_{-n}\ldots\psi^\nu_{-m}|0\rangle, \tag{18}$$

$$\text{R:} \qquad \Phi = \sum_{m,j,i\in\mathbb{N}} \Psi^A{}_{i\ldots}(x)\, \beta_{-i}\ldots\gamma_{-j}\ldots b_{-k}\ldots c_{-l}\ldots\alpha^\mu_{-n}\ldots\psi^\nu_{-m}|0\rangle_{-1/2}. \tag{19}$$

The characteristic feature of the action (17) is the choice of the 0 picture for the string field Ψ. The vacuum $|0\rangle_{-1/2}$ in the R sector is defined as $|0\rangle_{-1/2} = e^{-\phi(0)/2}|0\rangle$, where $|0\rangle$ stands for the $SL(2,\mathbb{R})$-invariant vacuum. In the description of the open NSR superstring the string field Ψ belongs to the GSO+ sector.

1.6. Feynman Diagrams in SFT

1.6.1. Siegel Gauge

The gauge invariance (8) of SFT action (7) requires a gauge fixing procedure. The Siegel gauge is

$$b_0\Psi = 0. \tag{20}$$

The advantage of this gauge is a link to the first quantized string due to the form of the propagator in this gauge (see below (23)).

We outline here the gauge-fixing for the free part of the action (7) (the nonlinear case is much more involved and can be found in the literature [25]). In this case the free action is simply invariant under $\delta\Psi = Q\Lambda$, $G_\Lambda = 0$. The Faddeev-Popov determinant $\det(b_0Q)$ calculated from (20) creates the ghost Ψ_0 and antighost string fields. The b_0 factor can be absorbed into the definition of the antighost field Ψ_2 resulting in the constraint $\delta(b_0\Psi_2)$ and the ghost action

$$\int \Psi_2 \star Q\Psi_0. \tag{21}$$

This action is obviously again gauge-invariant under $\delta\Psi_0 = Q\Lambda$. The Siegel gauge-fixing of this invariance creates one more term of the form (21) with one more ghost-antighost pair Ψ_1, Ψ_3. This infinite procedure can be packed into a compact form if one introduces the string field containing all ghost numbers with the action

$$\int \Psi \star Q\Psi \tag{22}$$

and the Siegel gauge condition (20). In this gauge:

$$\int \Psi \star Q\Psi = \int \Psi \star c_0L_0\Psi$$

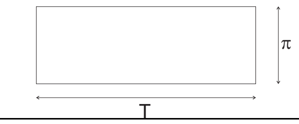
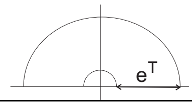
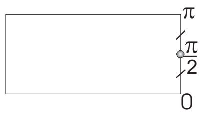
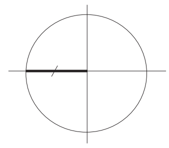
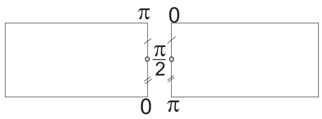
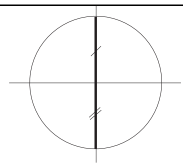
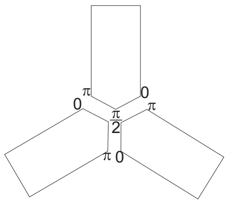
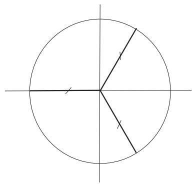
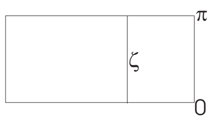
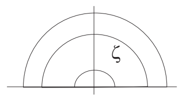
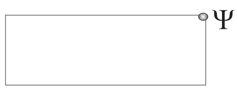
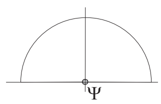
and the propagator is obtained to be

$$\Delta = \frac{b_0}{L_0}. \tag{23}$$

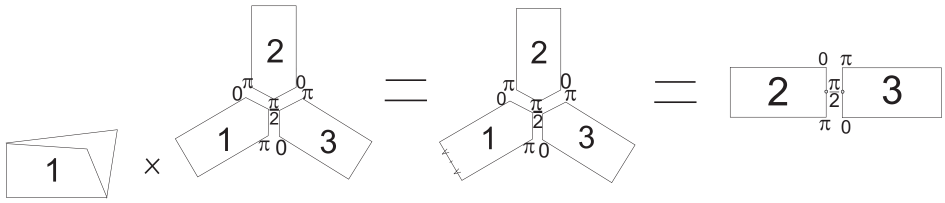
1.6.2. Elements of Diagram Technique

As in quantum field theory [17] different processes describing string interactions correspond to diagrams in SFT.

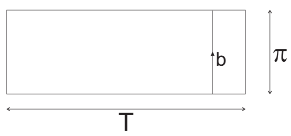
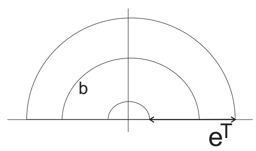
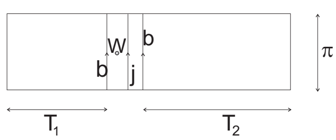
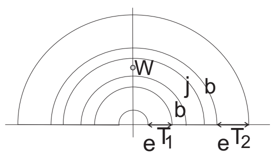
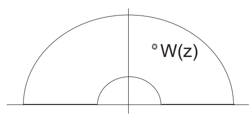
Below we present the table in which a correspondence between analytical expressions of the theory elements and their graphical presentation is established.

No	Analytical expression	Graphical presentation	Conformal presentation
1	$\frac{1}{L_0} = \int_0^\infty e^{-L_0 T} dT$		
2	Folded strip $\langle V_1\| \equiv \langle I\|$		
3	Two string interaction $\langle V_2\|$		
4	Three string interaction $\langle V_3\|$		
5	Integral operator $\xi = \frac{1}{2\pi}\int_0^\pi \zeta(\sigma)d\sigma$		
6	External state $\|\Psi\rangle$ in the infinite past $\tau = -\infty$		

Note that using the relation $\langle I|V_3\rangle = |V_2\rangle$ we may rewrite a diagram with the vertex $|V_2\rangle$ as a diagram with the vertices $|I\rangle$ and $|V_3\rangle$.



All the above rules formally work both for the bosonic and the fermionic strings. The difference in the fermionic string is in the a form of propagator and a presence of additional insertions in the point.

No	Analytical expression	Graphical presentation	Conformal presentation
1	Bosonic string propagator $\Delta_B = \frac{b_0}{L_0}$		
2	Fermionic string propagator $\Delta_F = \frac{b_0}{L_0} W Q \frac{b_0}{L_0}$		
3	Local operator $W(\sigma, \tau)$		

1.6.3. Feynman diagrams

Using the rules above one can write a matrix element of any diagram.

In particular an N-point tree diagram for superstring can be schematically presented in the form [16]

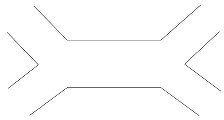
$$\langle V_3|Y_{-2}^{(1)}...\langle V_3|Y_{-2}^{(N-2)}\Delta^{(1')}|V_2\rangle...\Delta^{(N-3')}|V_2\rangle|\Psi_1\rangle...|\Psi_N\rangle. \tag{24}$$

An N-point one-loop diagram is a connected part of

$$\langle V_3|Y_{-2}^{(1)}...\langle V_3|Y_{-2}^{(N)}\Delta^{(1)}|V_2\rangle...\Delta^{(N)}|V_2\rangle|\Psi_1\rangle. \qquad (25)$$

The simplest tree diagram has the form

$${}_{12i}\langle V_3|_{34j}\langle V_3|Y_{-2}^{i}Y_{-2}^{j}\Delta^{i}|V_2\rangle_{ij}|\Psi_1\rangle_1|\Psi_2\rangle_2|\Psi_3\rangle_3|\Psi_4\rangle_4 \Longleftrightarrow$$

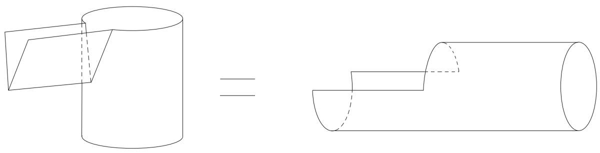


Here and above $\Psi_1,\ \Psi_2, ...,\ \Psi_N$ are external states. This diagram was calculated in [14].

The simplest one-loop diagram is the tadpole diagram (for the bosonic string this diagram was considered in [18])

$${}_3\langle\Psi|_{12}\langle V_2|\Delta Y_{-2}|V_3\rangle_{123}. \qquad (26)$$

The tadpole diagram is topologically equivalent to the diagram which describes the open-closed transition:



To calculate this diagram we simplify it using algebraic properties of vertices and picture-changing operators. The NS propagator can be represented in the form [16]

$$\Delta = A + Q\frac{b_0}{L}A\frac{b_0}{L}Q - Q\frac{b_0}{L}A - A\frac{b_0}{L}Q. \qquad (27)$$

Here $A \equiv \xi(i)Q\xi(-i) = \xi Q\xi^*$ ($\xi(z)$ appears after bosonization of $\beta-\gamma$ system), and we drop the index of operator L_0. Inserting this propagator into (26) we obtain a representation for the tadpole as a sum of 4 terms

$${}_{12}\langle V_2|\Delta Y_{-2}|V_3\rangle_{123} = \mathcal{T}, \qquad \mathcal{T} = \sum_{i=1}^{4}\mathcal{T}_i, \qquad (28)$$

$$\mathcal{T}_1 = {}_{12}\langle V_2|AY_{-2}|V_3\rangle_{123}, \quad \mathcal{T}_2 = {}_{12}\langle V_2|Q\frac{b_0}{L}A\frac{b_0}{L}QY_{-2}|V_3\rangle_{123},$$

$$\mathcal{T}_3 = -{}_{12}\langle V_2|Q\frac{b_0}{L}AY_{-2}|V_3\rangle_{123}, \quad \mathcal{T}_4 = -{}_{12}\langle V_2|A\frac{b_0}{L}QY_{-2}|V_3\rangle_{123},$$

here we put the propagator on the line number one and omit label of line where it does not lead to misunderstanding.

Let us transform $\mathcal{T}_3$ to the form of $\mathcal{T}_4$. To this purpose we use the explicit form for A, $\mathcal{T}_3 = {}_{12}\langle V_2|Q\frac{b_0}{L}\xi Q\xi^* Y_{-2}|V_3\rangle_{123}$, and carry it to the left. Changing the line (Fock space) index for ξ^* using overlap (i.e. $\xi^{(1)}|V_3\rangle = \xi^{(2)}|V_3\rangle$), then moving $\xi^{(2)}$ to the left as the second line operators (anti)commut with first line operators:

$$ {}_{12}\langle V_2|Q\frac{b_0}{L}\xi Q\xi^{(2)*} Y_{-2}|V_3\rangle_{123} = {}_{12}\langle V_2|\xi^{(2)*}Q\frac{b_0}{L}\xi Q Y_{-2}|V_3\rangle_{123} $$

and using the overlap for $\langle V_2|$ we get

$$ \mathcal{T}_3 = {}_{12}\langle V_2|\xi^{(1)*}Q\frac{b_0}{L}\xi Q Y_{-2}|V_3\rangle_{123}. $$

BRST invariance of $|V_2\rangle$ gives

$$ \mathcal{T}_3 = -{}_{12}\langle V_2|\xi^{(1)*}Q\frac{b_0}{L}\xi Y_{-2}(Q^{(2)}+Q^{(3)})|V_3\rangle_{123} $$

and moving $Q^{(2)}$ and $Q^{(3)}$ to the left we get $\mathcal{T}_3 = \mathcal{T}_{3,1} + \mathcal{T}_{3,2}$, where

$$ \begin{aligned} \mathcal{T}_{3,1} &= -Q^{(3)}{}_{12}\langle V_2|\xi^{(1)*}Q\frac{b_0}{L}\xi Y_{-2}|V_3\rangle_{123}\,, \\ \mathcal{T}_{3,2} &= -{}_{12}\langle V_2|Q^{(2)}\xi^{(1)*}Q\frac{b_0}{L}\xi Y_{-2}|V_3\rangle_{123}. \end{aligned} $$

By the same trick we present $\mathcal{T}_{3,2}$ as

$$ \mathcal{T}_{3,2} = {}_{12}\langle V_2|Q^{(1)}\xi^{(1)*}Q\frac{b_0}{L}\xi^{(2)} Y_{-2}|V_3\rangle_{123} $$

move $\xi^{(2)}$ to the left, and change $(2) \to (1)$:

$$ \mathcal{T}_{3,2} = {}_{12}\langle V_2|\xi^{(1)}Q^{(1)}\xi^{(1)*}Q\frac{b_0}{L} Y_{-2}|V_3\rangle_{123}. $$

Here we also used BRST invariance of Y_{-2}, $[Q, Y_{-2}] = 0$ and $[\xi, Y_{-2}] = 0$.

Combining $\mathcal{T}_4$ and $\mathcal{T}_{3,2}$ we obtain

$$ \mathcal{T}_4 + \mathcal{T}_{3,2} = -{}_{12}\langle V_2|AY_{-2}|V_3\rangle_{123} $$

that cancels $\mathcal{T}_1$ in the $\mathcal{T}$ (see (28)).

The second term in (28) can be modified by the same trick as above to give

$$ \begin{aligned} \mathcal{T}_2 &= -{}_{12}\langle V_2|Q\frac{b_0}{L}A\frac{b_0}{L}(Q^{(2)}+Q^{(3)})Y_{-2}|V_3\rangle_{123} \\ &= -{}_{12}\langle V_2|Q^{(2)}Q\frac{b_0}{L}A\frac{b_0}{L}Y_{-2}|V_3\rangle_{123} - Q^{(3)}{}_{12}\langle V_2|Q\frac{b_0}{L}A\frac{b_0}{L}Y_{-2}|V_3\rangle_{123} \\ &= -Q^{(3)}{}_{12}\langle V_2|Q\frac{b_0}{L}A\frac{b_0}{L}Y_{-2}|V_3\rangle_{123}\,. \end{aligned} $$

So by using formal algebraic manipulations based of the properties of the vertices we prove the tadpole graph to be BRST exact

$$\mathcal{T} = -Q^{(3)}{}_{12}\langle V_2|\xi^{(1)*}Q\frac{b_0}{L}\xi Y_{-2}|V_3\rangle_{123} - Q^{(3)}{}_{12}\langle V_2|Q\frac{b_0}{L}A\frac{b_0}{L}Y_{-2}|V_3\rangle_{123}.$$

2. Lecture 2

2.1. Identity over a physical vacuum

Identity over a physical vacuum is built in [19] and is given by

$$|I^{bc}\rangle = b(\frac{\pi}{2})b(-\frac{\pi}{2})|I^{ghost}\rangle = b(\frac{\pi}{2})b(-\frac{\pi}{2})e^{U}|+\rangle, \tag{29}$$

where $U = \sum_{n=1}^{\infty}(-)^n c_{-n}b_{-n}$.

By definition $|I^{ghost}\rangle$ solves the mode overlap[2] [19]

$$[c_n + (-)^n c_{-n}]|I^{ghost}\rangle = 0, \qquad [b_n - (-)^n b_{-n}]|I^{ghost}\rangle = 0. \tag{30}$$

In particular, from (30) it follows that $b(\pm\frac{\pi}{2})|I^{ghost}\rangle \neq 0$.

The field overlap conditions on $|I^{ghost}\rangle$ is

$$[c(\sigma) + c(\pi - \sigma)]|I^{ghost}\rangle = \sum_{n=-\infty}^{\infty}[e^{in\sigma}c_n + (-)^n e^{-in\sigma}c_n]|I^{ghost}\rangle =$$

$$= \sum_{n=-\infty}^{\infty} e^{in\sigma}[c_n + (-)^n c_{-n}]|I^{ghost}\rangle = (\text{ due to (30)}) = 0. \tag{31}$$

Hence, $|I^{ghost}\rangle$ solves the mode and the field overlap conditions.

2.2. Overlap verification for $|I^{bc}\rangle$

To verify the overlap conditions on $|I^{bc}\rangle$ we calculate the commutator of the insertion $b_+b_- \equiv b(\frac{\pi}{2})b(-\frac{\pi}{2})$ with the overlap condition.

$$[c(\sigma) + c(\pi - \sigma), b_+b_-] = \tag{32}$$
$$= \sum_{k=-\infty}^{\infty}(i^{-k}e^{ik\sigma} + i^k e^{-ik\sigma})b_- - \sum_{k=-\infty}^{\infty}(i^k e^{ik\sigma} + i^{-k}e^{-ik\sigma})b_+.$$

From (32) we may try to get the mode overlap condition multiplying both parts by $e^{ik\sigma}$ and integrating by σ. We get:

$$[c_n + (-)^n c_{-n}]b_+b_-|I^{ghost}\rangle = 2i^n\,[(-)^n b_- - b_+]\,|I^{ghost}\rangle. \tag{33}$$

[2]A tradition makes no difference between the "mode overlap" and the "field overlap". In this text it will become clear that these are two different conditions.

Vanishing of this expression is equal to the conditions:

$$b_+|I^{ghost}\rangle = 0, \quad b_-|I^{ghost}\rangle = 0$$

which are not satisfied as remarked above. We conclude the mode overlap condition is not true for $|I\rangle$.

We continue the examination of (32) It is clear that the RHS of (32) requires a regularization. We use a small parameter ε to make a shift $\lambda \equiv i^{-1}e^{i\sigma} \to -ie^{i\sigma-\varepsilon} \equiv \lambda_\varepsilon$. So we get:

$$\xi \equiv \sum_{k=-\infty}^{\infty} i^{-k}e^{ik\sigma} \to \xi^\varepsilon = \left(\sum_{k=0}^{\infty}(\lambda_\varepsilon^k + \bar{\lambda}_\varepsilon^k) - 1\right) = \frac{1-\lambda_\varepsilon\bar{\lambda}_\varepsilon}{1-\lambda_\varepsilon-\bar{\lambda}_\varepsilon+\lambda_\varepsilon\bar{\lambda}_\varepsilon}.$$

We take the limit $\lim_{\varepsilon\to 0}\xi^{(\varepsilon)}$ and using $\lim_{\varepsilon\to 0}\lambda_\varepsilon\bar{\lambda}_\varepsilon = 1$ and $\sigma \neq \frac{\pi}{2}, \frac{3\pi}{2}$ we get that ξ is equal to zero.

Performing the second summation by the same trick it can be shown that it also vanishes. So

$$[c(\sigma) + c(\pi - \sigma), b_+b_-] = 0, \tag{34}$$

and we conclude that the field overlap commutes with the insertion. Therefore $|I^{bc}\rangle$ solves the field overlap.

Verification of overlaps for the antighost field b

$$[b_n - (-)^n b_{-n}]|I^{bc}\rangle = 0, \qquad [b(\sigma) - b(\pi - \sigma)]|I^{bc}\rangle = 0. \tag{35}$$

can be easily made because $\{b_n - (-)^n b_{-n}, b_+b_-\} = 0$ and $\{b(\sigma) - b(\pi - \sigma), b_+b_-\} = 0$ and the overlaps commute with the insertion.

2.3. Identity over conformal vacuum

In this subsection we establish the equivalence of the vertex built over the physical vacuum and the vertex built in [19, 20] over the conformal vacuum.

2.3.1. Operator derivation of expression without insertion

Our goal is to move the insertion b_+b_- through the exponential (29) and pass to the conformal vacuum.

We begin with computation of the commutator (see (29)):

$$[b(\sigma), U] = [\sum_{n=-\infty}^{\infty} e^{in\sigma}b_n, \sum_{k=1}^{\infty}(-)^k c_{-k}b_{-k}] = \sum_{k=1}^{\infty}(\mp i)^k b_{-k},$$

and

$$b_\pm e^U = e^U\left(b_\pm + \sum_{k=1}^{\infty}(\mp i)^k b_{-k}\right),$$

Moving the insertion through the exponent we get

$$b_+ b_- e^U = e^U (b_+ + \sum_{n=1}^{\infty} (-i)^n b_{-n})(b_- + \sum_{k=1}^{\infty} i^k b_{-k}).$$

Over $|+\rangle$ we may drop b_k with $k > 0$ and we have

$$b_+ b_- e^U |+\rangle = e^U (2 \sum_{n=1}^{\infty} (-i)^n b_{-n} + b_0)(2 \sum_{k=1}^{\infty} i^k b_{-k} + b_0)|+\rangle =$$

$$= 2e^U (2 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} (-i)^n i^k b_{-n} b_{-k} + \sum_{k=1}^{\infty} ((-i)^k - i^k) b_{-k} b_0))|+\rangle.$$

The structure $(-i)^k - i^k$ is not zero only for odd k ($k = 2m+1$) and we rewrite the answer as:

$$-4ie^U (i \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} i^{k-m} b_{-m} b_{-k} + \sum_{k=1}^{\infty} \sum_{m=0}^{\infty} (-)^m \delta_{k,2m+1} b_{-k} b_0)|+\rangle.$$

Now this expression may be rewritten with the conformal vacuum. We extract b_{-1} modes out of the sums:

$$-4ie^U (i \sum_{k=2}^{\infty} \sum_{m=2}^{\infty} i^{k-m} b_{-m} b_{-k} + i \sum_{k=2}^{\infty} i^{k-1} b_{-1} b_{-k} + i \sum_{m=2}^{\infty} i^{1-m} b_{-m} b_{-1} +$$

$$+ \sum_{k=3}^{\infty} \sum_{m=1}^{\infty} (-)^m \delta_{k,2m+1} b_{-k} b_0 + b_{-1} b_0)|+\rangle.$$

Introducing the notations:

$$B^0 \equiv \sum_{k=3}^{\infty} \sum_{m=1}^{\infty} (-)^m \delta_{k,2m+1} b_{-k}, \qquad B^1 \equiv -2 \sum_{k=2}^{\infty} \sum_{m=1}^{\infty} (-)^m \delta_{k,2m} b_{-k},$$

$$B^2 \equiv \sum_{k=2}^{\infty} \sum_{m=2}^{\infty} i^{k-m+1} b_{-m} b_{-k},$$

we have $b_+ b_- e^U |+\rangle = -4ie^U (B^2 + B^1 b_{-1} + B^0 b_0 + b_{-1} b_0)|+\rangle$. Taking into account that the conformal vacuum $|0\rangle$ is related to $|+\rangle$ as $|+\rangle = c_0 c_1 |0\rangle$, and separating the term $1 - c_{-1} b_{-1}$ from e^U we get

$$b_+ b_- e^U |+\rangle = -4i \exp\left\{ \sum_{n=2}^{\infty} (-)^n c_{-n} b_{-n} \right\} \cdot$$

$$\cdot (1 - c_{-1} b_{-1})(B^2 + B^1 b_{-1} + B^0 b_0 + b_{-1} b_0) c_0 c_1 |0\rangle.$$

Moving the braces $(1-c_{-1}b_{-1})$ to the conformal vacuum and using $b_{-1}|0\rangle = 0$ we get

$$(1-c_{-1}b_{-1})(B^2+B^1b_{-1}+B^0b_0+b_{-1}b_0)c_0c_1|0\rangle =$$
$$=(1-B^1c_0+B^0(c_1-c_{-1})-B^2(c_1-c_{-1})c_0)|0\rangle.$$

Finally:

$$b_+b_-\exp U|+\rangle = -4i\exp\left\{\sum_{n=2}^{\infty}(-)^n c_{-n}b_{-n}\right\}\times$$
$$\times\left(1-B^1c_0+B^0(c_1-c_{-1})+B^2(c_1-c_{-1})c_0\right)|0\rangle.$$

Now it is needed to prove that the quadratic in c term is the product of the two linear ones, i.e. we have to prove that $B^2 = B^0B^1$. To this aim the form of the quadratic term can be modified.

$$B^2=\sum_{m,k=2}^{\infty} i^{k-m-1}b_{-k}b_{-m} = (\text{due to } \{b_{-k},b_{-n}\}=0) =$$
$$=\frac{1}{2i}\sum_{m,k=2}^{\infty} i^{k-m}(1-(-)^{k-m})b_{-k}b_{-m} = \sum_{l=-\infty}^{\infty}\sum_{m,k=2}^{\infty}(-)^l\delta_{k-m,2l+1}b_{-k}b_{-m}.$$

For B^0B^1 we have

$$B^0B^1 = 2\sum_{\substack{p\geq 1,\\ n\geq 1}}\sum_{\substack{k\geq 2,\\ m\geq 2}}(-)^{n-p}\delta_{k,2n}\delta_{m,2p+1}b_{-k}b_{-m}$$

Then we make the following replacement:

$$l=n-p= \text{ from } \delta\text{ -s } =(k-m+1)/2 \quad\Rightarrow\quad k-m=2l-1,$$

here l is arbitrary. Therefore,

$$B^0B^1 = 2\sum_{l}\sum_{\substack{k\geq 2(even),\\ m\geq 2(odd)}}(-)^l\delta_{k-m,2l-1}b_{-k}b_{-m} = (\text{due to } \{b_{-k},b_{-n}\}=0) =$$
$$=\sum_{l}\sum_{\substack{k\geq 2,\\ m\geq 2}}(-)^l\delta_{k-m,2l-1}b_{-k}b_{-m} = B^2.$$

So, the preexponential factor may be written as:

$$(1-B^1c_0+B^0(c_1-c_{-1})-B^2(c_1-c_{-1})c_0) = \exp(-B^1c_0+B^0(c_1-c_{-1})).$$

After gathering the pieces together the full identity becomes:

$$|I^{bc}\rangle = b(\frac{\pi}{2})b(-\frac{\pi}{2})\exp\left\{\sum_{n=1}^{\infty}(-)^n c_{-n}b_{-n}\right\}|+\rangle \equiv -4i\exp W|0\rangle, \qquad (36)$$

$$W = \sum_{\substack{n\geq 2,\\ m\geq -1}} b_{-n}C_{nm}c_{-m}|0\rangle.$$

This equality shows that the identity over the physical vacuum can be rewritten as the identity over the conformal vacuum.

Changing the indexes: $m\to k,\ k\to n$ in the formulae for $B^0,\ B^1$ we get [20]

$$\begin{aligned} C_{nm} &= -(-1)^n\delta_{n,m} + \tilde{C}_{nm}; \\ \tilde{C}_{nm} &= \sum_k (-1)^k[\delta_{n,2k+1}(\delta_{m,-1}-\delta_{m,1}) + 2\delta_{n,2k}\delta_{m,0}]. \end{aligned} \qquad (37)$$

2.3.2. Overlap verification

Let us show once more that $|I^{bc}\rangle$ does not solve the mode overlap. It is easy to prove that if only creation operators stand in the mode overlap then overlap is not satisfied. The overlap operator moves freely and gives nonzero acting on the vacuum, because there are only creation operators in the exponent . It takes place in our case:

$$[c_{n_0} + (-)^{n_0}c_{n_0}]\exp W|0\rangle \neq 0, \quad n_0 = \pm 1, 0, \qquad (38)$$

because $c_{n_0}|0\rangle \neq 0$ for $n_0 = \pm 1, 0$.

To check the field overlap we start with a calculation of $c(\sigma)|I^{bc}\rangle$. Dividing $c(\sigma)$ the creation and the annihilation parts with respect to the conformal vacuum we get using eq. (36):

$$\begin{aligned} c(\sigma)|I^{bc}\rangle &= \sum_{n=-\infty}^{\infty} e^{in\sigma}c_n|I^{bc}\rangle = (\sum_{n\leq 1} + \sum_{n\geq 2})e^{in\sigma}c_n e^W|0\rangle = \\ &= (e^W\sum_{n\leq 1}e^{in\sigma}c_n|0\rangle + \sum_{n\geq 2}e^{in\sigma}e^W(c_n + \sum_{m\geq -1}C_{nm}c_{-m}))|0\rangle; \end{aligned}$$

and because the first term in the brackets is the annihilation operator acting on vacuum:

$$c(\sigma)|I^{bc}\rangle = e^W(\sum_{n\leq 1}e^{in\sigma}c_n + \sum_{n\geq 2}e^{in\sigma}\sum_{m\geq -1}C_{nm}c_{-m})|0\rangle. \qquad (39)$$

It is convenient to separate out the diagonal part of $C_{nm} = -(-)^n\delta_{nm} + \tilde{C}_{nm}$. Than

$$c(\sigma)|I^{bc}\rangle = e^W(\sum_{n\leq 1}e^{in\sigma}c_n - \sum_{n\leq -2}e^{-in\sigma}(-)^n c_n + \sum_{n\geq 2}e^{in\sigma}\sum_{m\geq -1}\tilde{C}_{nm}c_{-m})|0\rangle.$$

Analogously for $c(\pi-\sigma)$:

$$\begin{aligned} c(\pi-\sigma)|I^{bc}\rangle &= e^W(\sum_{n\le 1} e^{-in\sigma}(-)^n c_n - \sum_{n\le -2} e^{in\sigma} c_n + \\ &+ \sum_{n\ge 2} e^{-in\sigma}(-)^n \sum_{m\ge -1} \tilde{C}_{nm} c_{-m})|0\rangle, \end{aligned}$$

and as the next result we obtain:

$$\begin{aligned} &(c(\sigma)+c(\pi-\sigma))|I^{bc}\rangle = \\ &= e^W(\sum_{n\le 1} e^{in\sigma} c_n - \sum_{n\le -2} e^{in\sigma} c_n - \sum_{n\le -2} e^{-in\sigma}(-)^n c_n + \\ &+ \sum_{n\le 1} e^{-in\sigma}(-)^n c_n + \sum_{n\ge 2}\sum_{m\ge -1}(e^{in\sigma}\tilde{C}_{nm}c_{-m} + e^{-in\sigma}(-)^n\tilde{C}_{nm}c_{-m}))|0\rangle. \end{aligned} \tag{40}$$

From the former four sums only 6 terms remain due to the difference of summation limits (omitting e^W):

$$e^{i\sigma}c_1 + c_0 + e^{-i\sigma}c_{-1} - e^{-i\sigma}c_1 + c_0 - e^{i\sigma}c_{-1} = 2c_0 + 2i\sin\sigma(c_1 - c_{-1}).$$

Using (37) the last two terms in (40) can be presented as

$$\begin{aligned} &(c_1 - c_{-1})\sum_{n\ge 2}(e^{in\sigma} + (-)^n e^{-in\sigma})\sum_k (-)^k \delta_{n,2k+1} + \\ &+2c_0\sum_{n\ge 2}(e^{in\sigma} + (-)^n e^{-in\sigma})\sum_k (-)^k \delta_{n,2k} = (c_1 - c_{-1})\zeta + 2c_0\eta, \end{aligned} \tag{41}$$

where

$$\zeta = \sum_{k=1}(-)^k[e^{i(2k+1)\sigma} - e^{-i(2k+1)\sigma}], \tag{42}$$

$$\eta = \sum_{k=1}(-)^k[e^{2ik\sigma} + e^{-2ik\sigma}]. \tag{43}$$

Let us consider the first sum in the RHS of (42). As before we see that the sum looks like the geometric series but here $|e^{i(2k+1)\sigma}| = 1$ (it must be smaller then one for converging series). Therefore we introduce the small parameter ε and deal with a regularized sum:

$$\zeta^\varepsilon \equiv \sum_{k=1}(-)^k[e^{i(2k+1)\sigma - k\varepsilon} - e^{-i(2k+1)\sigma - k\varepsilon}].$$

Introducing the notation $\lambda_\varepsilon = e^{-2i\sigma-\varepsilon}$ we have

$$\zeta^\varepsilon = \frac{e^{i\sigma} - e^{-i\sigma} + \lambda_\varepsilon e^{i\sigma} - \bar{\lambda}_\varepsilon e^{-i\sigma}}{1 + \lambda_\varepsilon + \bar{\lambda}_\varepsilon + \lambda_\varepsilon\bar{\lambda}_\varepsilon} - e^{i\sigma} + e^{-i\sigma}.$$

Removing the regularization we obtain $\zeta = \lim_{\varepsilon\to 0}\zeta^{\varepsilon} = -2i\sin\sigma$. Using the same procedure as above we have

$$\eta^{\varepsilon} \equiv \sum_{k=1}(-)^k[\bar{\lambda}_{\varepsilon}^k + \lambda_{\varepsilon}^k] = -\frac{2\lambda_{\varepsilon}\bar{\lambda}_{\varepsilon}}{1+\lambda_{\varepsilon}+\bar{\lambda}_{\varepsilon}+\lambda_{\varepsilon}\bar{\lambda}_{\varepsilon}},$$

$\varepsilon \to 0$ limit gives

$$\eta = \lim_{\varepsilon\to 0}\eta^{\varepsilon} = \lim_{\varepsilon\to 0}\frac{2+e^{-2i\sigma-\varepsilon}+e^{2i\sigma-\varepsilon}}{1+e^{-2i\sigma-\varepsilon}+e^{2i\sigma-\varepsilon}+e^{-2\varepsilon}} - 2 = -1. \tag{44}$$

Combining the pieces together we get for (41)

$$2c_0 + 2i\sin\sigma(c_1 - c_{-1}) - (c_1 - c_{-1})2i\sin\sigma - 2c_0 = 0. \tag{45}$$

This equation implies that the field overlap condition takes place. Keeping ε – regularization one writes the field overlap condition as

$$[c(\sigma + i\frac{\varepsilon}{2}) + c(\pi - \sigma + i\frac{\varepsilon}{2})]|I^{bc}\rangle = 0. \tag{46}$$

2.4. BRST invariance of identity

The string action (7) proposed by Witten [1] has the symmetry under the gauge transformation (8). Besides this symmetry it is invariant under transformation $\delta\Psi = A\Psi$, where A is any differentiation of algebra, i.e.

$$A(\Psi_1 \star \Psi_2) = A\Psi_1 \star \Psi_2 + (-)^{grading}\Psi_1 \star A\Psi_2, \qquad \int A\Psi = 0. \tag{47}$$

Moreover it must obey $[A, Q] = 0$. An appropriate operator is $K_n \equiv L_n - (-)^n L_{-n}$ where L_n is Virassoro operator. According to [19, 22] K_n has the above-listed properties. From the second property $\int K_n\Psi = \langle I|K_n|\Psi\rangle = 0$ it follows that $\langle I|K_n = 0$. Note that the given expression is obeyed only on the full vertex, i.e. the vertex containing the matter and ghost parts of bosonic string.

2.4.1. K_n anomaly

We consider some properties of operator K_n. The K_n algebra [19]

$$[K_n, K_m] = (n-m)K_{n+m} - (-)^m(n+m)K_{n-m}$$

is the consequence of Virassoro algebra. If we introduce f, the eigenvector for K_n for each n, such that $K_n f = k_n f$, then k_n (which is a C-number) must satisfy

$$0 = (n-m)k_{n+m} - (-)^m(n+m)k_{n-m}.$$

The unique solution is $k_{2N} = k(-)^N N$, $k_{2N+1} = 0$ N=1,2,...; k is a number is just the anomaly.

2.4.2. Matter

The identity for the matter sector reads [19, 22]

$$|I^x\rangle = \exp\left\{-\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-)^n}{n}\alpha_{-n}\alpha_{-n}\right\}|0\rangle \tag{48}$$

We get the anomaly acting on it by K^x_{2N} [19]:

$$K^x_{2N}|I^x\rangle = -\frac{D}{2}N(-)^n|I^x\rangle. \tag{49}$$

2.4.3. Ghosts

Here we will use the first representation of the identity for the ghosts i.e. we'll write the insertions.

$$|I^{bc}\rangle = b_+b_-\exp\left\{\sum_{n=1}^{\infty}(-)^n c_{-n}b_{-n}\right\}|+\rangle = b_+b_-e^U|+\rangle. \tag{50}$$

The operator K_n has the form $K^{bc}_{2N} = L^{bc}_{2N} - L^{bc}_{-2N}$ and therefore

$$\begin{aligned} K^{bc}_{2N} &= \sum_m(2N+m)b_{2N-m}c_m - \sum_m(-2N+m)b_{-2N-m}c_m = \\ &= \sum_m(2N+m)b_{2N-m}c_m + \sum_m(2N-m)b_{-2N-m}c_m. \end{aligned} \tag{51}$$

Acting by this operator on the identity we calculate the following commutator

$$[K^{bc}_{2N}, b_\pm] = 8N(-)^N b_\pm - (-)^n\sum_m(\pm i)^m m b_m + (-)^n\sum_l(\pm i)^l l b_l,$$

i.e $[K^{bc}_{2N}, b_\pm] = 8N(-)^N b_\pm$. Next we act on the exponential by K_n

$$K^{bc}_{2N}e^U|+\rangle = -3N(-)^N e^U|+\rangle.$$

Gathering the pieces we get the anomaly [19]

$$K^{bc}_{2N}|I^{bc}\rangle = [2\cdot 8N(-)^N - 3N(-)^N]|I^{bc}\rangle = 13N(-)^N|I^{bc}\rangle. \tag{52}$$

Recall that, the same anomaly for D=26 but with the opposite sign presents in the matter identity (49). So by taking into account the matter and the ghosts we get the total cancellation of anomalies,

$$K_{2N}|I\rangle = (-13N(-)^N + 13N(-)^N)|I\rangle = 0,$$

$K_{2N}=K^x_{2N}+K^{bc}_{2N}$, $|I\rangle=|I^x\rangle\otimes|I^{bc}\rangle$. One can write Q as (we drop the terms which contain c_0 because they annihilate the physical vacuum $|+\rangle$)

$$\begin{aligned} Q &= \sum_{n=1}^{\infty}[c_{-n}(\mathcal{L}_n-(-)^n\mathcal{L}_{-n})+\mathcal{L}_{-n}(c_n+(-)^n c_{-n})-3n(-)^n c_{-2n}]= \\ &= \sum_{n=1}[\frac{1}{2}c_{-n}K^x_n+\frac{1}{2}c_{-n}K_n+\mathcal{L}_{-n}(c_n+(-)^n c_{-n})-3n(-)^n c_{-2n}], \end{aligned} \quad (53)$$

where $\mathcal{L}_n\equiv L^x_n+\frac{1}{2}L^{(bc)}_n$, and $K^{x,\ ghost}_n=L^{x,\ ghost}_n-(-)^n L^{x,\ ghost}_{-n}$, $K_n=K^x_n+K^{ghost}_n$. Hence, the verification of BRST invariance is reduced to the verification of K_n invariance.

Now we can examine BRST invariance of the identity. We start with Q acting it on the insertion b_+b_-. Taking into account $\{Q,b_n\}=L^x_n+L^{(bc)}_n\equiv L_n$, we get

$$\begin{aligned} [Q,b_+b_-] &= \{Q,b_+\}b_- - b_+\{Q,b_-\}=\sum_k i^k\{Q,b_k\}b_- \\ &-\sum_k (i)^k b_+\{Q,b_k\}=\sum_k i^k L_k b_- - \sum_k (i)^k b_+ L_k = L_+b_- - b_+L_-, \end{aligned} \quad (54)$$

where $L_\pm=\sum_k(\pm i)^k L_k$. Using

$$\sum_k i^k k b_k|I^{ghost}\rangle=e^U(-\sum_{k=0}^{\infty}i^{-k}kb_{-k}+\sum_{k=1}^{\infty}i^{-k}kb_{-k})|+\rangle=0$$

we get

$$[Q,b_+b_-]e^U|+\rangle=-8b_+b_-\sum_{k=1}^{\infty}(-)^k kc_{2k}e^U|+\rangle. \quad (55)$$

Gathering together matter and ghosts we obtain

$$\begin{aligned} Qb_+b_-e^U|+\rangle|I^x\rangle &= (b_+b_-\sum_{n=1}[\frac{1}{2}c_{-n}K^x_n+\frac{1}{2}c_{-n}K_n+\mathcal{L}_{-n}(c_n+(-)^n c_{-n})- \\ &-\frac{3}{2}\,n(-)^n c_{-2n}]-8b_+b_-\sum_{k=1}^{\infty}(-)^k kc_{2k})e^U|+\rangle|I^x\rangle. \end{aligned}$$

The overlap conditions (30) for the terms with $\mathcal{L}_{-n}$ and (61) gives that the RHS of the above formula is equal to

$$b_+b_-(\sum_{n=1}[-c_{-2n}\frac{13}{2}n(-)^n-\frac{3}{2}n(-)^n c_{-2n}]-8\sum_{k=1}^{\infty}(-)^k kc_{2k})e^U|+\rangle|I^x\rangle$$

Therefore,

$$Qb_+b_-e^U|+\rangle|I^x\rangle = -8b_+b_-\sum_{n=1} n(-)^n(c_{-2n} + c_{2n})e^U|+\rangle|I^x\rangle = 0.$$

In the last equation we used the overlap condition. Q.E.D.

3. Lecture 3

3.1. Basic conformal maps

In SFT there is an alternative calculation method based on the conformal field theory (CFT). The idea is to present any given matrix element of SFT calculated according to the diagram technique presented in the table as a correlation functions on a Riemann surfaces. An information about the diagram is encoded in the geometry of the corresponding Riemann surface. To calculate a given matrix element one conformal by maps a corresponding nontrivial surface where correlation functions are unknown to a simple one say, on upper half plane (UHP), or a disk, where correlation functions are known.

To view the string world-sheet as a Riemann surface let us assemble the string coordinate τ and σ into a complex coordinate ρ

$$\rho = \tau + i\sigma.$$

The world-sheet of a freely propagating open string is a strip. The string at a given time is a vertical segment of constant τ. Let us map this strip to the UHP using the exponential function (see Fig. 1). Note that the full string

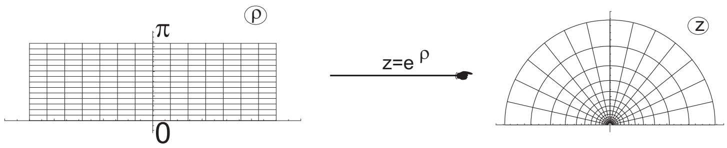


Figure 1: The map the strip into the UHP.

in the infinite past is mapped to the point $z = 0$. The two boundaries of the strip are mapped to the real line, i.e. the boundary $\sigma = 0$ is mapped to the positive half of the real axis and $\sigma = \pi$ is mapped to the negative part of the real axis. At fixed original time τ the string appears to be semicircle.

One can map the UHP into the unit disk in the w plane via the conformal transformation (see Fig. 2)

$$w = \frac{1 + iz}{1 - iz}.$$

This transformation maps the real line into the unit circle. The two boundaries are the left and right semicircles. In terms of the initial time τ fixed time positions of the string are now the arcs centered at $w = 1$ ($\tau < 0$) and at $w = -1$ ($\tau > 0$), $\tau = 0$ corresponds to a vertical line $Re\ w = 0$, the infinite past is at $w = 1$ and the infinite future is at $w = -1$. To describe

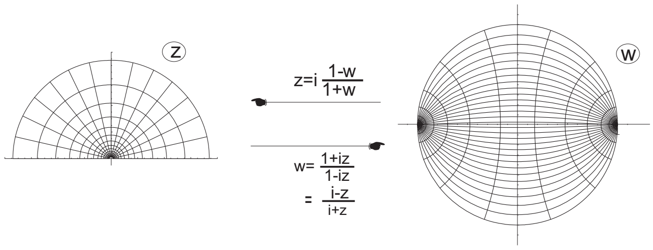


Figure 2: The map w(z) of the UHP into the disk.

the Witten's $\star$-multiplying and string $\int$-integral we need to explore more out of the map $w(z)$. In particular, we need to know images of the interior of the unit half-disk and its exterior. Separately these maps look like presented on Fig.3. Combining the map from Fig.1 with the map 1 on Fig.3

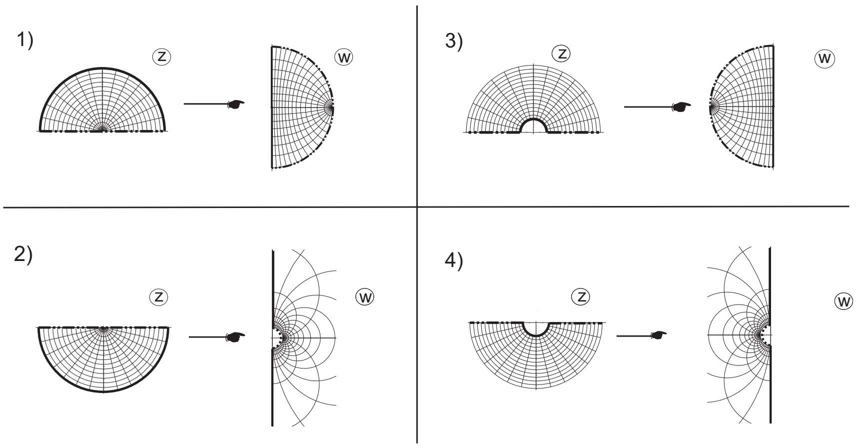


Figure 3: The separate pieces of the map $w = \frac{1+iz}{1-iz}$.

we get the map of the half-strip $\tau \leq 0$ to the right half-disk.

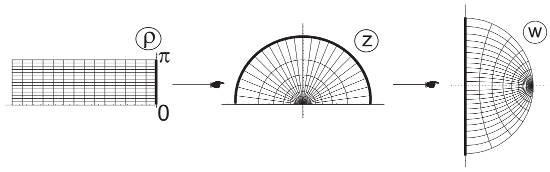


Figure 4: The map the strip into the right half-disk.

We meet the first nontrivial map (see Fig.5) when discuss the Riemann surface of string integral. It maps a half-strip with identified segments $[0, i\pi/2]$, $[i\pi/2, i\pi]$ into the unit disk and is obtained from $w(z(\rho))$ by adding one arrow $w \to w^2 \equiv u$ (see Fig.4). For interacting strings more

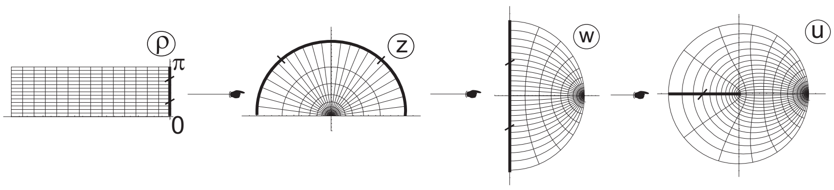


Figure 5: The map of the string integral Riemann surface.

complicated maps are of actual interest. Let us remaind that there are two different pictures for string interaction. The old light-cone picture deals with end gluing picture and in this picture there are two possible interactions of open strings: an open string can split into two open strings and two open string can join to form a single open string.

To map string with cuts into UHP one uses the Schwarz-Christoffel map that transform UHP into polygons [2].

3.2. SFT action in CFT language

To use the conformal field theory we turn to complex variables $z = e^{\tau + i\sigma}$, $\bar{z} = e^{\tau - i\sigma}$, z is a coordinate on the upper-half complex plane. In these variables the equation of motion for fields $X^\mu(z, \bar{z})$ is

$$\partial_z \partial_{\bar{z}} X^\mu(z, \bar{z}) = 0. \tag{56}$$

And the solution to this equation is $X^\mu(z, \bar{z}) = X^\mu_L(z) + X^\mu_R(\bar{z})$ where

$$X^\mu_L(z) = \frac{1}{2}x^\mu - \frac{i}{2}\alpha' p^\mu \log z^2 + i\left[\frac{\alpha'}{2}\right]^{1/2} \sum_{m\neq 0} \frac{\alpha^\mu_m}{m z^m} \quad \text{and} \quad X^\mu_R(\bar{z}) = X^\mu_L(\bar{z}).$$

The holomorphic part of the correlation function for X's on the UHP is of the form

$$\langle X^{\mu}_{L}(z) X^{\nu}_{L}(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w).$$

Whereas for the full string one has

$$\langle X^{\mu}(z,\bar{z}) X^{\nu}(w,\bar{w})\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}(\log|z-w|^2+\log|\bar{z}-w|^2). \tag{58}$$

Another representation of the relation (58) is the OPE

$$X^{\mu}(z) X^{\nu}(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w). \tag{59}$$

Here we dropped the indexes "L". The symbol "$\sim$" means that l.h.s. and r.h.s. are equal up to regular in $z-w$ terms. For ghosts we have the following OPE

$$c(z)b(z) \sim \frac{1}{z-w}. \tag{60}$$

Remind that only operators which cancel the background ghost charge -3 survive in the brackets $\langle c(z_1)c(z_2)c(z_3)\rangle = (z_1-z_2)(z_1-z_3)(z_2-z_3)$. In order to represent n-string interactions via correlation functions on special Riemann surfaces R_n (the so-called string configuration) we put [24]

$$\int \Phi_1 \star \dots \star \Phi_n = \langle V_n|\Phi_1\rangle_1 \otimes \dots \otimes |\Phi_n\rangle_n = \langle \Phi_1, \dots, \Phi_n\rangle_{R_n}$$

Efficiency of this method is a possibility to reduce calculations of correlation functions on n-string configuration to calculations of correlation functions on the upper half-disk, or the upper half-plane using the equality

$$\langle \Phi_1, \dots, \Phi_n\rangle_{R_n} = \langle F^{(n)}_1 \circ \Phi_1 \, \dots \, F^{(n)}_n \circ \Phi_n\rangle, \tag{61}$$

where $F^{n}_{k}(w) = (P_n \circ f^{(n)}_k)(w)$, explicit formulae for P_n and $f^{(n)}_n$ can be founding [9].

$(f \circ \Phi)$ in (61) means the conformal transform of Φ by f. In more details, if Φ is a primary field of conformal weight h, then $f \circ \Phi(z)$ is given by

$$(f \circ \Phi)(w) = (f'(w))^h \Phi(f(w)). \tag{62}$$

For a derivative of a primary field of conformal weight h one has

$$(f \circ \partial\Phi)(w) = \partial_w (f \circ \Phi)(w). \tag{63}$$

3.3. Neumann function method to solve overlap conditions

In addition to direct operator calculations in [19] another method is used based on fundamental properties of Neumann function is used.

Quadratic form W which stands in the exponent in $|I^{bc}\rangle$ can be rewritten as

$$W = \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} \int_{-\pi}^{\pi} \frac{d\sigma'}{2\pi} b(\sigma) \hat{C}(\sigma, \sigma') c(\sigma'), \tag{64}$$

where

$$\hat{C}(\sigma, \sigma') = \sum_{\substack{n \geq 2, \\ m \geq -1}} e^{in\sigma} e^{im\sigma'} C_{nm}. \tag{65}$$

We hold C_{nm} as unknown and weil find them in Neumann function language.

The overlap condition can be rewritten as conditions on Neumann functions. In our notations formula (39) can be rewritten as:

$$c(\sigma)|I^{bc}\rangle = e^W \left(\int_{-\pi}^{\pi} \frac{d\sigma'}{2\pi} C(\sigma, \sigma') c(\sigma') \right) |0\rangle$$

and

$$c(\pi - \sigma)|I^{bc}\rangle = e^W \left(\int_{-\pi}^{\pi} \frac{d\sigma'}{2\pi} C(\pi - \sigma, \sigma') c(\sigma') \right) |0\rangle,$$

where

$$C(\sigma, \sigma') \equiv \sum_{n=-1}^{\infty} e^{-in(\sigma - \sigma')} + \hat{C}(\sigma, \sigma').$$

Hence the overlap condition for the field $c(\sigma)$ takes the form

$$\int_{-\pi}^{\pi} \frac{d\sigma'}{2\pi} (C(\sigma, \sigma') + C(\pi - \sigma, \sigma')) c(\sigma')|0\rangle = 0$$

and respectively for $b(\sigma)$

$$\int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} (C(\sigma, \sigma') - C(\sigma, \pi - \sigma')) b(\sigma)|0\rangle = 0.$$

The coefficients C_{nm} will be found provided the function $C(\sigma, \sigma')$ built with the properties:

$$C(\sigma, \sigma') = -C(\pi - \sigma, \sigma'), \quad C(\sigma, \sigma') = C(\sigma, \pi - \sigma'). \tag{66}$$

Notice that the conditions on $C(\sigma, \sigma')$ reproduce the b, c overlap conditions with right conformal weights of fields.

As it has been shown in [19] the solution for (66) is given by the b, c Neumann function on the half-strip with identified halves at $\tau = 0$ (see Fig. 5).

To find it we use the map $\rho(u) = \log \frac{\sqrt{u}-i}{\sqrt{u}+i} + \frac{1}{2}i\pi$ which maps the disk in to the half strip (see Fig. 1 and 2):

$$C(\rho, \rho') = \left(\frac{\partial u}{\partial \rho}\right)^2 \frac{1}{u - u'} \left(\frac{u'}{u}\right)^{\frac{3}{2}} \left(\frac{\partial u'}{\partial \rho'}\right)^{-1}, \tag{67}$$

where $C(u, u') = \frac{1}{u-u'} \left(\frac{u'}{u}\right)^{\frac{3}{2}}$ is the Neumann function on unit disk over conformal vacuum. Using $u = w^2$ we get the Neumann function of strip at $\tau = 0$ as:

$$C(w(\sigma), w'(\sigma')) = \left(\frac{w(w^2+1)}{i}\right)^2 \frac{1}{w^2 - w'^2} \left(\frac{w'(w'^2+1)}{i}\right)^{-1} \left(\frac{w'}{w}\right)^3, \tag{68}$$

where (see Fig. 4) $w = (i - e^{i\sigma})/(i + e^{i\sigma})$. For $\sigma \to \pi - \sigma$ the function $w = w(\sigma)$ is transformed as

$$w(\pi - \sigma) = \frac{i - e^{i\pi - i\sigma}}{i + e^{i\pi - i\sigma}} = \frac{i + e^{-i\sigma}}{i - e^{-i\sigma}} = \frac{e^{i\sigma} i + 1}{e^{i\sigma} i - 1} = -\frac{i - e^{i\sigma}}{i + e^{i\sigma}} = -w(\sigma).$$

Let us insert the function $w(\pi - \sigma) = -w(\sigma)$ into $C(w, w')$. We get

$$C(\pi - \sigma, \sigma') = i \frac{(w(w^2+1))^2}{w'(w'^2+1)} \frac{1}{w^2 - w'^2} \left(\frac{w'}{w}\right)^3 = -C(\sigma, \sigma'),$$

and we see that $C(\pi - \sigma, \sigma') = -C(\sigma, \sigma')$, similar to the second equation of (66).

Quadratic form (64) can be rewritten through the full Neumann function $C(\sigma, \sigma')$ and if fields c and b are replaced by creations, c^{cr} and b^{cr}, part with respect to the conformal vacuum

$$W = \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} \int_{-\pi}^{\pi} \frac{d\sigma'}{2\pi} c^{cr}(\sigma) C(\sigma, \sigma') b^{cr}(\sigma'). \tag{69}$$

The diagonal part of function C does not make an contribute to W.

Now we verify directly that the Neumann function (68) has expansion (66) ($\tau = 0$) with the coefficients (37)

$$C_{nm} = -(-1)^n \delta_{n,m} + \sum_k (-1)^k [\delta_{n,2k+1}(\delta_{m,-1} - \delta_{m,1}) + 2\delta_{n,2k}\delta_{m,0}]. \tag{70}$$

In terms of x related with σ as $x = ie^{i\sigma}$ and, therefore, with w as $w = (1+x)/(1-x)$ we have

$$C(x, x') = \frac{x^2}{x'} \frac{1}{x - x'} - \frac{1}{1 - xx'} + 1 + \frac{x^3/x' + 2x^2 + xx'}{1 - x^2}. \tag{71}$$

On the other hand let us write the series expansion of this function is:

$$C(x,x') = \sum_{n\geq -1} \left(\frac{x'}{x}\right)^n + \sum_{\substack{n\geq 2,\\ m\geq -1}} (-i)^{n+m} x^n x'^m C_{nm}. \tag{72}$$

For the first term in the RHS of (72) we have:

$$\sum_{n\geq -1} \left(\frac{x'}{x}\right)^n = \frac{x}{x'} \sum_{n\geq 0} \left(\frac{x'}{x}\right)^n = \frac{x}{x'} \frac{1}{1-x'/x} = \frac{x^2}{x'(x-x')}, \tag{73}$$

(in this calculation a regularization is assumed; this regularization is removed in the last step). This is exactly the first term in (71). Inserting (70) in (81) we get

$$\sum_{\substack{n\geq 2,\\ m\geq -1}} (-i)^{n+m} x^n x'^m C_{nm} = -\frac{1}{1-xx'} + 1 + \frac{x^3/x' + 2x^2 + xx'}{1-x^2}. \tag{74}$$

That is the remaining part of the expression (71).

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