Proc. 8th Int. Conf. Quarks and Nuclear Physics (QNP2018)
JPS Conf. Proc. **26**, 031026 (2019)
https://doi.org/10.7566/JPSCP.26.031026

Critical Dynamics of Massless QCD with the Chiral Magnetic Effect

Masaru HONGO[1], Noriyuki SOGABE[2], and Naoki YAMAMOTO[2]

[1] *iTHEMS Program, RIKEN, Wako 351-0198, Japan*
[2] *Department of Physics, Keio University, Yokohama 223-8522, Japan*

E-mail: nori.sogabe@keio.jp

(Received January 31, 2019)

We study the critical dynamics of the second-order chiral phase transition in massless two-flavor QCD under an external magnetic field. We show that inclusion of the chiral magnetic wave in the low-energy modes, which is dictated by the chiral magnetic effect, modifies its dynamic universality class to model A from model E.

KEYWORDS: QCD Phase Diagram, Dynamic Universality Class, Chiral Magnetic Effect

1. Introduction

One of the big aims of the Beam Energy Scan program at Relativistic Heavy Ion Collider (RHIC) is the search for the QCD critical point; see Ref. [1] for a review. One most important property attached to the QCD critical point is the dynamic universality class and the dynamic critical exponents. It has been previously found that the dynamic universality class of the QCD critical point at high temperature T at finite baryon chemical potential $\mu_{\rm B}$ with finite quark mass $m_{\rm q}$ is the same as that of model H [2–5] within the conventional classification by Hohenberg and Halperin [7]. On the other hand, the second-order chiral phase transition in massless two-flavor QCD at finite T and $\mu_{\rm B} = 0$ is the same as that of O(4) antiferromagnet [8]. This difference originates from the presence of the massless pions and the absence of the mixing between the chiral condensate and the baryon number density in the latter [4].

Meanwhile, it is expected that a gigantic magnetic field is produced in noncentral heavy-ion collisions, which may potentially affect the QCD critical dynamics as well. Of particular relevance among others is the anomaly-induced transport phenomena in the magnetic field, called the chiral magnetic effect (CME) [9–11] and the chiral separation effect (CSE) [12, 13], and the resulting propagation wave of the vector and axial charges, called the chiral magnetic wave (CMW) [14, 15].

In this proceedings, we report on our recent study of the dynamic universality class of the second-order chiral phase transition in massless two-flavor QCD in the magnetic field [16]. We demonstrate that the presence of the magnetic field with or without the CME modifies the dynamic universality class as shown in Table I. (Here C is the transport coefficient of the CME defined in Eq. (5).)

Table I. Summary of dynamic universality classes (massless two-flavor QCD).

$\boldsymbol{B} = \boldsymbol{0}$, $C = 0$ [8]	$\boldsymbol{B} \neq \boldsymbol{0}$, $C = 0$	$\boldsymbol{B} \neq \boldsymbol{0}$, $C \neq 0$
O(4) antiferromagnet	model E	model A

2. Formulation

We consider massless two-flavor QCD in the presence of the magnetic field $\boldsymbol{B}$. In this case, chiral symmetry is explicitly broken down to $\mathcal{G} \equiv \mathrm{U}(1)^3_{\mathrm{V}} \times \mathrm{U}(1)^3_{\mathrm{A}}$, and the action is invariant under $q \to q' = e^{i\alpha_{\mathrm{V}}\tau^3} e^{i\alpha_{\mathrm{A}}\tau^3\gamma^5} q$. Due to this explicit chiral symmetry breaking, charged pions become massive, and the order parameter of the chiral phase transition is given by the two-component field composed of the chiral condensate and neutral pion, $\{\phi_\alpha\} = \{\bar{q}q, \bar{q}i\gamma^5\tau^3 q\}$ ($\alpha = 1, 2$). To describe the low-energy dynamics near the second-order chiral phase transition, we also take into account the conserved charge densities associated with $\mathcal{G}$, i.e., the isospin density $n = \bar{q}\gamma^0\tau^3 q$ and the axial isospin density $n_5 = \bar{q}\gamma^0\gamma^5\tau^3 q$ as low-energy degrees of freedom. Then, the generalized Lagevin equations for ϕ_α, n, and n_5 with macroscopic dissipation are given by [16]

$$\frac{\partial\phi_\alpha(\boldsymbol{r},t)}{\partial t} = -\Gamma\frac{\delta F}{\delta\phi_\alpha(\boldsymbol{r},t)} - g\int \mathrm{d}\boldsymbol{r}' \,[\phi_\alpha(\boldsymbol{r},t), n_5(\boldsymbol{r}',t)]\,\frac{\delta F}{\delta n_5(\boldsymbol{r}',t)} + \xi_\alpha(\boldsymbol{r},t), \tag{1}$$

$$\frac{\partial n(\boldsymbol{r},t)}{\partial t} = \lambda\boldsymbol{\nabla}^2\frac{\delta F}{\delta n(\boldsymbol{r},t)} - \int \mathrm{d}\boldsymbol{r}' \,[n(\boldsymbol{r},t), n_5(\boldsymbol{r}',t)]\,\frac{\delta F}{\delta n_5(\boldsymbol{r}',t)} + \zeta(\boldsymbol{r},t), \tag{2}$$

$$\begin{aligned}\frac{\partial n_5(\boldsymbol{r},t)}{\partial t} &= \lambda_5\boldsymbol{\nabla}^2\frac{\delta F}{\delta n_5(\boldsymbol{r},t)} - g\int \mathrm{d}\boldsymbol{r}' \,[n_5(\boldsymbol{r},t), \phi_\alpha(\boldsymbol{r}',t)]\,\frac{\delta F}{\delta\phi_\alpha(\boldsymbol{r}',t)} \\ &\quad - \int \mathrm{d}\boldsymbol{r}' \,[n_5(\boldsymbol{r},t), n(\boldsymbol{r}',t)]\,\frac{\delta F}{\delta n(\boldsymbol{r}',t)} + \zeta_5(\boldsymbol{r},t),\end{aligned} \tag{3}$$

where $F = F[\phi_\alpha, n, n_5]$ denotes the Ginzburg-Landau-Wilson functional obtained by the systematic expansions with respect to ϕ_α, n and n_5 and their derivatives; ξ_α, ζ, and ζ_5 denote the noise terms that are assumed to satisfy fluctuation-dissipation relations (FDR). We here omit the explicit forms of F and the FDR, since they are not important in the following arguments (see Ref. [16] for the details).

The essential ingredient in our analysis is the following Poisson brackets (PBs) resulting from the classical limit of the corresponding quantum commutators:

$$[n_5(\boldsymbol{r},t), \phi_\alpha(\boldsymbol{r}',t)] = \varepsilon_{\alpha\beta}\phi_\beta\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{4}$$

$$[n(\boldsymbol{r},t), n_5(\boldsymbol{r}',t)] = C\boldsymbol{B}\cdot\boldsymbol{\nabla}\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{5}$$

Here $\varepsilon_{\alpha\beta}$ denotes the antisymmetric tensor, and C denotes the coefficient of the CME, which is related to the anomaly coefficient away from the second-order chiral phase transition, $C = 1/(2\pi^2)$. Note that Eq. (4) represents the symmetry at the classical level while Eq. (5) is postulated based on the anomalous commutation relation concerning the triangle anomalies in quantum field theories [17,18] and the CME [19], so that the CME is introduced into the Langevin theory.

3. Renormalization group analysis

In order to apply the renormalization group (RG) analysis to the classical stochastic dynamics considered in the previous section, we convert the generalized Langevin equations into the path-integral formalism originally developed by Martin-Siggia-Rose, Janssen, and de Dominicis (MSRJD) [20–22]. Then, we can employ the usual techniques in field theory, such as the perturbation theory (with respect to the small anomalous dimension $\epsilon = 4-d$), Feynman rules, and the RG equations. We however note that one of the differences from the usual relativistic field theories is that the characteristic frequency is also rescaled as $\omega \to \omega' = b^z\omega$ under RG. Here, b is the scale factor, and z is the dynamic critical exponent. We derive the RG equations for the following dimensionless parameters:

$$f \equiv \frac{g^2\Lambda^{-\epsilon}}{8\pi^2\lambda_5\Gamma}, \quad w \equiv \frac{\Gamma\chi}{\lambda}, \quad w_5 \equiv \frac{\Gamma\chi_5}{\lambda_5}, \quad h \equiv \frac{C|\boldsymbol{B}|}{\sqrt{\lambda\lambda_5}\Lambda}, \tag{6}$$

and look into fixed-point solutions. Here, χ and χ_5 are the isospin and axial isospin susceptibilities defined by $\chi \equiv \partial n/\partial\mu$ and $\chi_5 \equiv \partial n_5/\partial\mu_5$, respectively, and Λ is the momentum cutoff. In Fig.1, we show certain sections of the RG flow diagrams in which two of the parameters, (w, h^2) or (f, w_5), are fixed.

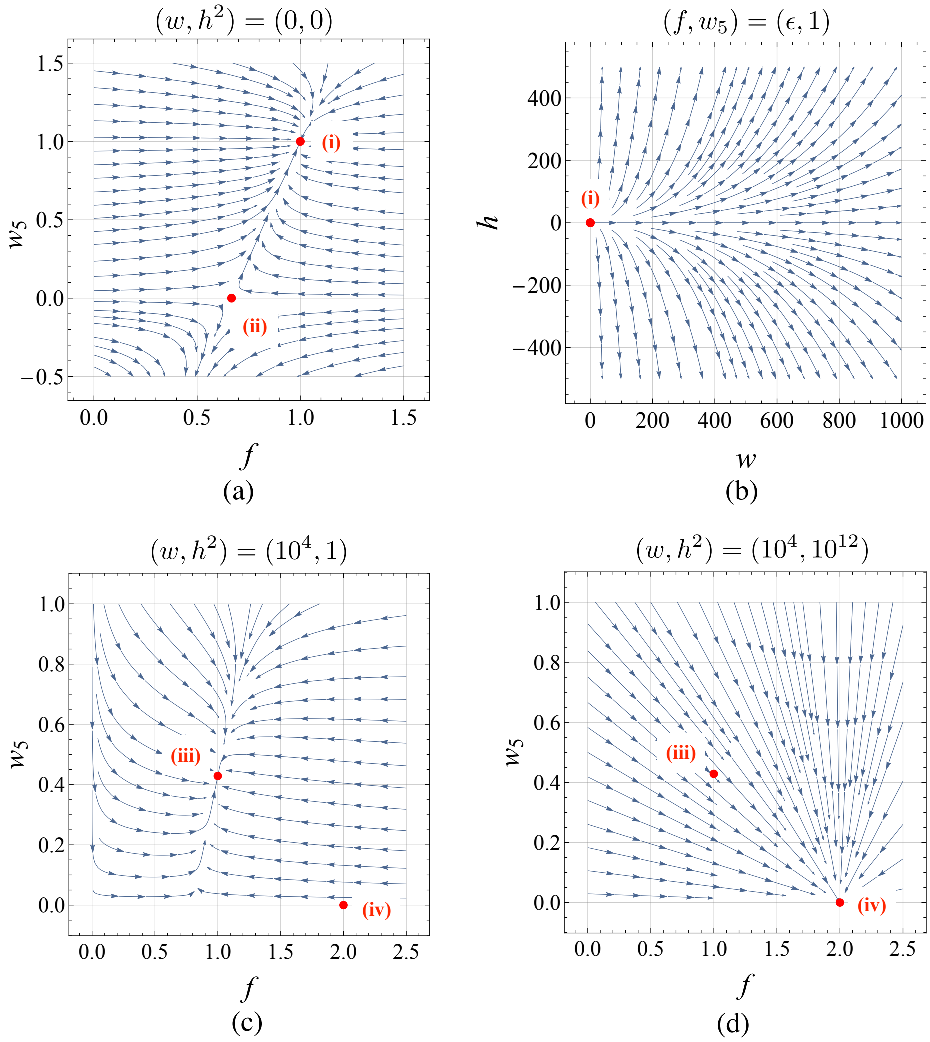

Fig. 1. Flow diagrams at $\epsilon = 1$. They are obtained by plotting the beta functions $(\beta_x(x, y), \beta_y(x, y))$ as vector fields on the $(x, y) = (f, w_5), (w, h)$ plane with fixing other parameters on the top.

When the CME is absent ($C = 0$ with fixed $\boldsymbol{B}$), the stable and unstable fixed points (i) and (ii) are obtained as shown in Fig. 1(a). Since $(w, h^2) = (0, 0)$ in this case, the critical dynamics is determined by the two-component order parameter field ϕ_α and the conserved density n_5 in the presence of the PB (4). These hydrodynamic variables can be mapped into those of the system described by model E, which is the same universality class as that for the magnetization (m_x, m_y) on the easy xy-plane and the conserved magnetization along the z direction, m_z with finite PB among them [7].

When the CME is taken into account ($C \neq 0$ with fixed $\boldsymbol{B}$), the fixed point (i) becomes unstable as shown in Fig. 1(b). Eventually, RG flows approach the fixed point (iii) or (iv), as one can find in Figs. 1(c) and (d). The former and latter fixed points correspond to the cases $w \gg h^2$ and $w \ll h^2$, respectively. Using the expression $h^2/w \propto C^2/\lambda_5$, the transition between these two cases is dictated by the competition between the diffusion of n_5 ($\omega \propto -\mathrm{i}\boldsymbol{k}^2$) and the propagation of the CMW ($\omega \propto |\boldsymbol{k}|$), with $\boldsymbol{k}$ being the characteristic momentum. Because higher derivative terms are irrelevant in the usual RG sense, the fixed point (iv) is more relevant than the fixed point (iii). Furthermore, one finds that the self-energy of the order parameter field ϕ_α, induced by the internal propagation (loop) of the conserved densities n and n_5, vanishes at the fixed point (iv) while it is some nonzero constant at the fixed point (iii); see Eq. (4.36) of Ref. [16]. Recalling that the CMW is the propagating wave of n and n_5, this vanishing self-energy means that not only n but also n_5 is decoupled from the critical dynamics. Therefore, the dynamic universality class in this case is model A [16] which is dictated by the dynamics of the order parameter alone [7].

4. Conclusion

We demonstrated that the presence of the CMW modifies the dynamic universality class of the second-order chiral phase transition in a magnetic field into that of model A. As the magnetic field in heavy ion collisions is expected to decay rapidly, it would be interesting to study the crossover between the two regimes $\boldsymbol{B} = \boldsymbol{0}$ (O(4) antiferromagnet) and $\boldsymbol{B} \neq \boldsymbol{0}$ with the CME (model A).

Acknowledgement M. H. is supported by the Special Postdoctoral Researchers Program and iTHES/iTHEMS Project (iTHEMS STAMP working group) at RIKEN. N. S. is supported by JSPS KAKENHI Grant No. 17J04047. N. Y. is supported by JSPS KAKENHI Grant No. 16K17703 and MEXT-Supported Program for the Strategic Research Foundation at Private Universities, "Topological Science" (Grant No. S1511006).

References

[1] M. A. Stephanov, Prog. Theor. Phys. Suppl. **153**, 139 (2004) [Int. J. Mod. Phys. A **20**, 4387 (2005)].
[2] H. Fujii, Phys. Rev. D **67**, 094018 (2003).
[3] H. Fujii and M. Ohtani, Phys. Rev. D **70**, 014016 (2004).
[4] D. T. Son and M. A. Stephanov, Phys. Rev. D **70**, 056001 (2004).
[5] It has been also found that the high-$\mu_{\rm B}$ critical point inside the nuclear/quark superfluid phases belongs to the new dynamic universality class beyond the conventional Hohenberg and Halperin's classification [6].
[6] N. Sogabe and N. Yamamoto, Phys. Rev. D **95**, no. 3, 034028 (2017)
[7] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).
[8] K. Rajagopal and F. Wilczek, Nucl. Phys. B **399**, 395 (1993).
[9] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A **803**, 227 (2008).
[10] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).
[11] H. B. Nielsen and M. Ninomiya, Phys. Lett. **130B**, 389 (1983).
[12] D. T. Son and A. R. Zhitnitsky, Phys. Rev. D **70**, 074018 (2004)
[13] M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)
[14] D. E. Kharzeev and H. U. Yee, Phys. Rev. D **83**, 085007 (2011).
[15] G. M. Newman, JHEP **0601**, 158 (2006).
[16] M. Hongo, N. Sogabe and N. Yamamoto, JHEP **1811**, 108 (2018)
[17] R. Jackiw, "Field theoretic investigations in current algebra," in *Lectures on Current Algebra and Its Applications,* ed. S. B. Treiman, R. Jackiw and D. J. Gross (Princeton University Press, Princeton, NJ, 1972).
[18] L. D. Faddeev, Phys. Lett. B **145**, 81 (1984).
[19] D. T. Son and N. Yamamoto, Phys. Rev. Lett. **109**, 181602 (2012).
[20] P. C. Martin, E. D. Siggia and H. A. Rose, Phys. Rev. A **8**, 423 (1973).
[21] H-K. Janssen, Z. Phys. B **23**, 377 (1976).
[22] C. De Dominicis, Phys. Rev. B **18**, 4913 (1978).