CALCULATION OF THE INITIAL REGION OF STABLE PHASE OSCILLATIONS IN SYNCHRO-CYCLOTRONS

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(Presented by I. B. Enchevich)

The analysis of the trapping and acceleration of charged particles within the central synchro-cyclotron region carried out usually on the basis of the phase equation [1] does not supply a sufficiently faithful model of the process. This is connected primarily to the fact that the maximum possible energy increment per turn is an increasing function of the radius and only approaches the energy increment existing between the gaps at radii 5--10 times larger than the dimensions of the dee apertures.

The solution of the problem of proton phase motion within the central synchro-cyclotron region in the present paper is based on the direct solution of the equations of motion for charged particles within magnetic and electric accelerating fields by utilizing fast electronic computers.

We limit our study to the motion of charged particles within the median plane of the magnetic field exhibiting an axial symmetry; we then find [2]:

$$\left.\begin{aligned} \ddot{r} &= A_0 (1-\beta^2)^{1/2} [A_r (1-\dot{r}^2) - A_\theta \dot{r}\alpha] + \frac{\alpha^2}{r^2}, \\ \ddot{\theta} &= \frac{1}{r}\left\{A_0(1-\beta^2)^{1/2}[A_\theta(1-\alpha^2) - A_r \dot{r}\alpha] - \frac{2\alpha\dot{r}}{r}\right\}, \end{aligned}\right\} \qquad (1)$$

where the dot indicates the degree of the differentiation; $A_0 = e/m_0c^2$; $\alpha = r\dot{\theta}$; $A_r = \mathcal{E}_r + \alpha Z_0 B_z$; $A_\theta = \mathcal{E}_\theta - \dot{r} Z_0 B_z$ (B_z is the magnetic induction, $\mathcal{E}_r$ and $\mathcal{E}_\theta$ are the components of the electric field, and Z_0 is the impedance of the free space).

Following a series of transformations, the dependence of the electric field strength on the radius may be represented in the following manner:

$$\mathcal{E}_r = \frac{\mathcal{E}_0 \sin\theta}{1 + \frac{\pi^2}{D^2} r^2 \sin^2\theta} \cos(1+\Delta)(1-\gamma\omega_0 t)\,\omega_0 t, \qquad (2)$$

where

$$\Delta = \frac{\dot{\varphi}}{\omega_0}, \quad \gamma = \frac{1}{2} \cdot \frac{d\omega_G}{dt} \cdot \frac{1}{\omega_0^2}; \tag{3}$$

$\mathcal{E}_0 = U_0/D$; U_0 is the amplitude of the accelerating voltage; D is the dee aperture; ω_0 is the rotational frequency of the ions at the center. We assume a linear relationship for the frequency changes of the accelerating voltage (generator) $\omega_G = \omega_{in} - (d\omega_G/dt)t$.

Usually, the magnetic induction within the central synchro-cyclotron region varies quadratically up to a certain radius r_1 while for radii larger than r_1 it decreases linearly, i.e.,

$$B_z = \begin{cases} B_0\left(1 - \frac{h}{2} r^2\right) & \text{at} \quad r \leqslant r_1, \\ a + br & \text{at} \quad r \geqslant r_1. \end{cases} \tag{4}$$

The magnetic induction B_1 in the OIYaI synchro-cyclotron is characterized by the following parameters:

$$r_1 = 0{,}10\ м; \qquad h_1 = 4{,}008 \cdot 10^{-1}\ м^{-2};$$
$$a_1 = 16{,}3455\ \text{kG}; \quad b_1 = -48{,}5 \cdot 10^{-2}\ \text{kG/m}.$$

For the electronic computer solution of equation (1) we chose the following initial conditions. The initial phase of the ions $\theta_{in} = 0$ [1], and the angular velocity $(d\theta/dt)_{in} = \omega_0$. The radial velocity $(dr/dt)_{in}$ was determined from the equation

$$\frac{dr}{dt} = \frac{eU_{\max}}{\pi e B_z r (1-n)} \tag{5}$$

at the radius $r_{in} = 1$ cm (eU_{max} is the maximum energy increment per turn). The choice of the value of the initial radius was conditioned by the singularity of equation (1) at the point $r = 0$. Figure 1 shows the r(t) and $\Delta\varphi(t)$ relationships ($\Delta\varphi$ is the phase of the ion relative to the phase of the hf voltage). It can be seen that the ion a stops accelerating and oscillates around a constant radius orbit since its phase continuously shifts in the negative direction with the increase in velocity. During the phase shifts in the positive direction beyond the limit $\pm\pi/2$ the ion b is slowed down and returns towards the center. If all the ions within the phase velocity interval enclosed by Δ_a and Δ_b are captured into the accelerating cycle as is the case, for instance, with c then the values Δ corresponding to the values a and b determine two points on the boundaries of the stability region.

Figure 2 shows the stability regions in the γ, Δ coordinate system. With an increase in γ the trapping efficiency characterized by the quantity Δ has an optimum value in all the cases. With an increase in the aperture of the dee the trapping efficiency increases only by a negligible amount. At the same time, the stability region is displaced in the direction of the positive value of Δ, apparently caused by the decrease in the energy increment per turn. An increase in the magnetic field index, n, causes a rise in the trapping efficiency which is proportional to $\sqrt{K}$, and its optimum value is shifted in the direction of larger γ's in a manner directly proportional to $K(K = 1 + n/(1 - n) \cdot 1/\beta^2)$. Doubling the amplitude of the accelerating voltage widens the stability region along the γ-axis by a factor of two, while it increases along the Δ-axis by one and a half times.

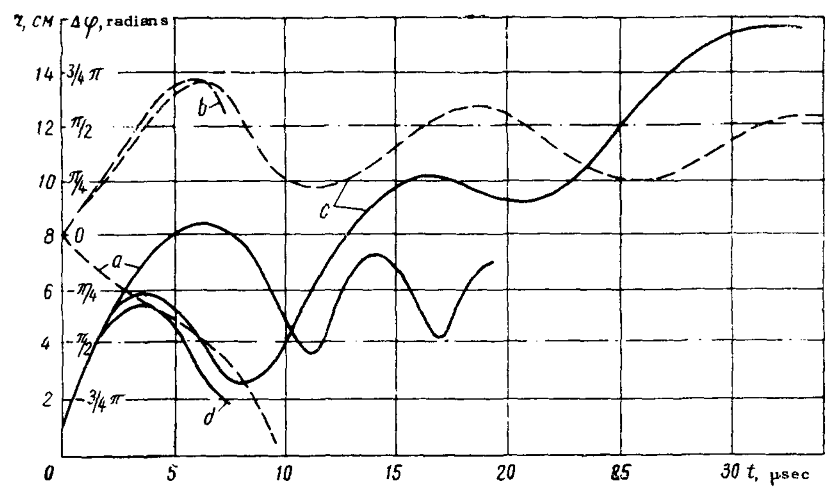

Figure 1. The radius (solid line) and phase (dashed line) of the ions as function of time:

a -- Ion oscillates relative to the orbit of constant radius; b -- ion returns towards the center; c -- ion is trapped into the accelerating cycle. Within the $-\pi/2 \leqslant \Delta\varphi \leqslant \pi/2$ region the ions are not decelerated.

The same transformation of the stability region with changes in accelerating voltage and with n may be obtained from the analysis of the equation quoted in the paper of Bohm and Foldy [1]. In addition, the values of $\cos \varphi_s$ calculated for the boundary points of the stability region along the γ-axis turned out to be very close to one:

Parameter of the stability region	D_1B_1	D_2B_1	D_3B_1	D_1B_2	D_2B_2	D_3B_2
Limiting values of $\cos\varphi_s$	0,975	0,990	0,985	0,995	0,920	0,880

The limiting values of $\cos \varphi_s$ were found utilizing the equation

$$\cos \varphi_s = \frac{\pi E_s}{e U_0 K} \gamma, \qquad (6)$$

within which we averaged the values of the coefficients.

As far as the boundary of the stability region along the Δ-axis is concerned, according to our calculation the lower limits agree almost completely with the calculations given in the paper [1] (Figure 3); however, the widths of these regions are essentially different. Such a disagreement may be explained first by the increases in energy due to absence of the primary turn slot. One should note that the value for the optimum $\cos \varphi_s$ is approximately equal to 0.65 and not 0.5 as was found during the analysis of the phase equation.

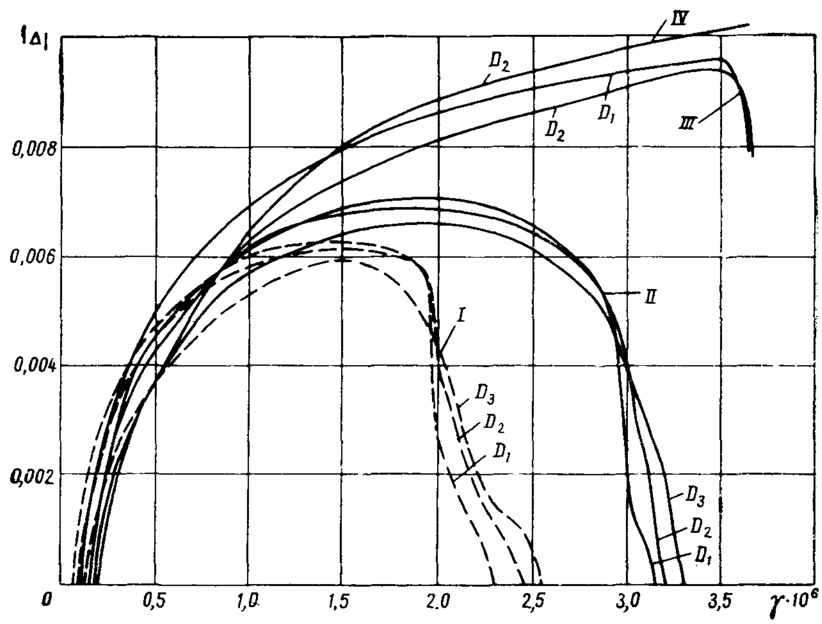

Figure 2. The trapping efficiency as function of the accelerator parameters: D_1 = 16 cm; D_2 = 12 cm; D_3 = 8 cm.

Curves	U_0, kV	h, m^{-2}	Magnetic induction
I	12	0,4	B_1
II	12	0,6	B_2^*
III	24	0,4	B_1
IV	24	0,6	B_2^*

* a^2 = 16.7792 kG (force), b_2 = 100.37 · 10^{-2} kG(force)/m.

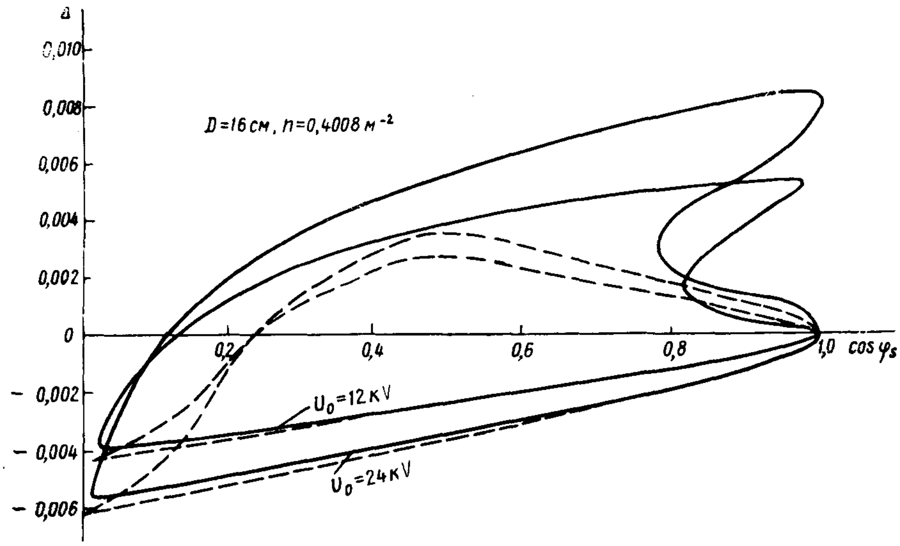

Figure 3. Trapping efficiency as function of cos φ_s:

——— - Region calculated from the radial-phase trajectories; - - - - region calculated using the phase equation.

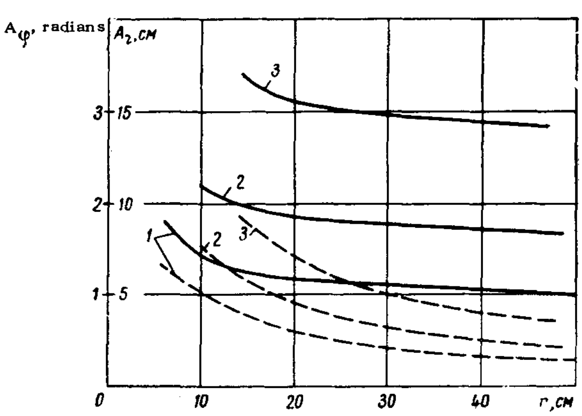

Figure 4. Damping of the radial-phase oscillation amplitudes with radius:

——— - Phase oscillation amplitude; - - - - radial oscillation amplitude;

1) $\Delta = 0.1 \cdot 10^{-2}$; 2) $\Delta = 0.1 \cdot 10^{-2}$; 3) $\Delta = -0.3 \cdot 10^{-2}$.

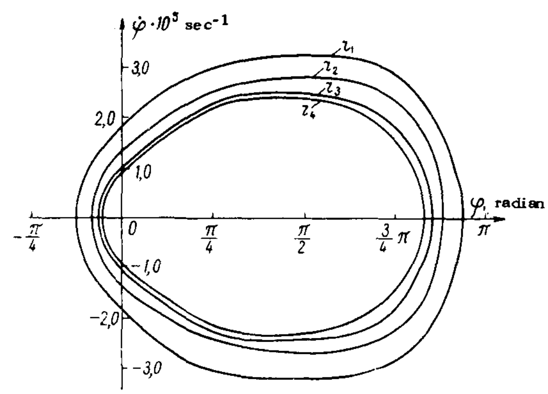

Figure 5. Stability regions $\dot{\varphi}(\varphi)$:

$r_1 = 12$ cm; $r_2 = 20$ cm; $r_3 = 32$ cm; $r_4 = 47$ cm.

From the analysis of the results obtained, we can explain the pattern of beam formation. Up to r = 12 cm the protons form a continuous rotating disc. After that, a train of protons with an angular extension of 1.2π and radial width of 12 cm separates from this disc. At about r = 50 cm, the beam compresses along the azimuth down to $0.9\,\pi$, and along the radius down to 7 cm. During the acceleration to r = 50 cm, the ions carry out about 8000 orbits and go through 14 radial-phase oscillations. The damping of the amplitudes of the radial-phase oscillations as function of radius is shown in Figure 4. It is interesting to note that during the initial stage of acceleration there are no particles with small amplitudes of the radial-phase oscillation.

The analysis of the radial-phase oscillations showed that the operational integral $J = \oint I\dot{\varphi}\, d\varphi$ [3] is invariant even during the initial stages of the acceleration although the changes in the parameters of the system cannot be considered small. Figure 5 shows the $\dot{\varphi}(\varphi)$ dependence for several radii. The ratio of the areas of the separatrices at radii 12 and 47 cm is in good agreement with the data shown in the paper [3].

In this manner, the study of the trajectories of the radial-phase oscillations led to the calculation of the trapping efficiency as function of various accelerator parameters and allowed us to establish the pattern of ion beam formation during the ion acceleration from the center up to the radius of 50 cm.

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DISCUSSION

P. Lapostolle

I would like to mention that in CERN we experienced the same beneficial effect of electrostatic focusing in the central region. As for the rf phase problem, we cannot change the frequency program of our tuning fork, but we modulate the rf voltage; that is not so efficient but also gives an increase of current.

R. S. Livingston

What is rf voltage on the cyclotron?

I. B. Enchevich

18 kV.