Nongaussian Features from Inflationary Particle Production

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Abstract. The inflaton field can be expected to couple to a number of additional fields whose energy density does not play any significant role in driving inflation. Such couplings may lead to isolated bursts of particle production during inflation, for example via parametric resonance or a phase transition, and leave observable imprints in the cosmological fluctuations. I illustrate this effect for a simple prototype interaction $g^2(\phi - \phi_0)^2\chi$ between the inflaton, ϕ, and iso-inflaton, χ. Using both classical lattice simulations and analytical quantum field theory computations, I show that this mechanism generates localized bump-like features in the power spectrum and also a completely new type of nongaussianity. Observations are consistent with relatively large features of this type and the nongaussianity from particle production may be observable in future missions.

1. Introduction

The inflationary paradigm has become a cornerstone of modern cosmology. As measurements of the Cosmic Microwave Background (CMB) radiation grow increasingly precise, it has become topical to look beyond the simplest single-field, slow-roll inflationary scenario. In particular, it is interesting to determine the extent to which non-minimal signatures, such as features in the primordial power spectrum or observable nongaussianities, can be accommodated by microscopically sensible inflation models. Efforts in this direction are valuable because they allow us to test our theoretical prejudices and they provide observers with well-motivated templates for departures from the standard scenario. If detected, such signatures might open an observational window into fundamental particle physics at extremely high energy scales.

The motion of the inflaton may trigger the production of some non-inflaton (iso-curvature) particle *during* inflation [1, 2, 3]. Inflationary particle production is a generic feature of models from string theory (*eg* - brane/axion monodromy [4]) and also supersymmetric field theory [5]. Examples have been studied where particle production occurs via parametric resonance [1, 2, 3, 6, 7, 8, 9], as a result of a phase transition [10, 11], or otherwise [12]. These constructions have attracted interest recently for a number of reasons, including the possibility to exploit the dissipative effect of particle production to slow the motion of the inflaton on a steep potential [9, 12] (see also [13]).

In this talk, which is based [1, 2, 3], I will discuss the observational signatures of inflationary particle production. This scenario provides a simple, microscopically well-motivated mechanism to generate features in the primordial power spectrum and also observably large nongaussianity. The bispectrum from particle production is particularly novel: we find a completely new kind of

nongaussian signature which has been overlooked in previous literature. For reasonable values of model parameters, this should be detectable in future missions.

2. Particle Production During Inflation

To illustrate the basic physics of inflationary particle production, we consider the following prototype action

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{\mu^2}{2}\chi^2 - \frac{g^2}{2}(\phi - \phi_0)^2\chi^2 \right] \qquad (1)$$

which is understood to be minimally coupled to gravity. Here ϕ is the inflaton, χ is the iso-inflaton and we leave the potential $V(\phi)$ unspecified expect to assume that it is sufficiently flat to drive a long period of inflation with $H \equiv \dot{a}/a \cong$ const where $a(t)$ is scale factor. Note that we have included a "bare" mass μ^2 for the iso-inflaton; even if χ is classically massless at $\phi = \phi_0$, then such a term will be generated by radiative corrections.

The scenario we have in mind is the following. Inflation starts with $\phi \gg \phi_0$ so that the iso-inflaton is extremely massive $m_\chi^2 \equiv \mu^2 + g^2(\phi - \phi_0)^2 \gg H^2$ and stays pinned in the vacuum, $\chi = 0$. At the moment when $\phi = \phi_0$ the iso-inflaton mass varies non-adiabatically $|\dot{m}_\chi|/m_\chi^2 \gtrsim 1$ and χ particles are produced quantum mechanically. To describe this burst of particle production one must solve the following equation for the χ-particle mode functions:

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\frac{k^2}{a^2} + \mu^2 + k_\star^4 t^2 \right] \chi_k = 0 \qquad (2)$$

Here we have approximated $\phi \cong \phi_0 + vt$ and introduced the scale $k_\star \equiv \sqrt{g|v|} = \sqrt{g/(2\pi\mathcal{P}_\zeta^{1/2})}H$ where $\mathcal{P}_\zeta^{1/2} = 5\times 10^{-5}$ is the amplitude of the vacuum fluctuations from inflation. For reasonable values of the coupling $g^2 \gtrsim 10^{-7}$ we have $k_\star > H$ and particle production is rapid as compared to the expansion time. In this regime one can solve (2) very accurately for the occupation number of the created particles [14, 15]

$$n_k = e^{-\pi(\mu^2 + k^2)/k_\star^2} \qquad (3)$$

Clearly particle production effects may be suppressed if μ^2 is very large. For string theory or supersymmetric models, one naturally has $\mu^2 \sim H^2 \ll k_\star^2$ and there is no suppression.

Following the initial burst of particle production, two distinct physical effects take place. First, the energetic cost of producing a gas of non-equilibrium χ particles drains energy from the inflaton, forcing $\dot{\phi}(t)$ to drop abruptly. This velocity dip is a *backreaction* effect and contributes a negligible "ringing" pattern to the power spectrum [8]. The second physical effect, which yields the dominant contribution to the cosmological fluctuations, is called *rescattering* [1] and will be the subject of the remainder of this talk.

3. Rescattering and Infra-Red Cascading

The importance of rescattering effects for the observable cosmological fluctuations in models with particle production was first recognized in [1]. Fig. 1 illustrates the key process: bremsstrahlung emission of long-wavelength $\delta\phi$ fluctuations from rescattering of the produced χ particles off the condensate $\phi(t)$. The time scale for such processes, $k_\star^{-1}$, is fast as compared to the expansion time, H^{-1}. Moreover, the production of inflaton fluctuations $\delta\phi$ deep in the IR is extremely energetically inexpensive, since the inflaton is nearly massless. The combination of the short time scale for rescattering and the energetic cheapness of radiating IR $\delta\phi$ leads to a rapid build-up of power in long wavelength inflaton modes: infra-red (IR) cascading. These long-wavelength inflaton modes freeze outside of the horizon and lead to a bump-like feature in the primordial power spectrum.

In [1] the model (1) was studied using lattice field theory simulations, without neglecting any physical processes (that is to say that full nonlinear structure of the theory, including backreaction and rescattering effects, was accounted for consistently). However, this same dynamics can be understood analytically [3] by solving the equation for the inflaton fluctuations $\delta\phi$ in the approximation that all interactions are neglected, except for the diagram Fig. 1:

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\vec{\nabla}^2}{a^2}\delta\phi + V_{,\phi\phi}\delta\phi \cong -g^2\left[\phi(t) - \phi_0\right]\chi^2 \qquad (4)$$

Figure 1. Rescattering diagram

The solution of (4) may be split into two parts: the solution of the homogeneous equation and the particular solution which is due to the source term. Schematically we have

$$\delta\phi(t,\mathbf{x}) = \underbrace{\delta\phi_{\mathrm{vac}}(t,\mathbf{x})}_{\text{homogeneous}} + \underbrace{\delta\phi_{\mathrm{resc}}(t,\mathbf{x})}_{\text{particular}} \qquad (5)$$

The former contribution is the homogeneous solution which behaves as $\delta\phi_{\mathrm{vac}} \sim H/(2\pi)$ on large scales and, physically, corresponds to the usual scale invariant vacuum fluctuations from inflation. The particular solution, $\delta\phi_{\mathrm{resc}}$, corresponds physically to inflaton fluctuations which are generated by rescattering. The abrupt growth of χ inhomogeneities at $t = 0$ sources the particular solution $\delta\phi_{\mathrm{resc}}$, leading to the production of inflaton fluctuations which subsequently cross the horizon and become frozen. We have studied the dynamics of rescattering and IR cascading in the model (1) using both fully nonlinear lattice simulations and also analytical quantum field theory computations. We have found remarkable agreement between these two independent approaches, for a wide range of model parameters [1, 3].

Notice that the particular solution $\delta\phi_{\mathrm{resc}}$ of equation (4) is bi-linear in the gaussian field χ, suggesting that the fluctuations from rescattering will be highly nongaussian. We have explored the nongaussianity of the inflaton modes from rescattering in two different ways: by analytically computing the bispectrum and also by numerical evaluation of the Probability Distribution Function (PDF) [3]. We define the PDF, $P(\delta\phi)$, as the probability that the inflaton field has a fluctuation of size $\delta\phi = \phi - \langle\phi\rangle$. This is plotted in the left panel of Fig. 2 for several time steps during the evolution. In order to make the physics of inflationary particle production clear, we have subtracted off the usual vacuum fluctuations of the inflaton. That is, the PDF in the left panel of Fig. 2 is associated only with the contribution $\delta\phi_{\mathrm{resc}}$ in (5).

We can understand physically the behaviour of PDF plotted in the left panel of Fig. 2. Shortly after the initial burst of particle production the inflaton perturbations $\delta\phi$ are extremely nongaussian, due to the sudden appearance of the source term $J \propto \chi^2$ in the equation of motion (4). Very quickly, in less than an e-folding, nonlinear interactions begin to drive the system towards gaussianity. A very similar behaviour has been observed in lattice simulations of out-of-equilibrium interacting scalar fields during preheating [16]. In the case of rescattering during preheating, the system will eventually become gaussian when the fields thermalize. However, in our case the universe is still inflating during the process of rescattering and IR cascading. As a result, nongaussian inflaton fluctuations generated by rescattering are stretched out by the quasi-de Sitter expansion and must freeze once their wavelength crosses the Hubble scale. Hence, at late times the PDF does not become completely gaussian, but rather freezes-in with some non-trivial skewness. Within a few e-foldings from the moment of particle production the time evolution of the PDF has become completely negligible.
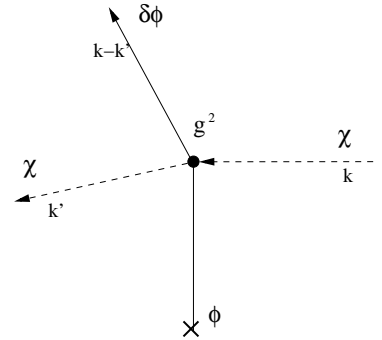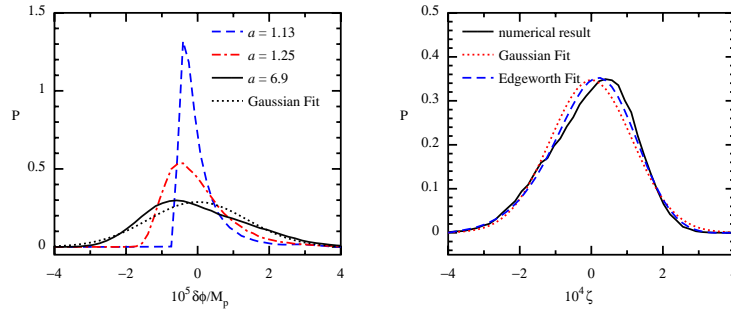
Figure 2. The left panel show the PDF of the inflaton fluctuations generated by rescattering and IR cascading, at a series of different values of the scale factor, a. The dotted black curve shows a Gaussian fit at late times and we have normalized the scale factor so that $a = 1$ at the moment when particle production occurs. The right panel shows the PDF of the *total* curvature fluctuation ζ, evaluated at late times (well after all relevant modes have crossed the horizon and frozen). The solid black curve is the exact result and the dotted red curve is a gaussian fit. We have also plotted the leading correction to the gaussian result in the Edgeworth expansion. For illustration, we have chosen $g^2 = 0.1$ and a standard chaotic inflation potential $V(\phi) = m^2\phi^2/2$ in both panels.

4. Features and Nongaussianity

Let us now consider the observational signatures associated with the dynamics described in the last section. The primordial power spectrum in the model (1) is well approximated by [2]

$$P_\zeta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s-1} + A_{\mathrm{IR}} \left(\frac{\pi e}{3}\right)^{3/2} \left(\frac{k}{k_{\mathrm{IR}}}\right)^3 e^{-\frac{\pi}{2}\left(\frac{k}{k_{\mathrm{IR}}}\right)^2} \tag{6}$$

The first term corresponds to the usual vacuum fluctuations from inflation (with amplitude A_s and spectral index n_s). The second term in (6) corresponds to the bump-like feature from particle production with amplitude A_{IR} (that depends on g^2) and location k_{IR} (that depends on ϕ_0). Current observational data are compatible with features as large as $A_{\mathrm{IR}}/A_s \sim 0.1$ for k_{IR} corresponding to CMB scales, whereas even larger features are allowed on smaller scales. In Fig. 3 we plot the power spectrum (6) for a representative choice of parameters.

Nongaussian statistics have attracted a considerable amount of interest recently, owing to their potential as a tool for observationally discriminating between the plethora of inflationary models in the literature. Nongaussianity is often characterized using the *bispectrum* [17], defined as the 3-point correlation function of the fourier transform of the curvature fluctuation:

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_\zeta(k_i) \tag{7}$$

Translational invariance ensures that $B_\zeta(k_i)$ depends on three wave-numbers which form a triangle: $\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3} = 0$. A general bispectrum $B_\zeta(k_i)$ may be characterized by specifying its size (amplitude of B_ζ), shape (whether B_ζ peaks on squeezed, equilateral or flattened triangles) and running (the dependence of B_ζ on the size of the triangle) [18].

As remarked above, the fluctuations generated by rescattering and IR cascading are highly nongaussian. This nongaussianity is very different from other models, such as the local, equilateral or enfolded shapes, which have been studied in the literature [3]. The bispectrum $B_\zeta(k_i)$ peaks strongly for triangles with a characteristic size $\sim k_{\mathrm{IR}}$, corresponding to the location of the bump in the power spectrum (6), and is therefore very far from scale invariant. The shape of $B_\zeta(k_i)$ is also novel [3].
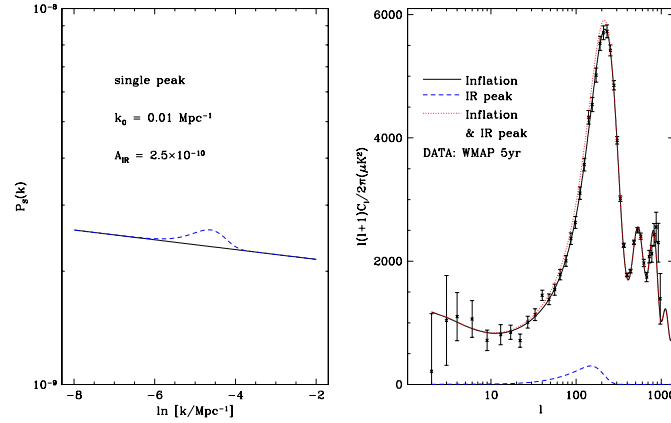
Figure 3. The left panel shows a sample bump in the power spectrum with amplitude $A_{\mathrm{IR}} = 2.5 \times 10^{-10}$, which corresponds to a realistic coupling $g^2 \sim 0.01$, and location $k_{\mathrm{IR}} = 0.01 \, \mathrm{Mpc}^{-1}$. This example represents a distortion of $\mathcal{O}(10\%)$ as compared to the usual vacuum fluctuations and is consistent with the data at 2σ. The right panel shows the corresponding CMB angular TT power spectrum.

The unusual shape/running of the bispectrum from particle production makes it difficult to compare the magnitude of the nongaussianity in our scenario to more familiar models, such as the local template. In order to quantify the nongaussianity from particle production we find it useful to compute the cummulants of the Probability Distribution Function (PDF). In the right panel of Fig. 2 we plotted the PDF of the inflaton fluctuations from rescattering. However, the quantity that is relevant for observations is the PDF of the *total* curvature perturbation, ζ, including *both* the contributions in (5). In the right panel of Fig. 2 we plot this quantity, evaluated at very late times, well after all relevant modes have crossed the horizon and become frozen.

Let us define the central moments of the PDF as

$$\langle \zeta^n \rangle \equiv \int \zeta^n P(\zeta) d\zeta \qquad (8)$$

These moments carry information about the correlation functions of ζ integrated over all wave-numbers, and therefore provide a useful tool to compare models with very different shape/running properties [19]. The dimensionless skewness, $\hat{\kappa}_3 \equiv \langle \zeta^3 \rangle / \langle \zeta^2 \rangle^{3/2}$, and kurtosis, $\hat{\kappa}_4 \equiv \langle \zeta^4 \rangle / \langle \zeta^2 \rangle^2 - 3$, encode departures from gaussianity. These are summarized in Table 1. We have also computed an "equivalent f_{NL}^{local}" which, for a given g^2, is the magnitude of f_{NL} necessary to reproduce the skewness $\hat{\kappa}_3$ with a local ansatz $\zeta = \zeta_g + \frac{3}{5} f_{NL} \left[\zeta_g^2 - \langle \zeta_g^2 \rangle \right]$.

Table 1. Moments of the Probability Distribution Function

g^2	skewness ($\hat{\kappa}_3$)	kurtosis ($\hat{\kappa}_4$)	"equivalent" f_{NL}^{local}
1	-0.51	0.2	-4500
0.1	-0.49	-0.1	-4300
0.01	-0.006	$< \mathcal{O}(10^{-3})$	-53

The coupling, g^2, controls both the nongaussianity and also the magnitude of the bump-like feature in (6). A key question is whether nongaussian effects can be observable in a regime where the feature is small enough to have evaded detection. The answer seems to be affirmative: for $g^2 = 0.01$ the spectrum (6) fits the data at 2σ [2] while $\hat{\kappa}_3 = -0.006$. This level of skewness would be produced by a local model with $f_{NL} \sim -53$, which is comparable to current observational bounds and is well within the accuracy of future missions.

5. Conclusions

Particle production during inflation is a simple and microscopically well-motivated mechanism that generates localized features in the primordial power spectrum and also significant nongaussianities. Such signatures provide a novel example of non-decoupling,[1] suggesting a possibility to probe extremely high scale physics with cosmology. The new type of nongaussianity that we have discovered is phenomenologically interesting and can be large even without tine-tuning the inflationary trajectory or appealing to re-summation of an infinite series of high dimension operators.

The key process that generates cosmological perturbations in our model, IR cascading, is interesting it its own right: it is qualitatively different from other mechanisms in the literature (in that we do not rely on the quantum vacuum fluctuations of some light iso-curvature fields) and the underlying dynamics are relevant also for preheating, moduli trapping and non-equilibrium quantum field theory more generally.

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[1] The produced χ particles are extremely massive for (almost) the entire history of the universe, however, their effect cannot be integrated out due to the non-adiabatic time dependence of the χ modes.