

Finite-Width Effects in Three-Body B Decays

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It is customary to apply the so-called narrow width approximation $\Gamma(B \rightarrow RP_3 \rightarrow P_1 P_2 P_3) = \Gamma(B \rightarrow RP_3) \mathcal{B}(R \rightarrow P_1 P_2)$ to extract the branching fraction of the quasi-two-body decay $B \rightarrow RP_3$, with R and P_3 being an intermediate resonant state and a pseudoscalar meson, respectively. However, the above factorization is valid only in the zero width limit. We consider a correction parameter η_R from finite width effects. Our main results are: (i) We present a general framework for computing η_R and show that it can be expressed in terms of the normalized differential rate and determined by its value at the resonance. (ii) We evaluate η_R in the theoretical framework of QCD factorization (QCDF) and in the experimental parameterization (EXPP) for three-body decay amplitudes. In general, η_R^{QCDF} and η_R^{EXPP} are similar for vector mesons, but different for tensor and scalar resonances. A study of the differential rates enables us to understand the origin of their differences. (iii) Finite-width corrections to $\mathcal{B}(B^- \rightarrow RP)_{\text{NWA}}$ obtained in the narrow width approximation are generally small, less than 10%, but they are prominent in $B^- \rightarrow \sigma/f_0(500)\pi^-$ and $B^- \rightarrow \bar{K}_0^{*0}(1430)\pi^-$ decays. The EXPP of the normalized differential rates should be contrasted with the theoretical predictions from QCDF calculation as the latter properly takes into account the energy dependence in weak decay amplitudes. (iv) It is common to use the Gounaris-Sakurai model to describe the line shape of the broad $\rho(770)$ resonance. After including finite-width effects, the PDG value of $\mathcal{B}(B^- \rightarrow \rho\pi^-) = (8.3 \pm 1.2) \times 10^{-6}$ should be corrected to $(7.9 \pm 1.1) \times 10^{-6}$ in EXPP and $(7.7 \pm 1.1) \times 10^{-6}$ in QCDF. (v) For the very broad $\sigma/f_0(500)$ scalar resonance, we use a simple pole model to describe its line shape and find a very large width effect: $\eta_{\sigma}^{\text{QCDF}} \sim 2.15$ and $\eta_{\sigma}^{\text{EXPP}} \sim 1.64$. Consequently, $B^- \rightarrow \sigma\pi^-$ has a large branching fraction of order 10^{-5} . (vi) We employ the Breit-Wigner line shape to describe the production of $K_0^*(1430)$ in three-body B decays and find large off-shell effects. The smallness of $\eta_{K_0^*}^{\text{QCDF}}$ relative to $\eta_{K_0^*}^{\text{EXPP}}$ is ascribed to the differences in the normalized differential rates off the resonance.

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In $B \rightarrow RP_3 \rightarrow P_1 P_2 P_3$ decay with R and P_3 an intermediate resonant state and a pseudoscalar meson, respectively, it is a common practice to apply the narrow width approximation (NWA),

$$\Gamma(B \rightarrow RP_3 \rightarrow P_1 P_2 P_3) = \Gamma(B \rightarrow RP_3) \mathcal{B}(R \rightarrow P_1 P_2), \quad (1)$$

to extract the branching fraction of the quasi-two-body decay, $\mathcal{B}(B \rightarrow RP_3)$. For the case when R has a finite-width, Eq. (1) does not hold. We defined a quantity [1, 2]

$$\eta_R \equiv \frac{\Gamma(B \rightarrow RP_3) \mathcal{B}(R \rightarrow P_1 P_2)}{\Gamma(B \rightarrow RP_3 \rightarrow P_1 P_2 P_3)}, \quad (2)$$

so that the deviation of η_R from unity measures the degree of departure from the NWA when the width is finite. After taking into account the finite-width effect, the branching fraction of the quasi-two-body decay should be corrected as

$$\mathcal{B}(B \rightarrow RP_3) = \eta_R \frac{\mathcal{B}(B \rightarrow RP_3 \rightarrow P_1 P_2 P_3)_{\text{expt}}}{\mathcal{B}(R \rightarrow P_1 P_2)_{\text{expt}}}, \quad (3)$$

which is suitable for a comparison with theoretical calculations.

In [1], we calculated the parameter η_R within the framework of QCD factorization [3] (QCDF) and the experimental parametrization [4] (EXPP) for various resonances and use these examples to highlight the importance of finite-width effects. We developed a general framework for the study of η_R and showed that η_R can be expressed in terms of a normalized differential decay rate,

$$\eta_R = \pi m_R \Gamma_R \frac{d\Gamma(m_R^2)}{dm_{12}^2} \left| \int \frac{d\Gamma(m_{12}^2)}{dm_{12}^2} dm_{12}^2 \right| = \pi m_R \Gamma_R \frac{d\tilde{\Gamma}(m_R^2)}{dm_{12}^2}, \quad (4)$$

and one can verify that η_R given in the above equation approaches unity in the narrow width limit, reproducing the well-known result of NWA. It turns out that η_R is nothing but the value of the normalized differential decay rate evaluated at the contributing resonance.

We compare between η_R^{QCDF} and η_R^{EXPP} for their width dependence in Fig. 1, while numerical results are summarized in Table 1 (with BW and GS stand for Breit-Wigner and Gounaris-Sakurai line shapes, respectively). In general, these two quantities are similar for vector mesons but different for tensor and scalar mesons. Note that the parameter η_R is proportional to the normalized

Resonance	$B^+ \rightarrow Rh_3 \rightarrow h_1 h_2 h_3$	Γ_R (MeV)	Γ_R/m_R	η_R^{QCDF}	η_R^{EXPP}
$f_2(1270)$	$B^+ \rightarrow f_2 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	$186.7^{+2.2}_{-2.5}$	0.146	$1.003^{+0.001}_{-0.002}$	$0.937^{+0.006}_{-0.005}$
$K_2^*(1430)$	$B^+ \rightarrow K_2^* \pi^+ \rightarrow K^+ \pi^- \pi^+$	109 ± 5	0.076	0.972 ± 0.001	1.053 ± 0.002
$\rho(770)$	$B^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	149.1 ± 0.8	0.192	0.93 (GS) 1.11 (BW)	0.95 (GS) 1.15 (BW)
$\rho(770)$	$B^+ \rightarrow K^+ \rho^0 \rightarrow K^+ \pi^+ \pi^-$	149.1 ± 0.8	0.192	0.95 (GS) 1.13 (BW)	0.93 (GS) 1.13 (BW)
$K^*(892)$	$B^+ \rightarrow K^* \pi^+ \rightarrow K^+ \pi^- \pi^+$	47.3 ± 0.5	0.053	1.067 ± 0.002	1.075
$\sigma/f_0(500)$	$B^+ \rightarrow \sigma \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	700 ± 26 [?]	≈ 1.24	2.15 ± 0.05	1.64 ± 0.03
$K_0^*(1430)$	$B^+ \rightarrow K_0^* \pi^+ \rightarrow K^+ \pi^- \pi^+$	270 ± 80	≈ 0.19	0.83 ± 0.04	1.11 ± 0.03

Table 1: A summary of the η_R parameter for various resonances produced in the three-body B decays. Note that η_R^{QCDF} are obtained in QCDF calculations, while η_R^{EXPP} from the experimental parameterization.

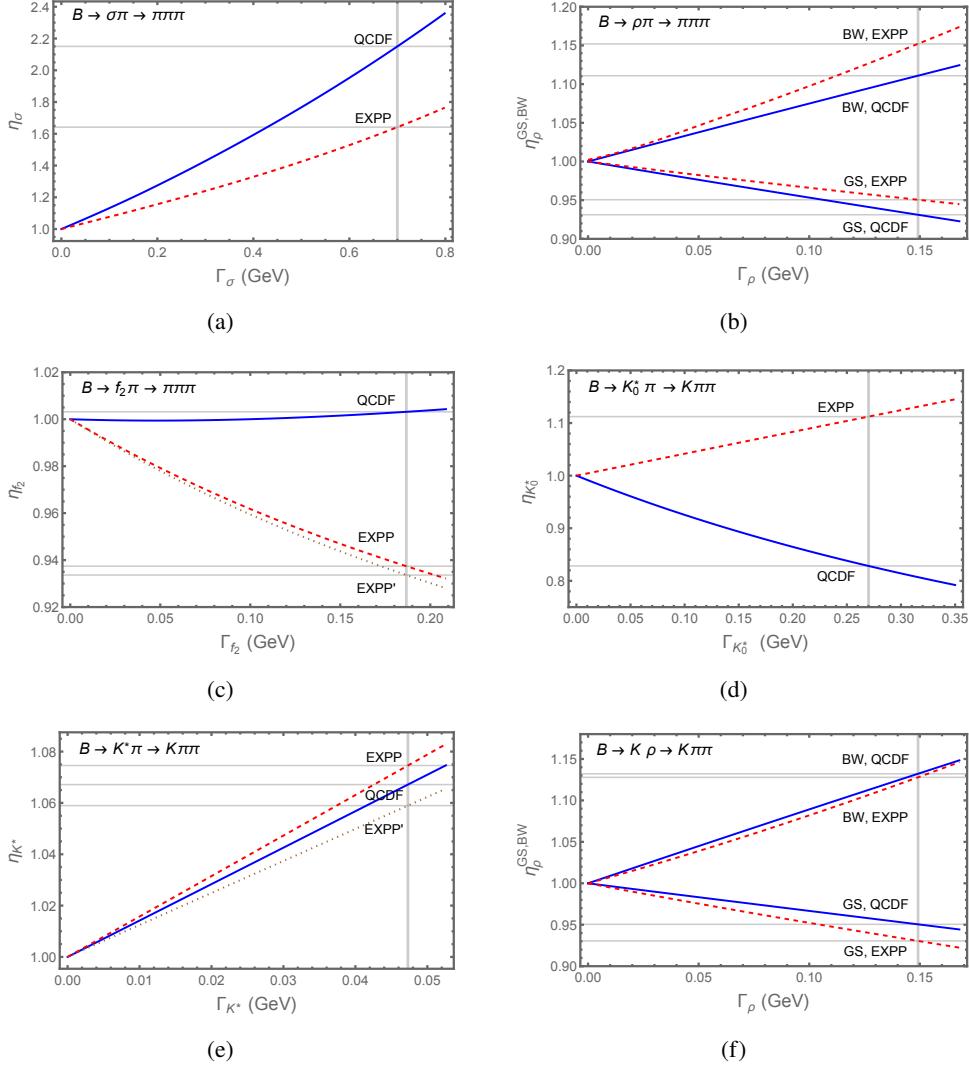


Figure 1: The parameter η_R as a function of the Γ_R in η_R of $B^- \rightarrow R\pi^- \rightarrow \pi^-\pi^+\pi^-$ and $B^- \rightarrow R\pi^- \rightarrow K^-\pi^+\pi^-$ decays. The positions of the central values of the physical widths are marked by vertical lines in the plots.

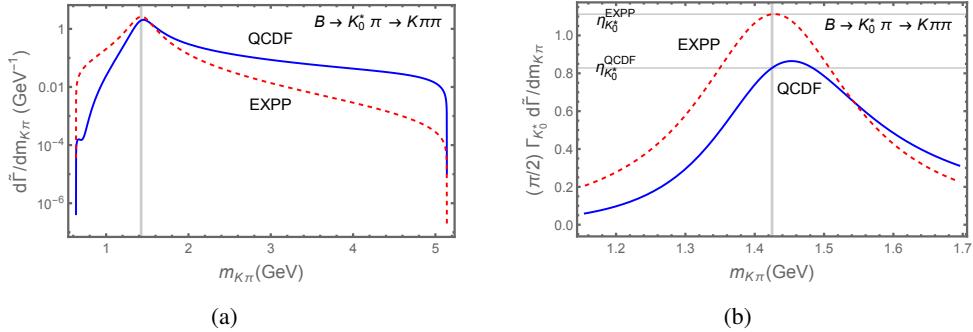


Figure 2: (a) The normalized differential rates in $B^- \rightarrow K_0^* \pi^- \rightarrow K^- \pi^+ \pi^-$ decay. The normalized differential rate obtained in the QCDF calculation is much larger than that using the EXPP scheme in the large $m_{K\pi}$ region. (b) The plot is scaled and blown-up in the resonance region. The heights at the resonance equal $\eta_{K_0^*}$.

Table 2: Branching fractions of quasi-two-body decays $B^+ \rightarrow RP_3$ (in units of 10^{-6}) derived from the measured $B^+ \rightarrow RP_3 \rightarrow P_1 P_2 P_3$ rates. $\mathcal{B}(B^+ \rightarrow RP_3)_{\text{NWA}}$ denotes the branching fraction obtained in the narrow width approximation.

Mode	$\mathcal{B}(B \rightarrow RP \rightarrow PPP)_{\text{expt}}$	$\mathcal{B}(B \rightarrow RP)_{\text{NWA}}$	$\eta_R^{\text{QCDF}} \mathcal{B}(B \rightarrow RP)_{\text{NWA}}$	$\eta_R^{\text{EXPP}} \mathcal{B}(B \rightarrow RP)_{\text{NWA}}$
$B^+ \rightarrow f_2 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	1.17 ± 0.20	2.08 ± 0.36	2.09 ± 0.36	1.95 ± 0.33
$B^+ \rightarrow K_2^{*0} \pi^+ \rightarrow K^+ \pi^- \pi^+$	$1.85^{+0.73}_{-0.50}$	$5.56^{+2.19}_{-1.50}$	$5.40^{+2.13}_{-1.46}$	$5.85^{+2.31}_{-1.58}$
$B^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	8.36 ± 0.77	8.36 ± 0.77	7.78 ± 0.72 (GS) 9.28 ± 0.86 (BW)	7.95 ± 0.73 (GS)
$B^+ \rightarrow K^+ \rho^0 \rightarrow K^+ \pi^+ \pi^-$	3.7 ± 0.5	3.7 ± 0.5	3.5 ± 0.5 (GS) 4.2 ± 0.6 (BW)	3.4 ± 0.5 (GS)
$B^+ \rightarrow K^{*0} \pi^+ \rightarrow K^+ \pi^- \pi^+$	6.71 ± 0.57	10.1 ± 0.8	10.7 ± 0.9	10.9 ± 0.9
$B^+ \rightarrow \sigma \pi^+ \rightarrow \pi^+ \pi^- \pi^+$	3.83 ± 0.84	5.75 ± 1.26	12.36 ± 2.71	9.44 ± 2.08
$B^+ \rightarrow K_0^{*0} \pi^+ \rightarrow K^+ \pi^- \pi^+$	$27.9^{+5.6}_{-4.3}$	45^{+9}_{-7}	37^{+8}_{-6}	50^{+10}_{-8}

differential rate at the resonance, see Eq. (4), which is anti-correlated to those off the resonance. A study of the differential rates enables us to understand the origin of the differences in η_R . For example, in $B^- \rightarrow \bar{K}_0^*(1430)\pi^- \rightarrow K^-\pi^+\pi^+$ decay, the $m_{K\pi}^2$ dependence associated with the penguin Wilson coefficients ($a_6^P - a_8^P/2$) yields a large enhancement in the QCDF differential rate in the large $m_{K\pi}$ distribution, rendering $\eta_{K_0^*}^{\text{QCDF}} < \eta_{K_0^*}^{\text{EXPP}}$, as shown in Fig. 2.

Finite-width corrections to the branching fractions of quasi-two-body decays obtained in the NWA, are summarized in Table 2 for both QCDF and EXPP schemes. In general, finite-width effects are small, less than 10%, but they are prominent in $B^+ \rightarrow \sigma/f_0(500)\pi^+$ and $B^+ \rightarrow K_0^{*0}(1430)\pi^+$ decays. In the presence of finite-width corrections, the PDG value of $\mathcal{B}(B^+ \rightarrow \rho\pi^+) = (8.3 \pm 1.2) \times 10^{-6}$ should be corrected to $(7.7 \pm 1.1) \times 10^{-6}$ using $\eta_{\rho}^{\text{QCDF}}(\text{GS})$ and $(7.9 \pm 1.1) \times 10^{-6}$ using $\eta_{\rho}^{\text{EXPP}}(\text{GS})$. We have found very large width effects: $\eta_{\sigma}^{\text{QCDF}} \sim 2.15$ and $\eta_{\sigma}^{\text{EXPP}} \sim 1.64$. Consequently, $B^- \rightarrow \sigma\pi^-$ has a large branching fraction of order 10^{-5} . We have employed the Breit-Wigner line shape to describe the production of $K_0^*(1430)$ in three-body B decays and found large off-shell effects. The smallness of $\eta_{K_0^*}^{\text{QCDF}}$ relative to $\eta_{K_0^*}^{\text{EXPP}}$ is ascribed to the fact that the normalized differential rate obtained in the QCDF calculation is much larger than that using the EXPP scheme in the off-resonance region.

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