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Chiral vacuum fluctuations in quantum gravity

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Abstract.
In this paper we investigate cosmological tensor modes in terms of the Ashtekar variables of loop quantum gravity, for complex values of the Immirzi parameter. While, on-shell, the classical Hamiltonian reduces to the usual expression found in cosmological perturbation theory, the quantum Hamiltonian displays significant differences. We can find a physical Fourier space Hamiltonian in terms of graviton creation and annihilation operators, after selecting out the non-physical modes through the inner product which itself is determined by the reality conditions. We are left with the usual graviton modes but with a chiral asymmetry in the the vacuum energy and fluctuations. The latter depends on γ (in particular it vanishes for purely real γ) and on the ordering of the 2-point function. Such an effect would leave a distinctive imprint in the polarisation of the cosmic microwave background, thus finally engaging quantum gravity in meaningful experimental test.

1. Introduction

Loop Quantum Gravity (see e.g. [1, 2, 3, 4]) is a promising framework for quantising gravity. It has a rigorous mathematical structure and a lot of progress has recently been made on specifying important field theoretical concepts such as the graviton propagator. However, as for any other theory of quantum gravity, currently missing is a distinct experimental test that could verify predictions made by LQG as well as distinguish it from other approaches. In a series of recent publications [5, 6, 7], we have provided such an experimental test by recasting the standard inflationary calculation for tensor perturbations in terms of Ashtekar variables. In these proceedings we will summarise this work.

In LQG, the Holst action [8] (which includes a topological term, dependent on the Immirzi parameter γ, that vanishes classically) instead of the standard Palatini-Kibble formalism is used and the connection is treated as the central gravitational variable. Using this approach and applying standard cosmological perturbation theory [9, 10], it turns out that the vacuum fluctuations of inflationary tensor modes, gravitational waves, exhibit a chirality, i.e. depend on the graviton polarisation. This is a complete novelty compared to the normal second order formalism [11]. Classically, the two formulations are equivalent, as they should be, with novelties only appearing in the quantum formulation. The observed chirality will be shown to depend on ordering prescriptions and in particular on the value of the Immirzi parameter γ (with the values $\gamma = \pm i$ which render the connection self-dual(SD)/anti-self-dual(ASD) being special cases) which is taken to be complex in general.

In Section 2 we set up the calculation by identifying the Ashtekar variables for tensor perturbations around an inflationary de Sitter background, deriving the perturbed Hamiltonian,

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and stating the classical solution. In Section 3 several issues that have to be taken into account to obtain the right theory are highlighted, and other novelties of the formalism are discussed. We will then set up the quantum theory in Section 4: we define reality conditions, commutation relations and a quantum Hamiltonian in terms of graviton creation and annihilation operators (of which only half are physical before reality conditions are imposed). Section 6 will show how this framework leads to a chirality in the gravitational vacuum fluctuations. We will summarise our results and outline plans for future work in the conclusion.

Throughout this paper we shall use units for which $\hbar = c = 1$ and we parameterise the strength of gravity with $l_P^2 = 8\pi G$. We'll be concerned with the real world, so the metric will invariably be Lorentzian; its signature is taken to be $-+++$.

2. Classical solution

In this section we provide all the ingredients for the calculation by defining the Ashtekar variables in cosmological perturbation theory, their Fourier expansions and the Hamiltonian formulation. We will show that the classical solution is the one we expect from the second order formalism. In Section 3, certain aspects of the approach used are explained in more detail and some steps that are needed to obtain the right results are highlighted.

2.1. Canonical variables

To describe inflationary tensor fluctuations, we consider perturbations around de Sitter space-time in the flat slicing:

$$ds^2 = a^2[-d\eta^2 + (\delta_{ab} + h_{ab})dx^a dx^b] \,, \tag{1}$$

where h_{ab} is a symmetric TT tensor, $a = -1/H\eta$, $H^2 = \Lambda/3$ and $\eta < 0$. Using the convention $\Gamma^i = -\frac{1}{2}\epsilon^{ijk}\Gamma^{jk}$ (where Γ^{ik} is the spin connection), the Ashtekar-Immirzi-Barbero connection is given by $A_a^i = \Gamma_a^i + \gamma\Gamma_a^{0i}$ (where $\gamma = \pm i$ corresponds to a SD/ASD connection, complex γ to the Immirzi connection and real γ to the Barbero connection). Its canonical conjugate is the densitised inverse triad, $E_i^a = \det\left(e_b^j\right) e_i^a$, where indices $i = 1, 2, 3$ are spatial Lorentz algebra indices and $a = 1, 2, 3$ are spatial indices for the base manifold. Making use of the torsion free condition

$$T^i = de^i + \Gamma^{ij} \wedge e^j = 0 \tag{2}$$

for the zeroth order solution, the canonical variables can be expressed as:

$$A_a^i = \gamma H a \delta_a^i + \frac{a_a^i}{a} \tag{3}$$

$$E_i^a = a^2\delta_i^a - a\delta e_i^a \,. \tag{4}$$

To make contact with cosmological perturbation theory and standard perturbative quantum field theory we need to expand the perturbed variables in terms of Fourier modes:

$$\begin{aligned}
\delta e_{ij} &= \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_r \epsilon_{ij}^r(\mathbf{k})\tilde{e}_{r+}(\mathbf{k},\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \epsilon_{ij}^{r\star}(\mathbf{k})\tilde{e}_{r-}^{\dagger}(\mathbf{k},\eta)e^{-i\mathbf{k}\cdot\mathbf{x}} \\
a_{ij} &= \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_r \epsilon_{ij}^r(\mathbf{k})\tilde{a}_{r+}(\mathbf{k},\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \epsilon_{ij}^{r\star}(\mathbf{k})\tilde{a}_{r-}^{\dagger}(\mathbf{k},\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}
\end{aligned} \tag{5}$$

where $\tilde{e}_{rp}(\mathbf{k},\eta) = e_{rp}(\mathbf{k})\Psi_e(k,\eta)$ and $\tilde{a}_{rp}(\mathbf{k},\eta) = a_{rp}(\mathbf{k})\Psi_a^{rp}(k,\eta)$, and ϵ_{ij}^r are polarization tensors. Amplitudes $\tilde{a}_{rp}(\mathbf{k})$ and $\tilde{e}_{rp}(\mathbf{k})$ have two indices (contrasting with previous literature, e.g. [13, 14]): $r = \pm 1$ for right and left helicities, and p for graviton ($p = 1$) and anti-graviton ($p = -1$) modes. The a_{rp} and e_{rp} can be chosen so as *not* to carry any time-dependence, and for simplicity we will assume that they are equal. After imposing on-shell conditions we'll find that functions $\Psi_a(k,\eta)$ must then carry an r and p dependence.

2.2. Hamiltonian and Hamilton's equations

In the Ashtekar formalism, the gravitational Hamiltonian constraint is given by

$$\mathcal{H} = \frac{1}{2l_P^2}\int d^3x N E^a_i E^b_j \Big[\epsilon_{ijk}(F^k_{ab} + H^2\epsilon_{abc}E^c_k) - 2(1+\gamma^2)K^i_{[a}K^j_{b]}\Big] \tag{6}$$

where $K^i_a = \frac{A^i_a - \Gamma^i_a(E)}{\gamma}$ is the extrinsic curvature of the spatial surfaces. The canonical variables have symplectic structure $\{A^i_a(\mathbf{x}), E^b_j(\mathbf{y})\} = \gamma l_P^2 \delta^b_a \delta^i_j \delta(\mathbf{x}-\mathbf{y})$. We can derive Hamilton's equations and by expanding them to first order get equations of motion for the perturbations in metric and connection variables. Combining them, we obtain

$$\delta e''_{ij} - \left(\partial^2 + \frac{2}{\eta^2}\right)\delta e_{ij} = 0\ . \tag{7}$$

This is exactly the same equation as obtained for the "v" variable used in cosmology to describe metric tensor perturbations. Classically, the Ashtekar approach therefore reduces to the second order formalism.

Our aim is to identify perturbative graviton states within a Fourier space Hamiltonian. Obviously gravitons should have dynamics, therefore we need to identify the part of the Hamiltonian that is not constrained to be zero. This is given by ${}^2_1\mathcal{H}$, the second order Hamiltonian in terms of products of first order variables. This is non-zero as only the full second order Hamiltonian, ${}^2\mathcal{H} = {}^2_1\mathcal{H} + {}^2_2\mathcal{H}$, must vanish on shell. The first term provides a candidate for the Hamiltonian to be identified with that of the second quantized QFT. The second contains the backreaction or compensation resulting from the non-linearity of the gravitational field, ensuring that the Hamiltonian constraint is satisfied. For general γ, the dynamical Hamiltonian to second order is given by (where we are being careful with the ordering with an eye on quantisation)

$$\begin{aligned}\mathcal{H}_{eff} = \frac{1}{2l_P^2}\int d^3x \quad & \Bigg[\frac{1}{\gamma^2}a_{ij}a_{ij} - 2H^2a^2\delta e_{ij}\delta e_{ij} + \left(1-\frac{1}{\gamma^2}\right)\epsilon_{ikl}(\partial_k\delta e_{lj})a_{ij} \\ & - \left(1+\frac{1}{\gamma^2}\right)\epsilon_{ikl}a_{ij}(\partial_k\delta e_{lj}) + \left(1+\frac{1}{\gamma^2}\right)\epsilon_{ikl}\epsilon_{jmn}(\partial_k\delta e_{lj})(\partial_m\delta e_{ni})\Bigg]\ .\end{aligned} \tag{8}$$

2.3. Classical solution

As opposed to the second order formalism, we treat connection and metric as independent variables initially. On-shell, i.e. when the equations of motion are satisfied, they are related by the torsion-free condition (2). Using this condition, we can find an expression for the first order connection on shell in terms of first order metric variables:

$$a_{ij} = \epsilon_{ikl}\partial_k\delta e_{lj} + \gamma\delta e'_{ij} \tag{9}$$

We can plug this into decomposition (5) to find the corresponding expression in Fourier space, $\Psi^{r+}_a = \gamma\Psi'_e + rk\Psi_e$ and $\Psi^{r-}_a = \gamma^*\Psi'_e + rk\Psi_e$. As the metric satisfies equation (7) on-shell, the functions $\Psi_e(k,\eta)$ will satisfy the corresponding Fourier space equation of motion, $\Psi''_e + \left(k^2 - \frac{2}{\eta^2}\right)\Psi_e = 0$, which has solution $\Psi_e = \frac{e^{-ik\eta}}{2\sqrt{k}}\left(1 - \frac{i}{k\eta}\right)$. We will be concerned with modes that were inside the horizon at the onset of inflation (for which $|k\eta| \gg 1$) as these are the modes we observe in CMB experiments now. On-shell, they satisfy

$$\Psi^{rp}_a = \Psi_e k\left(r - i\gamma_R + p\gamma_I\right)\ , \tag{10}$$

so for a γ with an imaginary part the connection is p-dependent and therefore made up of graviton and anti-graviton states.

3. Issues to be addressed

Classically, the Ashtekar formailsm is equivalent to the standard second order calculation. We should therefore rediscover all known classical results for the tensor perturbations when using the Ashtekar variables. This provides us with a good test of whether the formalism is consistent. It turned out that there were quite a few subtleties that had to be taken into account before the right result was reobtained. Furthermore, there have been a few misconceptions in previous literature, in particular related to Fourier expansions, that need to be addressed.

In this section we will summarise the main issues that have to be considered when doing cosmological perturbation theory in the Ashtekar framework.

- **Helicity and duality** There has been a belief (although proven to be wrong in [12]) that helicity (right and left handed) and duality (SD and ASD) states of the graviton should align. However, it should be obvious that this cannot be true: the two types of states can never align because helicity states are real whereas duality states must be complex for a Lorentzian space. Reality conditions therefore relate SD and ASD states; but they can never impose a constraint upon helicity states. The cause of the confusion lies in the Fourier expansion of the canonical variables: they should include positive and negative frequency states. This was missed, for example, in the works of [13, 14]. This can be easily seen by looking at the on-shell condition (10) which vanishes for certain r and p if $\gamma_I = \pm 1$. The surviving contributions are summarised in table 1. In the case $\gamma = \pm i$, the connection is therefore made up of the right (left) graviton and the left (right) anti-graviton.

Table 1. Helicity and Duality

		$r = +$ [R]	$r = -$ [L]
$p = +$	$[G]$	SD	ASD
$p = -$	$[\overline{G}]$	ASD	SD

- **Fourier space expansion** The expansions (5) are chosen to fulfil the following criteria:
  - As reality conditions are yet to be enforced there must be graviton and anti-graviton modes, so it's essential not to forget the negative frequencies in all expansions, and ensure that they are initially independent of the positive frequencies.
  - For a clearer physical picture, it is convenient to use the quantum field theory convention stipulating that for free modes the spatial vector $\mathbf{k}$ points in the direction of propagation *for both positive and negative frequencies.*

  If the above is employed, the physical Hamiltonian will not contain couplings between $\mathbf{k}$ and $-\mathbf{k}$ modes inside the horizon. The presence of such couplings in the formalism [13, 14] merely reflects not having properly identified the direction of propagation (and thus the polarisation). As the modes leave the horizon, couplings between $\mathbf{k}$ and $-\mathbf{k}$ may appear, and represent the production of particle pairs by the gravitational field (where the particles in each pair move in opposite directions) [15].
- **Other constraints** In Hamiltonian GR, the full Hamiltonian constraint contains two further constraints, Gauss and diffeomorphism. Whereas the diffeomorphism constraint is automatically satisfied to all orders considered, solving for the perturbations to first order allows for a non-vanishing Gauss constraint to second order (in the form of expressions quadratic in linear perturbation variables). This is also true for the torsion. To use the notations defined in the text, ${}^2_1G_i \neq 0$ and ${}^2_1T^a \neq 0$. As with the Hamiltonian constraint,

it is the second order (or backreaction) terms ${}^2_2G_i$ and ${}^2_2T^a$, that enforce these constraints to second order. In deriving the Ashtekar Hamiltonian from the ADM formalism [4] these two constraints are used, and therefore ${}^2_1\mathcal{H}$ acquires extra terms. However, these turn out to be irrelevant full divergences.

- **Perturbation variables as a canonical transformation** If one expands the full Ashtekar variables in terms of their perturbations and naively evaluates ${}^2_1\mathcal{H}$, then, contrary to expectations, one does *not* obtain the standard cosmological Hamiltonian. The reason is that the second order Hamiltonian needs to acquire an additional term: the perturbative expansions (3) and (4) can be regarded as a canonical transformation into new variables, a^i_a and δe^a_i, which happen to be "small". As this transformation is time dependent, as in standard Hamiltonian mechanics [16], an additional term has to be added to ${}^2_1\mathcal{H}$ which is given by the time derivative of the generating function of the transformation. Not only does this procedure ensure to give us the correct Hamiltonian, it also gives a more rigorous meaning to the perturbative quantisation procedure. We are not quantising the fluctuations whilst "freezing" the quantum mechanics of the background; we are merely quantising the full theory in new variables, which happen to be "small" in some circumstances.
- **Boundary term** Boundary terms [17, 18, 19] are often ignored in the literature, usually by invoking suitable fall-off conditions [13, 19]. However, it turns out that this leads to the wrong result for the cosmological Hamiltonian. The reason is that planes waves, the central tool of cosmological perturbation theory, do not satisfy the fall-off conditions, say, in a deSitter background. Therefore the boundary term has to be included to the right order in the expansion in order to obtain the correct Hamiltonian to be employed in quantising the graviton modes.
- **Hamiltonian is complex off shell** Whereas on shell the Hamiltonian is weakly zero and therefore real, off-shell, for a general γ, we have an intrinsically complex Hamiltonian. In particular this is true for its perturbative expression. Note, however, that the Hamiltonian is still Hermitian *with respect to the inner product used*, i.e. when taking the reality conditions into account. This looks very similar to the non-Hermitian Hamiltonians studied by Bender and collaborators [20], which are initially complex but become Hermitian with respect to a non-trivial inner product.

4. The Quantum Hamiltonian

4.1. Reality conditions and commutation relations

To set up the quantum theory, we need to determine the reality conditions for the Fourier space variables (which are present as γ, and therefore the connection, is complex in general). These will let us determine the physical inner product. The reality of the metric translates to the simple condition $e_{r+}(\mathbf{k}) = e_{r-}(\mathbf{k})$ in terms of modes. For the connection in real space, we have $\Re A^i = \Gamma^i + \gamma_R \Gamma^{0i}$ and $\Im A^i = \gamma_I \Gamma^{0i}$. We can write the spin connection in terms of the metric by again imposing the torsion-free condition (2). The reality conditions are supposed to embody the non-dynamical part of the torsion-free condition, and, in terms of modes, turn out to be

$$i\gamma^* \tilde{a}_{r+}(\mathbf{k},\eta) - i\gamma \tilde{a}_{r-}(\mathbf{k},\eta) = 2rk\gamma_I \tilde{e}_{r+}(\mathbf{k},\eta) \tag{11}$$

$$-i\gamma \tilde{a}^\dagger_{r+}(\mathbf{k},\eta) + i\gamma^* \tilde{a}^\dagger_{r-}(\mathbf{k},\eta) = 2rk\gamma_I \tilde{e}^\dagger_{r-}(\mathbf{k},\eta)\ . \tag{12}$$

Note that for a purely real γ the connection is real and therefore there are no reality conditions (instead $a_{r+} = a_{r-}$); though the torsion-free condition is still needed on shell to relate metric and connection. We also need to derive the commutation relations between the metric and connection in Fourier space, starting from the Poisson bracket of the real theory. They are given

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by [6, 7]

$$[\tilde{a}_{rp}(\mathbf{k}), \tilde{e}^{\dagger}_{sq}(\mathbf{k}')] = -i(\gamma_R + pi\gamma_I)\frac{l_P^2}{2}\delta_{rs}\delta_{p\bar{q}}\delta(\mathbf{k}-\mathbf{k}') \ , \tag{13}$$

where $\overline{q} = -q$. All other commutators are zero.

4.2. The Hamiltonian

We can find the quantum Hamiltonian by plugging the mode expansion (5) into the second order dynamical Hamiltonian (8). We will consider modes inside the horizon, $|k\eta| \gg 1$, so terms containing Ha can be neglected. We can then write $\mathcal{H}$ in terms of graviton creation and annihilation operators:

$$\begin{aligned}\mathcal{H}_{eff} = \frac{1}{2l_P^2}\int d^3k \sum_r & -(1+i\gamma r)G_{r\mathcal{P}_+}(\mathbf{k})G_{r\mathcal{P}_-}(-\mathbf{k}) - (1-i\gamma r)G_{r\mathcal{P}_-}(\mathbf{k})G_{r\mathcal{P}_+}(-\mathbf{k}) \\ & +(1+i\gamma r)G_{r\mathcal{P}_+}(\mathbf{k})G^{\dagger}_{r\mathcal{P}_+}(\mathbf{k}) + (1-i\gamma r)G^{\dagger}_{r\mathcal{P}_+}(\mathbf{k})G_{r\mathcal{P}_+}(\mathbf{k}) + (1-i\gamma r)G_{r\mathcal{P}_-}(\mathbf{k})G^{\dagger}_{r\mathcal{P}_-}(\mathbf{k}) \\ & +(1+i\gamma r)G^{\dagger}_{r\mathcal{P}_-}(\mathbf{k})G_{r\mathcal{P}_-}(\mathbf{k}) - (1-i\gamma r)G^{\dagger}_{r\mathcal{P}_+}(\mathbf{k})G^{\dagger}_{r\mathcal{P}_-}(-\mathbf{k}) - (1+i\gamma r)G^{\dagger}_{r\mathcal{P}_-}(\mathbf{k})G^{\dagger}_{r\mathcal{P}_+}(-\mathbf{k})\end{aligned} \tag{14}$$

Note that this contains physical, donated by $\mathcal{P}_+$, and unphysical graviton states donated by $\mathcal{P}_-$. They are summarised in table 2. Using the reality conditions, we can see that $G_{r\mathcal{P}}$ and $G^{\dagger}_{r\mathcal{P}}$ are indeed conjugates of each other (this is trivially true for a real γ). The unphysical modes can be seen to have negative energy from their commutators, and they vanish classically, i.e. when imposing the on-shell condition (10). This means that on-shell, we don't have any spurious $\mathbf{k}$, $-\mathbf{k}$ couplings that would signify particle production (which does not occur in the inside horizon limit $H \to 0$, which corresponds to Minkowski space). Quantum mechanically, the unphysical

Table 2. Physical and unphysical graviton modes

Physical $\mathcal{P} = \mathcal{P}_+ = 1$	Unphysical $\mathcal{P} = \mathcal{P}_- = -1$
$G_{r\mathcal{P}_+} = \frac{-r}{i\gamma}(\tilde{a}_{r+} - k(r+i\gamma)\tilde{e}_{r+})$	$G_{r\mathcal{P}_-} = \frac{-r}{i\gamma}(\tilde{a}_{r+} - k(r-i\gamma)\tilde{e}_{r+})$
$G^{\dagger}_{r\mathcal{P}_+} = \frac{r}{i\gamma}(\tilde{a}^{\dagger}_{r-} - k(r-i\gamma)\tilde{e}^{\dagger}_{r-})$	$G^{\dagger}_{r\mathcal{P}_-} = \frac{r}{i\gamma}(\tilde{a}^{\dagger}_{r-} - k(r+i\gamma)\tilde{e}^{\dagger}_{r-})$
$\left[G_{r\mathcal{P}_+}(\mathbf{k}), G^{\dagger}_{s\mathcal{P}_+}(\mathbf{k}')\right] = kl_P^2\delta_{rs}\delta(\mathbf{k}-\mathbf{k}')$	$\left[G_{r\mathcal{P}_-}(\mathbf{k}), G^{\dagger}_{s\mathcal{P}_-}(\mathbf{k}')\right] = -kl_P^2\delta_{rs}\delta(\mathbf{k}-\mathbf{k}')$

modes can be removed by determining the inner product through the reality conditions. We choose a holomorphic representation diagonalising $G^{\dagger}_{r\mathcal{P}}$,

$$G^{\dagger}_{r\mathcal{P}}\Phi(z) = z_{r\mathcal{P}}\Phi(z) \quad \Rightarrow \quad G_{r\mathcal{P}}\Phi = \mathcal{P}kl_P^2\frac{\partial\Phi}{\partial z_{r\mathcal{P}}} \tag{15}$$

(using the commutation relations to determine the action of $G_{r\mathcal{P}}$) and we can impose the reality conditions through $\langle\Phi_1|G^{\dagger}_{r\mathcal{P}}|\Phi_2\rangle = \overline{\langle\Phi_2|G_{r\mathcal{P}}|\Phi_1\rangle}$, fixing the inner product [2, 13, 21]. The measure μ of a general inner product $\langle\Phi_1|\Phi_2\rangle = \int dz d\bar{z} e^{\mu(z,\bar{z})}\bar{\Phi}_1(\bar{z})\Phi_2(z)$ can then be determined as

$$\mu(z,\bar{z}) = \int d\mathbf{k}\sum_{r\mathcal{P}}\frac{-\mathcal{P}}{kl_P^2}z_{r\mathcal{P}}(\mathbf{k})\bar{z}_{r\mathcal{P}}(\mathbf{k}) \ . \tag{16}$$

From this we see that the variables corresponding to the unphysical modes $\mathcal{P}_- = -1$ actually lead to a divergent measure and should therefore be removed from the physical Hilbert space. We can also use this representation to determine a perturbative ground state (just a constant in this case) as well as particle states (which are monomials in the physical z variable).

5. Vacuum fluctuations

From the analysis of the last section, we know that the physical Hamiltonian needed to determine the vacuum fluctuations is given by

$$\mathcal{H}^{ph}_{eff} \approx \frac{1}{2l_P^2}\int d\mathbf{k}\sum_r [G_r^{ph}G_r^{ph\dagger}(1+ir\gamma) + G_r^{ph\dagger}G_r^{ph}(1-ir\gamma)] \tag{17}$$

where $G_r^{ph} = G_{r\mathcal{P}_+}$. When we normal order this expression, we get a γ-independent graviton spectrum which is symmetric for right and left particles, however we are left with a chiral vacuum energy $V_r = 1 + i\gamma r$. Note that this contains imaginary terms in general, in particular even for a purely real γ (although the total vacuum energy, $V_R + V_L = 2$ is γ independent). This could be traced back to the fact that the physical Hamiltonian is in fact only Hermitian (under the chosen inner product) if γ is purely imaginary. It turns out that if we choose a symmetric ordering of the Hamiltonian, the vacuum energy is real and symmetric for both right and left gravitons. Whether this is a hint towards a preferred ordering of the Hamiltonian is up to debate.

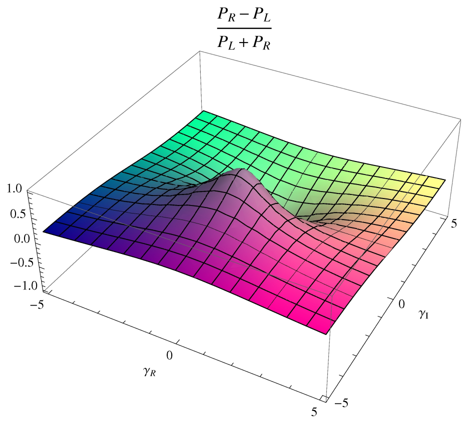

Figure 1. Power spectrum asymmetry as a function of a generally complex Immirzi parameter γ.

Whereas the vacuum energy is not physically measurable, the vacuum fluctuations are. These are given by the 2-point functions of the connection, $\langle 0|A_r^\dagger(\mathbf{k})A_r(\mathbf{k}')|0\rangle = P_r(k)\delta(\mathbf{k}-\mathbf{k}')$, where $A_r(\mathbf{k})$ includes the Fourier components of the connection, $A_r(\mathbf{k}) = a_{r+}(\mathbf{k})e^{-ik\cdot x} + a_{r-}^\dagger(\mathbf{k})e^{ik\cdot x}$. We can use our on-shell condition to express this 2-point function in terms of physical graviton states only,

$$\langle 0|A_r^{ph\dagger}(\mathbf{k})A_r^{ph}(\mathbf{k}')|0\rangle = \frac{(r+i\gamma)(r-i\gamma^*)}{4}\langle 0|G_{r\mathcal{P}_+}(\mathbf{k})G_{r\mathcal{P}_+}^\dagger(\mathbf{k}')|0\rangle\ , \tag{18}$$

which lets us determine the power spectrum asymmetry between right and left-handed states:

$$\frac{P_R - P_L}{P_R + P_L} = -\frac{2\gamma_I}{1+|\gamma|^2}\ . \tag{19}$$

Equation (19) shows that the chiral asymmetry vanishes for a purely real γ, but is present in general. This is in contrast with the second order theory, where right and left gravitons are completely symmetric. While the ordering of the Hamiltonian does not affect the 2-point function, the ordering of the connection components does. If we use $A_r A_r^\dagger$ instead of $A_r^\dagger A_r$, expression (19) flips sign, and for a symmetric ordering we get no chirality at all. The power spectrum asymmetry is shown in figure 1. It shows that the maximum chirality occurs at the SD/ASD values $\gamma = \pm i$ and that the chirality vanishes for $\gamma_I = 0$ as well as in the limit $|\gamma| \to \infty$, representing the Palatini-Kibble theory.

Conclusion

We have shown that using the Ashtekar formulation of canonical GR leads to a chiral gravitational wave background. The effect depends on the value of the free parameter of the theory, the Immirzi parameter γ: chirality vanishes for a purely real γ and is maximised for $\gamma = \pm i$, corresponding to a SD/ASD connection. Additionally, it also depends on the ordering of the 2-point function. In [22] it was shown that parity violation, which leads to a non-vanishing TB (temperature and magnetic mode) correlator, would render the detection of gravitational waves much easier. In [6] we argued that we can expect a siginificant TB signal for a range of $\frac{1}{800} < |\gamma| < 800$. Upcoming CMB polarisation experiments like PLANCK will therefore potentially be able to measure this chiral effect, assuming there is a sufficiently large tensor to scalar ratio.

Other mechanisms to produce gravitational chirality have been proposed (e.g. [23, 24, 25]), however the one pointed out here is by far the simplest. It would be useful to bridge our result to the one of [26], where a chirality in the graviton propagator was observed. However, as their approach relies on a Euclidean signature and real γ, the relationship is not obvious.

In addition to comparing different methods of producing gravitational chirality, there are a few other interesting lines of investigation to follow. Instead of using a holomorphic representation for the graviton operators (dependent on metric and connection), a representation purely in terms of the connection can be found. This will allow us to analyse the perturbative Kodama state [27] (a non-perturbative solution to the gravitational constraints) which turns out (as shown in [28]) *not* to be normalisable in terms of our inner product, and therefore cannot describe gravitons. Furthermore, it would be interesting to investigate the process of horizon crossing and the behaviour of modes outside the horizon. The dynamics are then fully driven by the metric, and we expect a complete reduction to second order theory. Understanding this transition might give us insight into the problem of decoherence in Cosmology and the nature of squeezed states.

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