PLENARY REPORT

ON

PROGRESS IN GAUGE THEORIES

J Iliopoulos

Laboratoire de Physique Théorique, Ecole Normale Supérieure, Paris*

CONTENTS

FOREWARD: A WARNING

An ass crossing a river with a load of salt lost his footing and slipped into the water, so that the salt was dissolved. He was mightily pleased at finding himself relieved of his burden when he got upon his legs again. So the next time he came to a river with a load on his back, he let himself go under on purpose. But this time he was loaded with sponges which absorbed so much water that he could not keep his head up and was drowned.[1]

INTRODUCTION

I was asked to report on the progress made recently in trying to apply the field theoretic methods, which have been proven so useful in quantum electrodynamics, to other areas of physics. I shall concentrate mainly on the most recent developments of the last year, since the earlier ones have already been reviewed several times[2 to 9].

The idea of unifying the weak and electromagnetic interactions is as old as the weak interactions themselves and already by the late 1950's several models were proposed[10] which incorporated most of the features that we find in present day theories. In particular the Yang-Mills[11] couplings were used with the photon, as one of the neutral gauge bosons. How ever at that time the gauge symmetry had to be explicitly broken by the vector meson mass terms and, as a result, these theories were not renormalizable. The last ingredient was discovered in 1964 with the study of spontaneously broken gauge symmetries[12].

It is remarkable that these two ingredients, namely Yang-Mills gauge invariance and spontaneously broken symmetries, each one taken separately, seem useless for weak interactions, both being hopelessly inflicted with zero mass bosons. However, when combined together in a spontaneously broken gauge symmetry, the two

* Laboratoire propre du CNRS, associé à l'Ecole Normale Supérieure et à l'Université Paris Sud.

diseases cure each other, and the resulting theory, although still gauge invariant, can be made to have the correct spectrum of states. The synthesis of the previous models with the Higgs mechanism was first proposed by S Weinberg[13] in 1967 and, although this model is the simplest one which seems to fit all the existing data today, it attracted little attention at the time it was published. The reason is that renormalizability of such a theory was suggested in the original papers[13] of Weinberg and Salam, but no proof was presented, so this model looked like one more, in a seemingly endless series of attempts to go beyond the Fermi theory of weak interactions.

Let us make a digression at this point and emphasize that the classical V-A theory had achieved by the late 1960's a degree of elegance which is rarely found in elementary particle physics. Of surprisingly simple and elegant form it could not fit a large variety of data, but also it incorporated such physical principles as C.V.C., Cabibbo's form of universality, $\mu - e$ universality and so on. Furthermore it was expressible in terms of the currents which people, through the successes of Current Algebra, had learnt to consider as very fundamental objects. In other words, if one could only forget for a moment that there was not much of a theory after all, and that the whole structure was just a phenomenological description of the data, one would have every reason to be satisfied. Especially if one compared it with the situation in strong interactions, in which there was also no consistent theory, but there was no elegance either. This explains the fact that no enthusiasm was manifested when renormalizable models of the weak interactions based on scalar rather than vector intermediaries were proposed[14] and which, on top of introducing a large number of exotic and unobserved particles, presented the V-A form as an accident of the lowest order calculations. Once more, when physicists were confronted with a consistent theory and an elegant structure, they unanimously chose the latter.

To return to the historical survey, there is one point which was realised by the late 1960's and which, although seems irrelevant in light of the subsequent discoveries, it helped to develop different techniques which proved useful later. It was the fact that, under very general assumptions about the equal time commutators of current densities, one could easily prove that strong interactions cannot provide a cut-off for removing the divergencies of weak interactions[15]. The argument is very simple and is actually used today in a more modern language. As a consequence a modified approach to perturbation theory was developed[16] which was inspired by models in statistical mechanics[17]

Weak interaction processes were classified according to their dependence upon a suitably introduced cut-off momentum Λ. Terms of the form $G^{\ell} \sum_{n=0}^{\infty} A_{\ell n} (G\Lambda^2)^n$ were called ℓth order.

(Logarithmic dependences on Λ were disregarded.) The usual selection rules of both strong and weak interactions yield a remarkably small cut-off of the order of 5 GeV[15]. This value was judged implausible and unrealistic, and several theorists tried to reformulate the traditional scheme of weak interactions[18]. The assumption that chiral SU(3) x SU(3) symmetry breaking belongs to a $(3,\bar{3}) + (\bar{3},3)$ representation solved the problem of the zeroth order terms, thus removing the spectre of strong violations of parity and hypercharge[19]. Since such a breaking was in any case the favored one to many theorists[20], this was considered a welcome result. In fact, based on some speculative stability requirements, attempts were made to calculate the Cabibbo angle[21]. However, the first order terms remained equally troublesome giving order G contributions to the $K_1 - K_2$ mass difference, or to processes like $K_2^o \to \mu^+\mu^-$. The way out of this difficulty turned out to require more drastic modifications of the conventional scheme. An enlargement from SU(3) to SU(4) was suggested[22], by the introduction of a new quantum number, conserved by the strong and e.m. interactions, called "charm"[23]. Furthermore for the first time, a consistent formulation of all weak interactions, leptonic as well as hadronic, into a Yang-Mills theory was proposed[22]. In fact, during the late 60's several theorists, and in particular M Veltman[24] were studying the renormalization properties of massive Yang-Mills theories, with the purpose of applying them to the weak interactions. It was shown[25] that serious cancellations occur in such theories. Today we know that these cancellations do not go far enough to make the theory renormalizable[26], but at that time this was unclear and the fundamental problem, namely a consistent way to control the higher order terms, remained unsolved[27]. The situation remained unchanged until the middle of 1971, when the solution was presented in two brilliant papers by G 't Hooft[28] followed by the works of B. Lee and Zinn-Justin[29] and 't Hooft and Veltman[30]. 't Hooft derived the correct Feynman rules for Yang-Mills fields in a large class of gauges[31] and explicitly constructed such gauges in which the theory was manifestly renormalizable by power counting. Furthermore, by a combinatoric use of the Ward identities, he gave a first proof of the gauge independence of the S-matrix. The detailed study of the renormalization program and the unitarity of the resulting theory was done by B. Lee and Zinn-Justin[29] and 't Hooft and Veltman[30]. This remarkable success opened the way into an avalanche of theoretical papers. It is impossible to mention all of them here[32], but there were two main lines of research. The first was, naturally, the construction of more or less realistic models of weak and electromagnetic interactions. The second aimed mainly at a better understanding of the field theory aspects. They developed parallel to one another and each one exercised considerable influence upon the other. Nor are they yet at an end. We are still actively involved in both of them and I shall try to review the most recent progress in the appropriate sections below.

II MODELS

It has become customary for any speaker who discusses this subject, to complain about the absence of the ultimate model of the world which the gauge theories were supposed to give us. I consider this attitude a very negative one. We have already forgotten that just three years ago we were still struggling to get all the divergencies of the Fermi theory under some sort of control, and we are complaining today because we do not yet know why the muon is so heavy and the pion so light, or why CP is violated in one place and isospin is not in another. Here I would like to adopt the opposite view and, taking for granted that with gauge theories we are on the right track, to make a list of the desired properties of this ultimate model, for which the gauge theories have given at least the glimmer of a possibility. I think that all these properties can be deduced from the requirement of maximum predictive power[33].

(i) Universality

This remarkable property of weak interactions can be understood, at least in principle, in the framework of non abelian gauge theories. It simply follows from the uniqueness of the coupling constant. All higher order corrections are necessarily finite and calculable. One is tempted to apply the same reasoning to electromagnetic interactions and this is in fact possible provided one uses a simple algebra, (or a direct product of isomorphic simple factors with discrete symmetries which interchange them) since any U(1) factor introduces arbitrary constants[34]. Unfortunately the condition is only necessary but not sufficient. The SU(2) model of Georgi and Glashow[35] had no arbitrary coupling constants, but there were mixing angles among the neutral lepton fields which destroyed universality.

(ii) The lepton spectrum

One expects; in the ultimate model, the ratio m_e/m_μ to come out calculable[36] and of order α. This, although again possible in principle, does not follow from any realistic scheme up to now.

As for other heavy leptons, there is certainly room for them in gauge theories and in fact they are required for the formulation of most models proposed so far[37]. The trouble with them is, first, that they grow heavier every day, and, second, that they introduce more mass ratios which one would like to determine inside the model. However there is no fundamental objection to their presence, although one may feel uneasy with the proliferation of elementary fields.

(iii) $1 + \gamma_5$ structure

Again it is strongly believed by many physicists that a satisfactory unified theory of weak and electromagnetic interactions should be able to account in a natural way for the simultaneous existence of a purely vector electromagnetic current and of V-A charged currents. There is no known solution to this very fundamental problem, but P Fayet[38] has formulated the SU(2) x U(1) model so that parity is spontaneously broken together with gauge invariance. A completely different approach assuems that (V-A) is a fundamental principle of Nature, and neutrino-like massless fermions are the building blocks of all matter[39].

(iv) The $|\Delta\vec{I}| = \frac{1}{2}$ rule

Of all the selection rules of weak interactions, the $|\Delta\vec{I}| = \frac{1}{2}$ non leptonic rule is the most difficult to implement. It does not follow from the ordinary current x current theory and the situation is basically the same in gauge theories. The simplest models do not obey it but it is possible to incorporate it in either exact or approximate form[40]. We still do not know for sure whether we are in the presence of a symmetry law or a dynamical enhancement. I would like to mention, however, the recent works of M K Gaillard and B W Lee and G Altarelli and L Maiani[41] who showed that such an enhancement, which could explain at least part of the observed effect, may follow form a non-abelian gauge theory of strong interactions.

(v) CP violation

Although it is not at all clear that the observed CP violation in K^0 decays is connected with the weak interactions, one feels unhappy with most models which simply ignore the effect. The truth is that model building is much simpler in a CP conserving world and usually one has to go out of his way to introduce CP violation. In the framework of gauge theories one can adopt two different attitudes: Either the violation is explicitly present in the Lagrangian[42], or it is introduced spontaneously through the Higgs system of scalar fields in the way suggested by T D Lee[43]. Along this second possibility it has been recently suggested[44] that the smallness of the observed effect is due to the fact that the spontaneous CP breaking does not occur in the tree approximation, but only when radiative corrections are taken into account[45]. This opens the possibility of connecting CP violation with other almost exact symmetries in the world. We see that here again gauge theories offer us a new line of thought.

(vi) Strong interaction symmetries

Each one of us has often been asked to give an introductory talk about elementary particle physics and standard way of starting was to explain the different interactions that have been observed. One was then inclined to write, in a schematic way, the Lagrangian of the world as a sum:

$$\mathcal{L} = \mathcal{L}_{strong} + \mathcal{L}_{em} + \mathcal{L}_{weak} + \ldots\ldots \quad (2.1)$$

The important point about (2.1) is that the different pieces were supposed to be independent from one another. Actually, this line of thought was not only used for purposes of vulgarization but it was the way most of us thought. There never was a question of compatibility of the different terms. We all knew that such an approach was limited, since terms which were experimentally of order α, like the mass splittings among the members of an isomultiplet, were not calculable, but we had learnt to live with it[45]. I think that one of the major successes of gauge theories is that, for the first time, the problem of the strong interaction symmetries can be put in a consistent field theoretic framework. The reason is that non abelian gauge theories are not renormalizable in the usual sense, like ϕ^4 or Yukawa theories. It follows that every term in the Lagrangian, no matter what its relative strength is, must respect the gauge symmetry. Obviously such a requirement severely restricts all the terms of $\mathcal{L}$. The ultimate goal is to find a non abelian gauge theory for which $\mathcal{L}$ is so restricted, that all symmetries of strong interactions arise naturally as zero order symmetries[47], (naturally in the sense that all higher order corrections are finite and calculable). Since the gauge coupling constant is supposed to be of order e, we expect the corrections to such symmetries to be, in general, of order α. Two distinct cases should be considered.

(a) Parity and strangeness conservation: In any acceptable model the weak interactions should not be allowed to introduce order α violations of these symmetries. It turns out that this can be achieved in a variety of ways. Let me give a simplified form of the argument for the case where the strong interactions are described by a gluon model. Let me notice first that this case enters the general class of a chiral symmetry breaking by a $(3,\bar{3}) + (\bar{3},3)$ term (the quark mass term) which was examined in the prehistoric paper of ref. 19. So we expect similar techniques to apply here as well. In order to make the argument simpler, let me however adopt a naive quark model and an abelian gluon and write the Lagrangian in the form[48]

$$\mathcal{L} = i\bar{q}\partial\!\!\!/ q - \bar{q}Mq + f\bar{q}\gamma_\mu qG^\mu + g\bar{q}\gamma_\mu C^i qW_i^\mu + \ldots \quad (2.2)$$

where q is a spinor representing the quark fields, M a certain diagonal matrix, G_μ and W_i^μ the vector fields for the gluon and the weak gauge bosons and C^i a set of matrices both in the group space and in ordinary space. In particular C^i contain γ_5 matrices. The dots stand for the kinetic energy and mass terms of the vector bosons, the terms with the Higgs scalars etc. The order g^2 corrections modify $\mathcal{L}$ into:

$$\mathcal{L}' = i\bar{q}\partial\!\!\!/ \left[1 - Z_2\right]q - \bar{q}\left[M - \Sigma\right]q + f\bar{q}\gamma_\mu\left[1 - Z_1\right]qG^\mu + \ldots.. \quad (2.3)$$

where Z_i and Σ are matrices of order g^2. In particular they have non zero off-diagonal matrix elements and they contain parts proportional to γ_5, therefore they are liable of introducing parity and strangeness violations to order α. However one can always perform a rotation of the quark fields of the form:

$$q = (A + B\gamma_5)q \quad (2.4)$$

where A and B are matrices, in other words one is allowed to perform independent right and left rotations, combined with arbitrary rescalings of the fields. It follows that one can simultaneously set $[\mathbb{1} - Z_2]$ equal to $\frac{\mathbb{1}}{\Lambda}$ and diagonalize $[M - \Sigma]$. The important point here is that, due to the gauge invariance of the gluon current, $Z_1 = Z_2$. Therefore the above transformation completely eliminates all parity and strangeness violation to order α. However, we see that in a different model for the strong interactions, this is not always the case. For example, if the strong interactions are mediated through scalar and pseudoscalar exchanges in the form of a generalized σ-model, this cancellation is not guaranteed.

This argument seems to plead in favour of a gauge theory of the strong interactions as well[49]. Since gauge theories have also other attractive properties (reggeization[50], asymptotic freedom, see below) this is a welcome result. I would like to notice however that it may be too early to exclude the presence of scalar or pseudoscalar intermediaries. As we shall show in the last section, there exist now models, including elementary scalar and pseudoscalar fields, which are assymptotically free. Furthermore these models have additional symmetry properties (super symmetries) which some of us believe that they will turn out to be relevant in elementary particle physics. For all these reasons it is interesting to examine under which conditions the presence of scalars in (2.2) does not violate to order α the strong interaction symmetries. I do not have a complete answer to this question, but, again, the conditions of ref. 19 apply here as well. In my simple quark model language, let me assume that all weak gauge bosons are coupled to the quarks either through a diagonal matrix (photon) or through a matrix proportional to $1 \pm \gamma_5$. Then it is easy to verify that the vertex corrections of Fig (1a) give contributions of order G and not order α, because all these diagrams are convergent. One has still to examine the other graphs, like the ones in Fig. (1b) and the answer will depend on the specific model. It would be interesting to examine this question using the general methods of ref (48), but since I have no particular model in mind, I do not want to push the point too far. Let me only notice that the presence of scalar fields may provide the solution to the η problem which will be discussed briefly below.

(b) There are other strong interaction symmetries, like isospin, for which order α corrections are welcome. However we would still like these symmetries to arise naturally in order for their corrections to

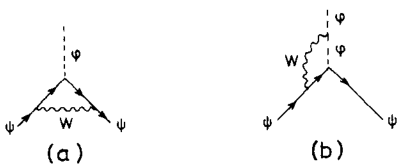

Fig. 1

be finite and calculable. I do not have anything to add here to the remarks made last year in Bonn and Aix[(3), (5)] except to point out some more recent calculations, using these ideas, of the $\pi^+ - \pi^0$ [(51)] and the p - n[(52)] mass difference.

The nice feature is that all these symmetries can be implemented naturally in the usual scheme in which the strong interactions are described by a colour SU(3) gauge group[(49)]. This scheme is very attractive but for one serious drawback. After the onset of the spontaneous symmetry breaking of the weak and e.m. gauge group, the strong interactions are described effectively by the twelve massive quark fields and the octet of the massless gluons. In such a theory the chiral symmetry is broken only by the quark masses. Let me forget for the moment, for simplicity, the strange and charmed degrees of freedom[(53)] and consider only the proton and neutron quarks. We can then construct an isovector and an isoscalar axial current which are both broken by the same terms, namely the proton and neutron quark masses.

$$\vec{J}_\mu^{\,5} = \sum_{\text{color}} \bar{q}\gamma_5\gamma_\mu\vec{\tau}q, \quad J_\mu^{\,5} = \sum_{\text{color}} \bar{q}\gamma_5\gamma_\mu q$$

In addition to this explicit breaking, one assumes that the chiral SU(2) x SU(2) is spontaneously broken giving rise to an almost Goldstone boson, which is identified with the pion. This picture has given very many good results (P.C.A.C., low energy theorems, etc.) and one wants absolutely to keep it. However the U(1) chiral group should be also equally broken since $\partial_\mu J_\mu^5$ is roughly equal to $\partial_\mu \vec{J}_\mu^{\,5}$. Still there is no light pseudoscalar isoscalar particle with mass comparable to m_π. In other words the question is why η is so heavy. This is basically the same old Sutherland problem[(54)] from the early days of current algebra. One can look at several ways out[(53)], none being particularly appealing, but I would like only here to repeat that the presence of fundamental, strongly interacting scalar fields gives a way to avoid the problem.

(vii) Baryon number conservation

The remarkable stability of the proton suggests that electric charge and baryon number are equally conserved. The first one is believed to be understood, in a field theory language, as a result of exact local gauge invariance. However there is no massless gauge field coupled to baryon number[(55)]. Trying to understand this difference one can adopt two opposite attitudes.

(a) Put baryon number on equal footing with electric charge and introduce a U(1) gauge group with a massless vector boson. By spontaneous breaking the vector meson acquires a mass, a hermitian scalar field, neutral with respect to baryon number, remains physical, and the resulting Lagrangian conserves baryon number[(56)]. Notice that this guarantees the lepton number conservation, but, if one wants to push the argument further, one should introduce an analogous mechanism in order to explain the separate conservation of the muonic and electronic numbers.

(b) A completely opposite point of view is to reject baryon number conservation altogether. This allows one to put in the same representation leptons and quarks. Since a three quartet quark scheme[(57)] is fashionable in gauge models, it is natural to use the lepton quartet as a "fourth color"[(58)] and put quarks and leptons in a 4 x 4 matrix. The resulting theory depends on the particular subgroup of $SU(4)_L \times SU(4)_R \times SU(4)'$ one is gauging. In particular Pati and Salam obtain exotic gauge bosons carrying both baryonic as well as leptonic quantum numbers and the possibility of baryon and lepton number violation. The amusing thing is that the model can be consistent with the extraordinary stability of the proton without introducing masses of the order of one gram! (For some versions of the model masses as heavy as 10^4GeV may be necessary.) The reason is that, if the proton is made out of three quarks, and if the one- and two-quark states (if they exist as asymptotic states) are assumed to be heavier, then the proton decay can occur only in third order in the interaction. One can easily see that baryon number violating coupling constants as large as $\sim 10^{-9}\, m_p^{-2}$ can be tolerated. Possible proton decay modes are $p \to 3\nu + \pi^+$, $p \to 4\nu + e^+$, $p \to 4\nu + \mu^+$, etc. No two-body decays are allowed. Notice that free quarks, if they exist, are unstable with life times ranging from 10^{-11} to 10^{-6} sec.

The Pati and Salam model is not the most economical one because we need four four-component lepton fields i.e. we must introduce also the right-handed neutrino fields. One may ask which is, if any, the simplest gauge group involving only the observed leptons in addition to the twelve quarks. The answer is obviously $SU(2) \times U(1) \times SU(3)$ where $SU(2) \times U(1)$ is the Glashow-Weinberg-Salam group and $SU(3)'$ is a strong interaction gauge group acting on the three quark quartets. This however has a low predictive power due to the U(1) factor. Is it possible to describe all interactions with a simple group? The answer to this question is given by Georgi and Glashow[(59)]. It is simple but yields again baryon number violation. It is shown that the smallest group is SU(5). Since it allows for only one coupling constant, it must be chosen of order e. It follows that the strong interactions must be enhanced by a separate mechanism. Georgi and Glashow use the conjecture made earlier[(60)] that such a mechanism can be provided by the infrared divergences of the strong gauge group SU(3) , which is assumed unbroken. The fact is that the infrared structure of non abelian Yang-Mills theories is very poorly understood at present[(61)] and, even at the classical level, the problem remains unsolved. However the idea of using an unbroken gauge symmetry for the strong interactions is quite attractive because then, the same conjecture, namely the infrared singularities, could also account for the absence of color non-singlet states from the physical spectrum[60].

Under this hypothesis the model is very simple and attractive. The fermions are placed in five- and ten-dimensional representations of SU(5). Baryon violation is strong and the corresponding gauge boson must be heavier than 10^{15} GeV $\sim 10^{-9}$g. Furthermore, since G is simple, there is only one arbitrary coupling constant which we call g. If we forget for the moment about renormalization effects, we can assume that g is constant independent of the momentum range we are measuring it. In this case a simple SU(5) calculation gives $\sin^2\theta_W = \frac{3}{8}$. The renormalization corrections will be examined below.

We see that all attempts to treat strong and weak interactions on equal footing result to non conservation of baryon number. In the absence of any experimental information, we have only our prejudices for guide.

(viii) Asymptotic freedom

The remarkable success of the simple parton model in predicting the behaviour of the structure functions in the deep inelastic lepton-nucleon scattering can only be described in a consistent field-theoretic language in the framework of non-abelian Yang-Mills theories[62]. The mechanism of asymptotic freedom is by now well understood and I won't describe it here[63]. I shall only make some remarks about the most recent developments.

The first is quite simple. Several people started worrying during the last year about the possibility of scaling violation in very high energy lepton-nucleon scattering. This was basically due to a misunderstanding of the meaning of asymptotic freedom. As it is often the case, whenever someone talks about freedom, it invariably turns out that he really means something else. The same is true here. The field theories in question are not really free and as a consequence the structure functions do not exactly scale. Logarithmic violations of scaling are expected and actually they can be used in order to make predictions about the μ-nucleon or ν-nucleon cross sections at N.A.L. energies. In D Gross' and A de Rujula's talks at the parallel sessions we heard in detail these predictions[64]. Let me only summarize the results here:

In an asymptotically free theory one can calculate the moments of the structure functions. They are given by[65]:

$$\int_0^1 F(x, Q^2)\, x^{n-2}\, dx = \sum_i C_i^{(n)}\, e^{-sA_i^{(n)}} \left| 1 + 0\left(\frac{\bar{g}^2}{4\pi}\right) \right| + 0\left(\frac{m^2}{Q^2}\right) \qquad (2.5)$$

where

$$s = \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_o^2}{\Lambda^2}} \qquad (2.6)$$

In (2.5) F is x F_1, F_2 or x F_3 and x is the Bjorken scaling variable. Q_o^2 is an arbitrary reference momentum and Λ sets the scale of approach to asymptotic freedom. $\bar{g}\,(g, \frac{Q^2}{\Lambda^2})$ is the effective coupling constant which tends to zero for $Q^2 \to \infty$. $A_i^{(n)}$ are calculable constants which depend only on the gauge group and the representations of the fields but $C_i^{(n)}$ are unknown. The sum over i corresponds to the different operators appearing in the operator product expansion of the two currents[66].

The form (2.5) satisfies all the parton model sum rules[65], but it does not allow to calculate the structure functions. However one can find linear combinations among them with the property that only a single operator contributes to each moment. In this case the logarithmic deviations from scaling can be computed, as functions of the parameter Λ[67]. They are more important for large values of Λ. By measuring the structure functions at different values of x and Q^2, one can measure Λ and also determine the gauge group of the strong interactions. These are the only arbitrary parameters in the theory. It is of course true that violations of scaling may be due to all sorts of other reasons (new thresholds, heavy particle production, etc.) but the ones predicted by asymptotic freedom have a very characteristic signature, especially at small values of ω[64] which makes them easy to distinguish. The idea of asymptotic freedom can therefore be tested experimentally in the near future.

Unfortunately this argument does not apply to e^+e^- annihilation, since we are really now looking at a Green function with all external momenta going to infinity[68]. Asymptotic freedom has a real meaning there, and the hadron production cross section, which absolutely refuses to fall, creates a serious problem. The best explanation may be that we are observing the opening of the charmed thresholds, in which case everything fits together very nicely.

My second remark concerns the recent calculations of the Callan-Symanzik β function in the two-loop approximation[69]. The general form is

$$\beta(g) = \frac{A\,g^3}{16\pi^2} + \frac{B\,g^5}{(16\pi^2)^2} \tag{2.7}$$

where A and B depend on the group and the fermion representations. The vector boson contribution to both A and B is negative and that of the fermions positive. In particular, by including enough fermion fields, one can make B large and positive, still keeping A negative. In this case β has an infrared stable zero very close to the origin. Notice also

that only the two first terms in (2.7) are physically relevant since all the higher ones can be changed arbitrarilly by changing the definition of the coupling constant[70].

My final remark concerns the possibility of introducing scalar fields. It was widely assumed, until very recently, that the presence of scalar fields destroys asymptotic freedom[71] because they introduce ϕ^4 couplings with new coupling constants. We learnt recently that this is not true! In the framework of supersymmetric theories (see below section VI) one can introduce scalar fields without new coupling constants. This by itself does not guarantee asymptotic freedom but a detailed calculation shows that the β function, in the one loop approximation, for a non abelian gauge and supergauge invariant theory based on the group SU(N), is given by[72]

$$\beta = -\frac{g^3}{16\pi^2}\,(3-n)\,N \tag{2.8}$$

where n is the number of scalar multiplets. Asymptotic freedom follows for $n < 3$. The $n = 3$ case is marginal. β vanishes identically in the one loop approximation and the behaviour of the theory will depend on the two loop contribution[73].

(ix) A general picture: Hierarchy of interactions

I believe that by now you are all convinced that gauge theories describe all interactions, from the strong down to the gravitational ones. Leaving the latter aside for the moment, since the renormalization problems are not yet understood, we see that a very simple and beautiful picture emerges, which has won the favour of most theorists. One starts from a large group G as the invariance group of the world[74]. G undergoes a two-step spontaneous breaking. During the first it is broken down to something like $SU(2) \times U(1) \times SU(3)'$ which is the observed symmetry group. This breaking is assumed to be superstrong, producing gauge bosons with superheavy masses. This is necessary in order to suppress all sorts of unobserved transitions (like proton decays!) which are allowed inside G. Then the second step is the usual breaking of the $SU(2) \times U(1)$ group which produces the ordinary massive intermediate bosons of the weak interaction, with masses of 10 - 100 GeV, leaving only a massless photon. As we said earlier, the strong interaction gauge group $SU(3)'$ is often assumed to be unbroken. The important thing is that, in this picture, all three interactions are supposed to be of comparable strength g, at momenta of the order of the superheavy masses. Let $g_i(\mu)$, $(i = 1, 2, 3)$ be the different coupling constants at a momentum range μ. Compared to the superheavy masses M, we are now exploring the infrared region[75], so let us choose $\mu << M$. The dependance of g_i on μ will be governed by a renormalization group equation of the form:

$$\mu \frac{\partial}{\partial\mu} g_i (\mu) = \beta_i \qquad (2.9)$$

and one can show[76] that one can calculate the functions β_i in an effective field theory based only on the observed group SU(2) x U(1) x SU(3)′, ie all superheavy particles can be omitted. Let us first, following Georgi, Quinn and Weinberg[77] assume that at some range of μ such that $m < \mu < M$, where m is a typical hadron mass (in practice we shall choose $\mu \sim 10$ GeV) all coupling constants are sufficiently small, so that one hopes to get sensible results from the one loop calculation. One then finds:

$$\beta_i (x) = b_i x^3 + \ldots\ldots \qquad (2.10)$$

where b_i are known constants. Therefore equation (2.9) gives:

$$g_i^{-2} (\mu) \sim \text{constant} - 2b_i \ln\mu \qquad (2.11)$$

The integration constants can be fixed by the assumption that at $\mu \sim M$, $g_i(M)$, assuming they are also small, are essentially equal to g, the coupling constant of G. Therefore we write (2.11) as:

$$g_i^{-2} (\mu) \sim g^{-2} + 2b_i \ln \frac{M}{\mu} \qquad (2.12)$$

The system (2.12), for $\mu \sim 10$ GeV, relates the values of the three "observed" coupling constants (which can be taken to be α, the Weinberg-Salam angle θ_W and the strong coupling g_3) with the parameters of G, namely g and M. Inserting numbers we get the following table:

g_3^2 (10 GeV)/4π	M (GeV)	$\sin^2\theta_W$
0.5	2.10^{17}	.17
0.2	2.10^{16}	.19
0.1	5.10^{14}	.21
0.05	2.10^{11}	.25

These results have been obtained under the assumption that at 10 GeV asymptotic freedom has already started and the effective value of the strong coupling constant is sufficiently small to allow perturbation calculations. This is substantiated by the observed scaling. If however we relax this assumption then we can only solve (2.9) for g_1 and g_2. The corresponding β functions must then be calculated to lowest order in g_1 and g_2, but to all orders in g_3. G Parisi[78] observed that, taking into account the strong interactions to all orders, relates β_1 and β_2 to R, the ratio of hadrons to $\mu^+\mu^-$ production cross sections in e^+e^- annihilation. Since asymptotic freedom is not assumed to have set, R can have any value. In this case one can only obtain a relation between θ_W, R and M. Actually Parisi[78] has a somehow different approach, which is closer to the spirit of Wilson's ideas[75]. He uses a theory with a cut-off which is finite, although very large, of the order of 10^{18} GeV. In the limit of infinite cut-off one expects all the coupling constants of non-asymptotically free theories to vanish. The equations (2.9) are now replaced by those obtained by using the leading logarithm method of Landau and Pomeranchuk[79]. Using the value of α we find $R \sim 19$ and $\sin^2\theta_W \sim .3$.

Let me summarize the whole story: At very short distances, much further than present day energies, one assumes that all three interactions have the same coupling constant. From this point one extrapolates down by using the renormalization group techniques. The resulting picture is shown in fig. (2). There are two main ways to look at it:

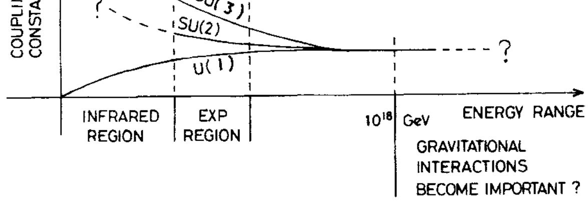

Fig. 2

(a) One can assume that the strong interactions are effectively described by an asymptotically free theory. In this case the common value g of the coupling constant at $\sim 10^{18}$ GeV is small but one can follow the evolution of the three coupling constants by using the equation (2.12)[77]. In the experimentally accessible region $\mu \sim 10$ GeV, the strong interactions are governed by asymptotic freedom, the coupling constant is small and we have approximate scaling.

(b) In the second point of view one again assumes that all couplings are equal but g is large, may be of order one. In the region from 10^{18} down to about 10 GeV, where the effective quark masses become important, the strong interactions are governed by an infrared stable fixed point[78]. (One assumes that we have enough fermion fields to create a second zero of the β function the way it was explained above.) Observed scaling is due to the smallness of anomalous dimensions. In both pictures the regions of very low (below ~ 1 GeV) and very high (above $\sim 10^{18}$ GeV), energies are essentially uncalculable, the first because it corresponds to the really infrared behaviour and the second because gravitational effects are expected to become important. The behaviour in the intermediate region is similar. The violations of scaling are logarithmic in the first picture, they obey a power law in the second.

Finally let me remark that the order of magnitude of the superheavy masses may sound enormous, however if one accepts the initial idea, namely that in some range all coupling constants are equal, there is no other possibility. In fact the dependance of the coupling constants on the momentum range predicted by the renormalization group is logarithmic (equation (2.12)) and therefore one needs these orders of magnitude in order to explain the observed differences of strength among the interactions in ordinary energies. Furthermore these are precisely the energies in which the gravitational interactions are expected to become important, which suggest some possible connections between superheavy breaking of G and gravitation[80].

I could continue my list of desired properties for the ultimate model of the world, but it must be obvious by now that no existing model satisfies all of them. However, continuing my optimistic view, I believe that all these conditions are so restrictive that the model will be quite unique. The reason why it has escaped discovery up to now, must certainly be attributed to insufficiency of experimental data (after all until some months ago there were people who did not believe in the existence of neutral currents). In any case, and I am sure everybody agrees, the fact that we can seriously discuss all these properties outside the science fiction conventions, is a tremendous progress which is solely due to the adventure of gauge theories.

III EXPERIMENTAL CONSEQUENCES

Now that we have a renormalizable theory of the weak interactions we should normally look for crucial experimental tests, the analog of the Lamb shift or the value of $g - 2$, which established quantum electrodynamics. Unfortunately things are not so easy. First of all, we should restrict ourselves to the leptonic world, if we want to have unambiguous predictions. The radiative corrections to μ decay, or the lepton-lepton scattering could be used for such a test and in fact the gauge theories allow for exact and detailed calculations[81]. However, the very fact that we are dealing with a renormalizable theory, tells us that all such corrections will be of order α. Therefore the experimental accuracy required to test the theory should be of at least $1^{o}/o$ or better, which is clearly out of reach at present. This situation forces us to look for less direct tests and we shall discuss some of them in this section. For more details one should look at the specialised talks of the parallel sessions.

The detailed predictions depend on the particular model we are assuming, but there are some which appear inescapably to all models:

(i) The existence of the intermediate vector bosons. They are obviously indispensible to any gauge model but their masses are expected to be large, of the order of $[\alpha G_F{}^{-1}]^{\frac{1}{2}}$, or $\sim$ 50 GeV, too heavy even for N.A.L. There exist indirect ways to look for them but they use extra assumptions, like scaling, which may be incorrect. Another possible test would be to look for prompt muons at I.S.R. but the sensitivity depends on the production cross section(82).

(ii) All gauge models at present use the Higgs mechanism to trigger spontaneous symmetry breaking. There remains always at least one physical scalar particle, but its mass is, in general, not restricted by the theory. Furthermore, the couplings with the leptons are small, proportional to the lepton masses.

(iii) The third prediction of almost all models concerns the hadronic states. Let us assume for the moment that all particles are built out of the three basic quarks p, n and λ. Then the Cabibbo current is given by:

$$J_\mu = \bar{q}\gamma_\mu (1 + \gamma_5)\, Cq$$

where

$$q = \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The adjoint $J_\mu{}^\dagger$ will simply involve the matrix $C^\dagger$. Now, in any gauge theory in which J_μ and $J_\mu{}^\dagger$ are coupled, their commutator will also appear somewhere.

$$J_\mu{}^{(o)} = \bar{q}\gamma_\mu (1 + \gamma_5) C^o q,$$

$$C^0 = [C, C^\dagger] = -\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\cos^2\theta & -\cos\theta\sin\theta \\ 0 & -\cos\theta\sin\theta & -\sin^2\theta \end{pmatrix}$$

We see that C^o has nonvanishing off diagonal matrix elements, ie it contains terms of the form $\bar{n}\lambda$. If the hadron and the lepton sectors are not completely disconnected(83), this current will eventually find its way to the leptons and it will induce strangeness changing neutral currents of the form $K^o \to \mu^+\mu^-$ or $K^+ \to \pi^+ \nu\bar{\nu}$, etc. It follows that the traditional SU(3) scheme is incompatible with the kind of theories we are discussing(84). The enlargement of the symmetry can go in different directions and from that point the predictions will depend on the particular model. In any case, new, as yet unobserved, hadronic states carrying new quantum numbers, are predicted. Their masses cannot be arbitrarily large(85) ($\sim$ 5 GeV is a reasonable guess) and although it is perfectly conceivable that they have escaped detection until now, their discovery, if they are there, is possible. I consider this prediction as the most crucial test of these ideas.

I will call these states collectively "charmed", although I do not restrict myself to the SU(4) model. I have won already several bottles of wine by betting for the neutral currents and I am ready to bet now a whole case that if the weak interaction sessions of this Conference were dominated by the discovery of the neutral currents, the entire next Conference will be dominated by the discovery of the charmed particles. Since "charm" is conserved by strong interactions, these particles are produced in pairs in hadronic reactions. On the other hand, due to their large mass, they are very short lived ($\sim 10^{-12}$ sec), do not make tracks in bubble chambers and they have a large number of decay channels. Their search in p-p collisions will parallel that of W's, ie by looking for prompt energetic muons(86). (I was told however that there exists a program to look for them directly in emulsions.)

It turns out that the best place to look for them is again neutrino reactions(86), (87). Let me take, as an example, the SU(4) charmed states. The charm changing current has the form $-\bar{p}'\, n \sin\theta + \bar{p}'\lambda \cos\theta$. It follows that charmed particles prefer to decay with a change of strangeness. On the other hand you do not expect them to be copiously produced since you either produce them on a neutron quark with a factor $\sin\theta$, or you excite the quark-antiquark sea in the nucleon which is supposed to be small. It follows that in either case the event has strange particles in the final state. Perhaps a detailed study of these events will provide us with some information about charmed particles.

More sensitive tests may be given by the fact that in "charming" theories the weak current is not charge-symmetric. Therefore precise tests of charge symmetry in high energy neutrino reactions may reveal the presence and the nature of charm[87]. One could also use the Adler and the Gross-Llewellyn Smith sum rules since their right hand sides change above the charm threshold. Since all these questions have been discussed in detail in A de Rujula's talk[88] in the parallel sessions, I will not elaborate any more here. All other predictions depend on the particular model and cannot be used to test the whole idea. Several models require the introduction of heavy leptons and we heard in B Barish's talk[89] everything about their present status. Detailed calculations about their production rates in e^+e^- annihilations and ν reactions have been performed[90]. The neutral currents have finally been observed and they have thus eliminated some of the early models. However a detailed study of their properties can still be used in order to discriminate among the remaining ones[91]. I will not review the different calculations in detail but I would like to correct here a wide spread error which we heard several times in this Conference. Some people think that in gauge theories, neutral currents are necessarily parity violating. This is absolutely wrong! One can construct gauge models with parity conserving neutral currents[92] and in fact this is one of the properties that can be used in order to choose the right theory. I would also like to mention some studies of neutral current effects outside the domain of elementary particle physics. Assuming they do violate parity, one place to look for them is to try to detect parity violation in radiative transitions between atomic levels[93]. It turns out that this is possible and a specific experiment is already in progress[93] which looks for such effects in twice forbidden magnetic dipole transitions, induced by a tunable laser beam, in heavy atoms. Another proposal suggests the study of similar effects in muonic atoms[93,94]. There is also a calculation of the influence of the neutral currents in the electric current flowing through two Josephson junctions[95]. I consider all these proposals very interesting because it is easy to construct models which differ appreciably only in the couplings of Z_μ to electrons and not to neutrinos. Therefore any information of neutral current effects outside the neutrino physics will be very useful.

Finally very interesting results have been obtained on the possible role of neutral currents in astrophysics. D Friedman[96] has calculated the coherent scattering of neutrinos on nuclei and his results show[97] that their role in the dynamics of a supernova explosion are very important.

IV FIELD THEORY AND RENORMALIZATION

(i) Some further results on renormalization

Although the problem of renormalization of spontaneously broken gauge symmetries is supposed to be solved since the pioneering works of 't Hooft[28], B Lee and Zinn-Justin[29] and 't Hooft and Veltman[30], several investigations have been performed recently in order to clarify certain points or to present more elegant and rigorous formulations. Furthermore the cancellation of divergences at the one loop level has been verified and the renormalization program has been worked out explicity in special models[98]. On the other hand similar results have been obtained while taking into account the strong interactions to all orders, using current algebra techniques[99]. An improvement to the original proofs of renormalization has been given by B Lee[100] who established and used the Ward identities for vertex functions instead of the ones for connected Green functions previously used. He thus obtained a more concise a general treatment which continued and completed the program started in the papers of ref.(29).

However I would like in this paragraph to mention especially a different line of approach, which uses the Zimmermann normal product algorithm[101], and which has made substantial progress during the last months. I will follow essentially the method of Becchi, Rouet and Stora[102] (B.R.S.) who have recently introduced an essentially new technique for the study of the Ward identities which, I believe, will be proven to be very useful.

The abelian gauge theories with spontaneous symmetry breaking had been studied[103] in the framework of the normal product algorithm, but so far the investigations were limited to the Stückelberg gauge[104]. In this gauge, like in ordinary quantum electrodynamics, the gauge fixing term is proportional to $\partial_\mu A_\mu$ and the theory is free from Faddeev-Popov ghosts[53]. However, in a non abelian theory, their appearance is unavoidable and, therefore, the study of the abelian models in the Stückelberg gauge could not be extended in a straight forward way to the Yang-Mills theories. The essential difficulty comes from the fact that the gauge invariance of the Lagrangian is now broken not only by the gauge fixing term, but also by the Faddeev-Popov compensating term. As a consequence the original Ward identities become more complicated, in a form found by Slavnov[105] and Taylor[106]. B.R.S. have found a very ingenious way to handle these identities.

Let me denote by A_μ^a (x) the gauge fields of the theory and by ϕ^i (x) the set of Higgs-Kibble scalars. Let me also introduce a gauge fixing term G^a which I shall choose to be of the form proposed by 't Hooft[28], since, in this case, I get the extra bonus that no massless propagators appear. The effective Lagrangian can now be written as:

$$\mathcal{L} = \mathcal{L}_{inv.} (\phi^i, A_\mu{}^a) - \tfrac{1}{2} (G^a)^2 - C^a M^{ab} \bar{C}^b \qquad (4.1)$$

where

$$M^{ab} = \frac{\delta G^a}{\delta \Lambda^b} \qquad (4.2)$$

and C and $\bar{C}$ are the anticommuting scalar ghost fields. $\mathcal{L}_{inv.}$ is the original gauge invariant Lagrangian.

B.R.S. observed that the Slavnor-Taylor identities follow formally from the invariance of $\mathcal{L}$ under the following transformations:

$$\begin{aligned} \delta\phi^i &= \lambda \frac{\delta\phi^i}{\delta\Lambda_b} \bar{C}^b \\ \delta A_\mu{}^a &= \lambda \frac{\delta A_\mu{}^a}{\delta\Lambda_b} \bar{C}^b \\ \delta C^a &= \lambda G^a \\ \delta\bar{C}^a &= \tfrac{1}{2} \lambda f^{abc} \bar{C}^b \bar{C}^c \end{aligned} \qquad (4.3)$$

where the infinitesimal parameter λ is not a c-number but an anticommuting parameter, ie an element of a Grassman algebra. The invariance of (4.1) under the transformations (4.3) can be checked directly remembering that c, $\bar{c}$ and λ all anticommute. From this invariance one obtains formally "Ward identities" which are nothing else but the Slavnov-Taylor identities of the original theory. Using this invariance B.R.S. were able to prove that these identities are true not only in the tree level but in all orders of the renormalized perturbation theory for arbitrary gauge groups. Furthermore they established the gauge invariance and the unitarity of the S-matrix for the abelian case. The extension of this result to non abelian groups is now in progress. However this is now only a matter of working through the algebra of the group and no essentially new techniques need to be developed.

I consider this result as a very important one, not only because it provides a simple and elegant proof of the renormalizability of Yang-Mills theories, but also because transformations of the type (4.3) may prove useful in other cases as well[107].

(ii) Yang-Mills theories in the many-field limit

Field theories in the many field limit often exhibit remarkably simple properties. This was known for the ϕ^4 theory and it was recently shown for Yang-Mills by G 't Hooft[108]. He studied a U(N) gauge theory with spinors belonging to the adjoint representation. The result is that, at the limit $N \to \infty$ with g^2N fixed, the dominant diagrams are the planar ones with the spinors at the edges. This set of diagrams can be summed exactly in a simplified model in two space-time dimensions[109]. They are actually reduced to self-energy and ladder diagrams. The resulting picture resembles that of a quantized dual string and the mass spectrum consists of a nearly straight Regge trajectory.

(iii) Is quantum theory of gravity renormalizable?

It is not the purpose of this talk to review the different attempts to quantize general relativity. The problem is still open and the results quoted here have to be taken with caution. Since non abelian gauge theories have been successfully quantized, it was normal to try the same functional methods on general relativity. Unfortunately the results were not so encouraging. One can easily obtain a set of covariant Feynman rules and study the renormalization properties at the one loop level. The results are the following:

(a) G 't Hooft and M Veltman[110] have found that pure gravity is renormalizable at that level.

(b) Coupling the Einstein field to matter destroys renormalizability. The systems studied include scalar fields[110], the electromagnetic field[111], fermions[112] and Yang-Mills fields[113]. The same is true for the Brans-Dicke theory which is again non renormalizable[111]. However spontaneously broken gauge theories in an external gravitational field remain renormalizable[114]. Finally let me mention the result obtained by Linde[115] who calculated the space-time curvature which results from the spontaneous symmetry breaking. Since this latter must be very large in order to generate the heavy intermediate vector bosons of the weak interactions, the effect on the curvature is enormous. However the existence of the effect depends on whether one switches on gravitation before or after the symmetry breaking.

(iv) Solutions of classical Yang-Mills theories

Although classical field theory has never ceased to attract the attention of theorists, the detailed study of classical models was considered until recently a somehow esoteric occupation by most elementary particle physicists. With the adventure of gauge theories there was first a revival of interest in quantum field theory and, through that, more people got interested in the study of the classical analogs. I am not going to report here on all the progress made in this general field, but I would like to mention one amusing result which was obtained in the classical Yang-Mills theory during the last year, by G. 't Hooft[116]. He studied the classical Lagrangian describing the interaction of a Yang-Mills and a scalar field:

$$\mathcal{L} = -\tfrac{1}{4}G_{\mu\nu}{}^{a}G_{a}{}^{\mu\nu} - \tfrac{1}{2}D_{\mu}\phi_{a}D^{\mu}\phi_{a} - \tfrac{1}{2}\mu^{2}\phi_{a}{}^{2} - \frac{\lambda}{8}(\phi_{a}{}^{2})^{2}$$

in the case $\mu^2 < 0$, ie when spontaneous symmetry breaking occurs. Let us take as an example the O(3) gauge group with ϕ_a belonging to a triplet. The interesting result is that a solution of the equations of motion exists in which the electromagnetic field $F_{\mu\nu}$ can be taken, at large distances, to be:

$$F_{\mu\nu} = -\frac{1}{e\tau^3}\varepsilon_{\mu\nu a}\tau_a$$

where τ is the distance from the origin and $\varepsilon_{\mu\nu a}$ is defined to be zero when any one of its indices takes the value zero. It follows that there is a radial magnetic field

$$B_i = \frac{\tau_i}{e\tau^3}$$

with a total flux $4\pi/e$. In other words the solution describes a magnetic monopole satisfying Schwinger's condition $eg = 1$ (117). The mass of the monopole can be easily calculated and it is given approximately by

$$M_m \sim \frac{M_W}{\alpha}$$

where M_W is the gauge boson mass. Since this last one in all unified theories is rather heavy, we see that this result is not very encouraging for the experimental search of the monopoles.
Solutions exhibiting monopole behaviour can also be found for other semi-simple gauge groups but not when a U(1) factor is present(118).

(v) Gauge fields on a lattice

Field theories on a lattice have been studied since very many years, but it is only recently that gauge theories have been included. Their study presents several interesting features, but one of the most important for physical applications is the fact that they give information about the strong coupling regime. By now we have good reasons to believe that gauge theories are relevant for strong interactions but the usual methods of perturbation theory tell us nothing about the region of large coupling constants. Therefore any method who provides such information, even at the price of sacrificing Poincaré invariance, is very welcome.
The program was initiated by K Wilson(119). He observed that one can quantize a gauge theory on a lattice still keeping exact gauge invariance at the Lagrangian level. The usual gauge fixing term is here replaced by a periodicity condition which restricts the range of integration over the gauge field. The results are physically very interesting. In particular it follows that an isolated fermion has infinite mass and a separated fermion-antifermion pair has a finite energy proportional to their separation. Hence Wilson speculates that such a theory may provide a model for quark confinement.
The model is quantized by the Feynman path integral method. In order to write down the action Wilson imposes exact gauge invariance on the lattice, not only at the continuum limit. The free fermion part can be written in a straight forward way. With the obvious changes from the continuum to a discreet space we write:

$$A \sim \sum_n \{ \frac{a^3}{2} \bar{\psi}_n i\gamma_\mu (\psi_{n+\hat{\mu}} - \psi_{n-\hat{\mu}}) - m_o a^4 \bar{\psi}_n \psi_n \} \quad (4.4)$$

where a is the lattice spacing, m_o is the bare fermion mass and $\hat{\mu}$ is a lattice vector of length a ("unit" vector) along the axis μ. Local gauge invariance is imposed the usual way, namely by introducing a gauge field and multiplying by the exponential of its line integral. This introduces an interaction term of the form:

$$\bar{\psi}_n \gamma_\mu \psi_{n+\hat{\mu}} e^{igaA_{n\mu}} \quad (4.5)$$

which is gauge invariant, provided $A_{n\mu}$ transforms like:

$$A_{n\mu} \to A_{n\mu} - \frac{\phi_{n+\hat{\mu}} - \phi_n}{ga} \quad (4.6)$$

The important point is that in (4.5) the variable $gaA_{n\mu}$ appears as an angle and the action is periodic with period 2π. K Wilson imposes this periodicity also to the free gauge field action and writes it as:

$$\frac{1}{2g^2} \sum_{n,\mu,\nu} e^{if_{n\mu\nu}} \quad (4.7)$$

where

$$f_{n\mu\nu} \equiv ga^2 F_{n\mu\nu},$$

$$F_{n\mu\nu} \equiv \frac{1}{a} \{ A_{n+\hat{\mu},\nu} - A_{n\nu} + A_{n+\hat{\nu},\mu} - A_{n\mu} \} \quad (4.8)$$

Notice that this requirement of periodicity, which is introduced by hand in the abelian model presented here, is imposed by the requirement of gauge invariance in the non-abelian cases. As it was noticed before, this periodicity property restricts the region of integration over the gauge field variable from $(-\infty \div +\infty)$ to $\frac{1}{ga^2}(-\pi \div \pi)$. Therefore no gauge fixing term needs to be introduced and consequently all gauge non invariant quantities vanish. In particular the two fermion propagator S(n) is given by:

$$S(n) \sim \langle o \mid T(\psi_n \bar{\psi}_o) \mid o \rangle \sim \delta_{no} \quad (4.9)$$

because it is not invariant for $n \neq 0$. We see therefore that the mass of an isolated fermion is infinite. This result may look surprising since S(n) may not be proportional to δ_{no} in the familiar weak coupling regime. The reason is that there, gauge invariance is expected to be broken spontaneously and a phase transition may occur when one goes from the weak to the strong coupling regions[120]. The behaviour of the theory in the strong coupling limit can be studied only for $m_o a >> 1$. In this limit the fermion mass term dominates and both the kinetic energy and the interaction can be treated as perturbations. Then one can prove (4.9) to all orders.

One can study next the minimum energy of a gauge invariant state (otherwise, as we just explained it has infinite energy) made out of a well separated fermion-antifermion pair. One finds that if τ is the separating distance

$$E \simeq \frac{\tau \ln g^2}{a^2} \quad (4.10)$$

ie it increases linearly with the separation. If such a model is an approximation of the bound quark system, one can understand why quarks scatter like free in deep inelastic lepton scattering but cannot still be produced as free particles.

V. SPONTANEOUS SYMMETRY BREAKING

(i) Genesis of spontaneous symmetry breaking:

The traditional way to induce spontaneous symmetry breaking (S.S.B), is to introduce a scalar field $\phi(x)$ and arrange the parameters of the theory so that the classical potential $V(\phi)$ has a minimum for $\phi \neq 0$. Coleman and Weinberg[45] observed that, even if the classical potential does not have such a minimum away from the origin, the radiative corrections may change the situation. They use the method of Jona-Lassinio[121] and they calculate the corrections to $V(\phi)$[44,122]. It follows that no such minimum for $\phi \neq 0$ exists in the case of a $\lambda\phi^4$ theory, at least for small values of λ where perturbation theory can hopefully be trusted. This situation is in fact common to all one-parameter theories. On the contrary such solutions exist for theories having more than one coupling constant. In particular they show that massless scalar electrodynamics is essentially unstable. The radiative corrections de-stabilize the vacuum and trigger a Higgs mechanism. The same is true for non-abelian gauge groups. Similar techniques can be used in order to study the possibility for a non-canonical field to induce S.S.B[123]. We thus hope to avoid the Higgs scalars which are usually introduced in a completely arbitrary way[124]. Let me choose a re-normalizable interaction between spinors and vector mesons. Obviously no canonical field will ever acquire a non-zero vacuum expectation value in such a theory. Suppose that I decide to probe the operator $\bar{\psi}\psi$. I introduce a source term $J(x)\bar{\psi}\psi$ in the Lagrangian and I calculate the effective potential as a function of η, the classical field corresponding to the operator $\bar{\psi}\psi$. The tree approximation of $V(\eta)$ obviously vanishes. The one loop contribution is given by the sum of all graphs of the form of Fig. 3. The crucial, though trivial, point is that these graphs correspond to a free field theory. It is only at the two loop level that the interactions appear for the first time. Therefore $V(\eta)$ can be written as:

$$V(\eta) = V^{(1)}(\eta) + \sum_i g_i^2 V_i^{(2)}(\eta) + \ldots\ldots \quad (5.1)$$

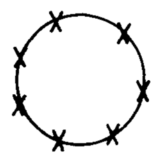

Fig. 3

where g_i's are the coupling constants of the various vector fields with the spinor ψ. It is easy to verify that $V^{(1)}(\eta)$ can be chosen to be a monotonic function of η which obviously vanishes at $\eta = 0$. Equation (5.1) now shows clearly that for all $\eta \neq 0$ $V'(\eta) \neq 0$ for small coupling constants at any finite number of loops. The same result holds obviously for any other scalar operator like A_μ^2 etc. Therefore perturbation theory cannot provide a mechanism for S.S.B. other than the classical one in which a canonical scalar field of the theory acquires a non-zero vaccum expectation value.

This result is certainly not surprising. S.S.B. is expected to occur, in the absence of elementary scalar fields, through the boundstates of the system. But it is well known that perturbation theory, in any finite order, does not describe bound state effects. Therefore, in order to draw any conclusions, one has the following alternative:

(a) To extract any information about bound states indirectly, for example by studying the Bethe-Salpeter equation,

(b) To investigate solvable field theory models.

The first method was used last year by Jackiw and Johnson[125] and Cornwall and Norton[126]. They presented the general formulation of S.S.B. through a bound state and they remarked the possibility of a zero energy bound state in a chiral U(1) x U(1) extension of quantum electrodynamics[127]. The second part of the alternative was examined recently by Gross and Neveu[128] who studied a class of two-dimensional fermion field theories, with quartic interactions[129], in the limit $N \to \infty$, where N is the number of components of the fermion field. Notice that these models are asymptotically free. In the limit $N \to \infty$ the model becomes solvable and they found that dynamical symmetry breaking occurs for any value of the coupling constant. However, due to the special kinematical properties of the two dimensions, the significance of the result is not clear, since it looks like it contradicts a general theorem by Coleman which states that there are no Goldstone bosons in two dimensions[130]. The authors remark that their effect is expected to disappear when higher orders in 1/N are taken into account. Therefore they introduce a chiral U(1) gauge group in which case a Higgs phenomenon occurs. Indeed, if Goldstone bosons are forbidden in two dimensions, spontaneous breaking of a gauge symmetry can still occur and in fact it happens in the well-known example of massless two-dimensional quantum electrodynamics[131].

To summarise, we see that dynamical symmetry breaking is perfectly conceivable, although no explicit study can be made in four-dimensional field theories[132]. What bothers me with this approach is that the connection with renormalized perturbation theory is totally obscure.

(ii) <u>Abnormal nuclear states and vacuum excitation</u>

A very simple classical example of a spontaneous symmetry breaking is given by a bent rod[133]. For a homogeneous cylindrical rod with an applied force along the z-axis, the problem has a rotational invariance around this axis. However, we all know, and in this case we can prove it by solving the corresponding eleasticity equations[133], that the resulting state is asymmetric and consists of a bent rod. The initial invariance is manifest in the degeneracy of the ground state. (We cannot predict which direction in the x - y plane the rod is going to bend.) The analog in quantum field theory is to say that the vacuum in degenerate. There exists however an important difference: By applying an infinitesimal lateral force to a bent rod one can rotate it and thus change from one ground state to another. On the other hand, in a quantum field theory, which is a system with an infinite number of degrees of freedom, this is not possible. Starting from each vacuum state one can build a corresponding Hilbert space and the different such spaces are supposed to be orthogonal to each other. T D Lee and G C Wick observed however[134] that such a change of state can still occur inside a finite though large domain. They found some very interesting phenomena which may have observable consequences. I will not give a detailed account of this work, because we heard T D Lee's talk in the parallel session[135], but I would like to summarise briefly the argument and present some of the results. As an example we consider a Lagrangian of a neutral scalar field $\phi(x)$:

$$L = -\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 - \frac{1}{3!}f\phi^3 - \frac{1}{4!}\lambda\phi^4 + \text{counter-terms.} \qquad (5.2)$$

and let me assume that all necessary translations have taken place and the counterterms are arranged so that

$$< 0 \mid \phi \mid 0 > = 0 \qquad (5.3)$$

Let me stay, for simplicity, at the tree level[136]. Condition (5.3) implies the inequality

$$3\mu^2\lambda \geqslant f^2 \qquad (5.4)$$

We see that if $3\mu^2\lambda > f^2$, there is only one absolute minimum at $\phi_c = 0$; for $3\mu^2\lambda = f^2$ a degenerate ground state appears. The result of T D Lee and G C Wick can be best stated if we consider the system quantized inside a finite volume Ω. Let me consider an excited stated in which, inside a region V, the system satisfies, instead of (5.3)

$$< 0 \mid \phi \mid 0 >_V = \phi_{vex} \qquad (5.5)$$

This excited state has a lifetime T (V). The result now can be stated as:

(a) Degenerated case: The two states (5.3) and (5.5) are completely symmetrical. When $\Omega \to \infty$ they become orthogonal to each other. Furthermore:

$$\lim_{\Omega \to \infty} T_\Omega(V) = \text{constant} \qquad (5.6).$$

where the limit is taken keeping V fixed. The r.h.s. depends on V and tends to infinite (stable state) when V becomes large.

(b) Non degenerate case:

$$\lim_{\Omega \to \infty} T_\Omega(\Omega) = 0 \qquad (5.7)$$

$$\lim_{\Omega \to \infty} T_\Omega(V) = \text{constant} \qquad (5.8)$$

We see therefore that in both cases the vacuum excitation can extend over a domain of macroscopic size. The phenomenon is analogous to the domain structure in a Heisenberg ferromagnet.

Now let us complicate the picture and add to (5.2) a Yukawa coupling to a fermion field ψ. Let us even assume that ψ is the nucleon field and V is the volume of a nucleus. Inside V the vacuum exp. value of ϕ can be equal to ϕ_{vex} and this, through the Yukawa coupling, will induce a mass-shift of the nucleon $\delta m \sim g\,\phi_{vex}$ where g is the Yukawa coupling constant. It may happen that such an "abnormal" nuclear state becomes stable for sufficiently high nuclear densities and this would result in the appearance of new objects which may be created with the help of high energy heavy ion beams.

(iii) <u>Finite temperature effects</u>

It is well-known that if we heat a piece of iron beyond the Curie point, the magnetization disappears and the rotational invariance is restored. Kirzhuits and Linde[137] have suggested that something analogous should happen in quantum field theory. Above a certain temperature the system should return to the symmetric state. In particular, if the s.b.s was a gauge symmetry, this means that, at sufficiently high temperatures, long range forces would appear. The cosmological implications of this phenomenon are obvious. Such high temperatures are expected to have occurred in the early period of the history of the universe, just after the big bang. At this time the weak interactions would have produced long range forces which could have affected the properties of our universe, such as isotropy or homogeneity[137]. Furthermore the considerations of the previous paragraph bring in mind again the analogy with ferromagnetism: When a piece of iron is cooled below the Curie point, in the absence of an external magnetic field, the spontaneous magnetization does not appear in the same direction over the whole piece, but one rather observes a domain structure. Does this suggest the possibility that our universe is in fact separated in such domains and inside each one the symmetry breaking has chosen a different direction? What sort of observable effects such a domain structure could have?

I shall not attempt to answer these questions here and I shall limit myself in giving rough estimations of the order of magnitude of the critical temperature in simple models[138]. Let me first notice that the phase transition is expected to occur at a temperature such that powers of T can compensate for the powers of the coupling constant of the perturbation expansion. For simplicity, I shall only consider a model of a neutral scalar field with a quartic interaction. The symmetry here is $\phi \to -\phi$. For a more detailed discussion as well as the consideration of continuous and gauge symmetries, I refer you to the recent paper by S Weinberg[139].

Let the bare Lagrangian of the system be:

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m^2\phi^2 + \lambda\phi^4 \tag{5.9}$$

Let me be very naive at first and ignore all renormal-renormalization effects. The condition for symmetry breaking in (5.9) is given by the mass term: $m^2 < 0$. At finite temperature T and ignoring renormalization, the critical region will be determined by the relative magnitude of the $m^2\phi^2$ and the $\lambda\phi^4$ terms. The volume of the system is $V \sim \left(\frac{1}{m}\right)^3$ and the momentum cut-off is $\sim kT$. We thus get:

$$\phi^2 \sim mkT \qquad\qquad \phi^4 \sim m^2(kT)^2$$

The condition therefore is:

$$1 > \frac{\lambda\phi^4}{kT\ m^3} \to kT < \frac{m}{\lambda} \tag{5.10}$$

However we know that renormalization effects are important, in particular because they change the value of the bare mass. Let m_R be the renormalized mass at $T = 0$ and $m_R(T)$ the renormalized mass at any T. The mass counterterms in a ϕ^4 theory are quadratically divergent. The divergent terms are absorbed by the renormalization from m to m_R, but the convergent parts of the corresponding loops give a correction of the form $\lambda(kT)^2$. We thus have:

$$m_R(T)^2 = m_R^2 - \lambda(kT)^2 \tag{5.11}$$

If the symmetry is broken at $T = 0$ the critical temperature will be given by the condition $m_R(T_c) = 0$ or

$$kT_c \sim \frac{m_R}{\sqrt{\lambda}} \tag{5.12}$$

The perturbation expansion is supposed to break down near the critical temperature. Let λ_{eff} be the "effective coupling constant", ie the value of the four point function at zero external momenta and any T. From (5.10) we get:

$$\lambda_{eff} \sim \lambda\frac{kT}{m_R(T)} \tag{5.13}$$

Perturbation theory breaks down when $\lambda_{eff} \sim 1$. Thus:

$$m_R(T) < \lambda kT$$

$$\left|\, m_R^2 - \lambda(kT)^2 \,\right| < \lambda^2(kT)^2$$

$$\frac{m_e^2}{\lambda} + m_R^2 > (kT)^2 > \frac{m_e^2}{\lambda} - m_R^2 \tag{5.14}$$

The inequalities (5.14) determine the width of the critical region. It is easy to show that it does not change when we take into account all the renormalization effects. The reason is that, as we noticed before, the important feature in these theories is the breaking at $T = 0$ by the negative square mass, so the important renormalization effect is the mass renormalization.

S Weinberg[139], by using a more detailed method of calculation finds similar results. For example, for the case of an O(n) global symmetry group he finds the critical temperature to be:

$$kT_c = \sqrt{\frac{6}{n+2}}\ \frac{m_R}{\sqrt{\lambda}} \qquad (5.15)$$

where m_R is the mass of the single non Goldstone boson (5.15) is essentially the same as our formula (5.12). For the case of a local O(n) symmetry with gauge coupling constant g he finds:

$$kT_c = m_R \left[\frac{1}{6}\ (n+2)\lambda + \tfrac{1}{2}\ (n-1)g^2 \right]^{\frac{1}{2}} \qquad (5.16)$$

For the weak and e.m. interactions (5.16) gives $kT_c \sim 300$ GeV. He finally notices some very amusing effects when he examines a global O(n) x O(n) symmetry.

VI. MORE SYMMETRY

One of the lessons we learnt by the study of vector meson theories, is that the actual degree of divergence encountered in the perturbation expansion of a Lagrangian field theory may be better than the one indicated by the power counting argument. Is this a feature characteristic of gauge theories? The answer is no, and I would like here to give you a simple example which exhinits remarkable cancellations of divergences[140]. The model consists of a scalar field A, a pseudoscalar B and a four component Majorana spinor ψ. The Lagrangian is given by:

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \tfrac{1}{2}m^2(A^2+B^2) - \frac{i}{2}m\bar{\psi}\psi$$
$$- mgA(A^2+B^2) - ig\bar{\psi}(A - \gamma_5 B)\psi + \frac{g^2}{2}(A^2+B^2)^2 \qquad (6.1)$$

(6.1) looks like a very ordinary Yukawa Lagrangian except that there are special relations among the masses and coupling constants. A priori one would not expect such relations to be preserved by renormalization, but the astonishing thing is that not only they are, but also they cause cancellations among the divergences of perturbation theory, such that at the end only one infinite wave function counterterm, common to all the fields, need to be introduced. No mass and no coupling constant counterterms are required. This remarkable property is not accidental. It is partly due to a very peculiar symmetry which is hidden in the model (6.1) and which is a special case of what we heard in B. Zumino's talk [141] under the name of "supersymmetry"[142]. I would like here to review briefly the properties of supertransformations and add some remarks about their possible applications. For more details you are referred to ref (141).

The general form of an infinitesimal transformation acting on a multiplet of fields $\phi^i(x)$ can be written as:

$$\delta\phi^i(x) = \varepsilon^\alpha (T_\alpha)_j^{\ i}\ \phi^j(x) \qquad (6.2)$$

where i and j denote the members of the multiplet and T_α are matrices characteristic of the representation in which the fields belong. In case of a gauge symmetry, there may be another term which is not proportional to any of the fields, if $\phi^i(x)$ are the gauge bosons. Usually the infinitesimal parameter ε^α are taken to be c-numbers (constants or functions of the space-time point x). It follows that a given irreducible representation contains either bosons or fermions, but never both. Let us however try to choose the ε's to be totally anticommuting spinors, elements of a sufficiently large Grassman algebra.

We see immediately that, if we succeed to assign some group properties to such transformations, and if representations exist, they must contain both fermions and bosons. These questions have been answered in four-dimensions by J Wess and B Zumino[143]. I shall only state some of the results: Let $\alpha(x)$ be the infinitesimal anticommuting Majorana spinor of the transformation. One can show that it has the form:

$$\alpha(x) = \alpha^{(0)} + \gamma.x\alpha^{(1)} \qquad (6.3)$$

where $\alpha^{(0)}$ and $\alpha^{(1)}$ are two arbitrary but constant infinitesimal Majorana spinors. The generators of the corresponding transformations are also spinors. The commutator of two supertransformations with parameters (6.3) is a conformal transformation in four dimensions combined with a γ_5 transformation. Actually the algebra among super-, conformal and γ_5 transformations closes[(144)]. An important subalgebra is obtained by considering only the supertransformations with constant parameters, ie by choosing $\alpha^{(1)} = 0$ in (6.3). In this case the algebra contains only the four dimensional translations and can be written in the form:[(145)]

$$\{Q_i, \bar{Q}_j\} = -2(\gamma^\mu)_{ij} P_\mu \tag{6.4}$$

$$[P_\mu, Q_i] = [P_\mu, P_\nu] = 0 \tag{6.5}$$

where Q_i are Majorana spinors. Notice that the (6.4) is an anticommutator.

We can find several representations of these symmetries[(143)] and I will only give here some examples. I will limit myself to restricted super-transformations with constant parameters. The simplest representation is a "scalar" one which contains a scalar field A(x), a pseudoscalar B(x), a Majorana spinor $\psi(x)$ and two auxiliary fields F(x) and G(x). Their transformation properties are given by:

$$\begin{aligned}
\delta A &= i\bar{\alpha}\psi \\
\delta B &= i\bar{\alpha}\gamma_5\psi \\
\delta\psi &= \partial_\mu(A-\gamma_5 B)\gamma^\mu\alpha + F\alpha + G\gamma_5\alpha \\
\delta F &= i\bar{\alpha}\gamma^\mu\partial_\mu\psi \\
\delta G &= i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\psi \ .
\end{aligned} \tag{6.6}$$

Another example is a "vector" representation containing four scalars D, C, M and N, a vector V and two Majorana spinors χ and λ. I shall not write the transformation properties[(143)], but I will only notice that the members F and G of a scalar multiplet and D of a vector one, transform under an infinitesimal super-transformation by four-derivitives.

Finally let me mention that there exists a tensor calculus in the sense that one can combine different prepresentations in order to obtain new ones[(143)]. These two properties (the transformation properties of F and D and the tensor calculus) can be used for the construction of invariant Lagrangians. One simply combines different representations and then chooses an F or a D member. Notice that, strictly speaking, one does not obtain invariant Lagrangians but invariant actions. The Lagrangians change by four-derivatives. Let me give an example of such a Lagrangian for the scalar multiplet (6.6):

$$\begin{aligned}
\mathcal{L} = &-\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \tfrac{i}{2}\bar{\psi}\not{\partial}\psi + \tfrac{1}{2}F^2 + \tfrac{1}{2}G^2 \\
&+ m(FA + GB - \tfrac{i}{2}\bar{\psi}\psi) \\
&+ g\left[F(A^2-B^2) + 2GAB - i\bar{\psi}(A-\gamma_5 B)\psi\right]
\end{aligned} \tag{6.7}$$

We see that, if we eliminate from (6.7) the auxiliary fields F and G, we obtain the Lagrangian (6.1).

Supersymmetries can be combined with internal symmetries, global or local. In particular Wess and Zumino[(146)] have written a supersymmetric extension of quantum electrodynamics and Ferrara and Zumino[(72)] and Salam and Strathdee[(147)] have shown that a non abelian Yang-Mills theory with massless Majorana spinors belonging to the regular representation of the group, is automatically invariant under super-transformations. Furthermore, even when scalar multiplets are introduced the theory can still be asymptotically free[(72), (148)] (see above section II). Supersymmetries have very attractive properties and I am sure that they will play an important role in particle physics. Except from the nice renormalization properties I mentioned before, which are specific to the scalar model, let me just point out that the notion of left- or right-handness can be given a very simple and natural interpretation in the framework of supersymmetries[(143)]. There is however an important difficulty although the progress made recently makes us hope that it will soon be completely solved. In order to see it, let me use the commutation relations (6.4) to obtain:

$$H = \frac{1}{4} \sum_i Q_i^2 \qquad (6.8)$$

where H is the Hamiltonian. It follows that all members of a multiplet must have the same mass. It is therefore necessary to find a suitable scheme of symmetry breaking, before applying these ideas to the physical world. In the second paper of ref (140) an explicit breaking by a linear term was introduced, which was sufficiently "soft", as to preserve all renormalization properties. But recently a mechanism for a spontaneous breaking has been found[149] which has several nice properties. The model was the Q.E.D. extension of Wess and Zumino[146]. It turns out that in this model the D member of the vector multiplet is both gauge and supersymmetry invariant and one can add it to the Lagrangian. Thus a spontaneous symmetry breaking can be induced which can also trigger a Higgs mechanism for the breaking of the gauge symmetry and the appearance of a massive vector particle. Notice that, since the supersymmetry current is spinorial, the corresponding Goldstone particle is a spinor[149,150]. The mechanism can be applied to other gauge groups provided they contain a U(1) factor, but until now no mechanism has been found for a spontaneous breaking of a semi-simple gauge group which is compatible with the requirements of renormalizability.

CONCLUSION

I attempted to describe to you in this talk, without any technical details, the physical ideas which are behind the gauge theories and the progress we have accomplished during the last year. I did not try, actually I did not want, to offer you an unbiased presentation. My aim was not to be critical, but rather to transmit to you a part of the excitement and enthusiasm that we enjoy, when we have the clear feeling of being on the right track. I wanted to share this excitement and enthusiasm with everybody, even the most sceptical and pessimistic. I wanted to convince you that gauge theories have come to stay. I am aware and I anticipate the criticism you may have. I am aware of the fact that when welcome results are obtained, the validity of the model is not examined, and when unwelcome results are obtained, we argue that we do not yet have the correct model. You are advised to keep that in mind in evaluating the significance of these ideas, but also to agree that, on the face of it, they are sufficiently successful to command the greatest attention. Thank you.

ACKNOWLEDGEMENTS

During the last year I have benefitted from the experience and advice of too many of my colleagues, both theorists and experimentalists, to name them here. I would like however to thank in particular those who helped me in the preparation of this report. I am indebted to Professors C Bouchiat, S Coleman P Fayet, B W Lee, A A Migdal, S Weinberg, K Wilson and B Zumino for numerous suggestions and clarifying remarks. I thank especially Professors C Bouchiat, S Coleman, B W Lee and S Weinberg for a critical reading of preliminary versions of the manuscript. Finally I would like to thank Drs. Ross and Danskin who did the final editing of the report.

REFERENCES

(1) "Fables of Aesop" Penguin Classics.

(2) B W Lee, in "Proceedings of the XVI International Conference on High Energy Physics" ed. by J D Jackson and A Roberts (NAL, Batavia, III 1972) Vol. IV p.249.

(3) M Veltman, "Gauge field theory" Invited talk presented at the International Symposium on Electron and Photon Interactions at High Energies. Bonn, 27-31 August 1973.

(4) C H Llewellyn Smith, "Unified models of weak and electromagnetic interations". Invited talk presented at the International Symposium on Electron and Photon Interactions at High Energies. Bonn, 27-31 August 1973.

(5) S Weinberg in "Proceedings of the 11^{e} Conference Internationale sur les Particules Elementaires" Aix-en-Provence 1973. Sup. au Journal de Physique, Vol. 34, Fasc. 11-12 C1-1973 p.45).

(6) E S Abers and B W Lee, "Gauge theories" Phys. Reports 9C Nb. 1.

(7) S Coleman, "Secret symmetry: An introduction to spontaneous symmetry breakdown and gauge fields" Lectures given at the 1973 Erice Summer School.

(8) S L Glashow, Lectures given at the 1973 Erice Summer School. (unpublished).

(9) M A B Beg and A Sirlin, "Gauge theories of weak interactions" to be published in Ann. Rev. Nucl. Sci. Vol. 24.

(10) J Schwinger, Ann. Phys. 2(1957)407; S A Bludman, Nuov. Cim. 9(1958)433; S L Glashow, Nucl. Phys. 22(1961)579; A Salam and J C Ward, Phys. Lett. 13(1964)168. For a more detailed historical review see M Veltman ref. (2) and S Weinberg ref. 5.

(11) C N Yang and R L Mills, Phys. Rev. 96(1954)191. See also R Utiyama, Phys. Rev. 101(1956)1597; M Gell-Man and S L Glashow, Ann. Phys. 15(1961)437

(12) F Englert and R Brout, Phys. Rev. Lett. 13(1964) 321; P W Higgs, Phys. Lett. 12(1964)132; Phys. Rev. Lett. 13(1964)508; Phys. Rev. 145(1966)1156; G S Guralnik, C R Hagen and T W B Kibble, Phys. Rev. Lett. 13(1964)585; T W B Kibble, Phys. Rev. 155(1967)1554. The phenomenon was known in non-relativistic physics, see for ex. P W Anderson, Phys. Rev. 112(1958)1900.

(13) S Weinberg, Phys. Rev. Lett. 19(1967)1264; A Salam in "Elementary particle physics" ed. by N Svartholm, Stockholm 1968 p.367.

(14) C Fronsdal, Phys. Rev. 136B(1964)1190; W Kummer and G Segre, Nucl. Phys. 64(1965)585; G Segre, Phys. Rev. 181(1969)1996; L F Li and G Segre, Phys. Rev. 186(1969)1477; N Christ, Phys. Rev. 176(1968)2086.

(15) B L Ioffe and E P Shabalin, Yadern Fiz. 6(1967)828 (Soviet J Nucl. Phys. 6(1968)603); Z Eksp i Teor. Fiz. Pis'ma U Redaktsiyu 6(1967)978(Soviet Phys. JETP Lett. 6(1967)390); R N Mohapatra, J Subba Rao and R E Marshak, Phys. Rev. Lett. 20(1968)1081; Phys.Rev. 171(1968)1502; F E Low, Comments Nucl. Part.Phys. 2(1968)33; R N Mohapatra and P Olesen, Phys. Rev. 179(1969)1917.

(16) T D Lee, Nuov. Cim. 59A(1969)579.

(17) T D Lee, K Huang and C N Yang, Phys. Rev. 106(1957) 1135.

(18) M Gell-Man, M L Goldberger, N M Kroll and F E Low, Phys. Rev. 179(1969)1518.

(19) C Bouchiat, J Iliopoulos and J Prentki, Nuov. Cim. 56A(1968)1150; J Iliopoulos ibid 62A(1969)209.

(20) S L Glashow and S Weinberg, Phys. Rev. Lett. 20(1968)224; M Gell-Man, R J Oakes and B Renner, Phys. Rev. 175(1968)2195.

(21) R Gatto, G Sartori and M Tonin, Phys. Lett. 28B(1968)128; Nuov. Cim. Lett. 1(1969)1; N Cabibbo and L Maiani, Phys. Lett. 28B(1968)131; Phys. Rev. D1(1970)707.

(22) S L Glashow, J Iliopoulos and L Maiani, Phys. Rev. D2(1970)1285.

(23) For earlier applications of SU(4) symmetry to strong interactions see also D Amati, H Bacry, J Nuyts and J Prentki, Nuov. Cim. 34(1964)1732; J D Bjorken and S L Glashow, Phys. Lett. 11(1964) 255.

(24) M Veltman, Nucl. Phys. B7(1968)637; B21(1970)288; J Reiff and M Veltman, Nucl. Phys. B13(1969)545; D Boulware, Ann. Phys. 56(1970)140.

(25) S L Glashow and J Iliopoulos, Phys. Rev. D3(1971) 1043; S L Glashow, J Iliopoulos and A Yao, Phys. Rev. D4(1971)1918.

(26) This is a corollary of a theorem derived recently by looking at the high energy behaviour of tree diagrams. The argument is based on the conjecture of "tree unitarity" i.e. that for every renormalizable theory the N-particle S-matrix elements in the tree approximation must grow no more rapidly than E^{4-N} in the limit of high energy E, at non-exceptional momenta (i.e. at angles such that all invariants $P_i P_j$ $i \neq j$ grow like E^2). It is proven that any Lagrangian of dimensions less or equal to four, involving scalar, spinor and vector fields, satisfies tree unitarity only if it is equivalent to a spontaneously broken gauge theory, with the possibility of adding arbitrary mass terms to vector fields associated with invariant abelian subgroups. See J M Cornwall, D N Levin and G Tiktopoulos, Phys. Rev. Lett. 30(1973)1268; (E) 31(1973)572; "Derivation of gauge invariance from high energy unitarity bounds on the S-matrix" to be published; C H Llewellyn Smith, Phys. Lett. B46(1973)233 J S Bell, to be published. The non-renormalizability of massive Yang-Mills theories was explicitly verified recently by R N Mohapatra, S Sakakibara and J Sucher, "Scattering of gauge bosons in sixth order and non-renormalizability of massive Yang-Mills theories" to be published. The results obtained in this paper agree with those previously obtained in ref. (24). They explicitly contradict the claim of J P Hsu and E C G Sudarshan, Phys. Rev. D9(1974)1678.

(27) An exception was the suggestion of T D Lee and C Wick, Nucl. Phys. B9(1969)209.

(28) G 't Hooft, Nucl. Phys. B33(1971)173; ibid B35(1971)167.

(29) B W Lee, Phys. Rev. D5(1972)823; B W Lee and J Zinn-Justin, Phys. Rev. D5(1972)3121; D5(1972)3137; D5(1972)3155; D7(1973)1049.

(30) G 't Hooft and M Veltman, Nucl. Phys. B44(1972)189; B50(1972)318; "Diagrammar" CERN Yellow report CERN 73-9

(31) This was a generalization of earlier work by R P Feynman Acta Phys. Pol. 24(1963)697; B S DeWitt, Phys. Rev. Lett. 12(1964)742; Phys. Rev. 162(1967)1195; L D Faddeev and V N Popov, Phys. Lett. 25B(1967)29; S Mandelstam, Phys. Rev. 175(1968)1580; ibid 1604(1968); L D Faddeev, Theor. I Mat. Fiz. 1(1969)3; (Theor. and Math. Phys. 1(1970)1); E.S. Fradkin and I V Tyutin, Phys. Rev. D2(1970)2841; R Mills, Phys. Rev. D3(1971) 2969.

(32) References (2) to (9) contain all the information on this period.

(33) See for instance H Georgi and A Pais "Calculability and Naturalness in gauge theories" to be published and references therein.

(34) For a detailed criticism of the SU(2) xU(1) model see S Weinberg ref. 5.

(35) H Georgi and S L Glashow, Phys. Rev. Lett. 28(1972)1494.

(36) H Georgi and S L Glashow, Phys. Rev. D7(1973)2457; See also R N Mohapatra, "Gauge model for chiral symmetry breaking and muon electron mass ratio" to be published.

(37) See for example, B W Lee, Phys. Rev. D6(1972)1188; J Prentki and B Zumino, Nucl. Phys. B47(1972)99; M A B Beg and A Zee, Phys. Rev. Lett. 30(1973)675; Y Achiman Nucl. Phys. B56(1973)635; W Alles, Heavy leptons and unified theories of weak and e.m. interactions" to be published.

(38) P Fayet, "A gauge theory of weak and electromagnetic interactions with spontaneous parity breaking" to be published.

(39) P Budini and P Furlan "On composite gauge fields and models" to be published, and references therein.

(40) B W Lee and S R Treiman, Phys. Rev. D7(1973)1211; A Pais Phys. Rev. Lett. 29(1972)1712; 30(1973)114 (E); Phys. Rev. D8(1973)625; T P Chang, Phys. Rev. D8(1973)496; M A B Beg, Phys. Rev. D8(1973) 664; P Fayet, Phys. Lett. 48B(1974)145; Fayyazuddin and Riazuddin, "The $\Delta I = \frac{1}{2}$ rule in non-leptonic weak decays" to be published.

(41) G Altarelli and L Maiani, "Octet enhancement of non-leptonic weak interactions in asymptotically free gauge theories" to be published; M K Gaillard and B W Lee, "Rare decay modes of the K mesons in gauge theories" to be published

(42) R N Mohapatra, Phys. Rev. D6(1972)2023; A Pais ref. 35, T P Cheng, ref. 35, A Pais and J. R Primack, Phys. Rev. D8(1973)3063.

(43) T D Lee, Phys. Rev. D8(1973)1226 and Phys. Rep. 9C (2). See also R N Mohapatra and J C Pati, "Left-right gauge symmetry and an "iso-conjugate" model of CP-violation" to be published; D Bailin and A Love, Nucl. Phys. B69(1974)142.

(44) A Zee, Phys. Rev. D9(1974)1772; R N Mohapatra ref. 42, H Georgi and A Pais, "CP violation as a quantum effect" to be published.

(45) S Coleman and E Weinberg, Phys. Rev. D7(1973)1888; see also S Coleman, Ref. 7.

(46) See however ref. 27.

(47) For early examples of zeroth order symmetries see G 't Hooft Nucl. Phys. B35(1971)167; S Weinberg, Phys. Rev. Lett. 29(1972)388 and 29(1972)1698; H Georgi and S L Glashow, Phys. Rev. D6 2977 and ref. 31. For further references see ref. 3 and ref. 5. For more recent contributions see P Schattner, Phys. Rev. D9(1974)2377; H Georgi and A Pais ref. 33 and S W MacDowell "Renormalizable gauge theories with non-linear constraints" to be published.

(48) For a more detailed argument taking into account the strong interations to all orders, and covering non-abelian gluon models see S Weinberg, Phys. Rev. D8(1973)605 and D8(1973)4482; see also R N Mohapatra, J C Pati and P Vinciarelli, Phys. Rev. D8(1973)3652.

(49) S Weinberg, Phys. Rev. Lett. 31(1973)494; D V Nanopoulos, Nuov. Cim. Lett. 8(1973)873.

(50) M T Grisaru, H J Schnitzer and H S Tsao, Phys. Rev. Lett. 30(1973)811 and Phys. Rev. D8(1973)4498. Also see S Y Lee, J M Rawls and D Y Wong, "Reggeization as spin 1 particles in general Lagrangian models" to be published.

(51) L Tataru, Phys. Lett. 46B(1973)221; D R T Jones, Nucl. Phys. B71(1974)111; Fayyazuddin and Riazuddin, Nucl. Phys. B74(1974)397; T C Yang, "Pseudo-Goldstone pion masses in a gauge model" to be published. G Pocsik, "Light-cone physics and e.m. mass shift of pions in unified gauge theories" to be published; S N Biswas and P Ghose, "Mass difference of pions in the symmetrical theory of weak and e.m. interactions" to be published. B de Witt, S Y Pi and S Smith, "Perturbative calculations in a Unified gauge field model of strong weak and e.m. interactions" to be published. For a review of these works see M K Gaillard, Proceedings of this Conference.

(52) J Lieberman, Phys. Rev. D9(1974)1749; B de Witt, S Y Pi and J Smith, Ref. 51.

(53) One can show that the inclusion of strangeness and charm can only make things worse. See S. Weinberg invited talk in this conference for a detailed discussion of the η problem.

(54) D Sutherland, Phys. Lett. 23(1966)384; J S Bell and D Sutherland, Nucl. Phys. B4(1968)315.

(55) The existence of such neutral massless field would result in a long range repulsive force between baryous, proportional to Γ^2 (the analog of α). Using the recent measurements on the gravitational to inertial mass ratio of P G Roll, R Krotkov and R H Dicke, Ann of Phys. 26(1964)442, one deduces $\frac{\Gamma^2}{4\pi\hbar c} \lesssim 10^{-43}\,\alpha$.

(56) A Pais, Phys. Rev. D8(1973)1844.

(57) C Bouchiat, J Iliopoulos and Ph. Meyer, Phys. Lett. 38B(1972)519.

(58) J C Pati and A Salam, Phys. Rev. D8(1973)1240; Phys. Rev. Lett. 31(1973)661 and Trieste preprint, to be published. Mixed quark-lepton representations have been also considered by C Itoh, T Minamikawa, K Miura, and T Watanabe, to be published, Y Achiman, "Weinberg-Salam angle in a gauge model of weak and strong interactions" to be published.

(59) H Georgi and S L Glashow, Phys. Rev. Lett. 32(1974)438.

(60) D Gross and F Wilczek, Phys. Rev. D8(1973)3633; S Weinberg, Phys. Rev. Lett. 31(1973)494; H Fritzsch, M Gell-Mann and H Leutwyler, Phys. Lett. B47(1973)365

(61) See however T Applequist, "The infrared structure of Yang-Mills theories" to be published. Some ideas that may be proved to be very useful are contained in K Wilson's treatment of the Kondo problem, K Wilson, Nobel symposium Vol. 24 and Adv. in Math. to be published.

(62) G 't Hooft, Nucl. Phys. B61(1973)455; B62(1973)444 H D Politzer, Phys. Rev. Lett. 30(1973)1346; D J Gross and F Wilczek, Phys. Rev. Lett. 30(1973)1343.

(63) For an introduction see for example C Callan Lectures in the 1973 Cargese Summer School and references therein.

(64) See the invited talks in this Conference, of D J Gross and A De Rujula and references therein.

(65) D J Gross and F Wilczek, Phys. Rev. D8(1973)3633; and D9(1974)980; H Georgi and H D Politzer, Phys. Rev. D9(1974)416; D Bailin, A Love and D Nanopoulos, Nuov. Cim. Lett. 9(1974)501.

(66) The question of the gauge invariance and the mixing of operators is very difficult and has not been completely solved. See ref. 65 as well as: S Sarkar, "Mixing of operators in Wilson Expansions" to be published. H Kluberg-Stern and J B Zuber, "Ward identities and some clues to the renormalization of gauge invariant operators" to be published; R Crewther, to be published; W Kainz, W Kummer and M Schweda, "Nonabelian gauge theories for hadrons and radiative corrections" to be published.

(67) G Parisi, "Detailed predictions for the p-n structure functions" to be published. A De Rujula, H Georgi and H D Politzer, "The breakdown of scaling in neutrino and electron scattering" to be published. H D Politzer, Phys. Rev. D9(1974)2174; D Gross, Phys. Rev. Lett. 32(1974)1071.

(68) T Appelquist and H Georgi, Phys. Rev. D8(1973)4000 A Zee, Phys. Rev. D8(1973)4038.

(69) D R T Jones, "Two loop diagrams in Yang-Mills theory" to be published; A A Belavin and A A Migdal, "Scale invariance and bootstrap in the non-abelian gauge theories - to be published. W E Caswell, Phys- Rev. Lett. 33(1974)244.

(70) K Symanzik: Unpublished.

(71) See for example T P Cheng, E Eichten and L F Li, Phys. Rev. D9(1974)2259.

(72) S Ferrara and B Zumino, "Supergauge invariant Yang-Mills theories" Nucl. Phys. B to appear.

(73) It seems that the two loop contribution is positive. D R T Jones, Private communications.

(74) When this scheme was first proposed by S Weinberg Phys. Rev. D5(1972)1962 the group G was only supposed to cover the weak and e.m. interations and it was taken to be SU(3) x SU(3) x P.

(75) This idea is due to K Wilson, Phys. Rev. 140(1965) B445; Phys. Rev. 179(1969)1944; Phys. Rev, D2 (1970)1438; Phys. Rev. B4(1971)3174.

(76) S Weinberg, ref. (53) and references therein.

(77) H Georgi, H R Quinn and S Weinberg, "Hierarchy of interactions in unified gauge theories" to be published; See also S Weinberg ref. (53).

(78) G Parisi, "On the value of fundamental constants" to be published, A A Belavin and A A Migdal, ref. 69.

(79) L D Landau and I Pomeranchuk, Dokl Akad Nank 95(1954)773.

(80) The connection of gravity with the scale of the other interactions has been emphasized in recent years by A Salam and J Strathdee, Nuov. Cim. Lett. 4(1970)101; C J Isham, A Salam and J Strathdee, Phys. Rev. D3(1971)1805 and D5(1972)2548.

(81) T Appelquist, J R Primack and H R Quinn, Phys. Rev. D6(1972)2998; Phys. Rev. D7(1973)2298; D A Ross, Nucl. Phys. B51(1973)116; A Sirlin, Nucl. Phys. B71(1974)29; T Hagiwara, Phys. Rev. D9(1974)1023.

(82) R Jaffe and J R Primack, Nucl. Phys. B61(1973)317.

(83) I Bars, M B Halpern and M Yoshimura, Phys. Rev. Lett. 29(1972)969; Phys. Rev. D7(1973)1233, B De Witt, Nucl. Phys. B51(1973)237; I Bars, "Lepton-quark symmetry" to be published.

(84) For a detailed list of possibilities see G G Ross and R K P Zia, "Gauge models without charmed hadrons" to be published.

(85) S L Glashow, J Iliopoulos and L Maiani ref. 22; B W Lee, J R Primack and S B Treiman, Phys. Rev. D7(1973)510; C E Carlson and P G O Freund, Phys. Lett. 39B(1972)349, B W Lee and M K Gaillard to be published.

(86) G Snow, Nucl. Phys. B35(1971)167.

(87) E A Paschos, "Theoretical interpretation of neutrino experiments" invited talk presented at the January 1973 meeting of the American Physical Society, and references therein; A De Rujula and S L Glashow, Phys. Lett. B46(1973)377; B46(1973)381 Phys. Rev. D9(1974)180; M A B Beg and A Zee, Phys. Rev. Lett.30(1973)675; Phys. Rev. D8(1973)2334; A Zee, Phys. Rev. D9(1974)1572; A De Rujula, H Georgi, S L Glashow and H Quinn, Revs. Mod. Phys. to be published, M K Gaillard "Notes on charmed particle searches in neutrino experiments" to be published; Fayyazuddin and Riazuddin, "Charge symmetry and neutral current in a unified model of weak and electromagnetic interactions" to be published.

(88) A De Rujula, Invited talk in this Conference and references therein.

(89) B Barish, Invited talk in this Conference and references therein.

(90) J D Bjorken and C H Llewellyn Smith, Phys. Rev. D7(1973)887; S Y Pi and J Smith, Phys. Rev. D9(1974)1498, A Soni, Phys. Rev. D9(1974)2092; and "Signatures of neutral currents and heavy leptons in weak inelastic processes" to be published, C H Albright and C Jarlskog, "Neutrino production of M+ and E+ heavy leptons - (I)" to be published. C H Albright, C Jarlskog and L Wolfenstein, "Neutrino production of M+ and E+ heavy leptons - (II)" to be published; A Ali, "Heavy neutral lepton production in the e^+e^- colliding beam experiments" to be published.

(91) There have been many detailed calculations and I shall only refer to a few of the most recent ones. For references to earlier work see ref. (2), (4) and (5). For a general discussion of the effects of neutral currents independently of any specific model see, A Pais and S B Treiman, Phys. Rev. D9(1974)1459; C H Llewellyn Smith and D V Nanopoulos, "Remarks on neutral currents phenomenology" to be published. For a different predictions of neutral current cross sections in deep inelastic region, or using parton ideas, see, C H Albright, Phys, Rev. D8(1973)3162;

Nucl. Phys. B70(1974)142. R B Palmer, Phys. Lett. 46B(1973)240; L M Sehgal, Nucl. Phys. B65(1973)141. Phys. Lett. 48B(1974)133; J Cleymans and L M Sehgal, Nucl. Phys. B74(1974)285; M A B Beg, Phys. Lett. 49B(1974)361; P N Scharback, Nucl. Phys. B49(1972)52; Nuov. Cim. Lett. 9(1974)173; and "Pion production by fragmentation of weak currents in the Weinberg theory" to be published; G Rajasekeran and K V L Sarma, "Analysis of the neutral current interaction in the inclusive neutrino reactions" to be published; Model independent analysis of the neutral current interaction in the inclusive neutrino reactions" to be published, "Bounds on the coupling constants of the neutral current" to be published.
For their influence in electromagnetic processes see, R Budny and A McDonald, Phys. Lett. 48B (1974)423, S M Berman and J R Primack, Phys. Rev. D9(1974)2171; L Palla and G Pocsik, "High energy e^+e^- anniliation in unified gauge theories" to be published, E Reya and K Schilcher, "Neutral current effects in elastic electron-nucleon scattering" to be published. Finally for estimations on purely leptonic processes see, H Terazawa, Phys. Rev. D8(1973)1817, L M Sehgal, Phys. Lett. 48B(1974)60. Nucl. Phys. B70(1974)61.
J E Kim, V S Mathur and S Okubo, "Electromagnetic properties of neutrino from neutral current experiments" to be published.

(92) See for ex. M A Beg, Phys. Lett. 49B(1974)361.

(93) M A Bouchiat and C Bouchiat, Phys. Lett. 48B(1974) 111 and "Parity violation induced by weak neutral currents in atomic physics" to be published.

(94) G Feinberg and M Y Chen, to be published, J Bernabeu, T E O Ericson and C Jarlskog, to be published.

(95) I B Khriplovich and A I Vainshtein, "Neutral currents of weak interactions and Josephson effect" to be published.

(96) D E Friedman, Phys. Rev. D9(1974)1389.

(97) J R Wilson, Phys. Rev. Lett. 32(1974)849.

(98) See for example T Appelquist, J Carazzone, T Goldman and H R Quinn, Phys. Rev. D8(1973)1747.

(99) V S Mathur and H C Yen, Phys. Rev. D8(1973)3569; R N Mohapatra and S Sakakibara, Phys. Rev. D9(1974)429; A Sirlin, Phys. Rev. Lett. 32(1974) 966.

(100) B W Lee, Phys. Lett. 46B(1973)214; Phys. Rev. D9(1974)933; M Tonin, Nuov. Cim. Lett. 9(1974)541.

(101) For a review of the Zimmermann formalism see W Zimmermann, Brandeis summer school 1971 edited by S Deser et al., MIT Press 1971; J H Lowenstein, Lecture notes at the University of Maryland, 1972.

(102) C Becchi, A Rouet and R Stora, "The abelian Higgs Kibble model. Unitarity of the S-operator" to be published; "Renormalization of the abelian Higgs Kibble model" to be published; "Renormalization of gauge field models"; presented by C Becchi in the 1974 Conference on Lagrangian field theories, Marseille, June 1974.

(103) J H Lowenstein, B Schroer, M Weinstein and W Zimmermann, to be published, J H Lowenstein and B Schroer, to be published, O Piguet, to be published.

(104) E C G Stuckelberg, Helv. Phys. Acta 11(1938)225; 11(1938)299; 30(1956)209.

(105) A Slavnov, Teor i Mat. Fiz. 10(1973)153; {Thoer. and Math. Phys. 10,99}

(106) J C Taylor, Nucl. Phys. B33(1971)436.

(107) The connection, if any, of these "super transformations" with those discussed in section VI is not clear.

(108) G 't Hooft, Nucl. Phys. B72(1974)461.

(109) G 't Hooft, Nucl. Phys. B75(1974)461.

(110) G 't Hooft and M Veltman, Ann. de l'Inst. H Poincaré to appear.

(111) S Deser and P Van Nieuwenhuizen, Phys. Rev. Lett. 32(1974)245 and "One loop divergences of quantized Einstein-Maxwell fields" to be published.

(112) S Deser and P Van Nieuwenhuizen "Nonrenormalizability of the quantized Dirac-Einstein system" to be published.

(113) S Deser, H S Tsao and P Van Nieuwenhuizen: "Nonrenormalizability of Einstein-Yang Mills interactions at the one loop level" to be published.

(114) D Z Freedman, I J Muzinich and E J Weinberg, to be published.

(115) A D Linde, JETP Lett. 19(1974)320; See also M Veltman, "Cosmology and the Higgs mechanism" unpublished.

(116) G 't Hooft, "Magnetic monopoles in unified gauge theories" to be published and proceedings of this Conference. Dr Migdal informed me that similar results were obtained by Polyakov (unpublished).

(117) Solutions satisfying Dirac's condition $eg = \frac{n}{2}$ can be obtained if one considers the group SU(2).

(118) Solutions of the classical Yang-Mills equations have also been obtained by R Dashen, B Hasslacher and A Neveu, to be published; M Veltman, to be published.

(119) K Wilson, "Quark confinement" to be published.

(120) For a study of phase transitions in the framework of these theories. See R Balian, J M Drouffe and C Itzykson. "Gauge fields on a lattice I and II" to be published; C Korthals-Altes to be published.

(121) G Jona-Lasinio, Nuov. Cim. 34(1964)1790; For field theoretical examples of "dynamical" symmetry breaking see, Y Nambu and G Jona-Lasinio, Phys. Rev. 122(1961)345.

(122) For a review of these questions as well as for explicit calculations of $V(\phi_c)$ see, J Iliopoulos and C Itzykson "Functional methods and perturbation theory" to be published, R Jackiw, Phys. Rev. D9(1974)1686; A Salam and J Strathdee, Phys. Rev. D9(1974)1129.

(123) J Iliopoulos, unpublished, The case of a ϕ^4 theory in which one probes the operator ϕ^2, has been studied in detail by G Altarelli, N Cabibbo, J Iliopoulos and L Maiani, unpublished.

(124) For a possible way to reduce the arbitrariness associated with the Higgs system of scalars see G B Mainland and L O' Raifeartaigh "Derivation of Unified Gauge Theory from a generalized universal principle" to be published.

(125) R Jackiw and K Johnson, Phys. Rev. D8(1973)2386.

(126) J H Cornwall and R E Norton, Phys. Rev. D8(1973) 3338.

(127) For a review of these results see S Weinberg ref. 5.

(128) D J Gross and A Neveu, "Dynamical symmetry breaking in asymptotically free field theories" to be published. See also R Dashen and Y Frishman, Phys. Lett. 46B(1973)439; P K Mitter and P H Weisz, Phys. Rev. D8(1973)4410.

(129) It is the two-dimensional analog of the model originally studied by Nambu and Jona-Lusinio ref (121). Their model was non-renormalizable (it was in four dimensions) and an explicit cut-off needed to be introduced. For finite values of the cut-off dynamical symmetry breaking was obtained.

(130) S Coleman, Comm. Math. Phys. 31(1973)259.

(131) Lowenstein and Swieca, Ann. Phys. 68(1971)172.

(132) For an attempt to apply the techniques of ref. (125) and (126) see, F Englert and R Brout, Phys. Lett. 49B(1974)77.

(133) See for ex. L D Landau and E M Lifsitz, "Theory of Elasticity" or S P Timoshenko, "Theory of elastic stability" McGraw-Hill, New York 1936.

(134) T D Lee and G C Wick, Phys. Rev. D9(1974)2291. T D Lee, invited talk at the January 1974 Annual Bevatron Users meeting, Berkeley, Cal.

(135) See T D Lee, invited talk at this Conference.

(136) For a discussion of the loop connections see ref. (134).

(137) D A Kizzhuits and A D Liude, Phys. Lett. 42B (1972)471; D A Kizzhuits, JETP Lett. 15(1972)745.

(138) K Wilson, private communications.

(139) S Weinberg, "Gauge and global symmetries at high temperature" to be published. See also L Dolan and R Jackiw, "Symmetry behaviour at finite temperature" to be published.

(140) J Wess and B Zumino, Phys. Lett. B49(1974)52, J Iliopoulos and B Zumino, Nucl. Phys. B (to appear); S Ferrara, J Iliopoulos and B Zumino, Nucl. Phys. B (to appear) H S Tsao, to be published.

(141) B Zumino, Invited talk in this Conference.

(142) We adopt here the term "supersymmetry" introduced by Salam and Strathdee in order to replace the misleading name "supergauge symmetry" used previously.

(143) J Wess and B Zumino, Nucl. Phys. B70(1974)39. For a formulation in two space-time dimensions see J L Gervais and B Sakita, Nucl. Phys. B34 (1971)633. For a systematic study of the theory of representations see, A Salam and J Strathdee, Trieste preprint IC/74/11 and IC/74/16, S Ferrara, J Wess and B Zumino, CERN preprint TH 1863(1974).

(144) We see that supersymmetries combine in a non-trivial way an interval symmetry with the Poincare group. The reason they avoid the consequences of the O' Raifeartaigh of Coleman and Mandula theorem is that they do not really form a Lie algebra since they involve both commutators and anticommutators.

(145) We choose a representation of real γ matrices and we define $\bar{\psi} = \psi\gamma^0$.

(146) J Wess and B Zumino, CERN preprint TH 1857(1974).

(147) A Salam and J Strathdee, Trieste preprint IC/74/36.

(148) One could arrive at the idea of the supersymmetry by imposing asymptotic freedom in a Yang-Mills theory with scalar fields. In general the $\lambda\Phi^4$ couplings of the scalars destroy asymptotic freedom but one can look for lines in the λ,g plane (g is the gauge coupling constant), along which the theory is asymptotically free. See N P Chang, "Eigenvalue conditions and asymptotic freedom for Higgs scalar gauge theories" to be published.

(149) P Fayet and J Iliopoulos, "Spontaneously broken supergauge symmetries and Goldstone spinors" to be published.

(150) See also A Salam and J Strathdee, Phys. Lett. 49B(1974)465.